Divide and Conquer – Quantum Style!

G. M. Crosswhite

Department of Physics
University of Washington

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The Grand Strategy

1. Divide
2. Conquer
3. Win

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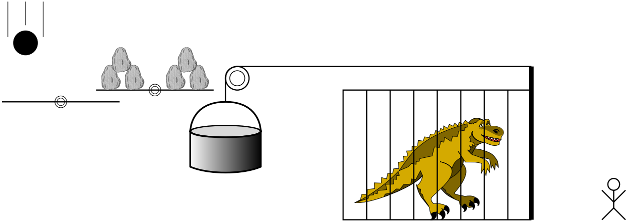
Will the Dinosaur Eat the Man?



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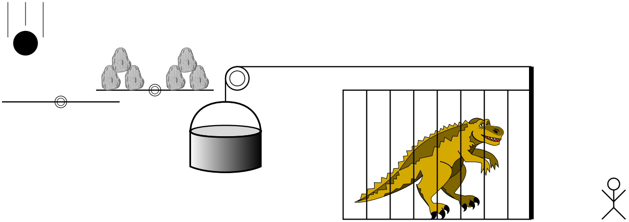

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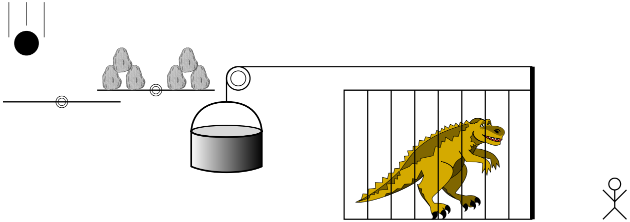

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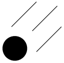

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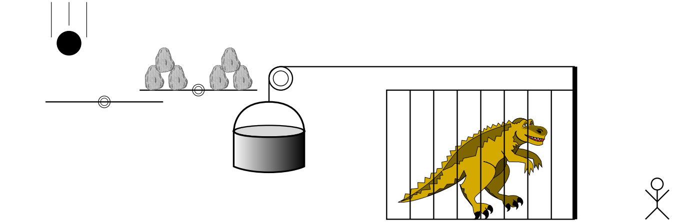

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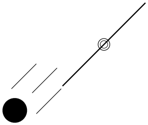

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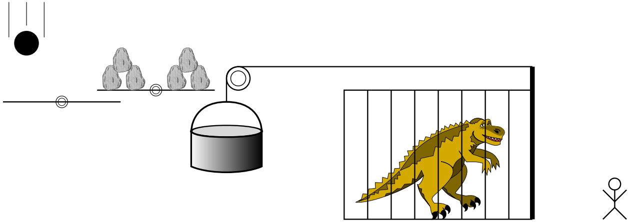

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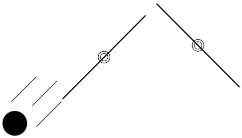

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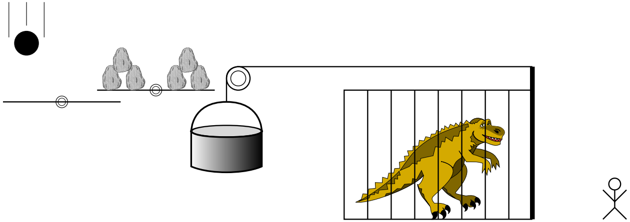

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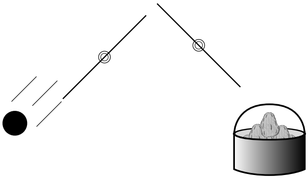

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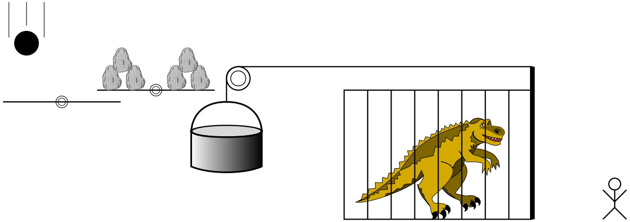

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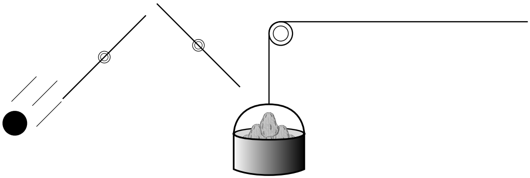

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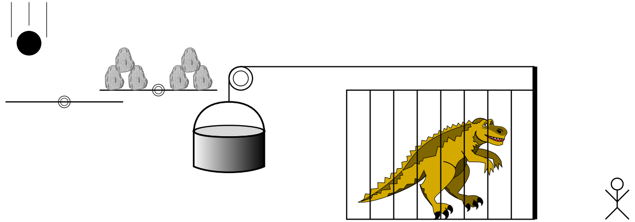

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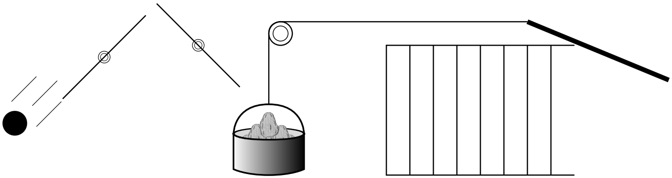

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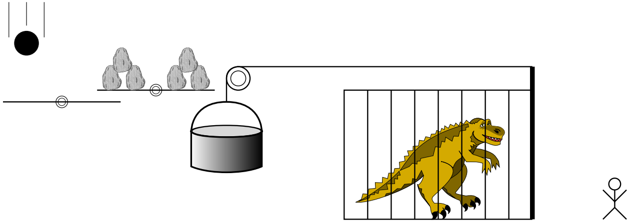

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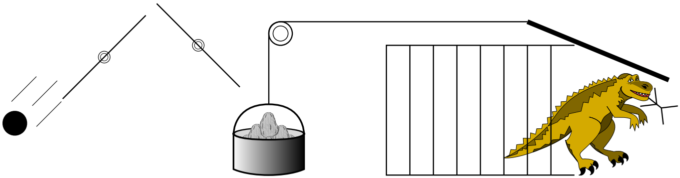
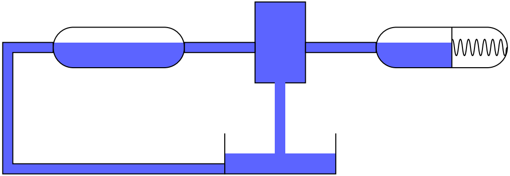


Outline

Outline

A Classical System



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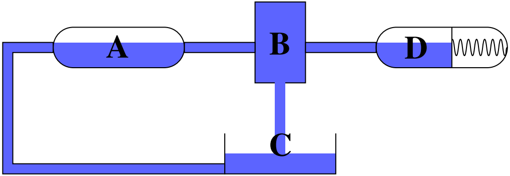

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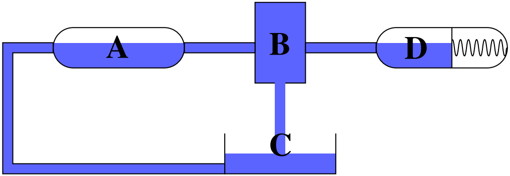
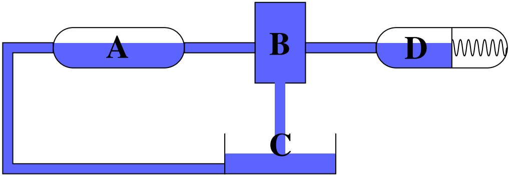


Step 1 – Divide!

$$A_0 = 27\,\mathrm{kL}$$
$$B_0 = 40\,\mathrm{kL}$$
$$C_0 = 22\,\mathrm{kL}$$
$$D_0 = 16\,\mathrm{kL}$$
$$\vec{S}_0 = \begin{bmatrix} A_0 & B_0 & C_0 & D_0 \end{bmatrix}$$

A Classical System



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Step 2 – Conquer!

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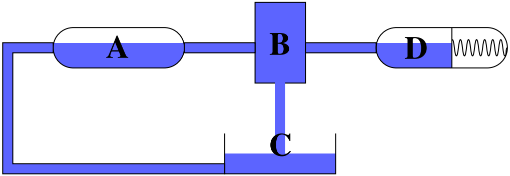

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Step 2 – Conquer!

$$\dot{A} = H_{AB}B + H_{AD}D$$
$$\dot{B} = H_{BA}A + H_{BC}C$$
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$$\dot{D} = H_{DA}A + H_{DB}B$$
$$\dot{\vec{S}} = \mathbf{H} \cdot \vec{S}$$

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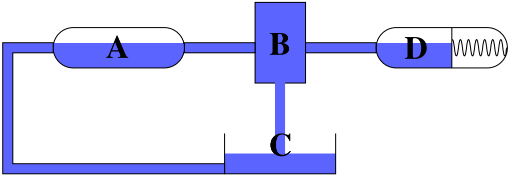

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Step 3 – Win!

