Divide and Conquer – Quantum Style!

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University of Washington

SIAM Conference on Computational Science and Engineering, 2009

The Grand Strategy

1. Divide
2. Conquer
3. Win

The Grand Strategy

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The Grand Strategy

1. Divide
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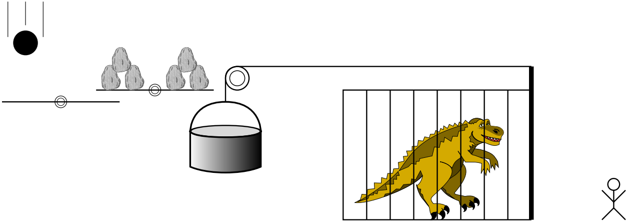
Will the Dinosaur Eat the Man?



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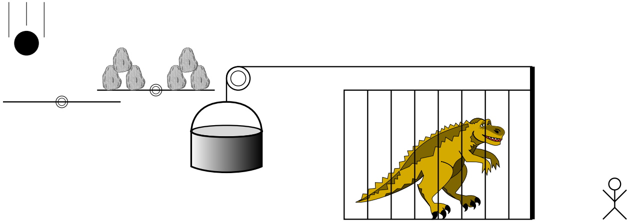

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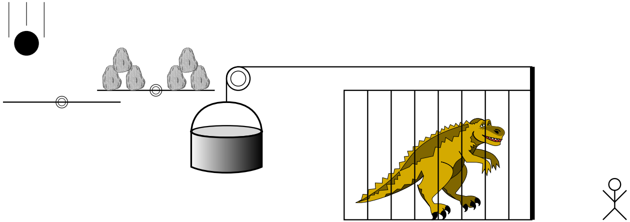

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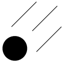

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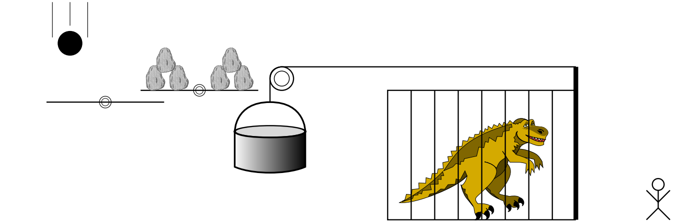

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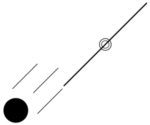

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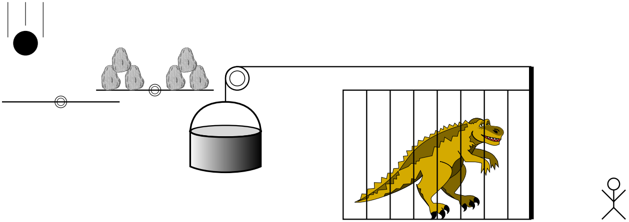

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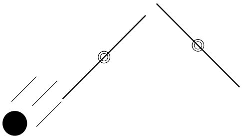

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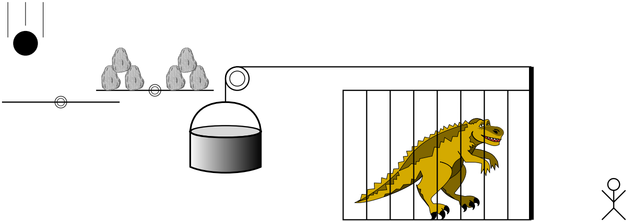

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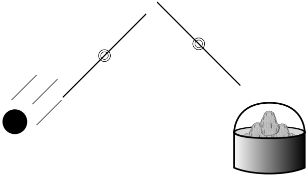

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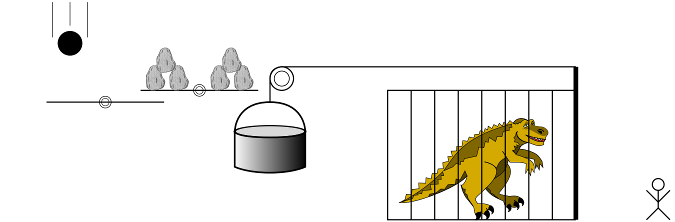

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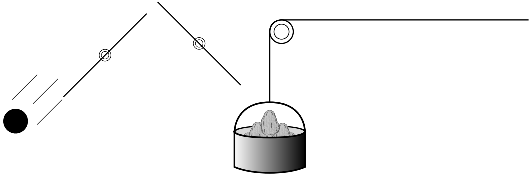

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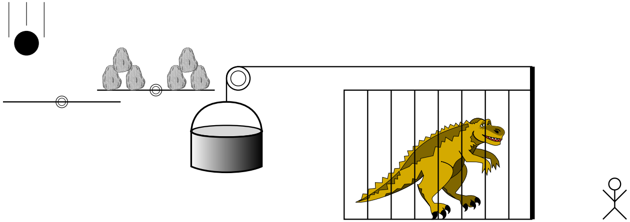

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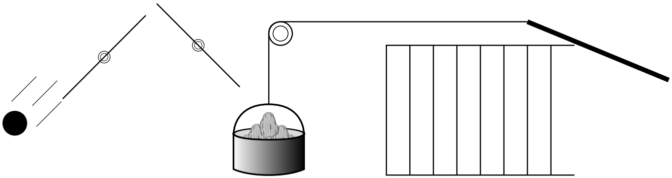

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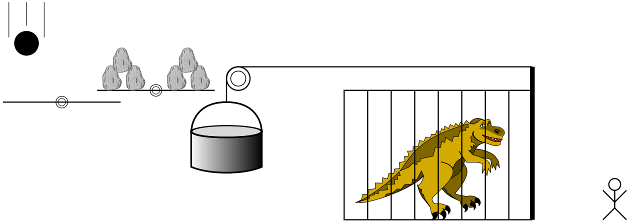

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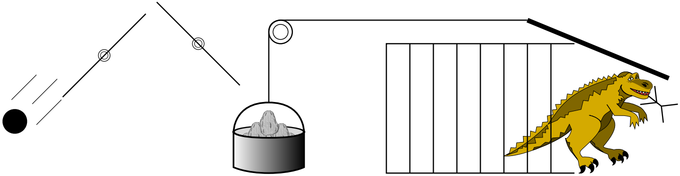

Outline

Outline

1. Motivation
   - Classic Divide and Conquer
   - Quantum Divide and Conquer

2. Application
   - Grand Objective
   - Using “Divide and Conquer”

A Classical System

fluiddiagram-1.pdf

A Classical System

fluiddiagram-2.pdf

A Classical System

fluiddiagram-2.pdf

Step 1 – Divide!

$$A_0 = 27\,\mathrm{kL}$$
$$B_0 = 40\,\mathrm{kL}$$
$$C_0 = 22\,\mathrm{kL}$$
$$D_0 = 16\,\mathrm{kL}$$
$$\vec{S}_0 = \begin{bmatrix} A_0 & B_0 & C_0 & D_0 \end{bmatrix}$$

A Classical System

fluiddiagram-2.pdf

Step 1 – Divide!

Step 2 – Conquer!

$$A_0 = 27\,\text{kL}$$
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Step 2 – Conquer!

$$\dot{A} = H_{AB}B + H_{AD}D$$
$$\dot{B} = H_{BA}A + H_{BC}C$$
$$\dot{C} = H_{CB}B$$
$$\dot{D} = H_{DA}A + H_{DB}B$$
$$\dot{\vec{S}} = \mathbf{H} \cdot \vec{S}$$

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Step 3 – Win!

