POLITECNICO DI TORINO

Electronic and Communications Engineering

Assignment Report 1

Applied Signal Processing Laboratory

01TUMLP

Academic Year 2019/20

Gloria DAL SANTO
s245220

Exercise 1

In the first exercise it was asked to generate a truncated sinusoidal signal $s(t)$ defined over a time period T of 1 s with random phase ϕ and frequency f_0 between 1 and 10 Hz, defined as follows:

$$s(t) = P_T(t) sin(2\pi f_0 t + \phi)$$

where $P_T(t)$ represents the rectangular pulse function.
To implement this function as well as all the other exercises in this assignment, I used Matlab development environment *App Designer*.
I chose to sample the signals at frequency $f_s = 1000\ Hz$, 50 times higher than the Nyquist frequency $2 \cdot f_0$ with $f_0 = 10\ Hz$. To generate the random phase I used rand Matlab function based on uniformly distributed probability density function. From the graphs it can be seen that the first signal has a phase shift around $-\pi/2$ while for the second this is almost null.
The two outcomes for a frequency of 1 Hz and 10 Hz are reported in Figures 1 and 2.

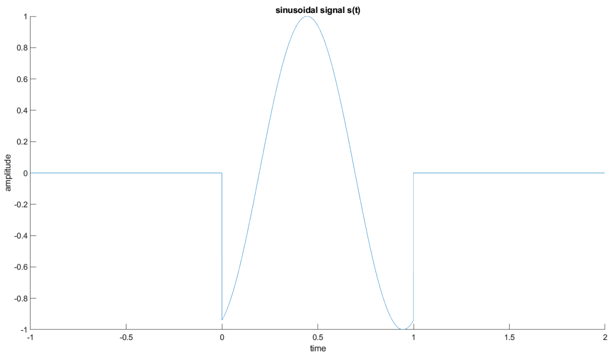

Figure 1: Exercise 1 *signal* $s(t)$ *at frequency* $f_0 = 1\ Hz$

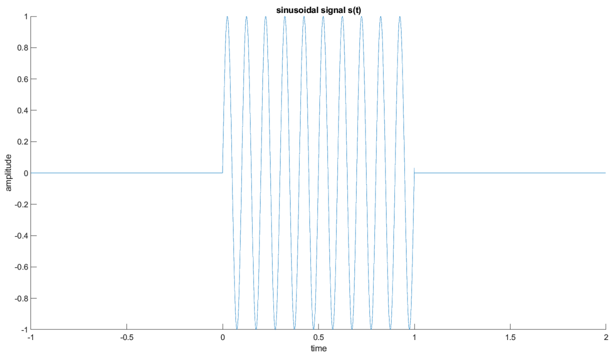

Figure 2: Exercise 1 *signal* $s(t)$ *at frequency* $f_0 = 10\ Hz$

Exercise 2

The second exercise involves the convolution of two rectangular signals $s_1(t)$ and $s_2(t)$ defined as follows:

$$s_1(t) = A_1 P_{T_1}(t)$$

$$s_2(t) = A_2 P_{T_2}(t - D_2)$$

$s_1(t)$ is defined with a unitary amplitude A_1 and a time window T_1 of 1 second, whereas $s_2(t)$ has an amplitude A_2 with integer value between 1 and 4, time window T_2 between 0 and 3 seconds and time delay D_2 from 0 to 4 seconds. To perform the convolution I used the Matlab function conv and then multiplied it by the sampling period $t_s = 1/f_s = 1/1000\ s$ to adjust the amplitude. Since the conv function implements the following sum:

$$\sum_j u(j)v(k - j + 1)$$

it is necessary to multiply the result with the sampling period t_s so that it correctly approximates the continuous time convolution function by means of rectangle rule.
Figure 3 shows the GUI displaying the two signals and the result of the convolution $s_3(t) = s_1(t) * s_2(t)$. The correctness of the result can be verified from the shape and extension of $s_3(t)$. The convolution of two rectangular pulses is indeed an isosceles trapezoidal signal with lower and upper limits of the extension that are the sum of the respective lower and upper limits of the extensions of the two input signals.

