

Statistical Inference Course Project - Part 1

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Overview

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. Set `lambda = 0.2` for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should

1. Show the sample mean and compare it to the theoretical mean of the distribution.
2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
3. Show that the distribution is approximately normal.

Data Simulation

```
require(ggplot2)

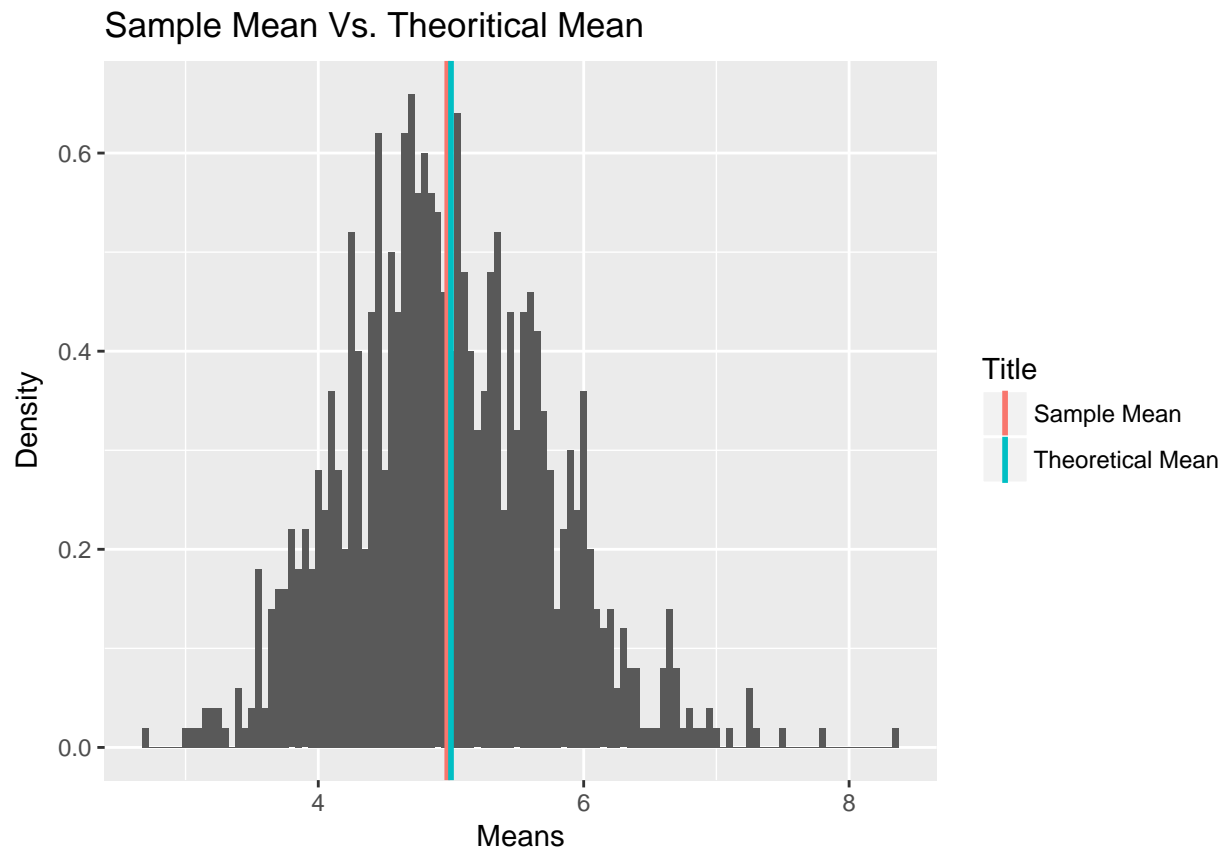
## Loading required package: ggplot2

lambda = 0.2
n = 40
simulations = 1:1000
set.seed(12345)
simDf <-
data.frame(x = sapply(simulations, function(x) {
  mean(rexp(n, lambda))
})))
```

Sample Mean Vs. Theoretical Mean

```
mean_data <- data.frame(Title=c("Sample Mean", "Theoretical Mean"), vals=c(colMeans(simDf), 1/lambda))
g = ggplot(data = simDf, aes(x = x))
g = g + geom_histogram(binwidth = 0.05, aes(y = ..density..))
g = g + geom_vline(data = mean_data, mapping=aes(xintercept = vals, colour=Title), size=1, show.legend=
```

```
g = g + labs(title = "Sample Mean Vs. Theoretical Mean")
g = g + labs(x="Means") + labs(y = "Density")
g
```



Conclusion

As shown below the theoretical mean of the distribution is **5.00** and sample mean is **4.97**

```
##           Title      vals
## x      Sample Mean 4.971972
##   Theoretical Mean 5.000000
```

Sample Vs. Theoretical Std.Dev & Variance

```
sampleSD <- sd(simDf$x)
sampleVariance <- sampleSD ^ 2

theoreticalSD <- (1/lambda) / sqrt(n)
theoreticalVariance <- theoreticalSD ^ 2

resultDF = data.frame(c("Sample", "Theoretical"),
                      c(sampleSD, theoreticalSD),
                      c(sampleVariance, theoreticalVariance))
```

```
colnames(resultDF) <- c("", "Std.Dev", "Variance")
```