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fluiddiagram-2.pdf

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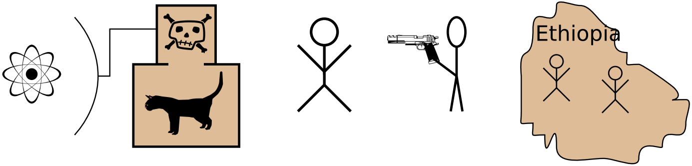
Step 3 – Win!

$$\vec{S}(t) = \int_0^t \dot{\vec{S}}\, dt + \vec{S}_0$$

Outline

A Quantum System



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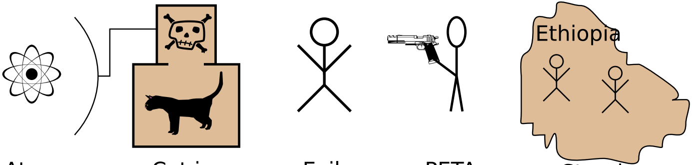


Atom
(Might decay)

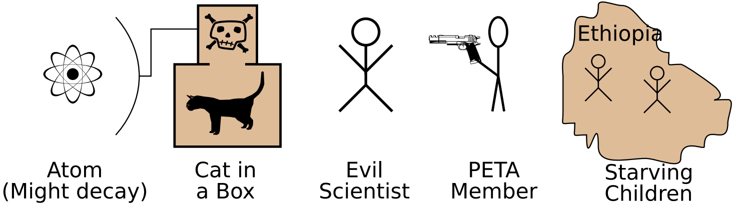
Cat in
a Box

Evil
Scientist

PETA
Member

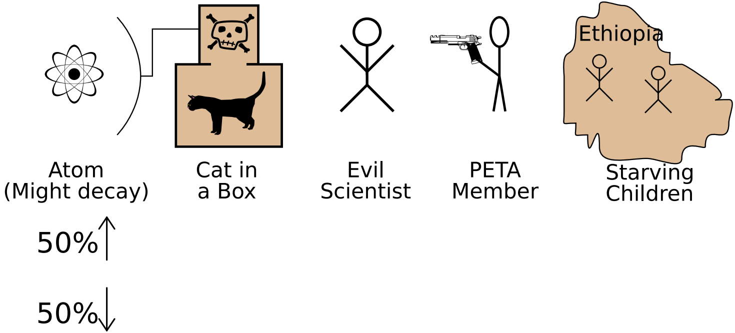
Starving
Children

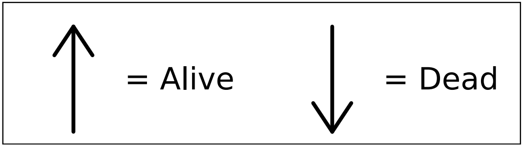
A Quantum System



$\uparrow$ = Alive $\qquad$ $\downarrow$ = Dead

A Quantum System





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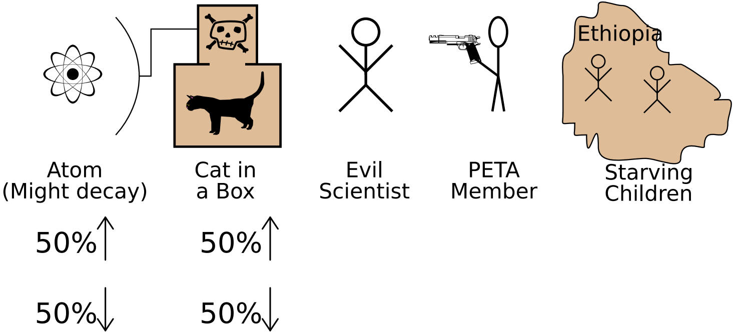
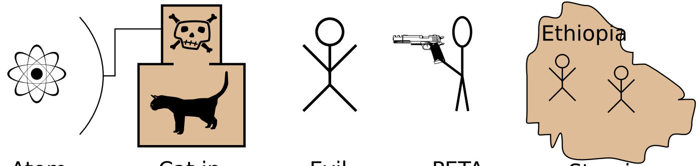


↑ = Alive ↓ = Dead

A Quantum System



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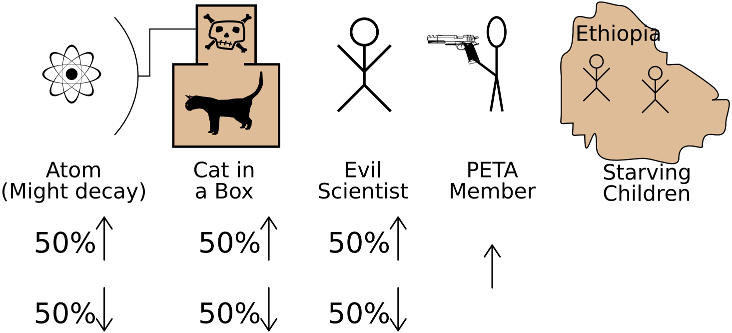
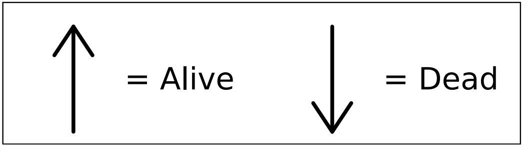
50%$\uparrow$ 50%$\uparrow$ 50%$\uparrow$

50%$\downarrow$ 50%$\downarrow$ 50%$\downarrow$

$\uparrow$ = Alive $\downarrow$ = Dead

A Quantum System





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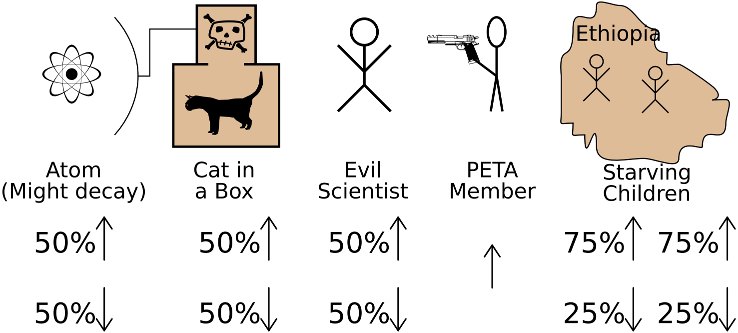

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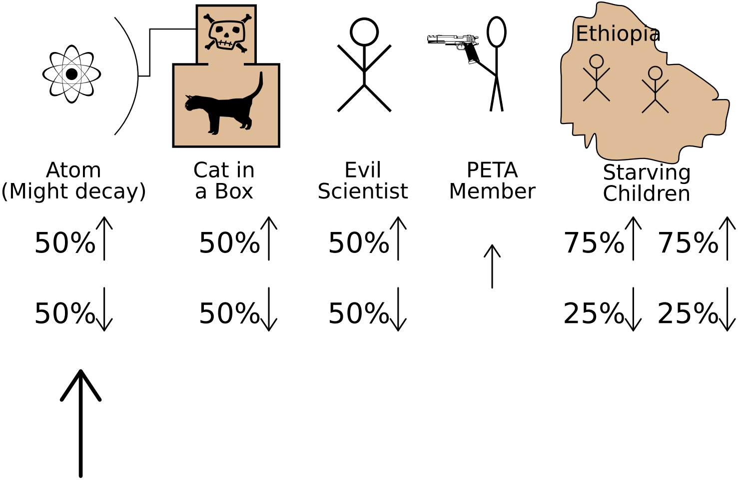
A Quantum System

quantumdiagram-8-5.pdf

A Quantum System



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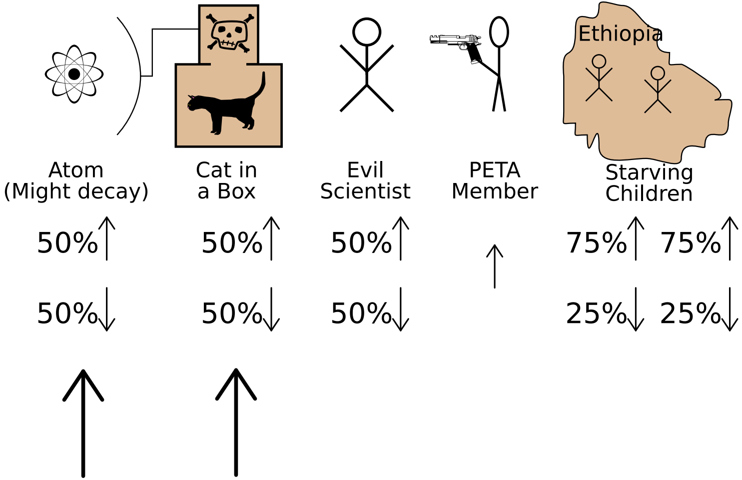