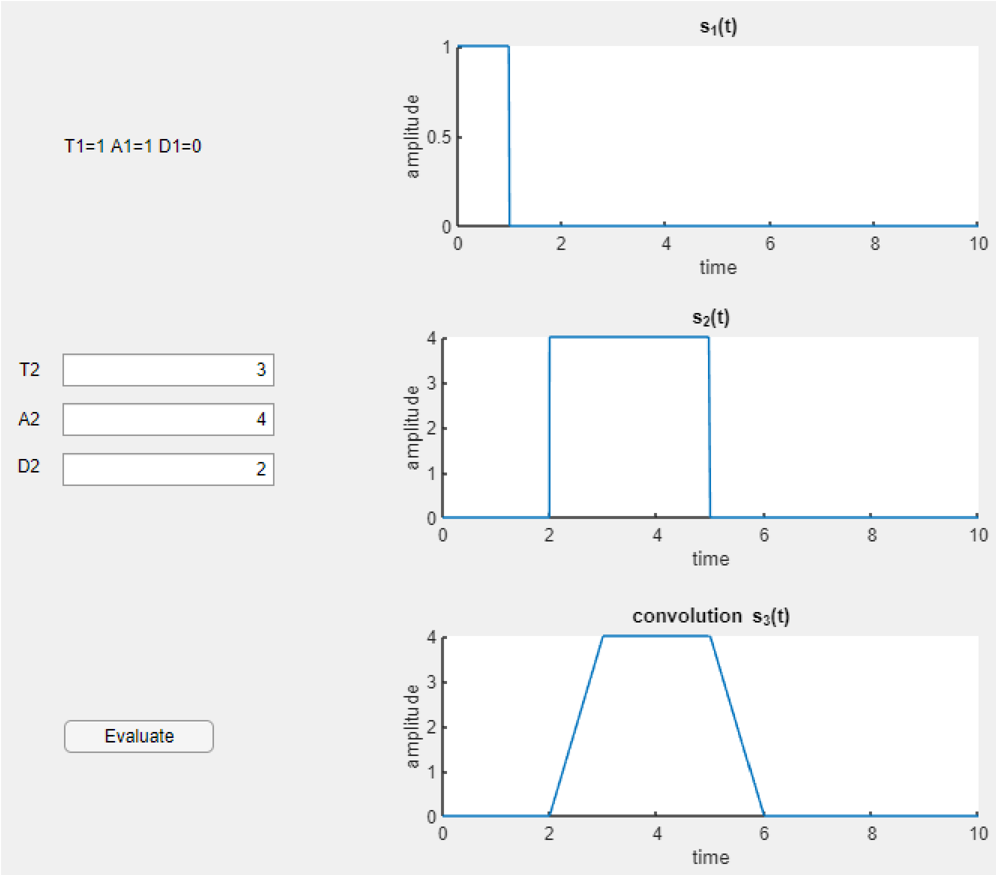

Figure 3: Exercise 2 *GUI with input parameters and output signals*

Exercise 3

The third exercise asked for the generation of 100 truncated sinusoidal signals of the type of those depicted in the first exercise, with increasing frequency from 1 to 100 Hz.
In Figure 4 are shown six different signals defined as follows:

$$s_{10}(t) = P_T(t)sin(2\pi 10t + \phi_{10})$$
$$s_{20}(t) = P_T(t)sin(2\pi 20t + \phi_{20})$$
$$s_{100}(t) = P_T(t)sin(2\pi 100t + \phi_{100})$$
$$s_a(t) = \sum_{i=1}^{10} s_i(t)$$
$$s_b(t) = \sum_{i=1}^{20} s_i(t)$$
$$s_c(t) = \sum_{i=1}^{100} s_i(t)$$

where $s_i(t) = P_T(t)sin(2\pi it + \phi_i)$.
From the figure it is evident that with an increasing number of sinusoidal signals that are added together the resulting signal becomes more and more chaotic and its amplitude higher, but still remaining centered around zero. Looking at the sums in time domain it is hard to derive the frequencies and phases of the original signals even though it is clear that they are sinusoidal functions.