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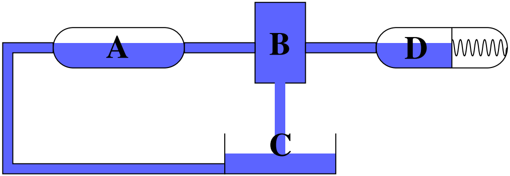

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Step 3 – Win!

$$\vec{S}(t) = \int_0^t \dot{\vec{S}}\, dt + \vec{S}_0$$

Outline

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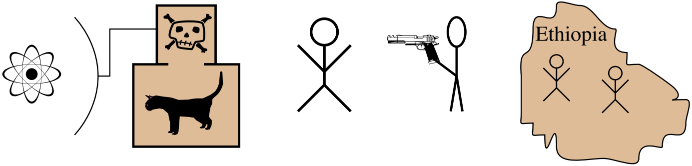

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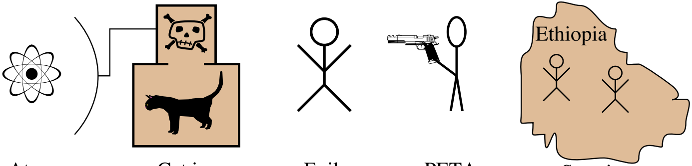
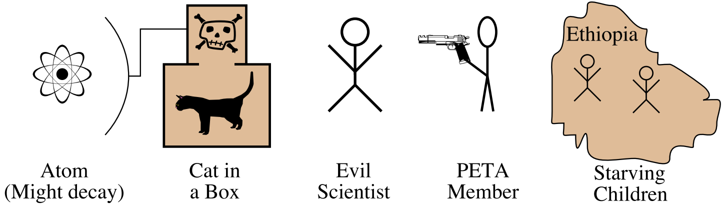
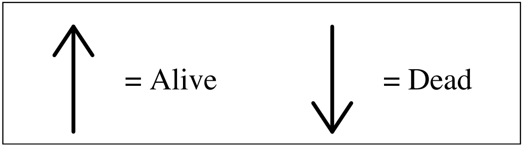


Atom (Might decay) | Cat in a Box | Evil Scientist | PETA Member | Starving Children

A Quantum System





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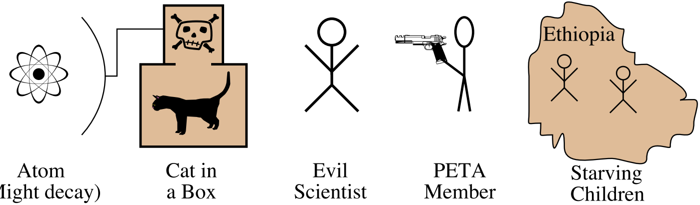
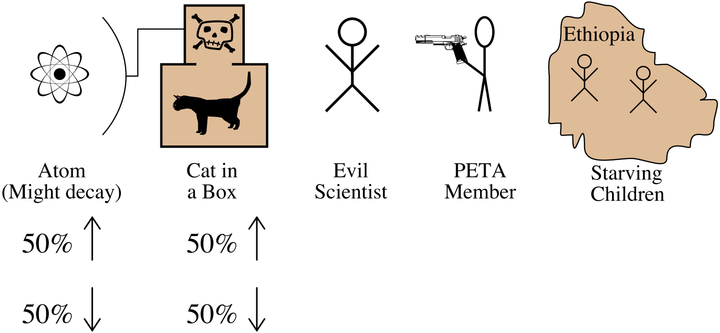


50% $\uparrow$

50% $\downarrow$

$\uparrow$ = Alive	$\downarrow$ = Dead

A Quantum System



↑	= Alive	↓	= Dead

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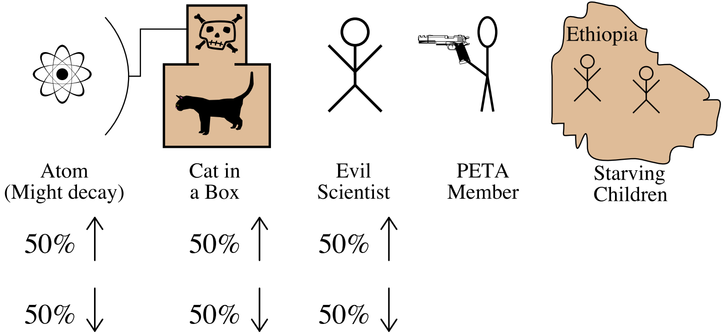

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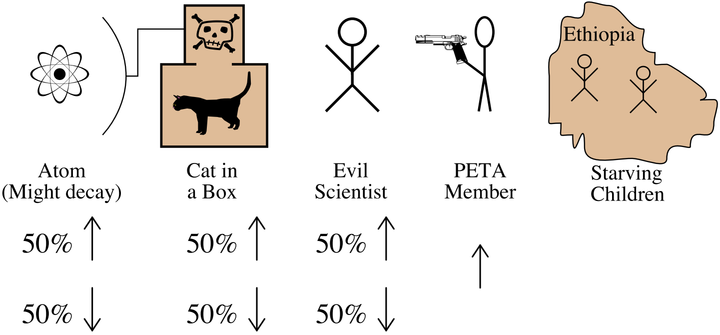

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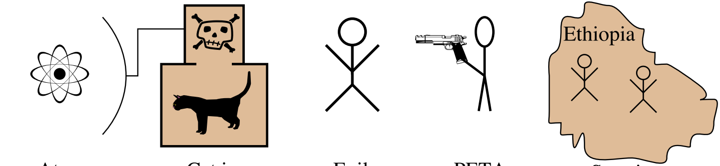

Atom (Might decay)	Cat in a Box	Evil Scientist	PETA Member	Starving Children	
50% $\uparrow$	50% $\uparrow$	50% $\uparrow$	$\uparrow$	75% $\uparrow$	75% $\uparrow$
50% $\downarrow$	50% $\downarrow$	50% $\downarrow$		25% $\downarrow$	25% $\downarrow$

$\uparrow$ = Alive $\qquad$ $\downarrow$ = Dead

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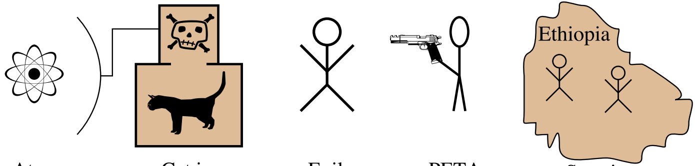

Atom (Might decay)	Cat in a Box	Evil Scientist	PETA Member	Starving Children	
50% $\uparrow$	50% $\uparrow$	50% $\uparrow$	$\uparrow$	75% $\uparrow$	75% $\uparrow$
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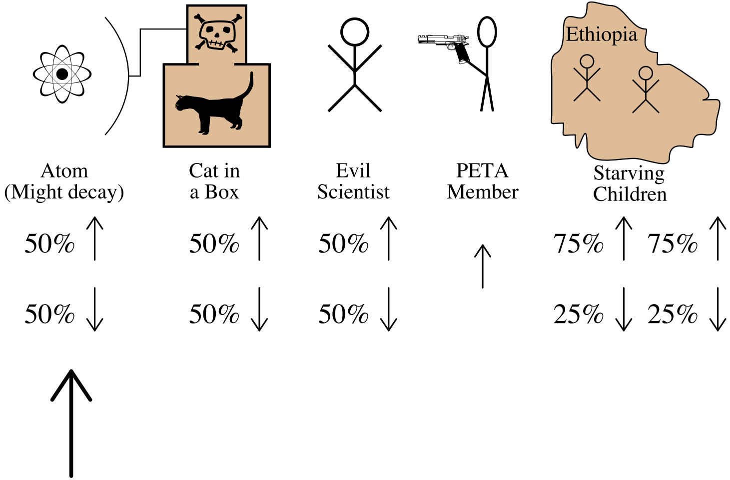

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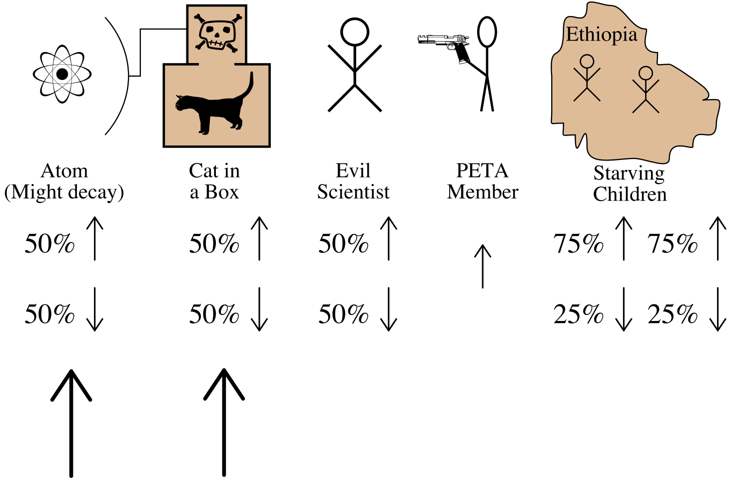

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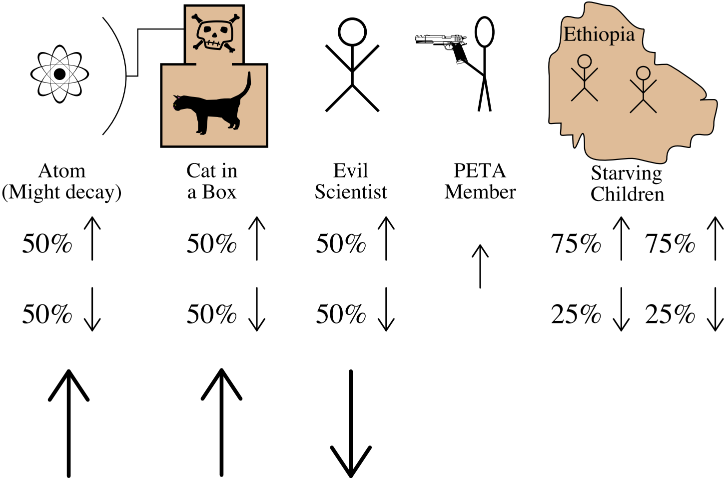

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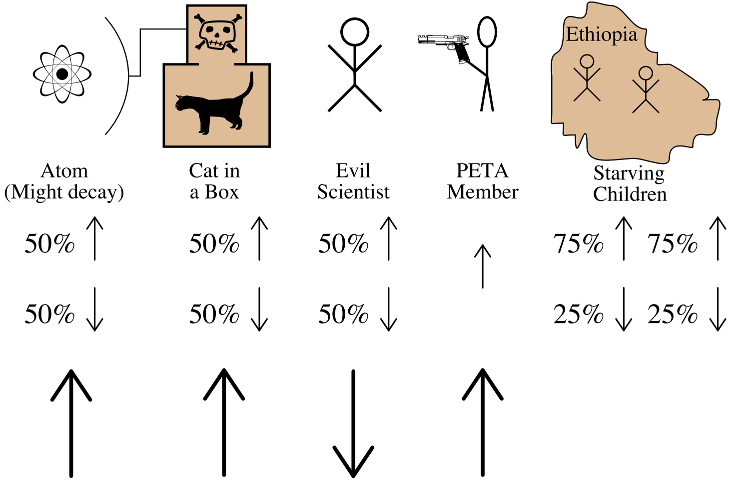