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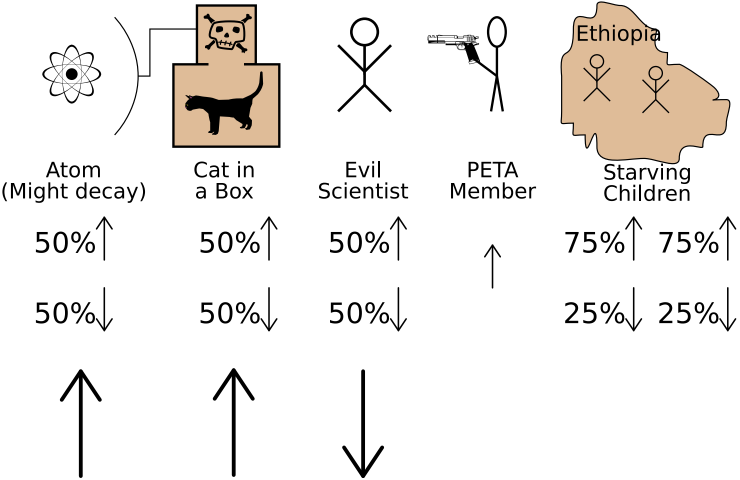

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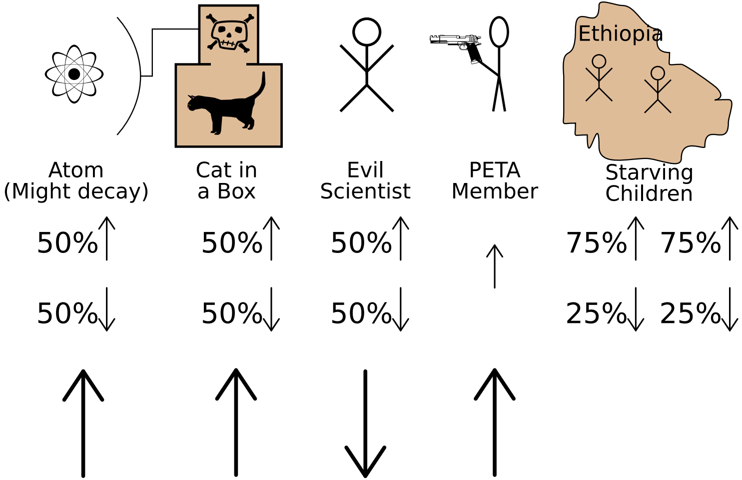

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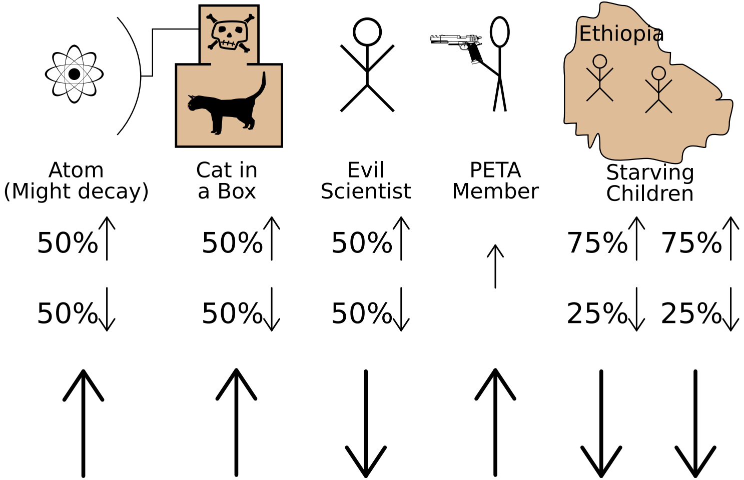

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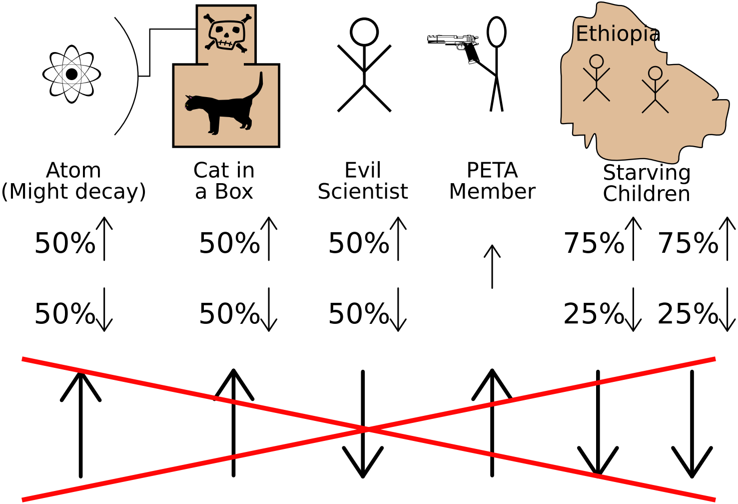

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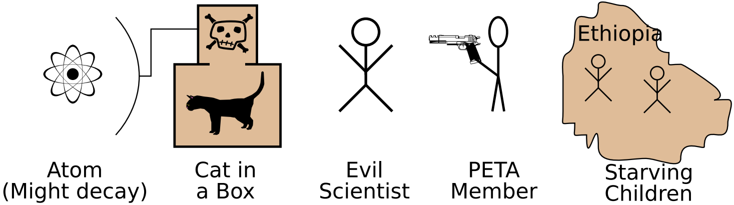

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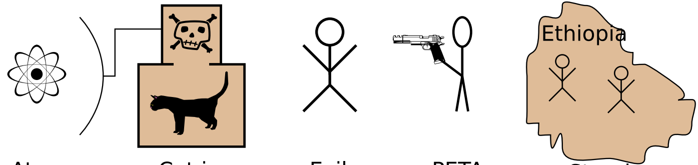

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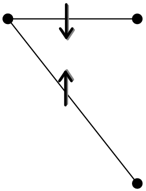

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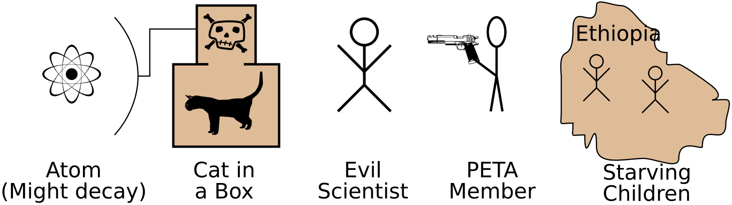

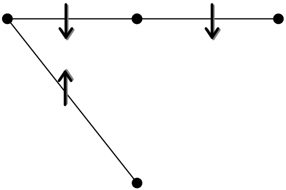

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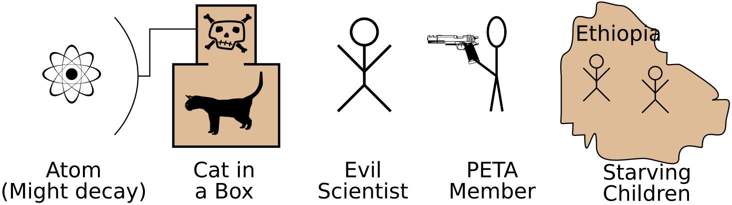