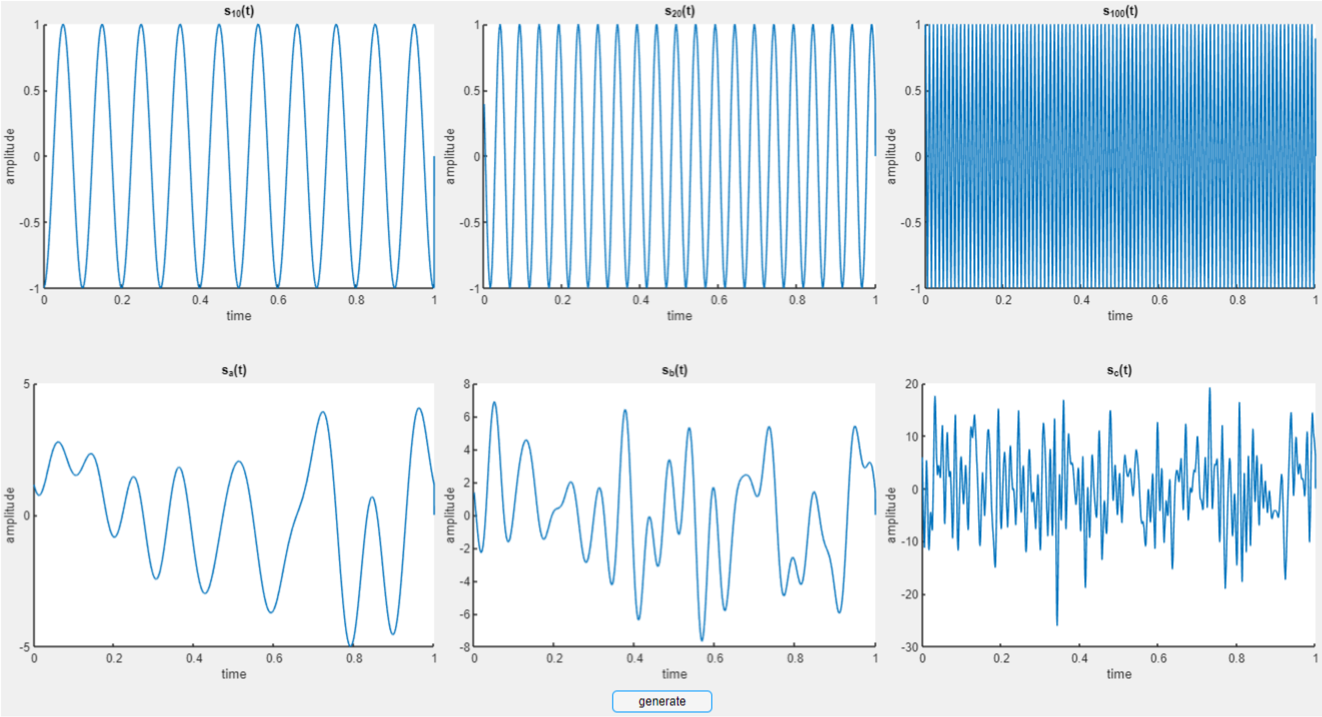

Figure 4: Exercise 3 *GUI with output signals*

Exercise 4

In the first phase of exercise 4 it was asked to compute the energy of a rectangular pulse $x(t) = AP_T(t)$ with arbitrary amplitude A and time window T in a range between 1 and 3 seconds, and to compare it with its theoretical value.
To compute the energy I used the discretized version of the formula of the energy for continuous time signals:

$$E\{x\} = \sum_{n=1}^{N} |x[n]|^2 \cdot t_s$$

where t_s coincides with the sampling period $t_s = \frac{T}{1000}$.
With amplitude $A = 2$ and period $T = 2$ s the energy calculated through the sum was $E\{x\} = 7.9960$ while from the theory it was expected a value of $E\{x(t)\} = A^2T = 8$. The difference between the two values is 0.004 which is due to the approximation performed by Matlab $\mathrm{rectangularPulse}$ function at point $n = 1$ and $n = T + 1$, where the function has value $A/2$. The theoretical value of the energy $E\{x(t)\} = A^2T$ assumes that at point $t = 0$ and $t = T$ the continuous function has amplitude A, while for $t > T$ it goes to 0, as a consequence using the discretized function I introduced an error of $|A/2|^2 \cdot 2t_s$.
Then it was asked to compute the Fourier transform of $x(t)$ using Matlab function fft. To apply a proper amplitude normalization I multiplied the result by a normalization factor $k = \frac{AT^2}{1000}$. Since the fft function organizes the frequency components in the period $[0\ f_s]$, I rearranged the elements using a circular shift of $f_s/2$ to center the frequency axis in the origin and to display the transform in the period $[-\frac{f_s}{2}\ \frac{f_s}{2}]$. Figure 5 shows the resulting Fourier transform limited in the frequency range of $[-\frac{5}{T}\ \frac{5}{T}]$. To compute the energy through the following approximation of the Parseval identity

$$E\{X\} = \sum_{n=1}^{N} |X[n]|^2 \cdot \frac{f_s}{N}$$

I used the continuous time Fourier transform of the sampled signal obtained by multiplying the fft with the sampling period t_s. For $A = 2$ and $T = 2\ s$ the energy calculated was $E\{x\} = 7.9960$, which has the same error as the one calculated before (in time domain).
The energy contained in the frequency range $[0\ 100/T]$, which corresponds to the energy contained in the first 100 lobes in the positive axis, has value 0.39957 and it accounts for the 99.94% of the total energy (evaluated in the positive frequency axis). The graphs in Figure 6 and 7 show the energy percentages contained in each of the 100 lobes. The first lobe alone includes 90.33% of the energy, while the remaining 9.61% is spread throughout the other 99 lobes with a decreasing contribution moving to higher frequencies.
The exact number of lobes that one should take into account depends on how finite the approximation should be. However it can be noticed that taking the first 10 lobes 99% of the total energy is conserved.

