POLITECNICO DI TORINO

Electronic and Communications Engineering

Assignment Report 2 - Signal Correlation

Applied Signal Processing Laboratory

01TUMLP

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Exercise 1

In the first exercise it was asked to generate two sequences $x[n]$ and $y[n]$ with periods $T_x = 5$ and $T_y = 10$ and to check their correlation properties through different discrete correlation functions. As first step, I construct the two vectors on the following periodic sequences:

$$x_T = [1\ 2\ 3\ 5\ 8]$$

$$y_T = [1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89]$$

Figure 1 and 2 shows the resulting signals. Vector $x[n]$ has $N = 50$ elements corresponding to 10 repetitions of its periodic segment, while vector $y[n]$ has $M = 80$ elements, hence 8 periods.

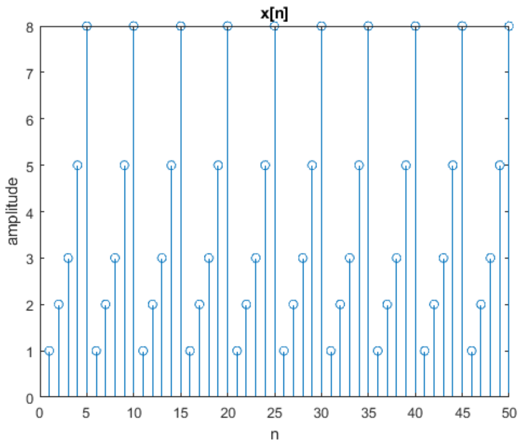

Figure 1: Exercise 1 *signal* $x[n]$ *with period* $T_x = 5$

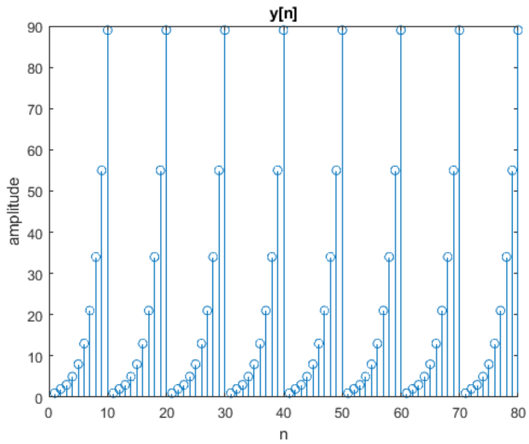

Figure 2: Exercise 1 *signal* $y[n]$ *with period* $T_y = 10$

I implemented the correlation function according to the following formula:

$$R_{xy}[n] = \sum_{k=0}^{N} x[k] \cdot y[k-n] \tag{1}$$

For simplicity I zero padded the signals to have the periodic sequences at the middle of the vectors, with a number of zeros at the beginning and at the end equal to the length of the original vectors. This step was meant to simulate in the function a shift towards the left that otherwise would have been difficult to implement since Matlab does not accept index values less than 1.
The resulting autocorrelation vector $R_{xx}[n]$ is shown in Figure 3 where are included also the output vectors of Matlab functions xcorr and conv. The obtained vectors are equal in length (each one has dimension $2 \cdot N - 1$) and in amplitude with the exception of the autocorrelation computed with xcorr function. Even if it is not visible from the graph, $R_{xx2}[n]$ has some of its elements that differ from those of the other two vectors $R_{xx}[n]$ and $R_{xx3}[n]$ for a negligible quantity of the order of $1e-14$. The local maxima of the function are located 5 elements apart, which corresponds to the period of $x[n]$. These maxima occur when in the definition of eq.(1) the lag n coincides with a multiple of T_x and, as a consequence, when the periods of $x[k]$ and $x[k-n]$ are aligned. Thanks to this result one could derive the periodicity of $x[n]$ directly from its autocorrelation. Regarding to the other properties of the autocorrelation function, it can be seen that the signals are even symmetric and their absolute maximum is exactly at the origin, as it was expected. The elements at the origin are all equal