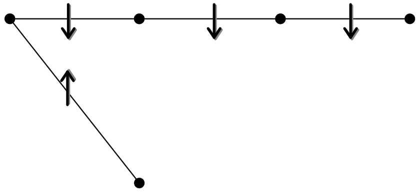

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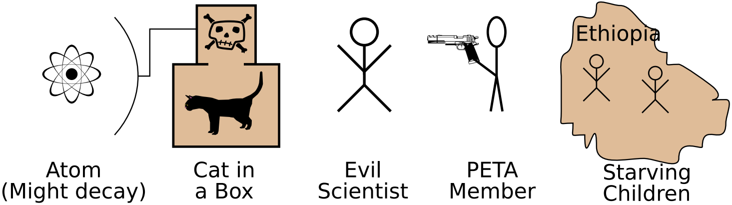

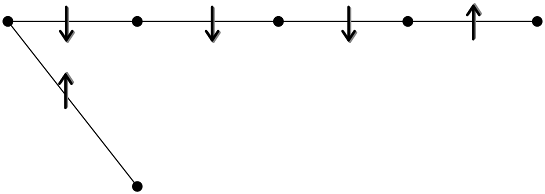

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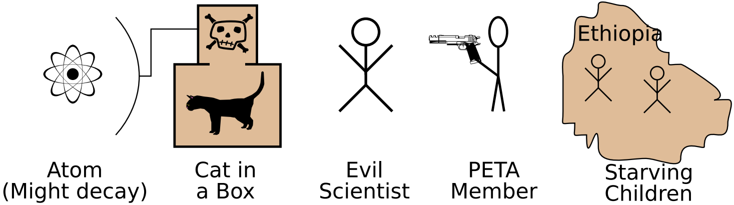

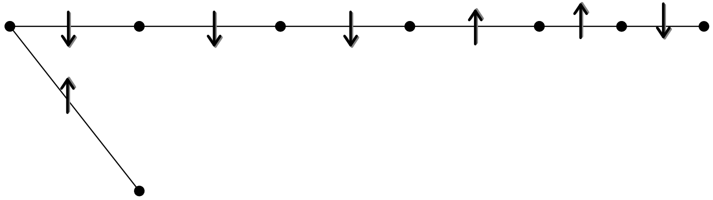

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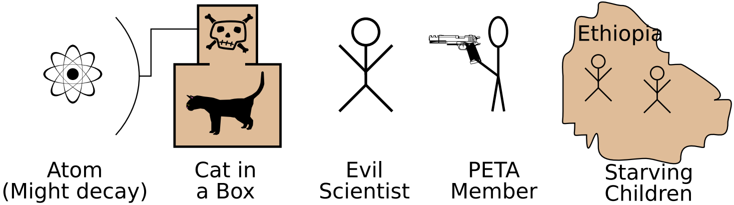

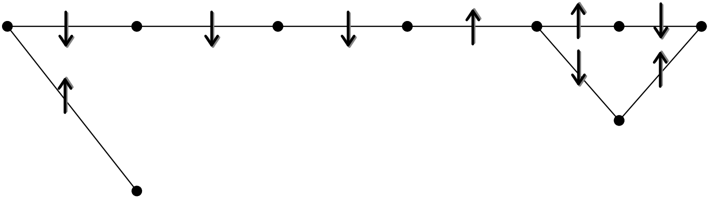

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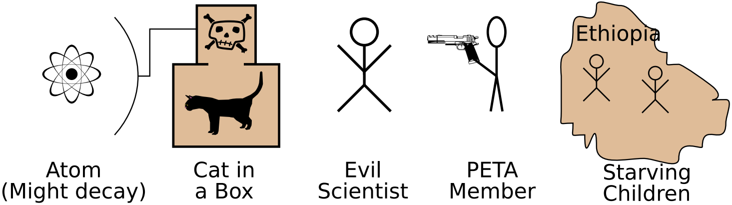

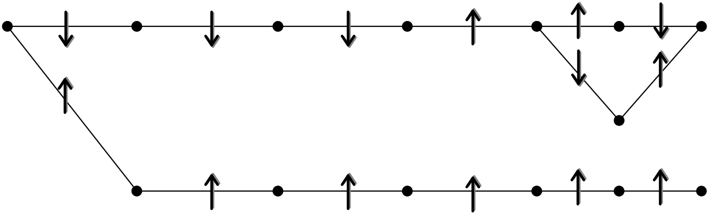

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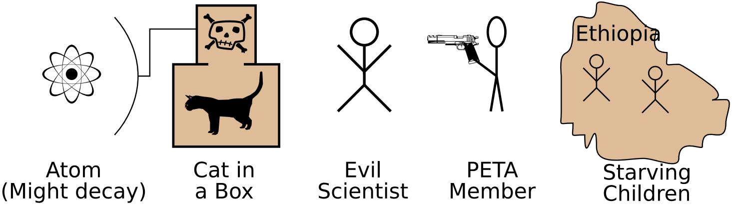

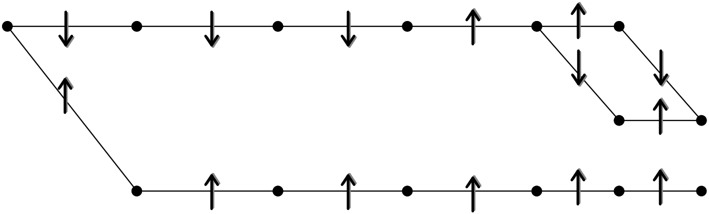

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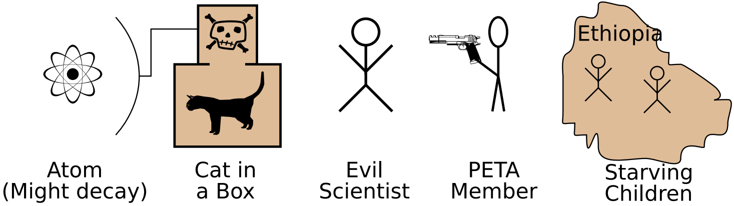

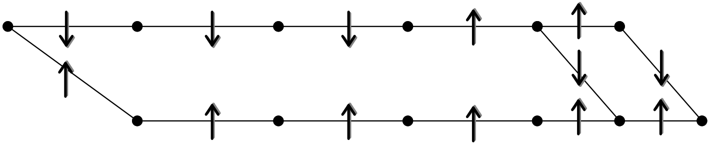

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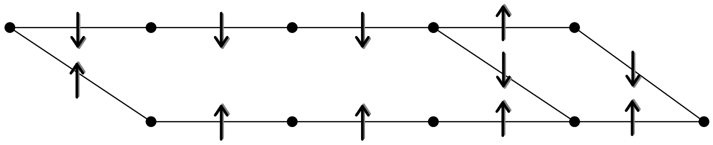

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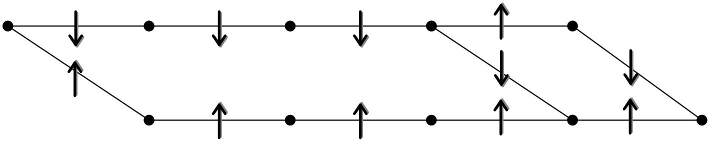
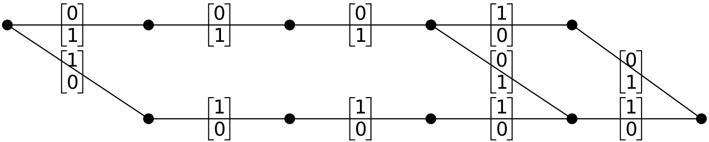


$$\begin{bmatrix}1\\0\end{bmatrix} = \uparrow \qquad \begin{bmatrix}1\\0\end{bmatrix} = \downarrow$$

$$\begin{bmatrix}a\\b\end{bmatrix} = a\uparrow + b\downarrow$$

A Quantum System



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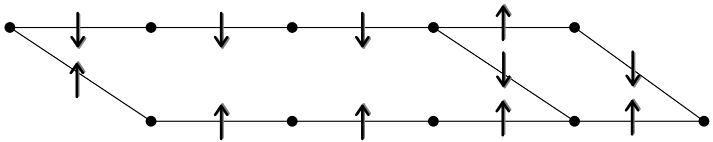