Conclusion

From the table shown below the difference between theoretical and sample values for Std.dev and variance are very less.

```
##              Std.Dev  Variance
## 1      Sample 0.7716456 0.5954369
## 2 Theoretical 0.7905694 0.6250000
```

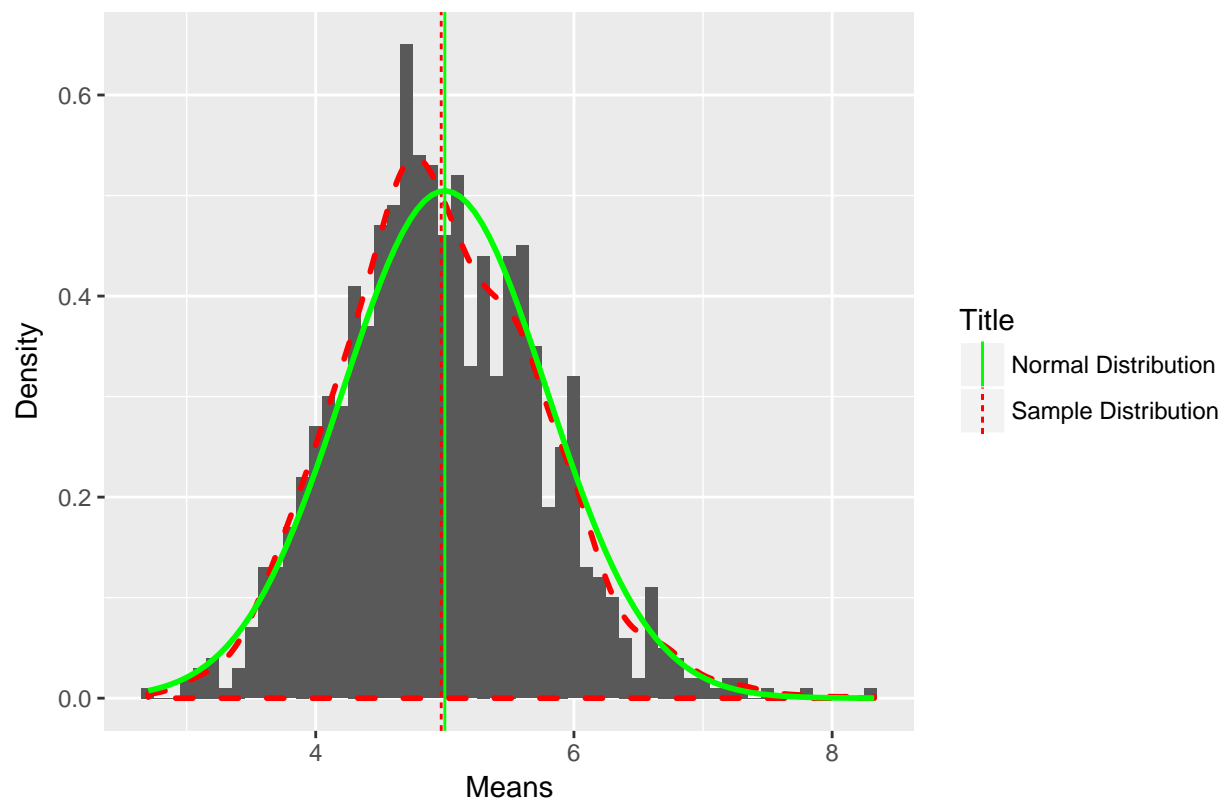
Distribution

```
df <- data.frame(Title=c("Sample Distribution", "Normal Distribution"), Vals=c(colMeans(simDf), 1/lambda))

g = ggplot(data = simDf, aes(x = x))
g = g + geom_histogram(binwidth=0.1, aes(y=..density..))
g = g + geom_density(color="red", size=1, linetype=2)
g = g + geom_vline(data=df[df$Title == "Sample Distribution", ], mapping=aes(xintercept=df[df$Title == "Sample Distribution", ]$Vals))
g = g + stat_function(fun=dnorm, args=list(mean=df$Vals[df$Title == "Normal Distribution"], sd=theoreticalSD))
g = g + geom_vline(data=df[df$Title == "Normal Distribution", ], mapping=aes(xintercept=df[df$Title == "Normal Distribution", ]$Vals))
g = g + guides(linetype=guide_legend(override.aes=list(colour = c("green", "red"))))
g = g + labs(title = "Distribution of Averages of Samples vs Theoretical Mean") + labs(x="Means") + labs(y="Density")

g
```

Distribution of Averages of Samples vs Theoretical Mean



Conclusion

The normal distribution line refers to $\lambda = 0.2$. The sample distribution refers to the averages of simulated samples. The graph shows that the two distribution lines are well aligned thus the distribution of simulated data is approximately normal.