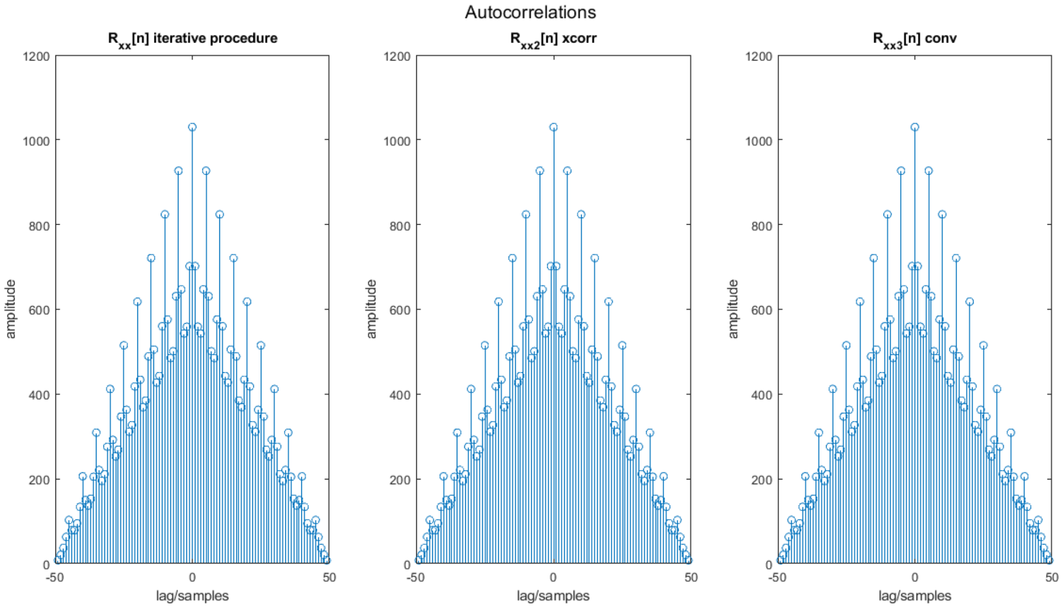

Figure 3: Exercise 1 *Autocorrelation functions*

to 1030, which corresponds exactly to the energy of $x[n]$ calculated through the following general formula:

$$E\{x[n]\} = \sum_{k=0}^{N} |x[k]|^2$$

Figure 4 shows the cross-correlation function $R_{xy}[n]$, computed with the same iterative procedure used for the autocorrelation, together with the signals resulting from Matlab xcorr and conv functions. Vector $R_{xy}[n]$ and $R_{xy3}[n]$ have length $N + M - 1$ while $R_{xy2}[n]$, due to the way xcorr function is implemented, has length $2 \cdot M - 1$. In fact, xcorr function appends zeros to the end of the shortest vector (which in this case is $x[n]$) so that the two signals have same length. However, this operation does not change the values taken by the cross-correlation. Because both $x[n]$ and $y[n]$ have monotonically increasing periods, the first peak occurs at $M - N = -30$ and all the local maxima are located 5 elements apart, which corresponds to the greatest common divisor of the two periods T_x and T_y.
Unlike the autocorrelation, the cross-correlation is not symmetric around the origin but it does exhibit conjugate symmetry as it is shown in Figure 5. A trivial conclusion is that the two signals $x[n]$ and $y[n]$ are not orthogonal.

Exercise 2

The second exercise involved the phase shift recovery between two sinusoidal signals by means of cross-correlation function xcorr.
The signals used are defined as follows:

$$s_1(t) = sin(2\pi f_1 t + \phi_1)$$
$$p(t) = sin(2\pi f_1 t + \phi_p)$$
$$s_a(t) = \sum_{i=1}^{N} sin(2\pi f_i t + \phi_i) \tag{2}$$

where each phase ϕ_i was randomly generated and the frequency is defined as $f_i = i$. All the signals are defined for $t \in [0, 100]$ s with a sampling period $T = 0.1$ ms.