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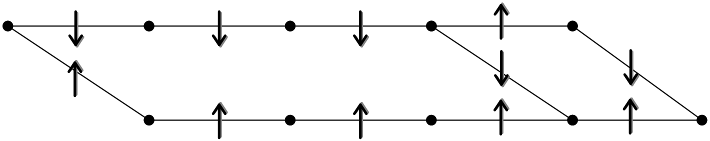
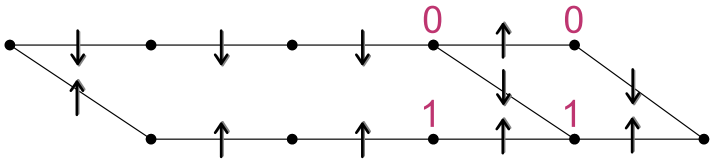


$$\begin{bmatrix} \uparrow & \downarrow \\ 0 & \uparrow \end{bmatrix}$$

A Quantum System



$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \uparrow & \downarrow \\ 1 & 0 & \uparrow \end{array}$$

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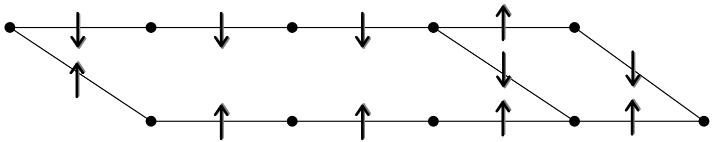

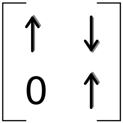

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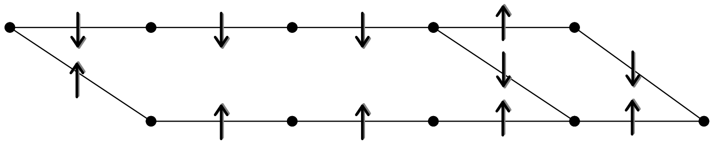
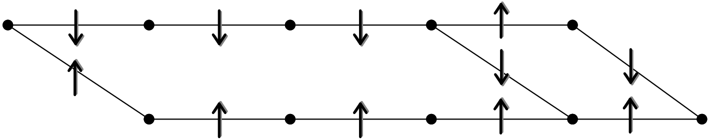


$$\begin{bmatrix} \downarrow & \uparrow \end{bmatrix} \begin{bmatrix} \downarrow & 0 \\ 0 & \uparrow \end{bmatrix} \begin{bmatrix} \downarrow & 0 \\ 0 & \uparrow \end{bmatrix} \begin{bmatrix} \uparrow & \downarrow \\ 0 & \uparrow \end{bmatrix} \begin{bmatrix} \downarrow \\ \uparrow \end{bmatrix}$$

A Quantum System



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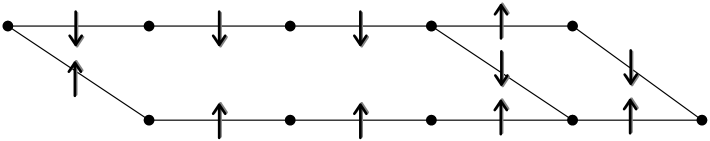

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$$B^{\beta}_{ij}$$

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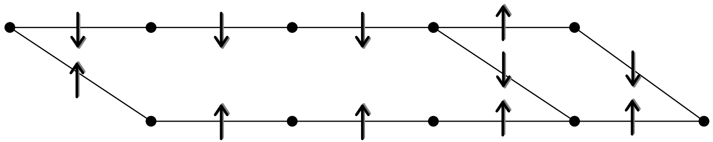

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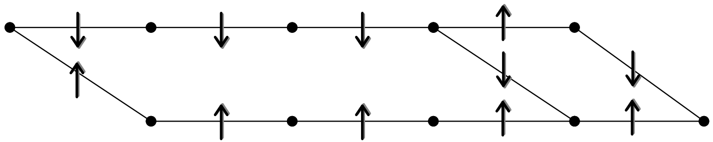

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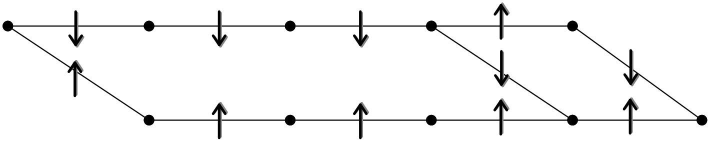

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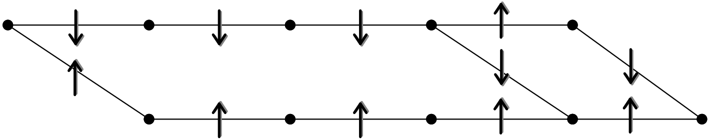
A Quantum System



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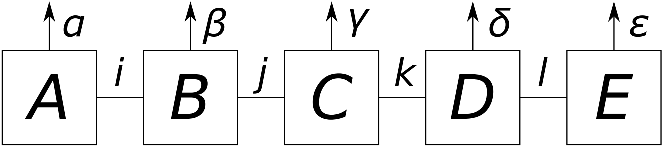
$$S^{\alpha\beta\gamma\delta\epsilon} = \sum_{ijkl} A_i^\alpha B_{ij}^\beta C_{jk}^\gamma D_{kl}^\delta E_l^\epsilon = A*B*C*D*E$$

A Quantum System



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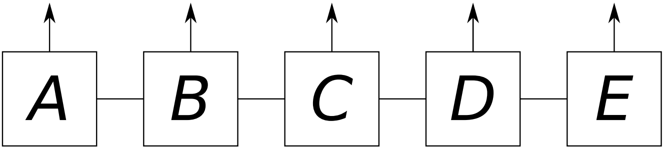

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Outline

Crash Course in Quantum Mechanics

1. The state of a quantum system is represented by a vector.

$$S^{\overbrace{\alpha\beta\gamma\delta\epsilon}^{X}} \equiv S^X \equiv \vec{S}$$

2. Time evolution of a quantum system is given by multiplication by a linear unitary operator.

$$\vec{S}(t) = \mathbf{U}(t) \cdot \vec{S}(0).$$

3. This unitary can be expressed as the exponentiation of a Hermiation operator called the Hamiltonian.

$$\mathbf{U}(t) = e^{i\mathbf{H}t}$$

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3. Goal of Quantum Mechanics: Find the eigenstates of **H**.
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Variational Approach

1. Objective: Find the lowest value of ω such that $\mathbf{H} \cdot \vec{S} = \omega \vec{S}$.
2. Equivalent to finding the state vector $\vec{S}$ to minimize the Rayleigh quotient $R(\vec{S}) := \frac{\vec{S}^* \cdot \mathbf{H} \cdot \vec{S}}{\vec{S}^* \cdot \vec{S}}$.
3. Since we don't know $\vec{S}$ *a priori*, make a guess with tunable parameters (an "ansatz") and vary these parameters to minimize $R(\vec{S})$.
4. The "divide and conquer" approach gives us an excellent ansatz for this purpose.
    1. $R(\vec{S})$ can be evaluated efficiently.
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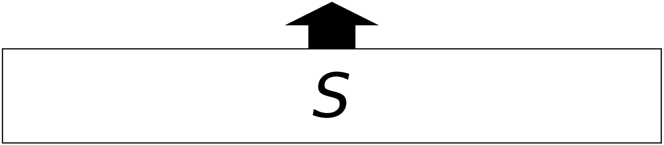
Outline

