

$$|w_n^2\rangle = \sum_{i=0}^{n-2} \sum_{j=0}^{n-2-i} |\uparrow\rangle^i |\downarrow\rangle |\uparrow\rangle^j |\downarrow\rangle |\uparrow\rangle^{n-i-j-2}$$