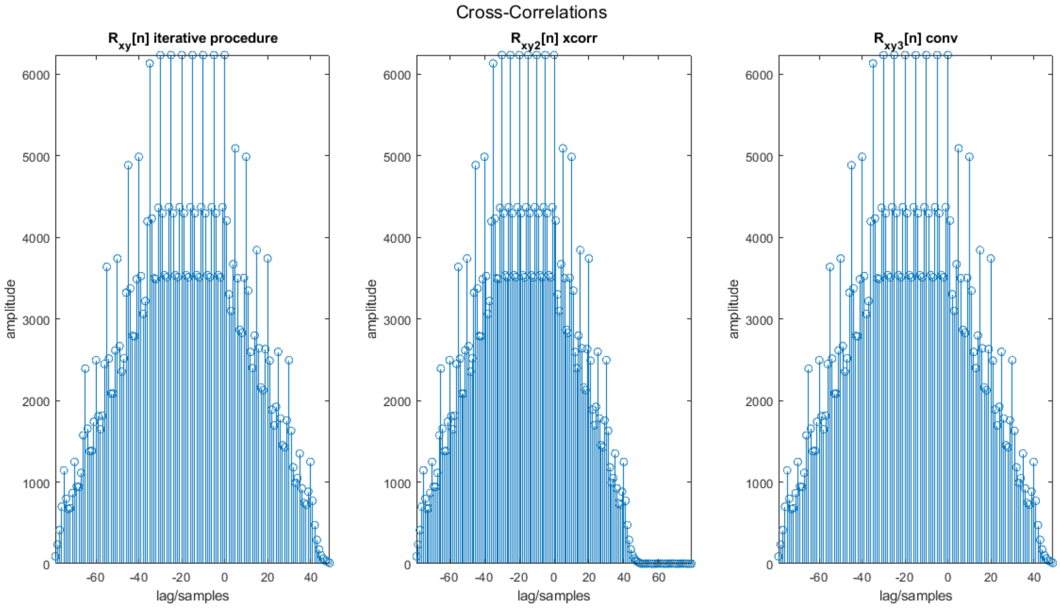

Figure 4: Exercise 1 *Cross-Correlation functions*

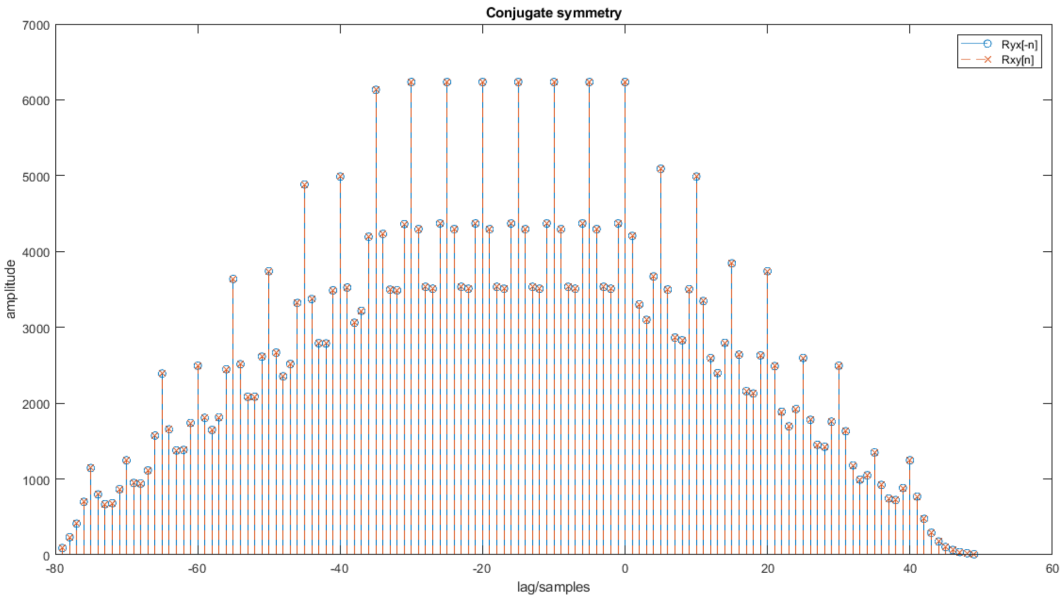

Figure 5: Exercise 1 *Cross-Correlation conjugate symmetry property*

In the first step, I computed the cross-correlation between signals $s_1(t)$ and $p(t)$, which have the same frequency $f_1 = 1\,Hz$ but different initial phases. In Figure 6 it can be seen that the cross-correlation of the two sinusoidal signals also oscillates at the same frequency f_1, indeed the peaks are equally spaced by 10^4 lags corresponding to the samples composing each period. All the pictures refer to a specific instance of the program in which the phase shift to be detect was $\Delta\phi = \phi_i - \phi_p = -61.2762°$. After computing the cross-correlation and detecting its absolute maximum a further step was required to overcome the ambiguity of the phase ϕ, which is a modulo 2π function. At the end, the detected phase shift was $\Delta\phi_{xcorr} = -60.8400$ which led to an error $\varepsilon = \Delta\phi_{xcorr} - \Delta\phi = 0.0762$ probably due to the fact that the elements of the cross-correlation do not show an "isolated" maximum value. Indeed, due to sampling limitations, the peak is represented by a short sequence of elements at the same value.

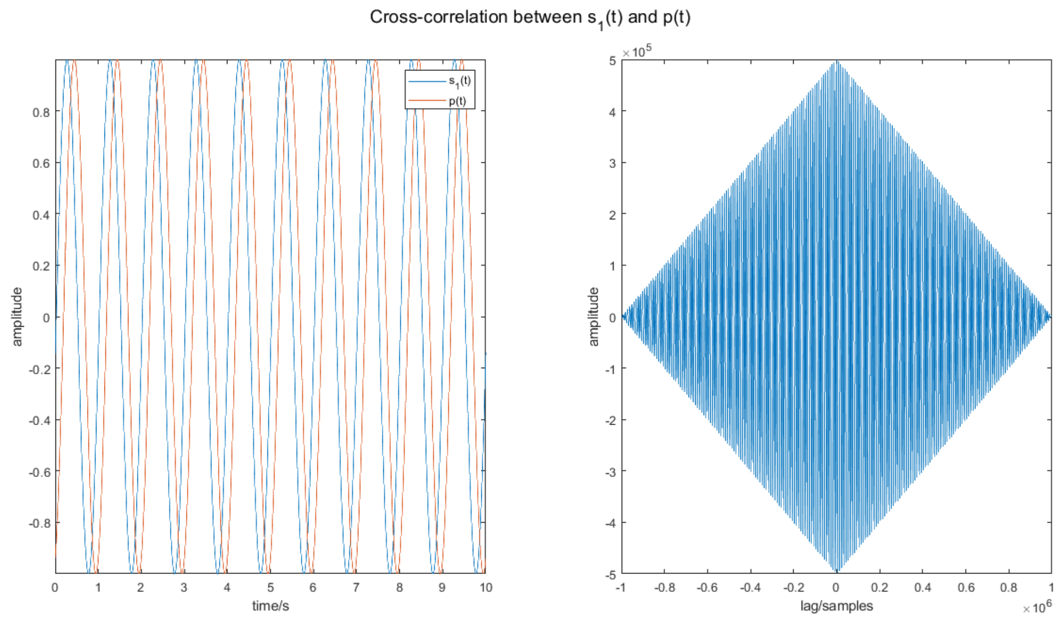

Figure 6: Exercise 2 *Signals* $s_1(t)$*,* $p(t)$ *and their cross-correlation*

In the second scenario, I summed $s_1(t)$ with other sinusoidal signals at higher frequencies to generate signal $s_a(t)$. The components at higher frequencies behave as noise applied on to the signal at $1\ Hz$, leading to higher errors during the phase recovery. Table 1 reports the values of the errors for different values of N in eq.(2). Even if the error increased, going from 0.12% up to 0.94%,