1. Motivation
   - Classic Divide and Conquer
   - Quantum Divide and Conquer
2. Application
   - Grand Objective
   - Using “Divide and Conquer”

Efficiently computing $\vec{S}^* \cdot \vec{S}$



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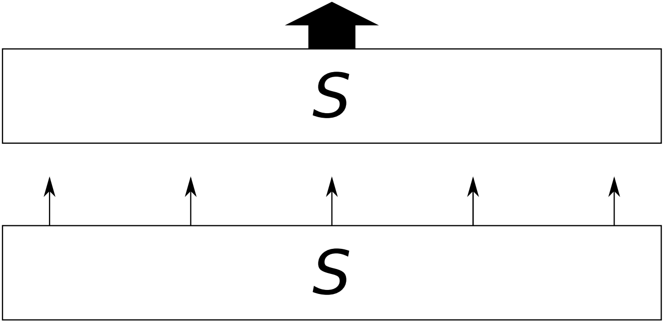

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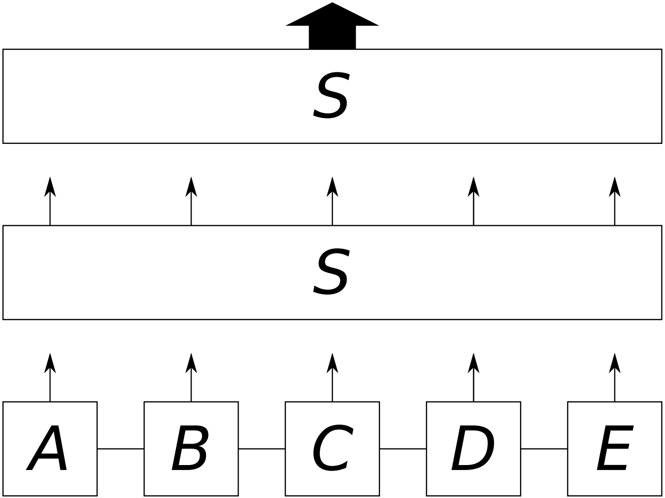

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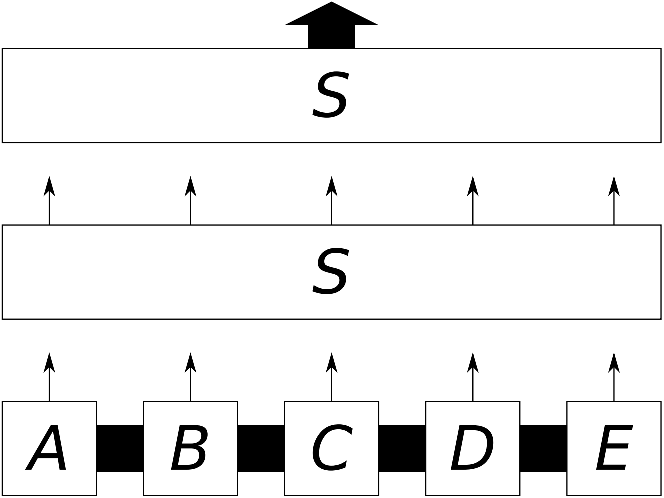

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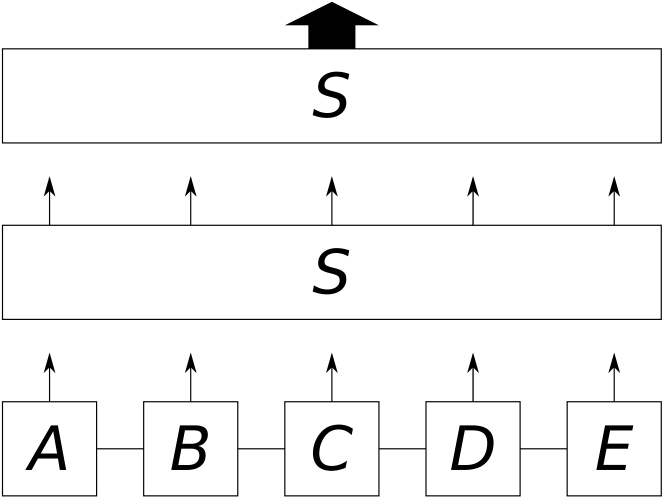

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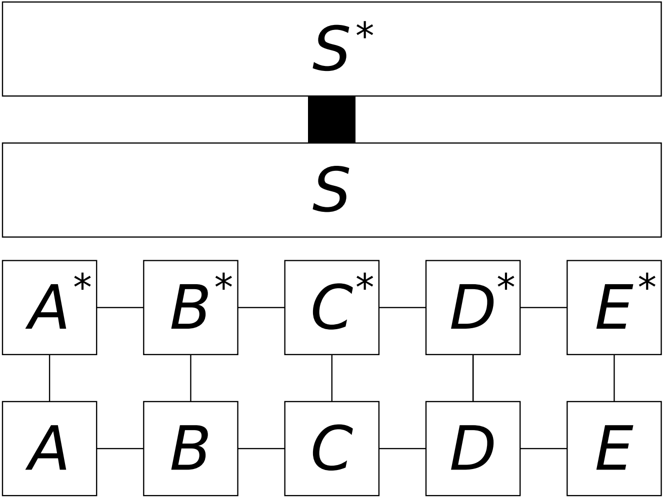

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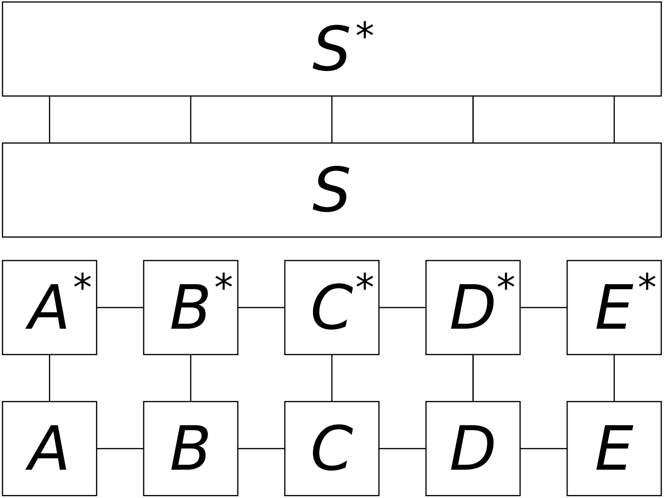

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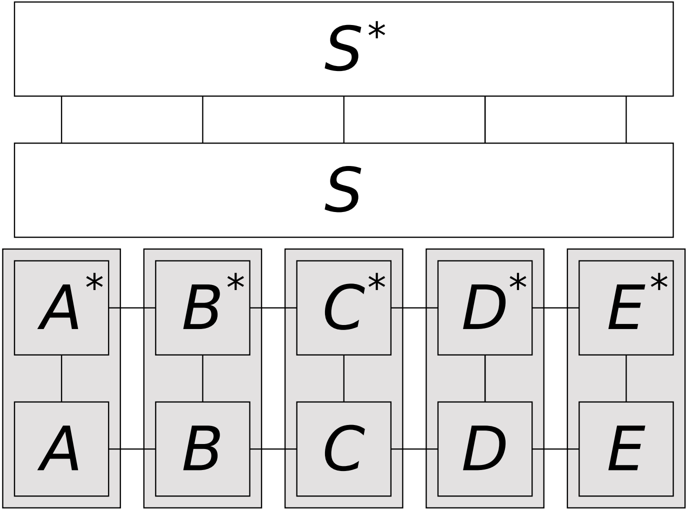