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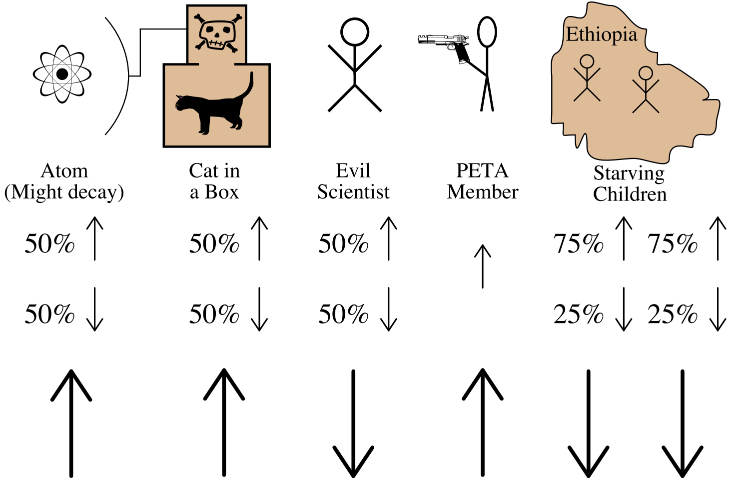

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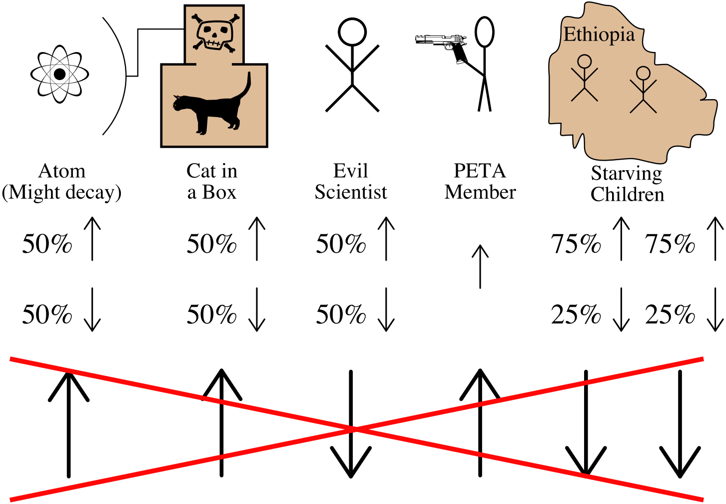

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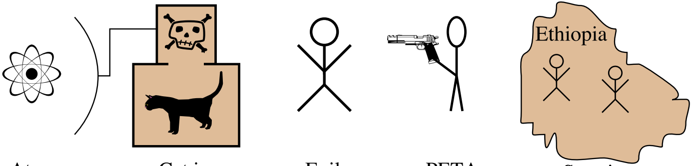

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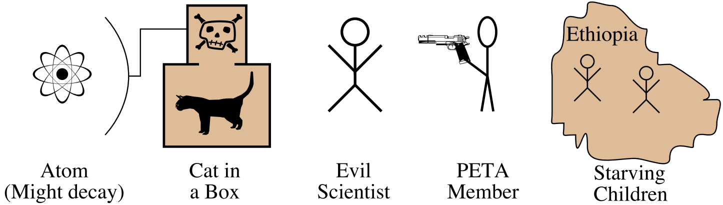

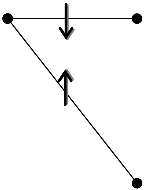

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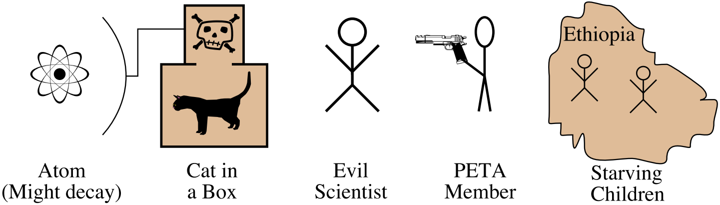

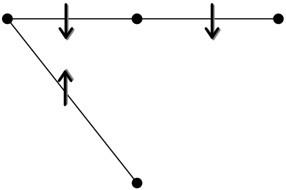

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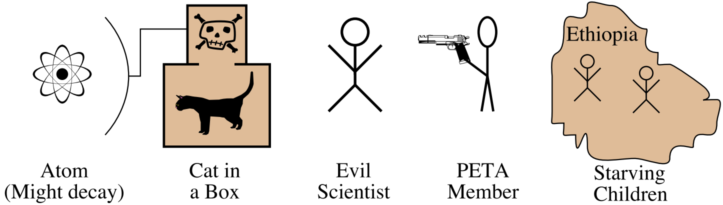

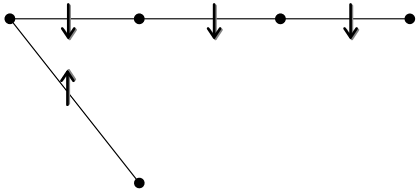

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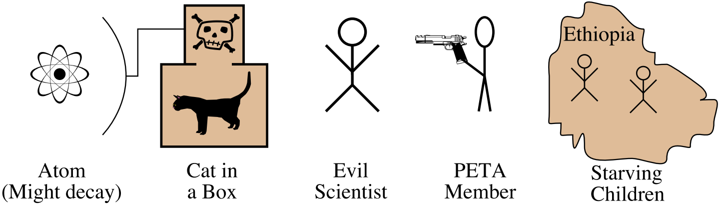

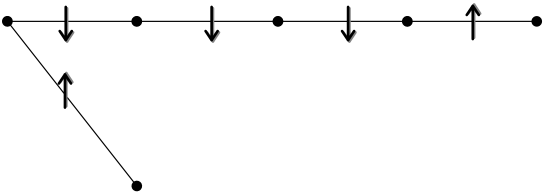

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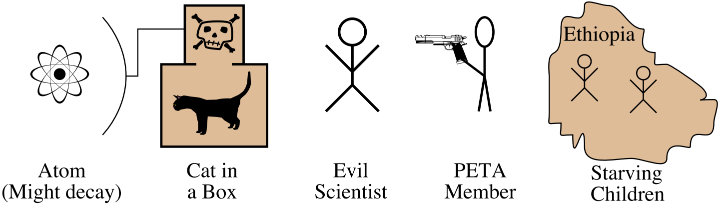

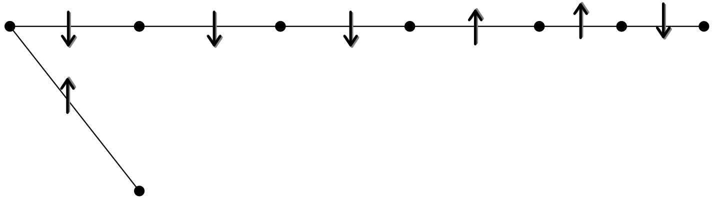

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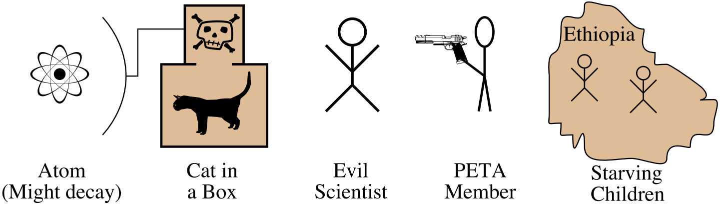

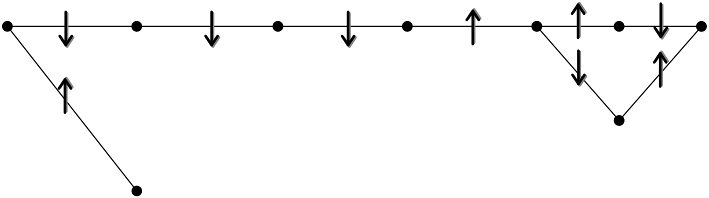

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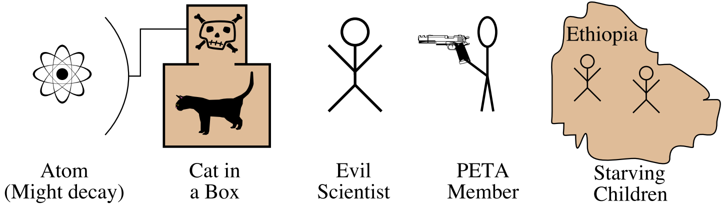

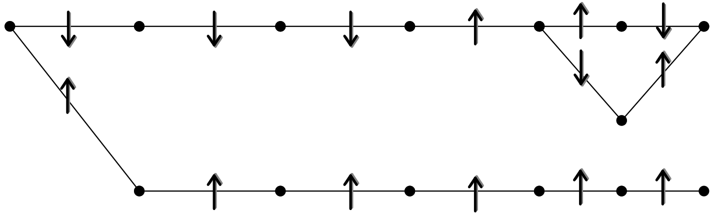

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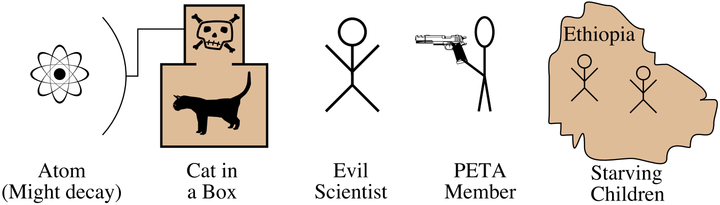