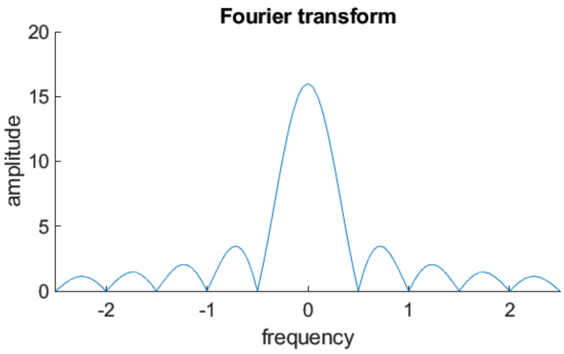

Figure 5: Exercise 4 fft *of a rectangular pulse for* $A = 2$ *and* $T = 2\ s$

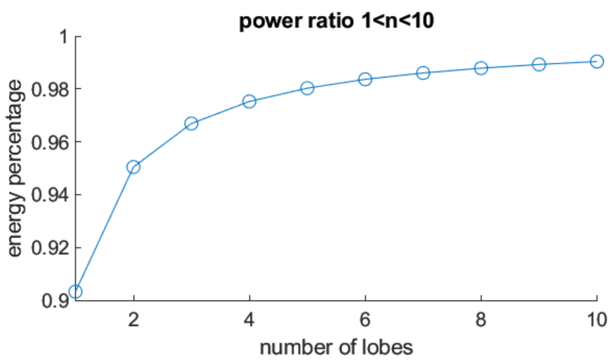

Figure 6: Exercise 4 *power ratios of* $x(t)$ *for the first 10 lobes*

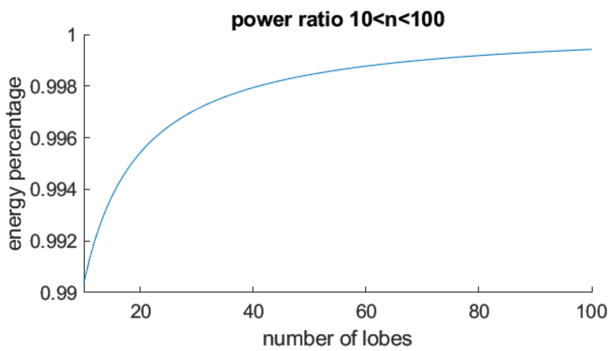

Figure 7: Exercise 4 *power ratios of* $x(t)$ *for the first 100 lobes*

Exercise 5

The last exercise involves an analysis of the effect on the energy spectral density $|X(f)|^2$ of different sampling frequencies applied on signal $x(t)$ defined as follows

$$x(t) = AP_T(t) \cdot [sin(2\pi f_1 t + \phi_1) + sin(2\pi f_2 t + \phi_2) + sin(2\pi f_3 t + \phi_3)]$$

where ϕ_i are random phases and the values of the frequencies are $f_1 = 10\ Hz$, $f_2 = 20\ Hz$ and $f_3 = 100\ Hz$. The amplitude of the rectangular pulse was set to 1. Figure 8 shows signal $x(t)$ for a set of randomly generated phases over a time window of 1 second.

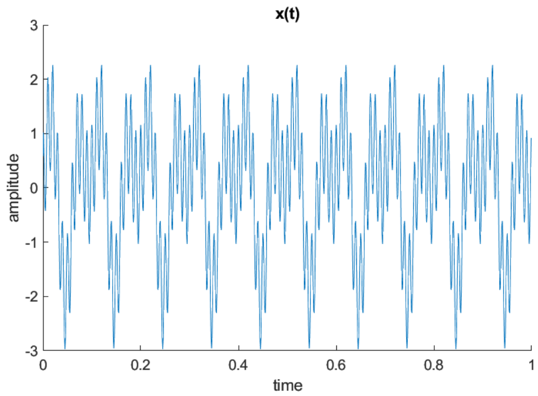

Figure 8: Exercise 5 *amplitude of signal* $x(t)$

$X(f)$ is the convolution between the Fourier transform of the rectangular pulse and 6 deltas with amplitude $A = 1/2$:

$$X(f) = \frac{A}{2} \cdot Sinc(f) * [\delta(f - f_1)e^{j\phi_1} + \delta(f + f_1)e^{-j\phi_1} + \\ \delta(f - f_2)e^{j\phi_2} + \delta(f + f_2)e^{-j\phi_2} + \delta(f - f_3)e^{j\phi_3} + \delta(f + f_3)e^{-j\phi_3}]$$

If sampled properly, the spectra of $x(t)$ in the frequency interval $[-f_s/2\ f_s/2]$ corresponds to 6 Sinc functions shifted to the frequencies were the deltas are different from zero. The infinite extension of the Sinc functions does not distort the shape of the neighboring Sinc because only the central lobes are relevant in amplitude. Figure 9 shows the energy spectral density $|X(f)|^2$ of $x(t)$ properly sampled at $f_s = 1000\ Hz$. The lobes are correctly located around $f_1 = \pm 10\ Hz$, $f_2 = \pm 20\ Hz$ and $f_3 = \pm 100\ Hz$ and their amplitude is $1/4$.
Figure 10 depicts the aliasing phenomena that occurs when signals $x(t)$ is sampled at a frequency lower that than $2f_3$ (Nyquist limit). With $f_s = 160Hz$ the frequency components of the alias spectrum fall into the range of the original spectrum, exactly at $f_s - f_3 = 60Hz$, $f_s - f_2 = 140Hz$ and $f_s - f_1 = 150Hz$. In the specific case of $f_s = 120\ Hz$ the frequency components of the alias around $f_2 = 20\ Hz$ overlap with those of the original signal. In this case $X(f)$ around $20\,Hz$ is multiplied by a factor of $e^{j\phi_2} + e^{j\phi_3}$ whose magnitude is not unitary but varies according to each different pairs of values of ϕ_2 and ϕ_3. As a consequence the amplitude of the overall energy spectral density at f_2 may vary from 0 to 1 (Figure 11 and 12).