N	$\Delta\phi_{xcorr}$	ε°	$\varepsilon\ \%$
10	-60.6960	0.5802	0.94
100	-61.0920	0.1842	0.30
1000	-60.8400	0.4362	0.71

Table 1: Phase shift detection errors wrt $\Delta\phi = \phi_i - \phi_p = -61.2762°$

there is no monotonic behavior of the error with respect to the number of samples N. This result suggests that higher frequency components have little effect on the phase shift recovery using the cross-correlation function that was implemented. Figure 7 and 8 displays the signals and their respective cross-correlation for each one of the three cases. The periodicity at frequency f_1 is clearly conserved in both $S_a(t)$ and the cross-correlation.

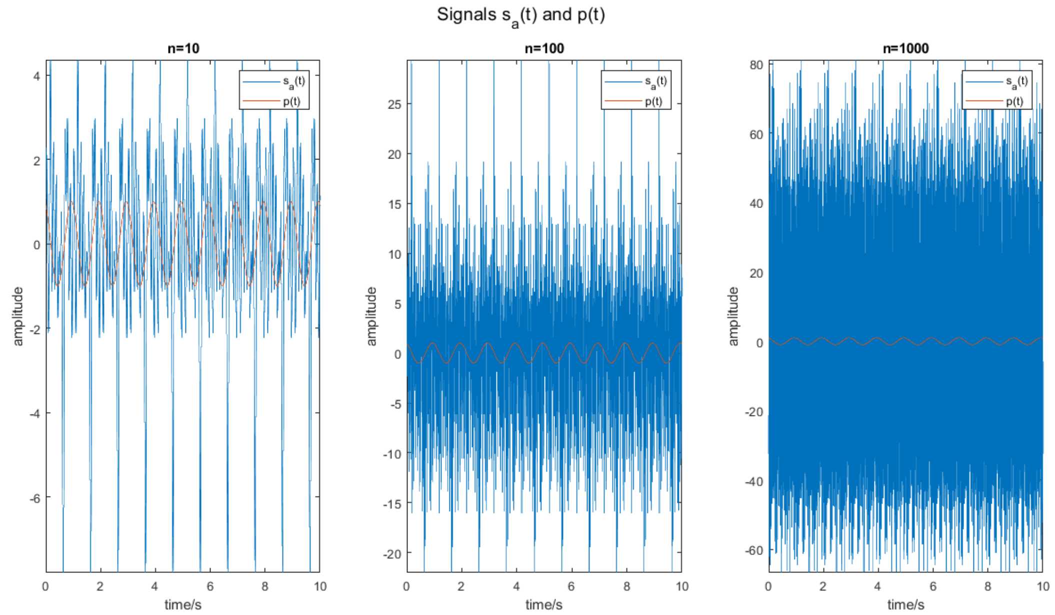

Figure 7: Exercise 2 *Signals* $s_a(t)$, $p(t)$ *for* $N = 10, 100, 1000$

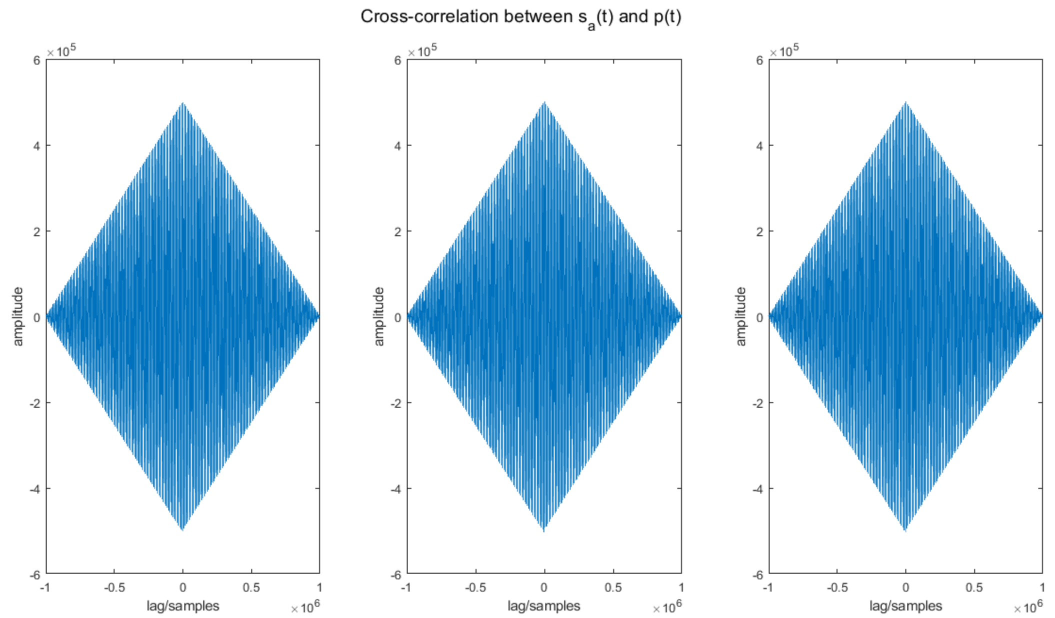

Figure 8: Exercise 2 *Cross-correlation between* $s_a(t)$ *and* $p(t)$ *for* $N = 10, 100, 1000$