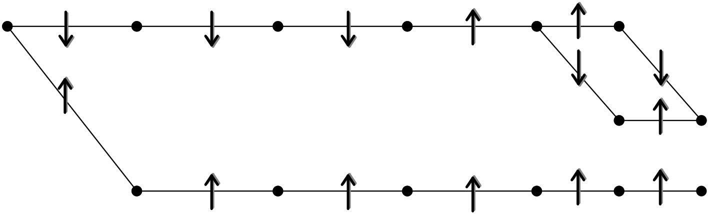

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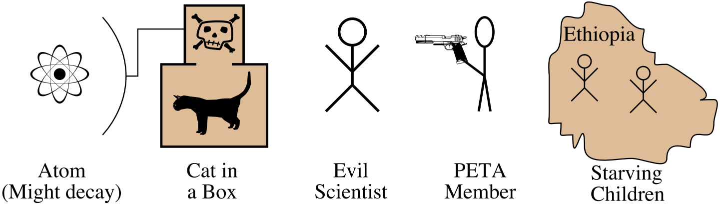

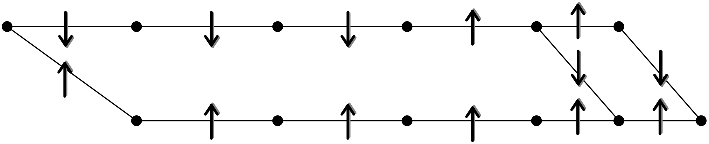

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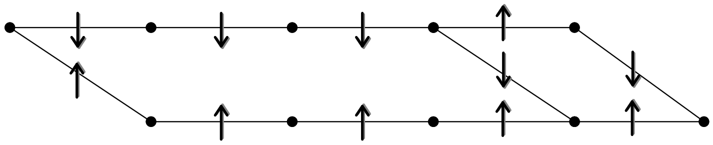

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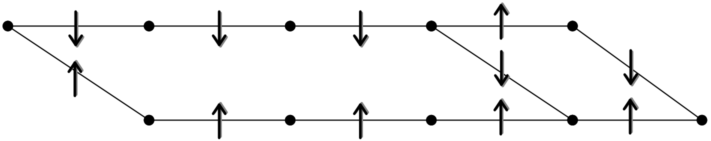

$$\begin{bmatrix}1\\0\end{bmatrix} = \uparrow \qquad \begin{bmatrix}1\\0\end{bmatrix} = \downarrow$$

$$\begin{bmatrix}a\\b\end{bmatrix} = a\uparrow + b\downarrow$$

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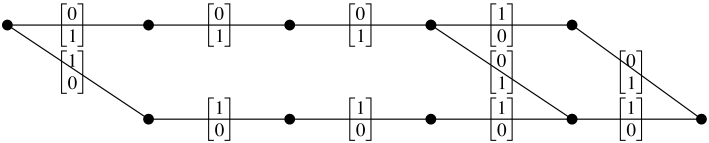

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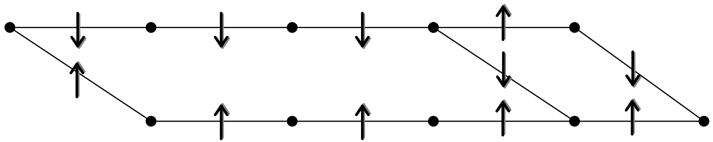
$$\begin{bmatrix}a\\b\end{bmatrix} = a\uparrow + b\downarrow$$

A Quantum System



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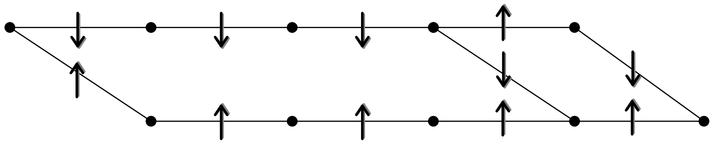

$$\begin{bmatrix} \uparrow & \downarrow \\ 0 & \uparrow \end{bmatrix}$$

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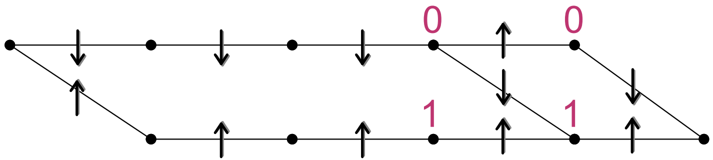
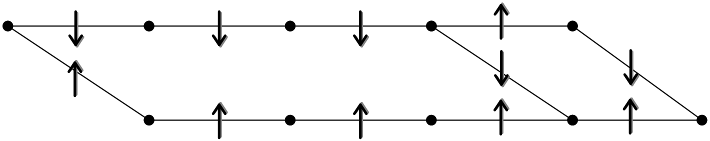


$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \uparrow & \downarrow \\ 1 & 0 & \uparrow \end{array}$$

A Quantum System



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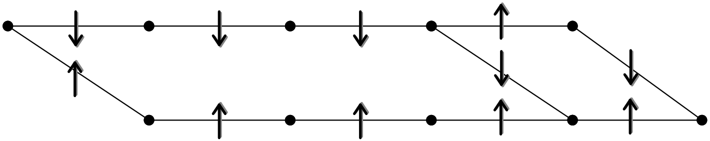

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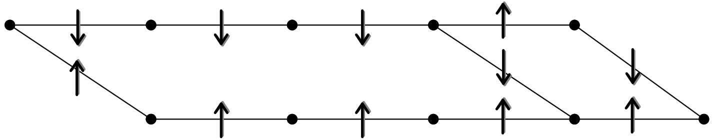

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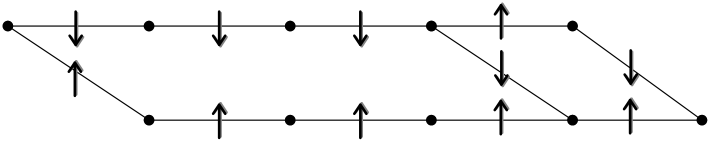
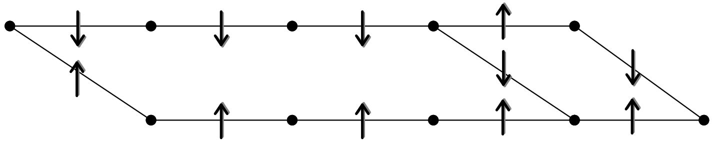


$$\begin{bmatrix}\downarrow & \uparrow\end{bmatrix} \begin{array}{cc} & j \\ i & \begin{bmatrix}\downarrow & 0\\ 0 & \uparrow\end{bmatrix}\end{array} * \begin{bmatrix}\downarrow & 0\\ 0 & \uparrow\end{bmatrix} * \begin{bmatrix}\uparrow & \downarrow\\ 0 & \uparrow\end{bmatrix} * \begin{bmatrix}\downarrow\\ \uparrow\end{bmatrix}$$

$$B_{ij}^{\beta}$$

A Quantum System



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$$B^{\beta}_{ij}$$

$$\boxed{\begin{array}{c}\vec{B}_{0,0} = \downarrow \\ B^{\uparrow}_{0,0} = 0,\ B^{\downarrow}_{0,0} = 1\end{array}}$$

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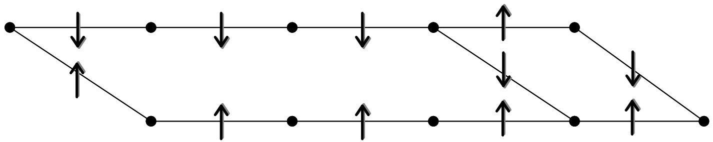

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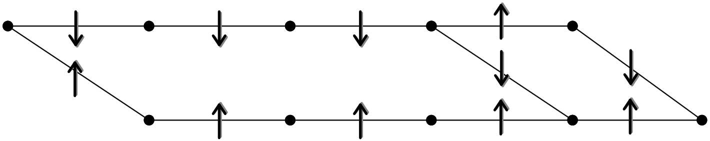

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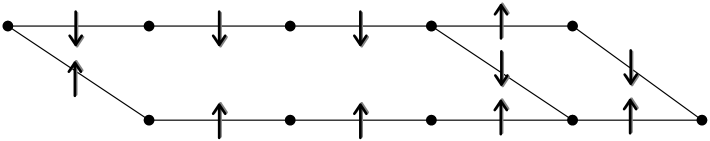
$$A_i^\alpha \quad B_{ij}^\beta \quad C_{jk}^\gamma \quad D_{kl}^\delta \quad E_l^\gamma$$

A Quantum System



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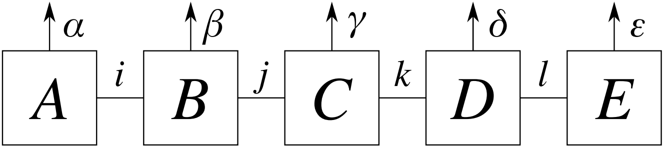
$$S^{\alpha\beta\gamma\delta\epsilon} = \sum_{ijkl} A_i^\alpha B_{ij}^\beta C_{jk}^\gamma D_{kl}^\delta E_l^\epsilon = A * B * C * D * E$$

A Quantum System



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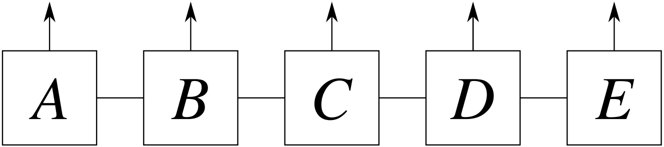

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Outline

Crash Course in Quantum Mechanics

1. The state of a quantum system is represented by a vector.

$$S^{\overbrace{\alpha\beta\gamma\delta\epsilon}^{x}} \equiv S^x \equiv \vec{S}$$

2. Time evolution of a quantum system is given by multiplication by a linear unitary operator.

$$\vec{S}(t) = \mathbf{U}(t) \cdot \vec{S}(0).$$

3. This unitary can be expressed as the exponentiation of a Hermiation operator called the Hamiltonian.

$$\mathbf{U}(t) = e^{i\mathbf{H}t}$$

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2. Eigenstates of **H** correspond to state vectors which are stationary over time except for an evolving overall phase.

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3. Goal of Quantum Mechanics: Find the eigenstates of **H**.
4. Starting Point: Find the eigenstate with the lowest eigenvalue–i.e., the "ground state".

