EE-550 | **Laboratory 2**
Dithering

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The images under analysis in this lab (figure 1) are in intensity format with dynamic range [0, 255].

1 Fixed threshold method

The fixed threshold method is the one that requires fewer operations as it needs only to compare directly each pixel with the threshold value which in this case is $T = 127$, half of the dynamic range. However, as it can be seen in figure 2 the image is heavily distorted. The dithered images lack depth and some of the items represented in 'tree.tif' are even lost, indeed the ribbons and the darker clew cannot be distinguished as they have been replaced by black pixels.

Figure 1: Original images without dithering

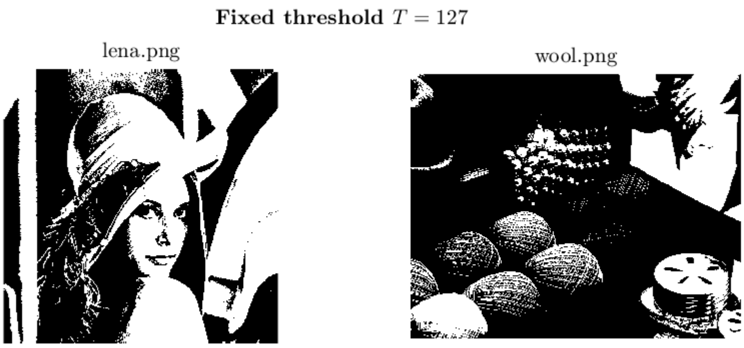

Figure 2: Dithered images with fixed threshold method

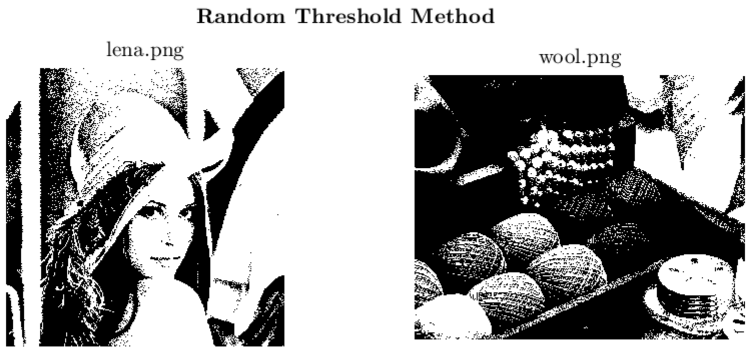

Figure 3: Dithered images with random threshold method

2 Random threshold method

The additive discrete uniform noise, generated with function "unidrnd", takes integer values in $[1, 32]$, a range that I have found to be a good compromise between the dithered image and the loss of quality.
It can be seen in figure 3 that some of the darker items that were previously hidden now can be distinguished. This is due to the additive noise which allows the pixels that reside just below the threshold to be detected as white pixels. On the other hand, some of the contours in image 'lena.png' are less visible since there is less contrast between darker and lighter regions in the dithered output. This method is also one of the easiest to implement as it requires only the generation of the noise and the fixed threshold operation.

3 Ordered threshold method

Note: The dithered images obtained by the following methods are not reported. The reduced dimension that the images would have taken introduces artifacts which make the reference meaningless.

The ordered threshold method is more complex in terms of number of operations as it requires two main steps before the thresholding, namely the construction of the threshold matrix form the elementary clustered dot matrix and the quantization of the input image. This directly affects the computational time, which is more than 100 times higher when compered to the fixed threshold method.
The quantization operation that I referred to is the following:

$$Q(x) = \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor$$

where Δ is the step size $\Delta = \frac{255}{N-1}$, N being the number of gray levels that the threshold matrix can represent. To compute the floor operation I used a combination of inequality relations and vector inner products. For each pixel, through the inequality equations I obtained a vector whose non zero element is located at the index correspondent to the quantization level that contains the pixel value within its range. The indicator vector then is multiplied, in terms of inner product, by a vector whose elements are the integer values between 0 and $N - 1$ to get the correct pixel value.
The images obtained after the thresholding operation have more depth and, even though the values taken by the pixels are either 0 or 1, it is perceived a wider range of intensities. The items are more detailed and there are no full black or full white regions. One disadvantage of this method is that it brings to an annoying artifact: the perception of a regular grid on top of the image.

The general considerations made in this exercise apply also for the next three exercises since the method that has been employed is the same.

4 Ordered matrix with centered points

Even if the differences between the results given by the two ordered matrices C_6 and E_6 are slight, when moving from one image dithered with matrix C_6 and the other dithered with E_6 it can be noticed that the pointed pattern of the latter appears to be a rotation towards the left of the pattern that characterize the former. The difference between these two methods is indeed the location of the point in the elementary matrix which may leave the brighter pixels toward the left (matrix C_6) or towards the right (matrix E_6).

5 Diagonal ordered matrix

With the diagonal ordered matrix the shapes and the edges of the subjects in the original image are slightly better represented. In the previous exercise there can be distinguished vertical and horizontal full black or full white lines that create a rather annoying grid on top of the image. Matrix O_8 does not produce this artifact and the output image appears smoother and more homogeneous.

6 Ordered matrix with dispersed dots

This method may be more computationally demanding with respect to the others ordered matrix methods since it is based on a recursive algorithm. The number of operations required depends on the number of recursions, thus on the size of the threshold matrix.
This method produces a denser grid of smaller points, which I think to be more disturbing. However when zooming in this annoyance disappears and the image appears more detailed than the previous ones.

7 Error diffusion method

To implement this method I wrote a function that reproduces the system presented in class. The error is computed from the difference between the original image, scaled to a dynamic range [0, 1], and the output of the threshold operation. This implementation requires the higher number of operations since it computes one subtraction and one matrix multiplication for each pixel, besides the thresholding operation.
The obtained result is the most pleasant and detailed among all dithered images. When the Floid & Steinberg matrix is used, in some regions the dots take a regular horizontal pattern leading to unwanted stripes as it can be seen above the hat of the woman in 'lena.png'. This did not occur when the Stucki matrix is employed.

8 Discussion

To compute the mean square error (MSE) I implemented a function that computes the following formula:

$$\mathrm{MSE} = \frac{1}{NM} \sum_{n=1}^{N} \sum_{m=1}^{M} \left[\hat{x}(n,m) - x(n,m) \right]^2 .$$

The images are kept in their dynamic range, without any normalization, which justifies values of MSE of the order 10^4. The obtained results are summarized in table 1 where it can be noticed that the values are similar among themselves. The method that produced lower MSE in both images is the fixed threshold with random noise while the ones that produced a higher MSE value are the ordered matrix S and the fixed threshold, respectively for 'lena.png' and 'wool.png'. To conclude I would say that the MSE is not the most suitable metric that can be used to evaluate the obtained visual quality and loss of details.

method	fixed	random	S	C_6	E_6	O_8	D_6	F & S	Stucki
lena.png	17412.4	17388.1	17440.1	17439.9	17439.9	17422.5	17439.5	17439.3	17440.0
wool.png	11089.1	11069.5	11087.8	11086.8	11086.8	11076.3	11086.7	11086.1	11087.7

Table 1: MSE of the different dithering methods