Exercise 3

In this exercise the cross-correlation was used to detect the sample delay between two binary sequences $x[n]$ and $y'[n]$ of 1000 random bits, where the latter is the shifted version of the first with additional noise. The noise is white Gaussian generated by randn Matlab function, whereas for the sample delay I used randi function. I generated sequence $x[n]$ only once while I updated the values of $y'[n]$ 50 times for each one of the following values of the standard deviation:

$$\sigma = [0.1\ 0.5\ 1\ 5\ 10\ 20\ 30].$$

Once the sample shift was detected through xcorr function, I generated a third sequence $x"[n]$ and defined it as the version of $y'[n]$ without additional noise, called $x'[n]$, compensated for the detected delay. Figure 9 compares a set of these signals for $\sigma = 0.1$ and a delay equal to 149 samples. In this case the shift was successfully detected and compensated, hence $x[n]$ and $x"[n]$ overlap. The cross-correlation between $x[n]$ and $y'[n]$ in Figure 10 has a distinguishable peak at lag 149 while in the cross-correlation between $x[n]$ and $x''[n]$ this is at zero-lag, meaning that the two signals are perfectly aligned. Figure 11 shows the obtained cross-correlation in a case where the noise added to

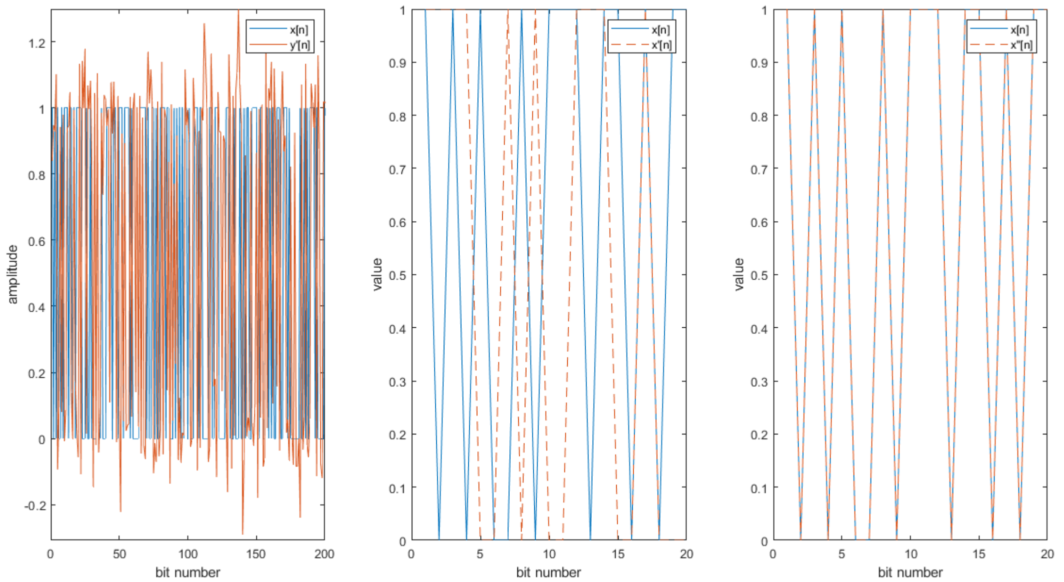

Figure 9: Exercise 3 *Set of generated sequences for* $\sigma = 0.1$

$y'[n]$ has standard deviation $\sigma = 5$. In this specific case the sample delay was equal to 587 samples, however the plot of $R_{xy'}[n]$ shows a maximum peak at lag -39 which led to a misreading of the sample delay. Indeed in the graph of $R_{xx''}[n]$ the leftmost peak coincides with the difference between the true delay and the one detected, indicating that the two sequences are still shifted with respect to each other. This error comes from the fact that, as highlighted in the figure, at the lag corresponding to the sample delay the effect of the noise is such that to reduce its amplitude and making it indistinguishable from the function. On the other hand, another peak comes out becoming the maximum of the function. In general, increasing the spread of the noise there will be an higher probability of retrieving a wrong value of the sample shift, as it can be seen in Figure 12 where $\sigma = 30$. In this case, xcorr function was not able to detect the correct shift in any of the 50 generated pairs of signals, leading to a Bit Error Rated (BER) always greater that 0. Compared to the case just discussed, when the signal $x[n]$ is periodic it is easier to detect the exact value for the sample shift from the cross-correlation. As shown in Figure 13, the cross-correlation has a number of peaks corresponding to the number of periods spaced by the number of samples that constitute one period. For the same value of $\sigma = 5$ just analysed, if $x[n]$ is periodic there is less probability of misreading, being the peaks higher in amplitude and more distinguishable from the other values taken by the vector. This behaviour is highlighted in Figure 14 where the BER remains null for the first four values of σ and has a smoother increase along the others.