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Variational Approach

1. Objective: Find the lowest value of ω such that $\mathbf{H} \cdot \vec{S} = \omega\vec{S}$.
2. Equivalent to finding the state vector $\vec{S}$ to minimize the Rayleigh quotient $R(\vec{S}) := \frac{\vec{S}^* \cdot \mathbf{H} \cdot \vec{S}}{\vec{S}^* \cdot \vec{S}}$.
3. Since we don't know $\vec{S}$ *a priori*, make a guess with tunable parameters (an "ansatz") and vary these parameters to minimize $R(\vec{S})$.
4. The "divide and conquer" approach gives us an excellent ansatz for this purpose.
   1. $R(\vec{S})$ can be evaluated efficiently.
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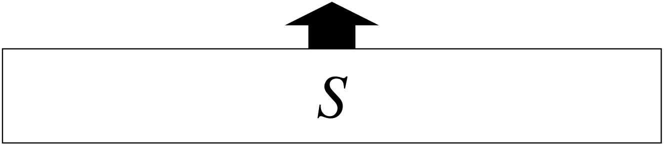

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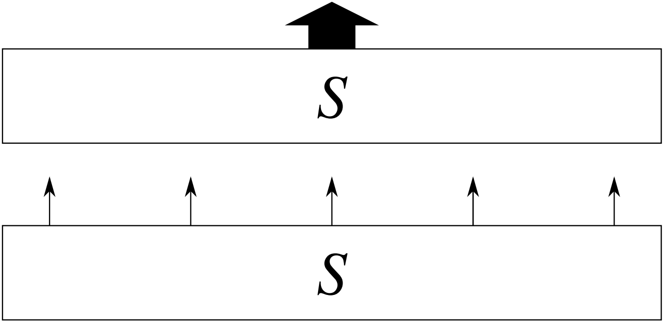

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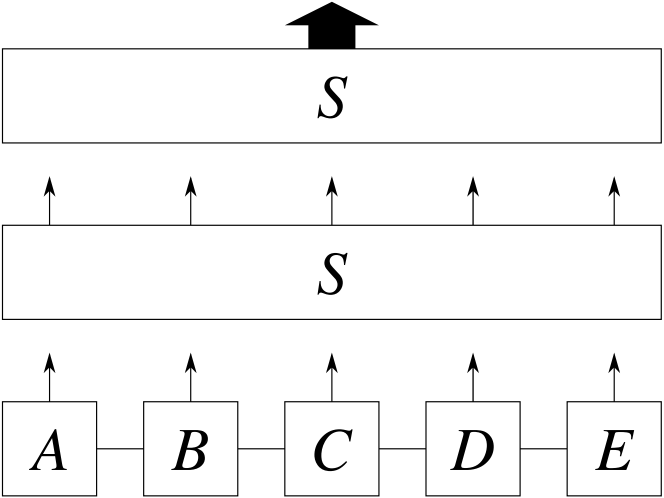

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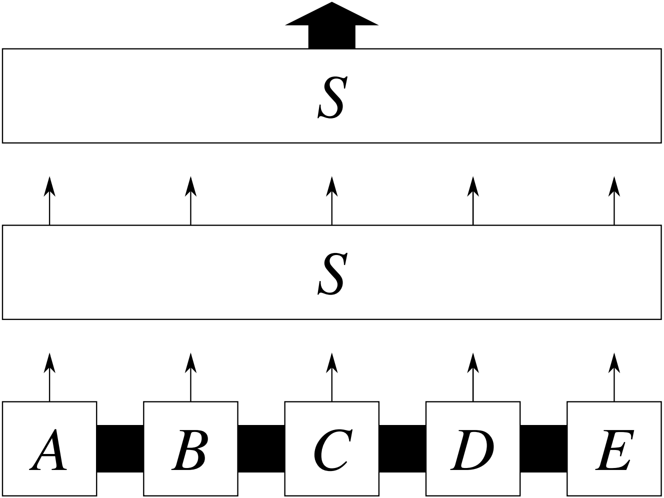

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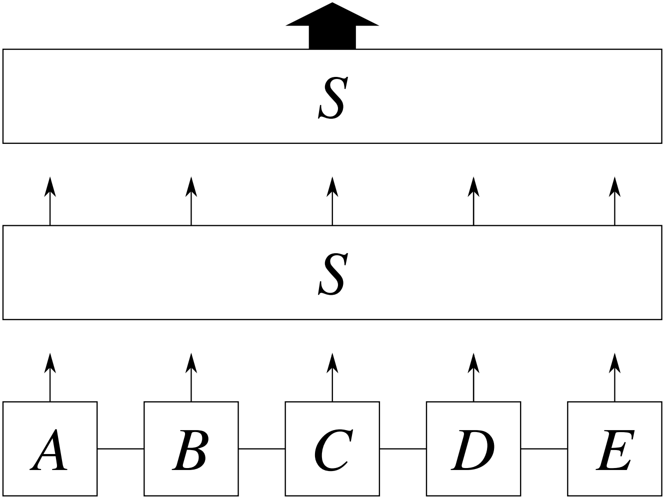

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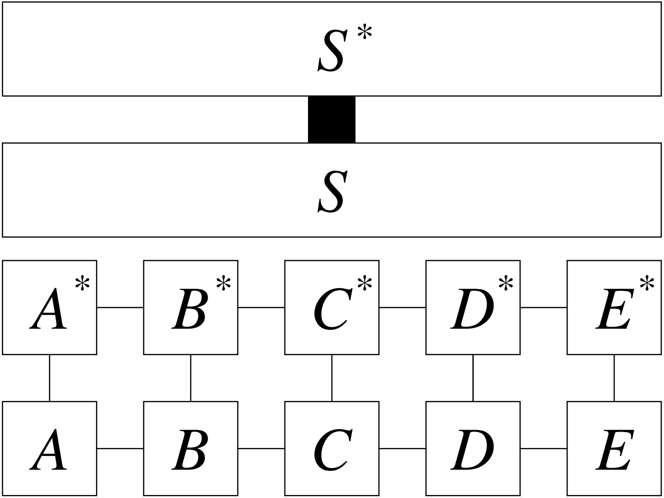

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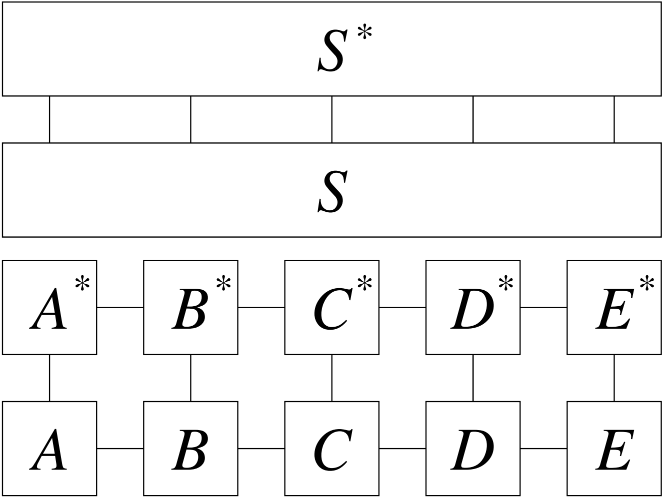

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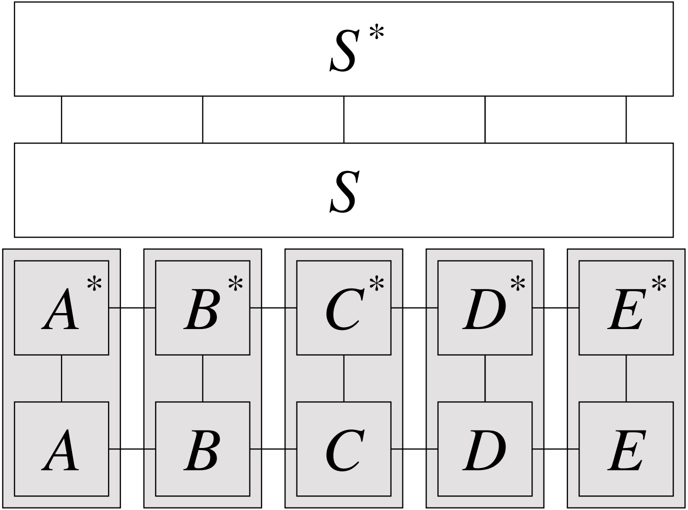

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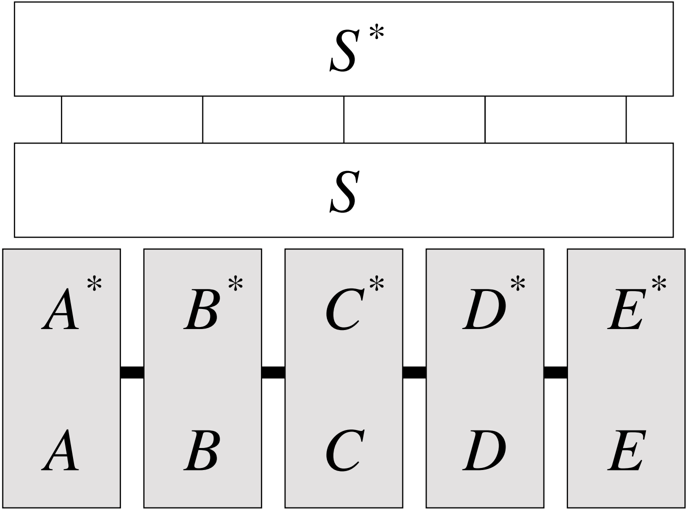

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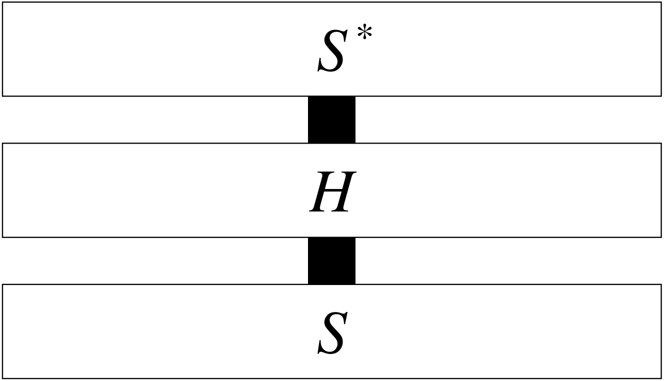