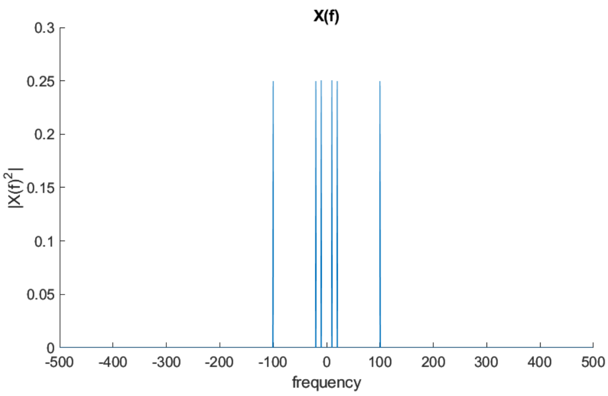

Figure 9: Exercise 5 $|X(f)|^2$ *with sampling frequency* $f_s = 1000\ Hz$

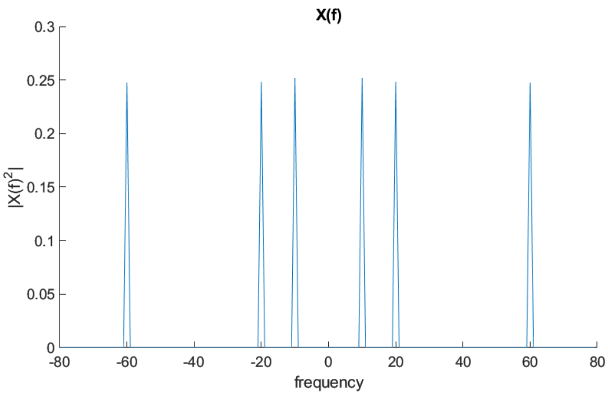

Figure 10: Exercise 5 $|X(f)|^2$ *with sampling frequency* $f_s = 160\ Hz$

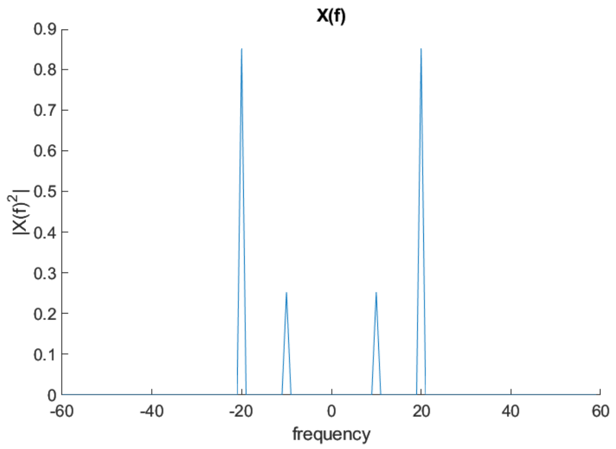

Figure 11: Exercise 5 $|X(f)|^2$ *with sampling frequency* $f_s = 120\ Hz$

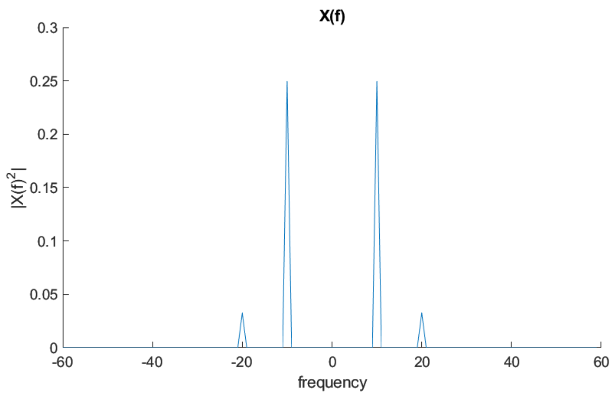

Figure 12: Exercise 5 $|X(f)|^2$ *with sampling frequency* $f_s = 120\ Hz$ *and different phases*