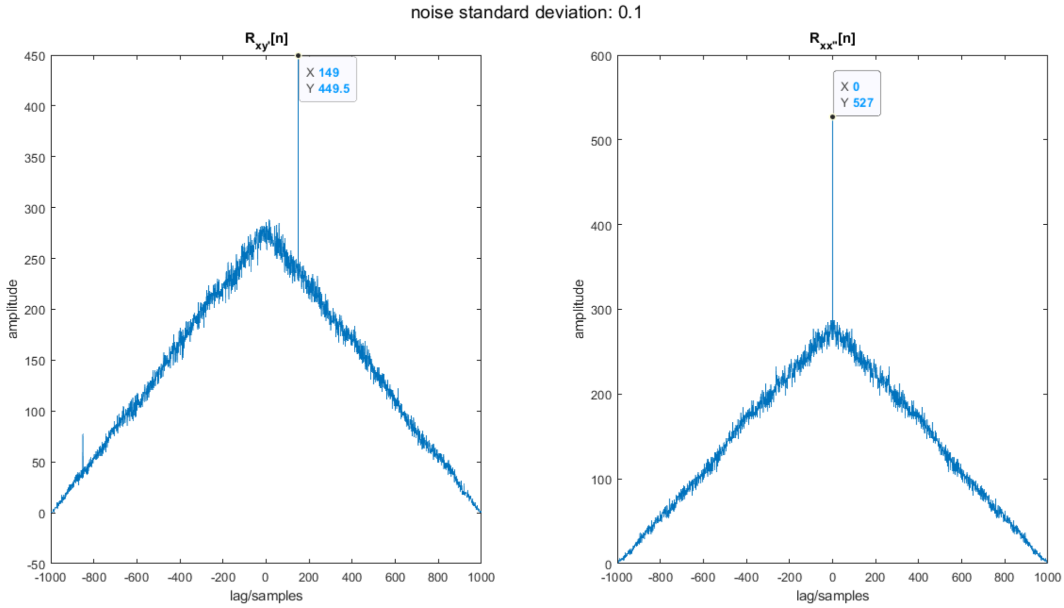

Figure 10: Exercise 3 *Cross-correlations for a sample shift of* 149

Exercise 4

In this last exercise it was asked to generate a set of Gold sequences from an m-sequence generated on the following primitive polynomial and to analyze their correlation properties:

$$x = x^6 + x + 1 \tag{3}$$

All the correlations were performed after a substitution of the elements of the vectors of interest from $[0, 1]$ to $[-1, 1]$ and invoking a function that I made which computes the following normalized circular correlation:

$$R_{ab}[d] = \frac{1}{2^n - 1} \sum_{m=1}^{2^n-1} a[m] \cdot b[m-d]$$

Having order $n = 6$, the polynomial in eq.(3) generates $2^6 - 1 = 63$ m-sequences each one with 63 elements. As first, I generated an m-sequence $a[n]$ starting from an initial vector of length n filled with ones and I checked that balance and run properties hold. Figure 15 shows two periods of its autocorrelation where it can be seen that it takes only two values, 1 for $n = 0$ and $\frac{1}{2^n-1} = 0.01587$ elsewhere, as expected. Then I generated the decimated version of $a[n]$, called $b[n]$. To be sure that I would obtain a preferred pair I choose $q = 2^k + 1$ with $k = 2$ so that the greatest common divisor of n and k was $gcd(n, k) = 2$. Then I obtained $b[n]$ by sampling a repeated version of $a[n]$ at sample period q. In Figure 16 the circular cross-correlation $R_{ab}[n]$ takes only three values, a property which confirms that $a[n]$ and $b[n]$ are a preferred pair. Finally, to obtain the set of Gold codes I binary added $a[n]$ to the cyclic shifted versions of $b[n]$ and obtained a matrix $M[n, m]$ that stored all the possible Gold codes generated from the preferred pair. Figure 16 shows the autocorrelation and cross-correlation of two arbitrarily selected Gold codes from matrix $M[n, m]$. At zero-lag the autocorrelation takes value 1, as expected. In contrast with the autocorrelation of m-sequences, a Gold codes does not show a two-valued function but in this case it takes the following three values:

$$\{-1,\ -17,\ 15\} \cdot \frac{1}{63} = \{-0.01587,\ -0.2698,\ 0.2381\}$$

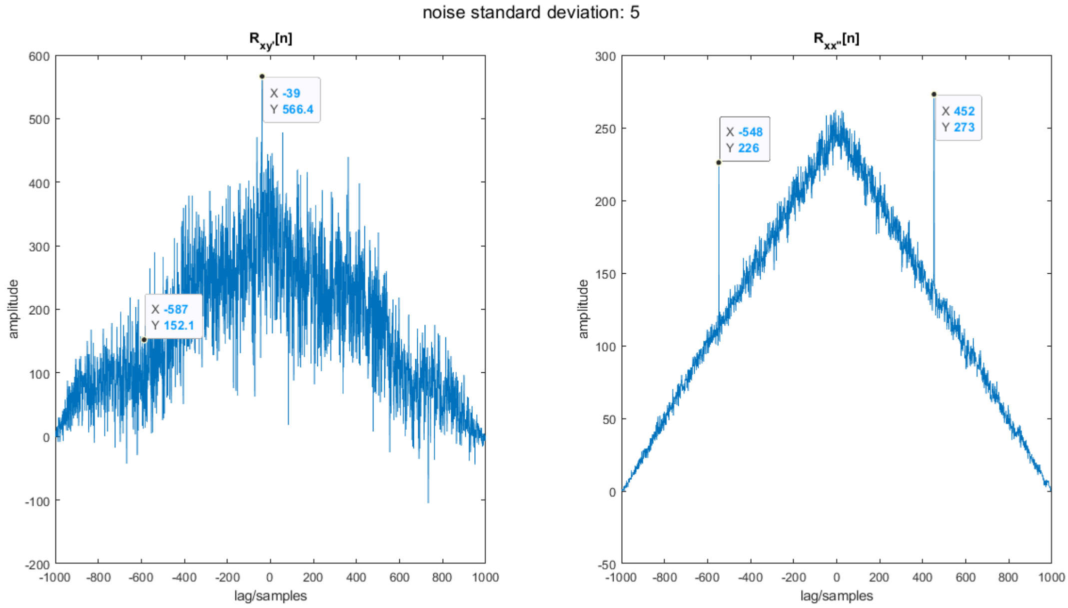

Figure 11: Exercise 3 *Cross-correlations when* $\sigma = 5$

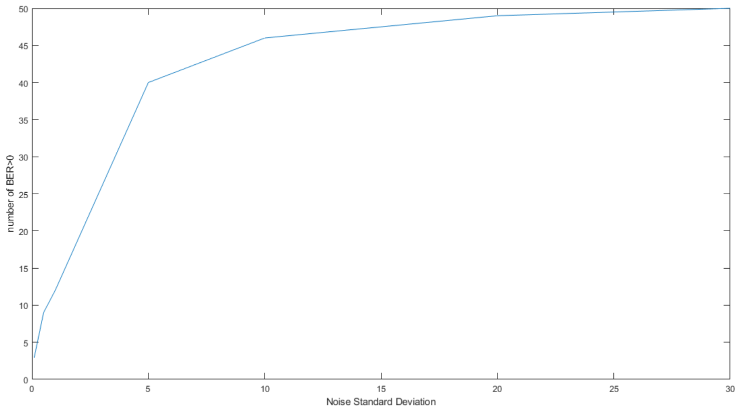

Figure 12: Exercise 3 *Number of BER>0 at each value of* σ

This result coincides with what has been studied in theory, that is the values taken by the cross-correlation are of the form $\{-1, -t(n), t(n) - 2\}$ where $t(n) = 2^{\frac{n+2}{2}} + 1$.

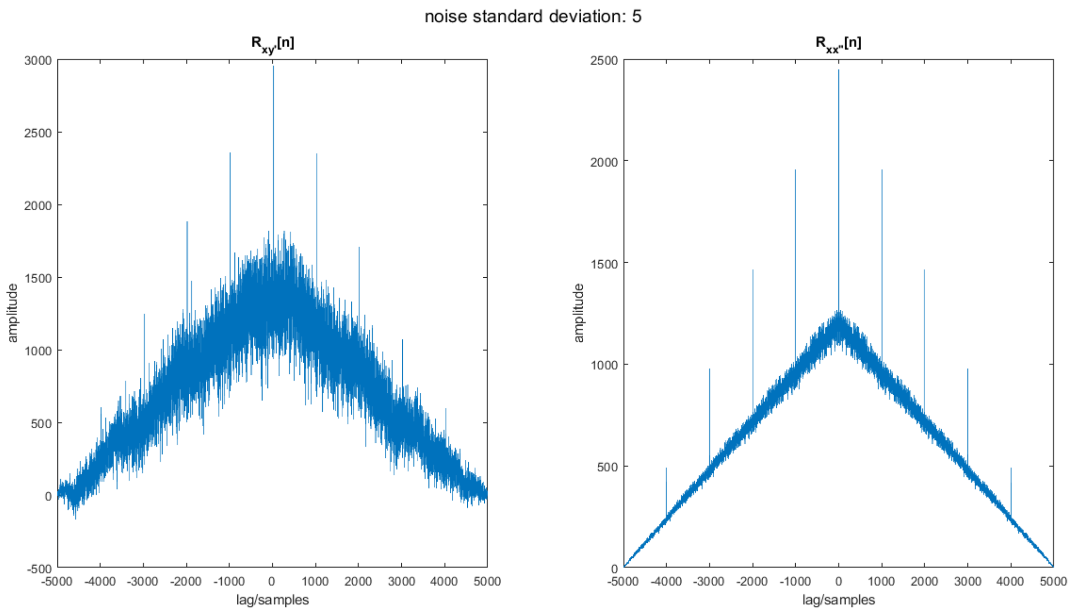

Figure 13: Exercise 3 *Cross-correlations when* $\sigma = 5$ *and* $x[n]$ *is periodic*