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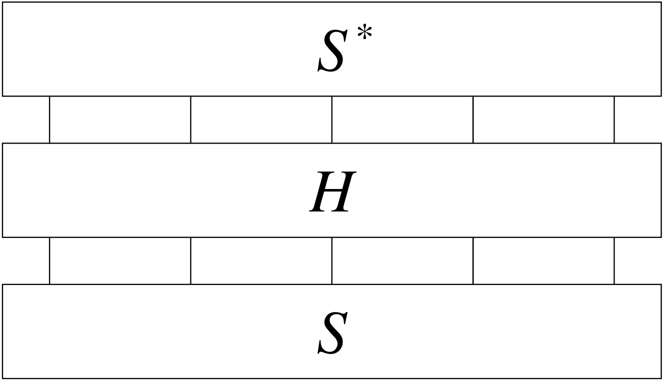

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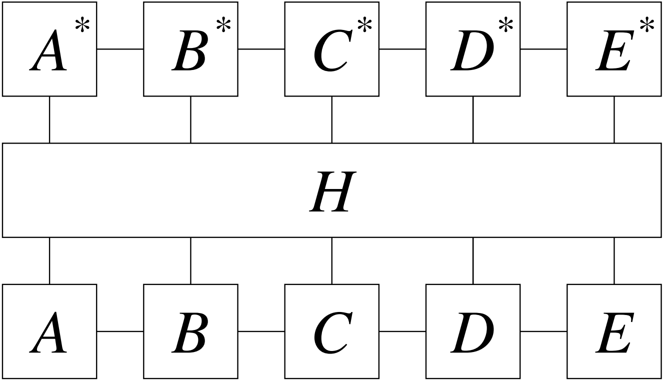

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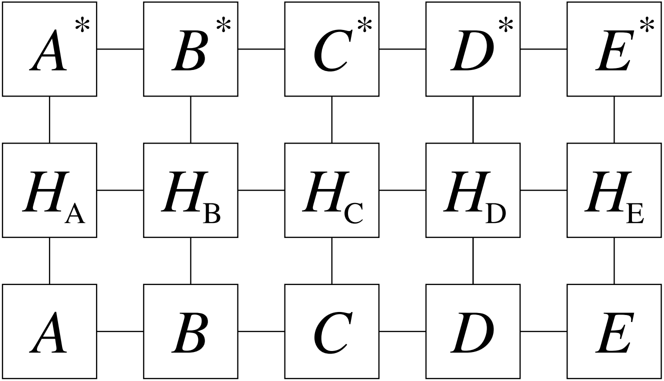

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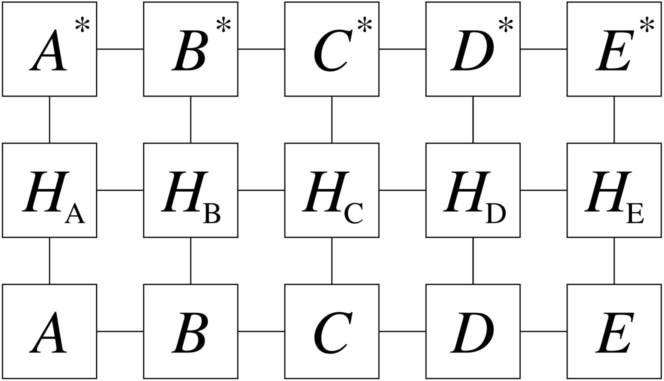

Look up "Crosswhite" on arxiv.org.

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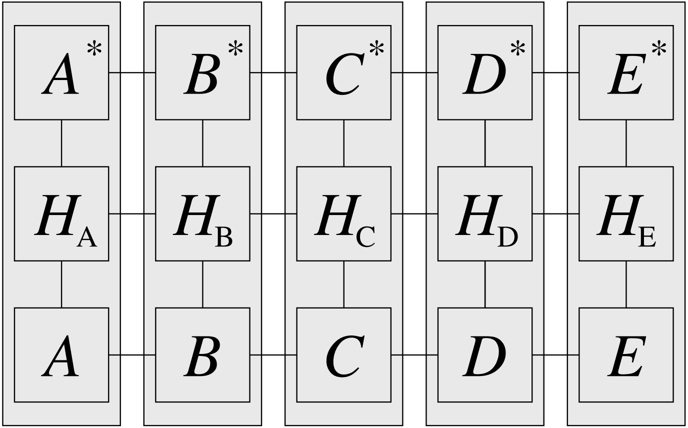

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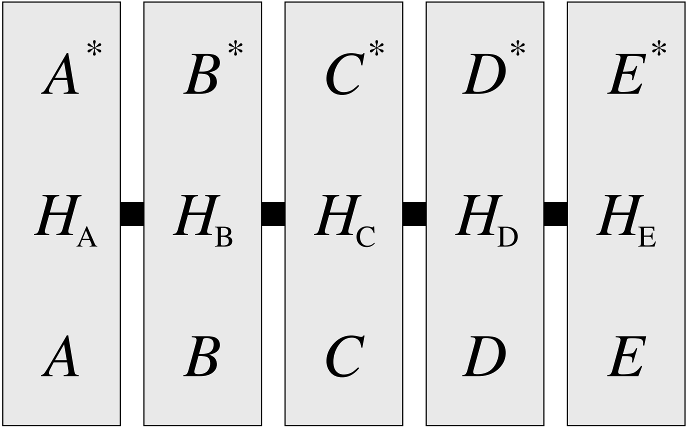
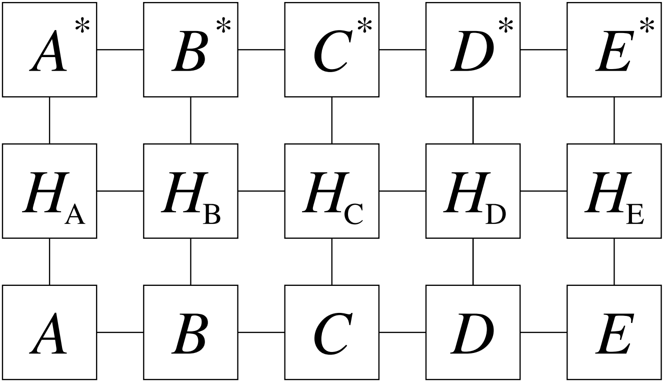


Efficiently varying a site



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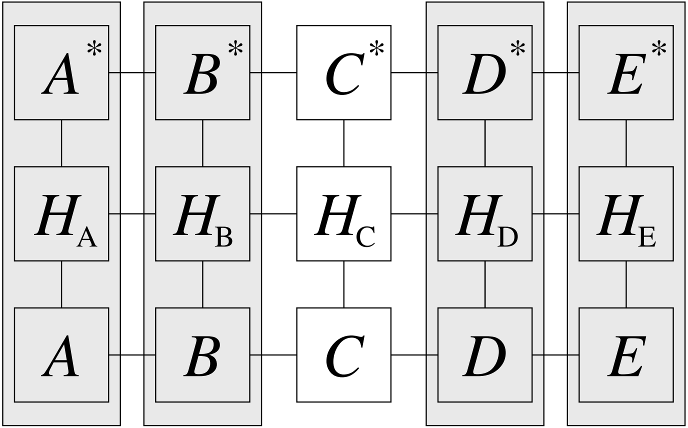
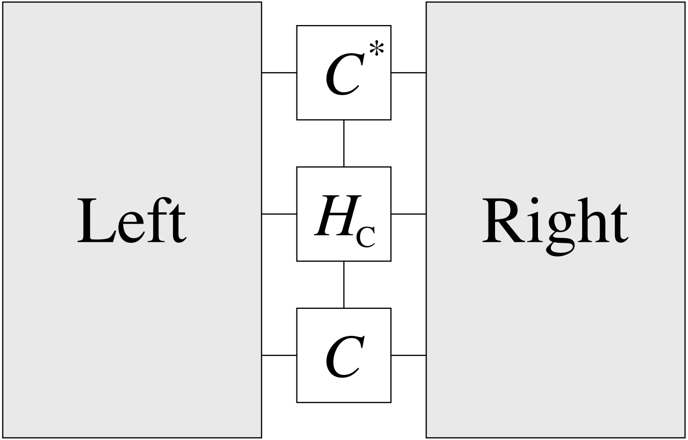


Efficiently varying a site



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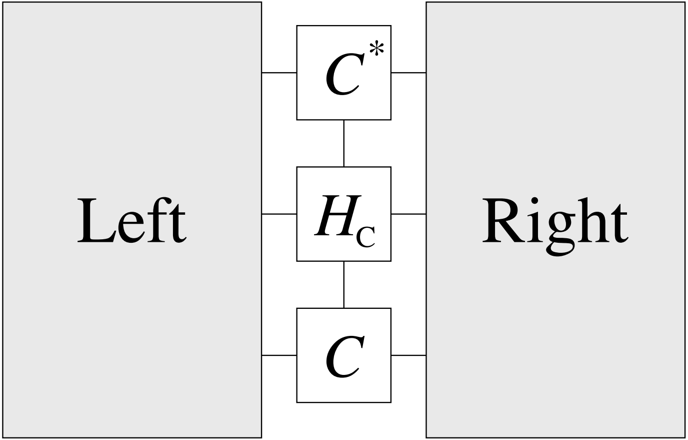