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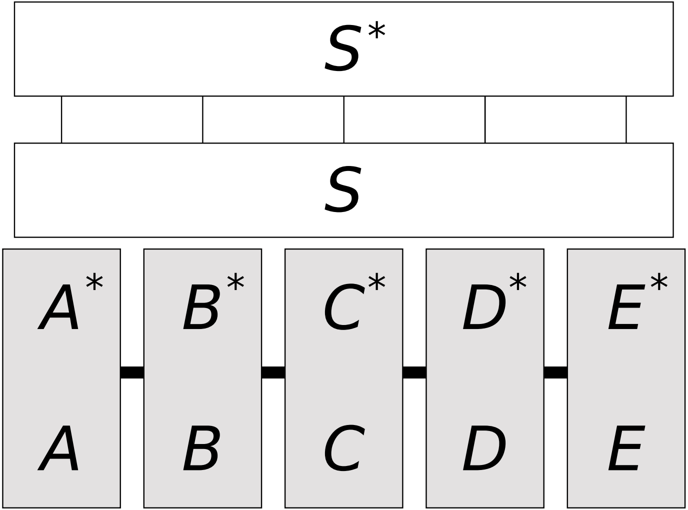

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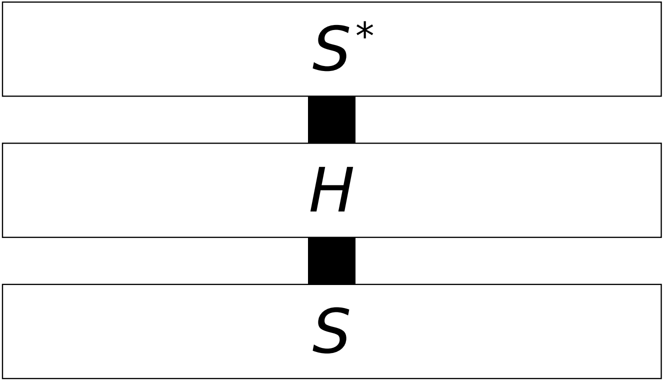

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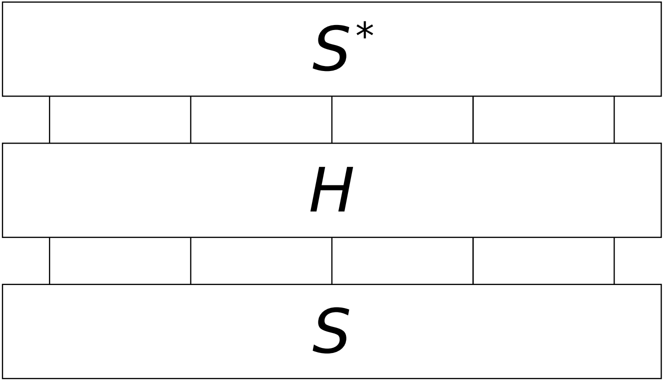

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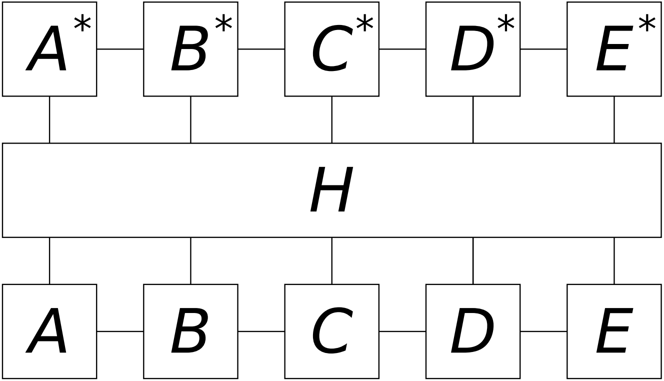

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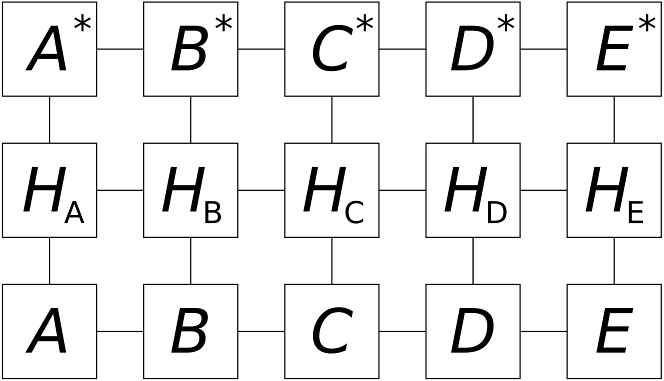

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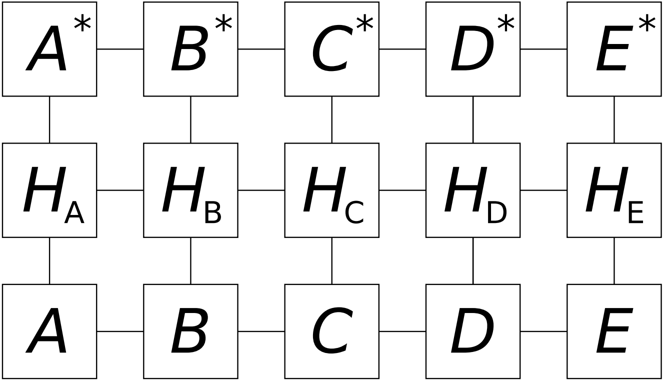

Look up "Crosswhite" on arxiv.org

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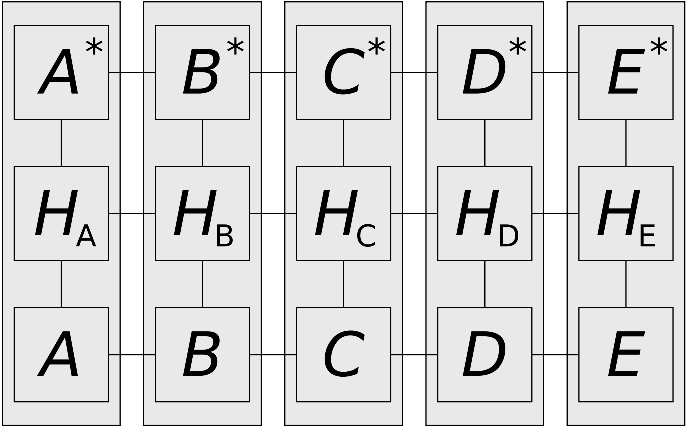

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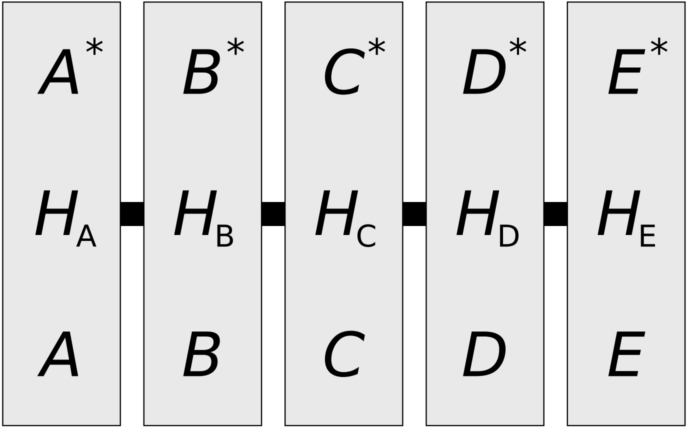
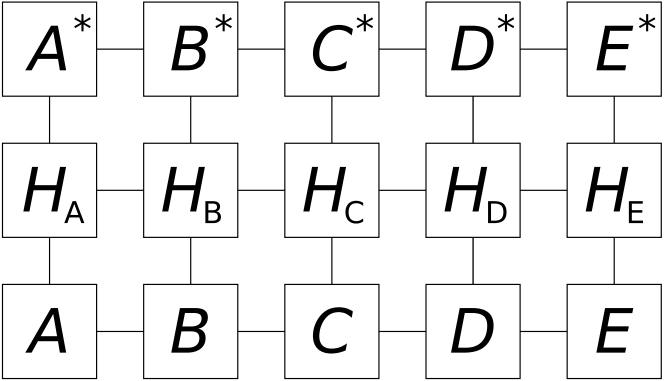