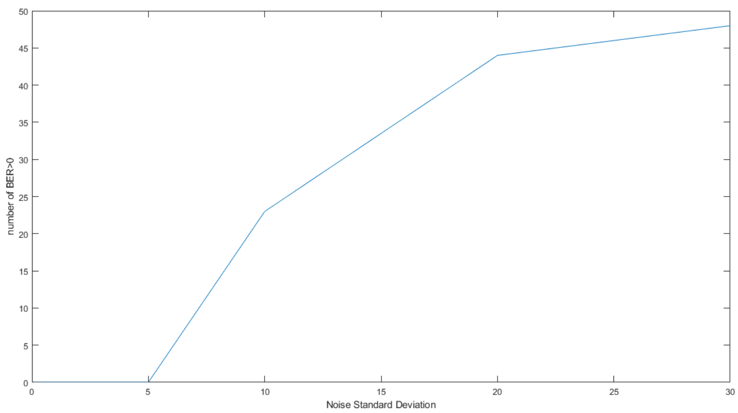

Figure 14: Exercise 3 *Number of BER>0 at each value of* σ *for periodic* $x[n]$

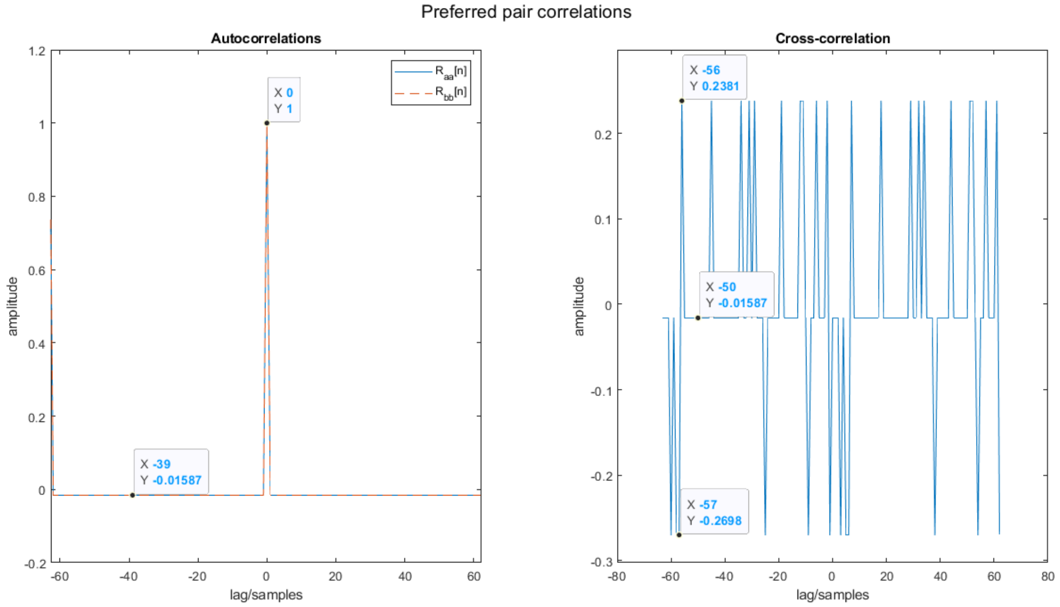

Figure 15: Exercise 4 *Preferred pair correlations functions*

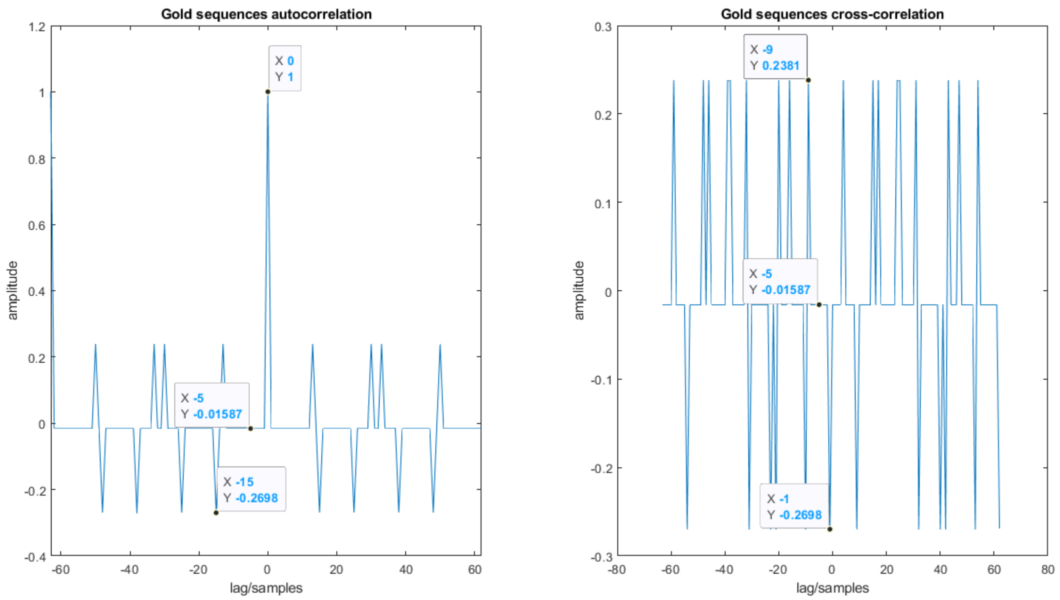

Figure 16: Exercise 4 *Gold codes correlation functions*