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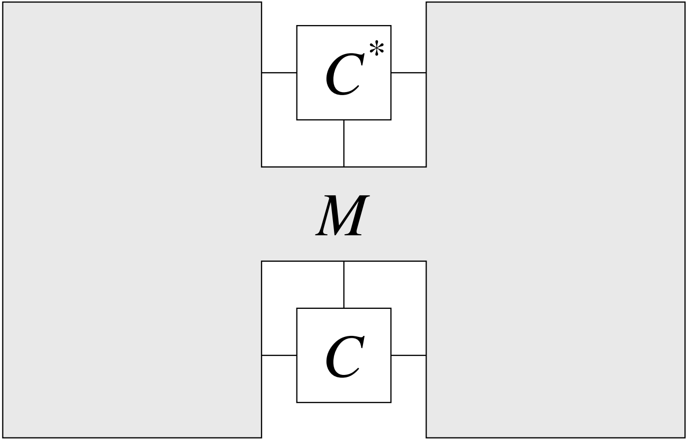

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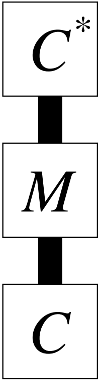

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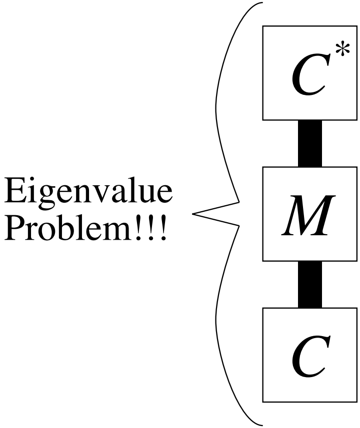
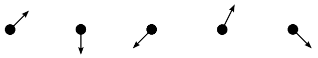


The Battle of Haldane-Shastry



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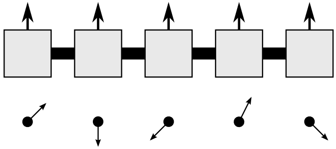

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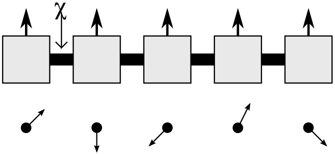

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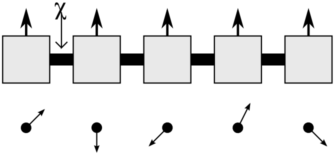
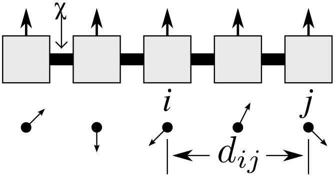


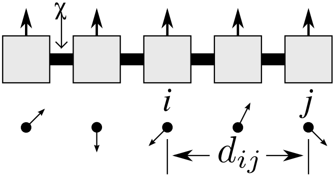
$$V_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j \; / \; d_{ij}$$

The Battle of Haldane-Shastry



$$V_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j \; / \; d_{ij}$$

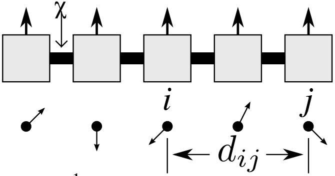
The Battle of Haldane-Shastry



$$V_{ij} = \vec{\sigma}_i \cdot \vec{\sigma}_j \;/\; d_{ij}$$

$$C_Z(d) = \langle \sigma_i^Z \sigma_{i+d}^Z \rangle_i$$

The Battle of Haldane-Shastry



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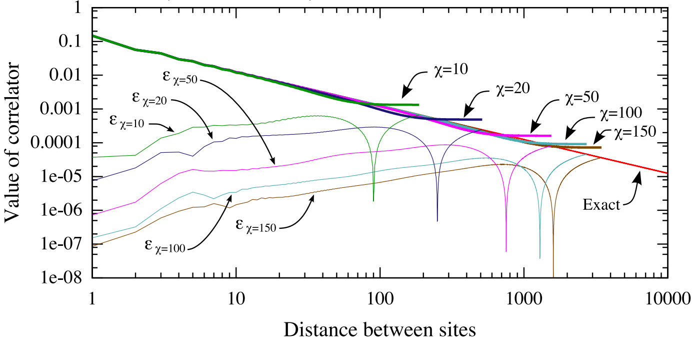
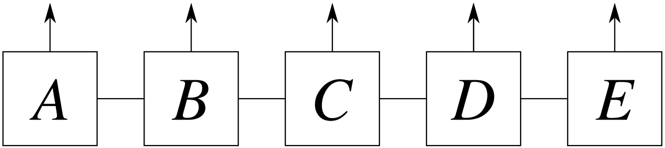
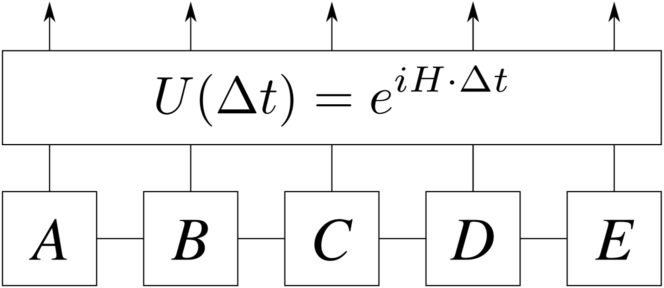