Efficiently varying a site



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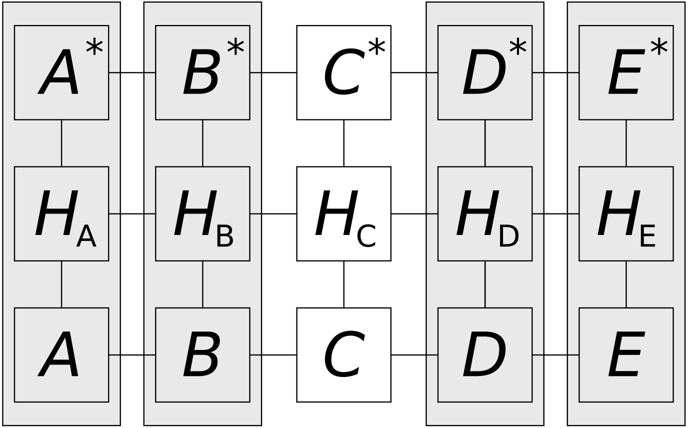

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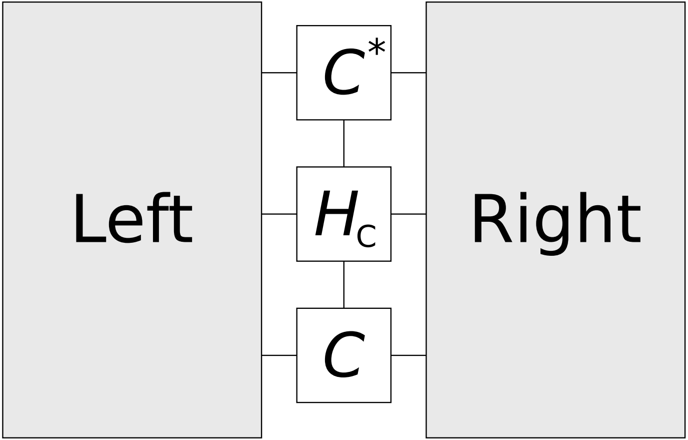

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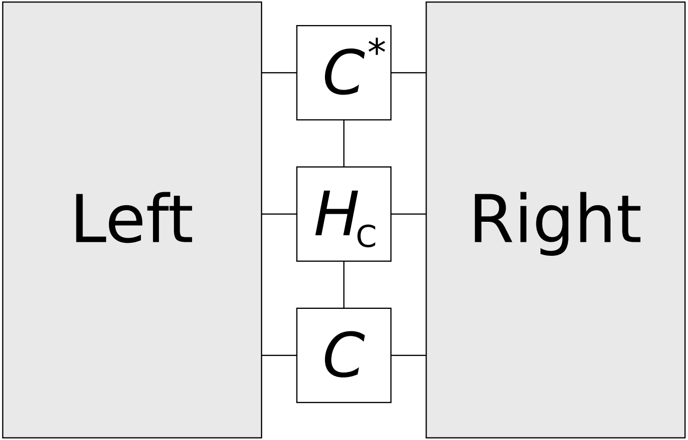

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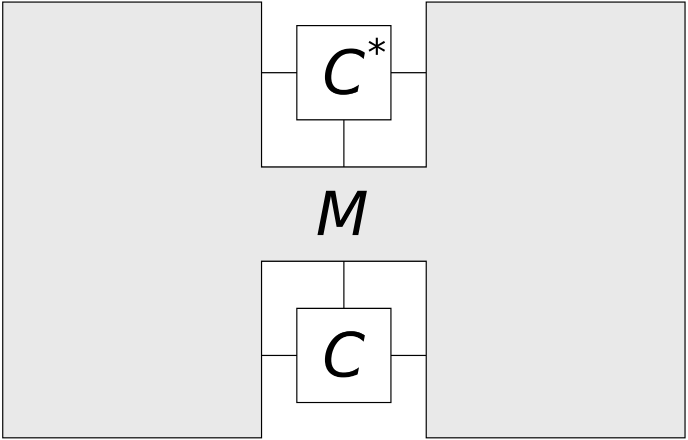

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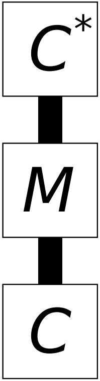
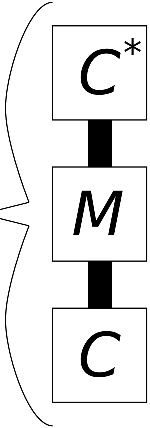
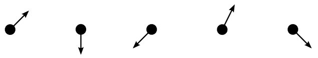


Efficiently varying a site

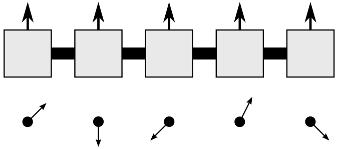
Eigenvalue
Problem!!!



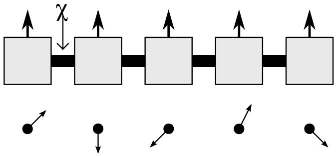
The Battle of Haldane-Shastry



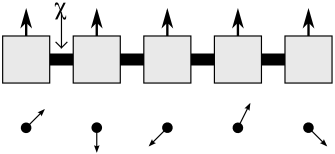
The Battle of Haldane-Shastry



The Battle of Haldane-Shastry

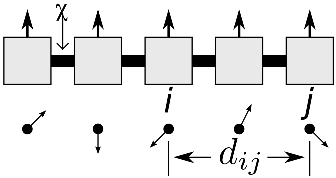


The Battle of Haldane-Shastry



$$V_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j \; / \; d_{ij}$$

The Battle of Haldane-Shastry



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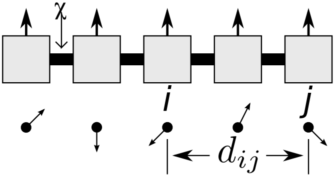
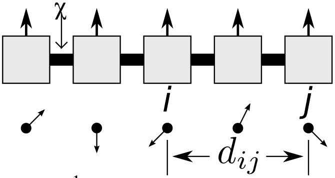
The Battle of Haldane-Shastry



$$V_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j \; / \; d_{ij}$$

$$C_Z(d) = \langle \sigma_i^Z \sigma_{i+d}^Z \rangle_i$$

The Battle of Haldane-Shastry



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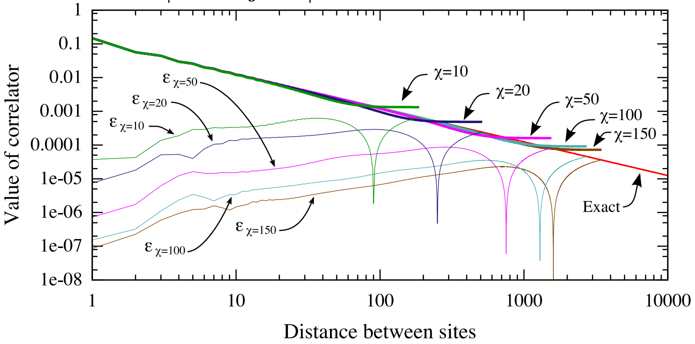

Summary

- Divide and Conquer is a general strategy vital to understanding complex systems.
- For systems with intrinsic nondeterminism, one must have a more nuanced sense of how to “divide” the system, but it still can be made to work.

References: Search for "Crosswhite" on arxiv.org.

Open Questions

- How well can these methods scale with inceased computing power?
- How can these ideas be made to apply to systems with more than one dimension?