Outline

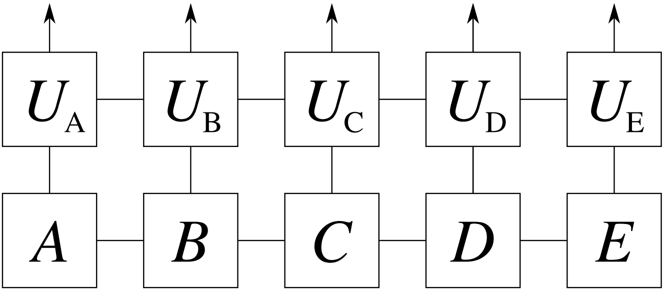
Time evolution: $T = 0$



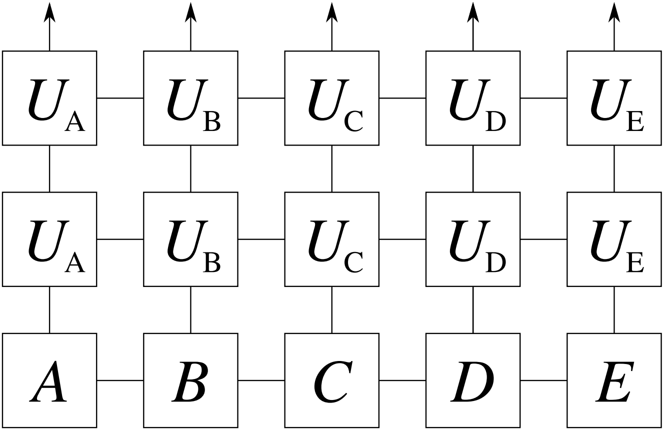
Time evolution: $T = \Delta t$



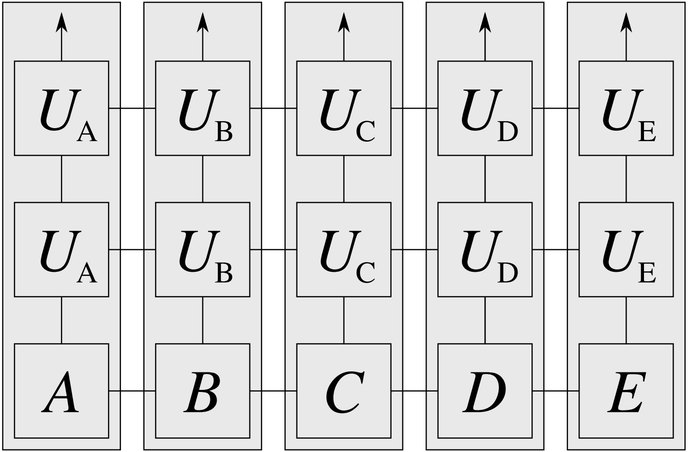
Time evolution: $T = \Delta t$



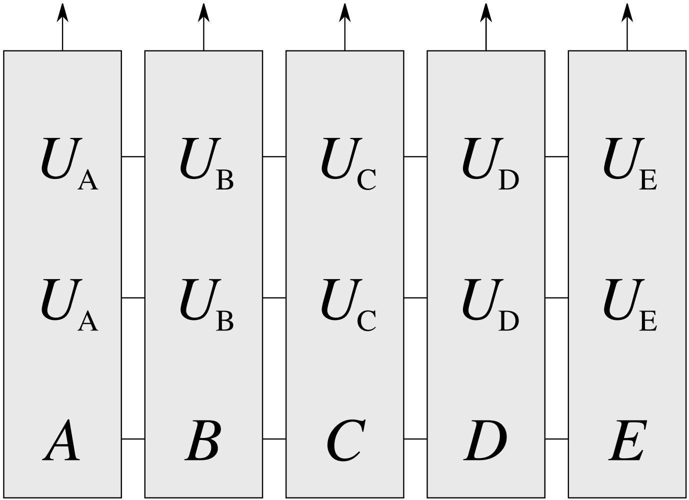
Time evolution: $T = 2\Delta t$



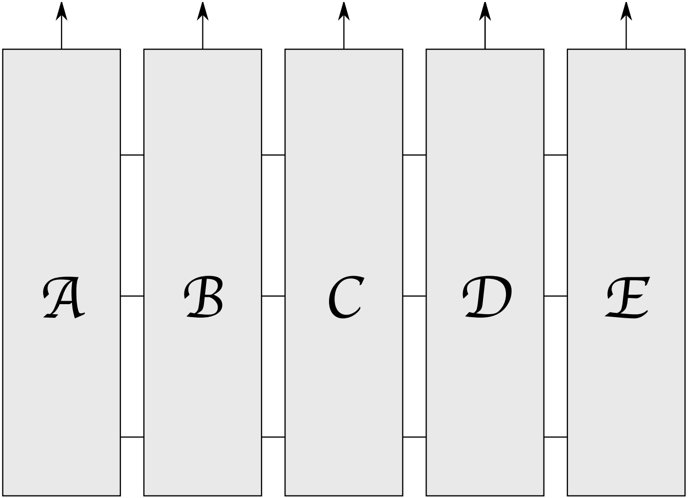
Time evolution: $T = 2\Delta t$



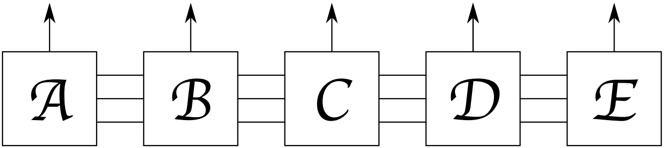
Time evolution: $T = 2\Delta t$



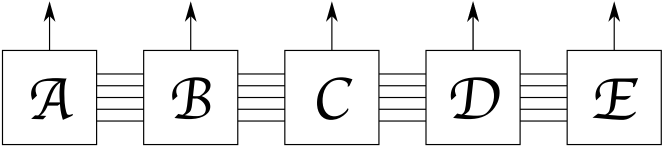
Time evolution: $T = 2\Delta t$



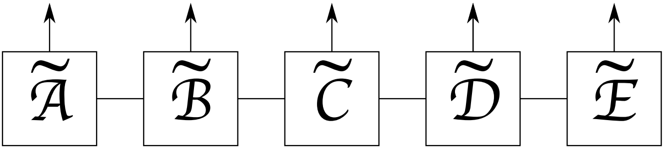
Time evolution: $T = 2\Delta t$



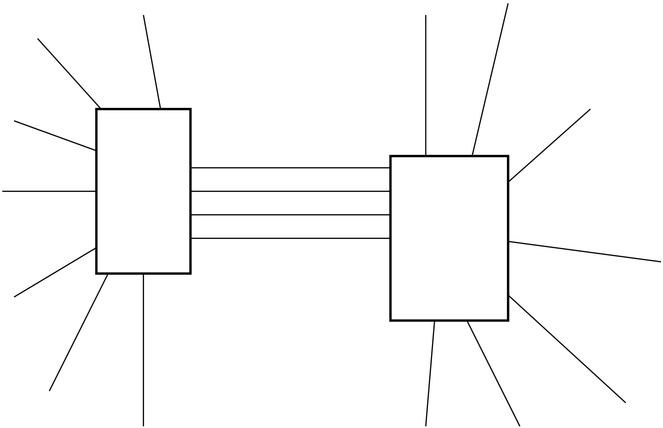
Time evolution: $T = 4\Delta t$



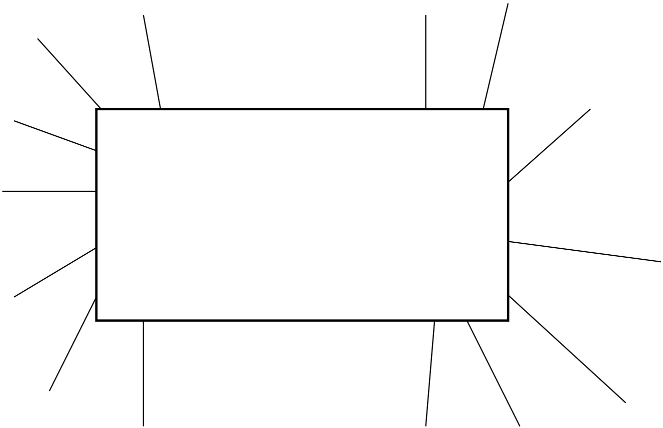
Time evolution: $T = 4\Delta t$



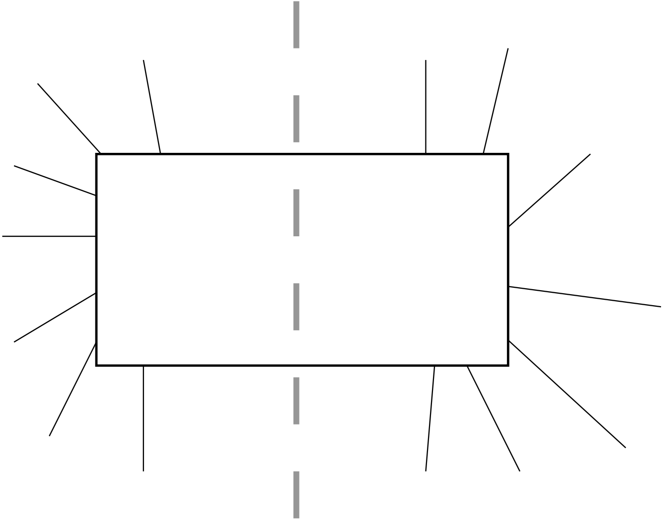
Singular Value Decomposition



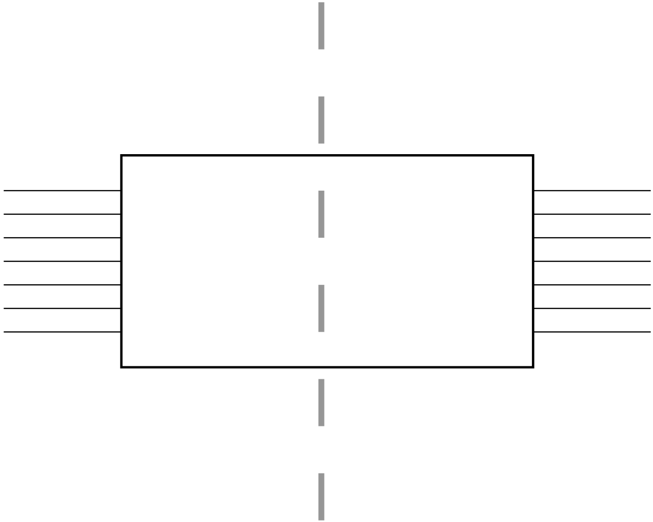
Singular Value Decomposition



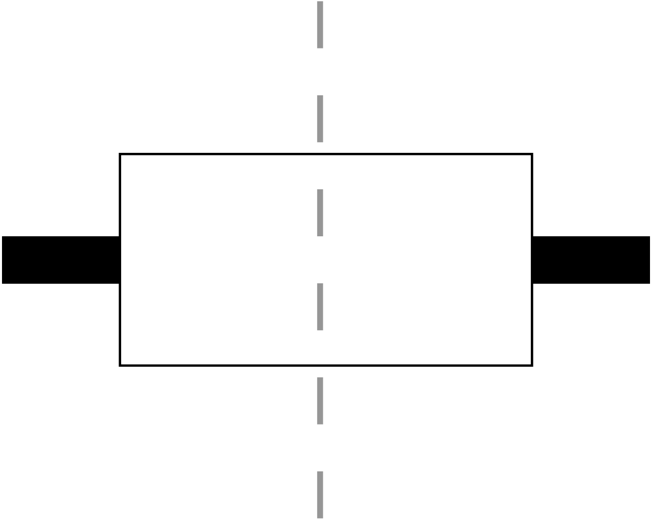
Singular Value Decomposition



Singular Value Decomposition



Singular Value Decomposition



Singular Value Decomposition

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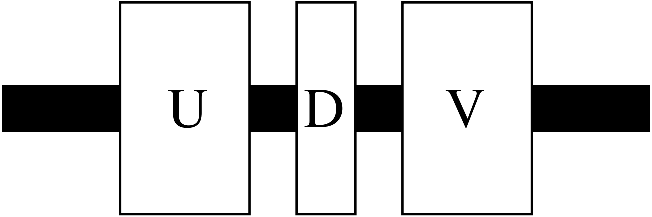

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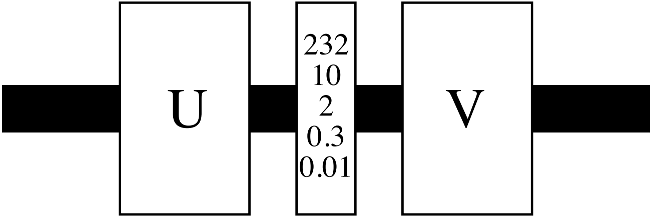
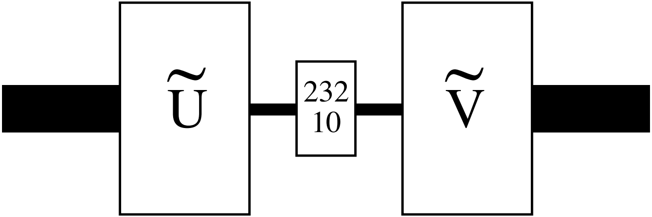
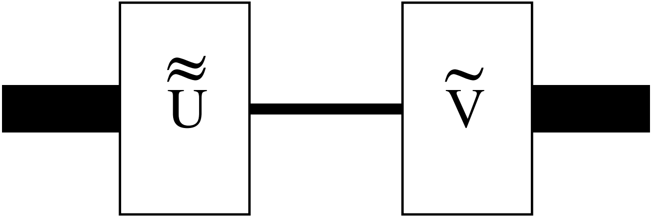


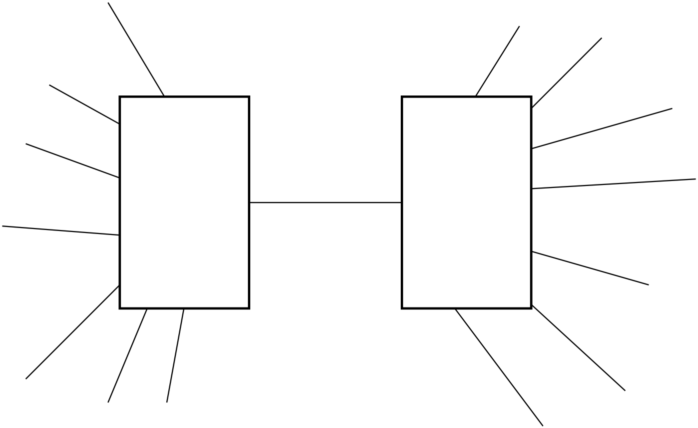
Singular Value Decomposition



Singular Value Decomposition



Singular Value Decomposition



Summary

- Divide and Conquer is a general strategy vital to understanding complex systems.
- For systems with intrinsic nondeterminism, one must have a more nuanced sense of how to “divide” the system, but it still can be made to work.

References: Search for "Crosswhite" on arxiv.org; arXiv:cond-mat/0403313

Open Questions

- How well can these methods scale with inceased computing power?
- How can these ideas be made to apply to systems with more than one dimension?