Analysis of Algorithms: Sorting algorithms (Selection sort and Quicksort)

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Course Notes: https://gdancik.github.io

What do we mean by Sorting?

- One of the most common operations in computer science is to sort data numerically or alphabetically
- We have seen previously that sorted data can be searched much more efficiently than unsorted data. Why?
- In addition, for presentation purposes, elements such as names, states, ages, GPAs, etc, are often displayed in sorted order (numeric data may be sorted from low to high or high to low; when we say that numeric data is sorted we will mean low to high)

11	21	18	3	15	19

- The list above in sorted order is: 3, 11, 15, 18, 19, and 21

Selection sort

- Find the maximum element in the list (all n elements)
  - Swap this maximum element with the last element in the list
- Find the next maximum element in the list (first $n - 1$ elements)
  - Swap this maximum element with the second to last element in the list
- Find the next maximum element in the list (first $n - 2$ elements)
  - Swap this maximum element with the third to last element in the list
- This process repeats until we are down to the first element. This is the minimum element, which is now the first element in the list

11	21	18	3	15	19

Selection sort (example)

- We search all n = 6 elements for the maximum, and swap this maximum element with the last one in the list (the 6^{th} one)

This is the max

Swap max with this element

11	**21**	**18**	**3**	**15**	**19**

The max is 21 → swap this with the last element

11	**19**	**18**	**3**	**15**	**21**

Selection sort (example)

- We search the first *n-1* = 5 elements for the maximum, and swap this maximum element with the 5th one (or the second to last one)

This is the max

Swap max with this element

11	19	18	3	15	21

The max is 19 → swap this with the 5th element

11	15	18	3	19	21

Selection sort (example)

- We search the first *n-2* = 4 elements for the maximum, and swap this maximum element with the 4th one

This is the max

Swap max with this element

11	15	18	3	19	21

The max is 18 → swap this with the 4th element

11	15	3	18	19	21

Selection sort (example)

- We search the first n – 3 = 3 elements for the maximum, and swap this maximum element with the 3rd one

This is the max

Swap max with this element

11	15	3	18	19	21

The max is 15 → swap this with the 3rd element

11	3	15	18	19	21

Selection sort (example)

- We search the first n – 4 = 2 elements for the maximum, and swap the maximum element with the 2nd one

This is the max

Swap max with this element

11	3	15	18	19	21

The max is 11 → swap this with the 2nd element

3	11	15	18	19	21

Selection sort (example)

- Once we have only 1 element left (we are finding the max of just the 1st element), then we are done. The list is now sorted.

3	11	15	18	19	21

Selection sort algorithm

- *end* = n – 1
- while *end* > 0 :
  - Set *max_index* to the index of the maximum element between *values[0]* through *values[end]*
  - Swap *values[max_index]* and *values[end]*
  - Set *end = end* - 1

Selection sort algorithm

- *end* = n – 1
- while *end* > 0 :
  - Set *max_index* to the index of the maximum element between *values[0]* through *values[end]*
  - Swap *values[max_index]* and *values[end]*
  - Set *end* = *end* - 1

- Set *max_index* to 0
- Set *i* to 0
- While *i* <= end:
  - If values[i] > values[max_index]*:*
    - set *max_index* to *i*
  - set *i* = *i +1*

Selection sort algorithm

- *end* = n – 1
- while *end* > 0 :
  - Set *max_index* to 0
  - Set *i* to 0
  - While *i* <= end:
    - If values[*i*] > values[*max_index*]*:*
      - set *max_index* to *I*
    - Set *i* = *i* + 1
  - Swap *values[max_index]* and *values[end]*
  - Set *end* = *end* - 1

Executed *n* – 1 times (while loop)

Executed up to *n* – 1 times each time (while loop)

This suggests an order of magnitude of n^2

Selection sort algorithm

- *end* = n – 1
- while *end* > 0 :
  - Set *max_index* to 0
  - Set *i* to 0
  - While *i* <= end:
    - If values[*i*] > values[*max_index*]*:*
      - set *max_index* to *I*
    - Set *i* = *i* + 1
  - Swap *values[max_index]* and *values[end]*
  - Set *end* = *end* - 1

Assume that $n = 4$

end	# iterations of inner while loop
3	4
2	3
1	2
0	-

In general, for a list of size *n*, the total number of inner loop iterations is:

$2 + 3 + 4 + ... + n$

This is $n(n+1)/2 - 1$, which has an order of magnitude of n^2.

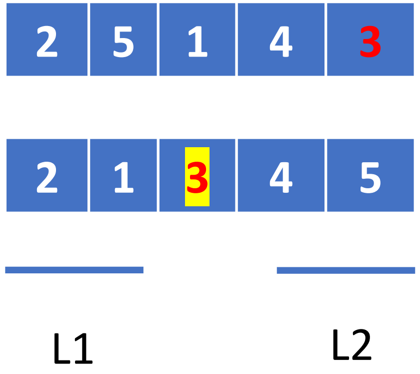
Quicksort algorithm

- Quicksort(arr, low, high) :
  - While low < high :
    - pi = partition (arr, low, high)
    - Quicksort(arr, low, pi – 1)
    - Quicksort(arr, pi + 1, high)

Partition step:
- select a *pivot*
- move *pivot to* correct location
  - all elements less than pivot are moved to its left
  - all elements greater than pivot are moved to its right
- Return partition index

Call Quicksort on L1

Call Quicksort on L2

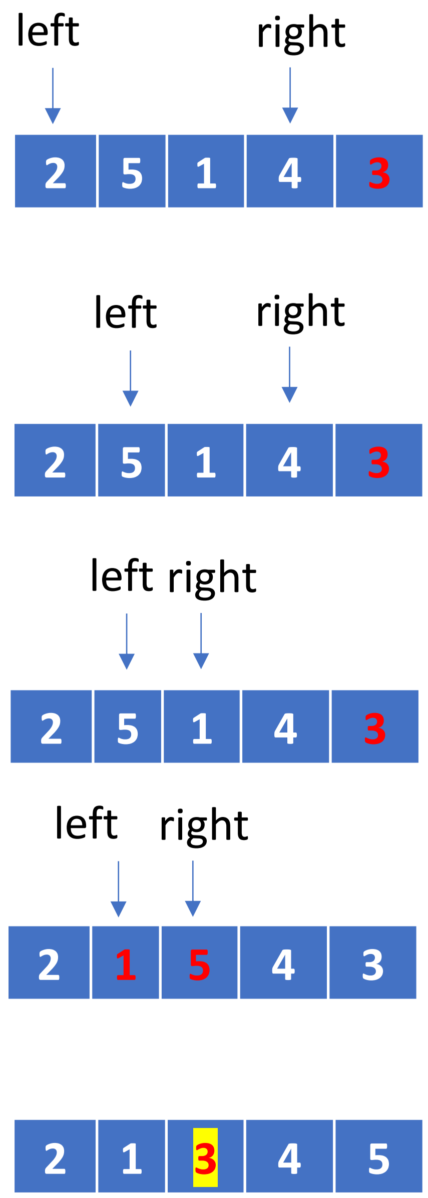


We select the last element as the pivot

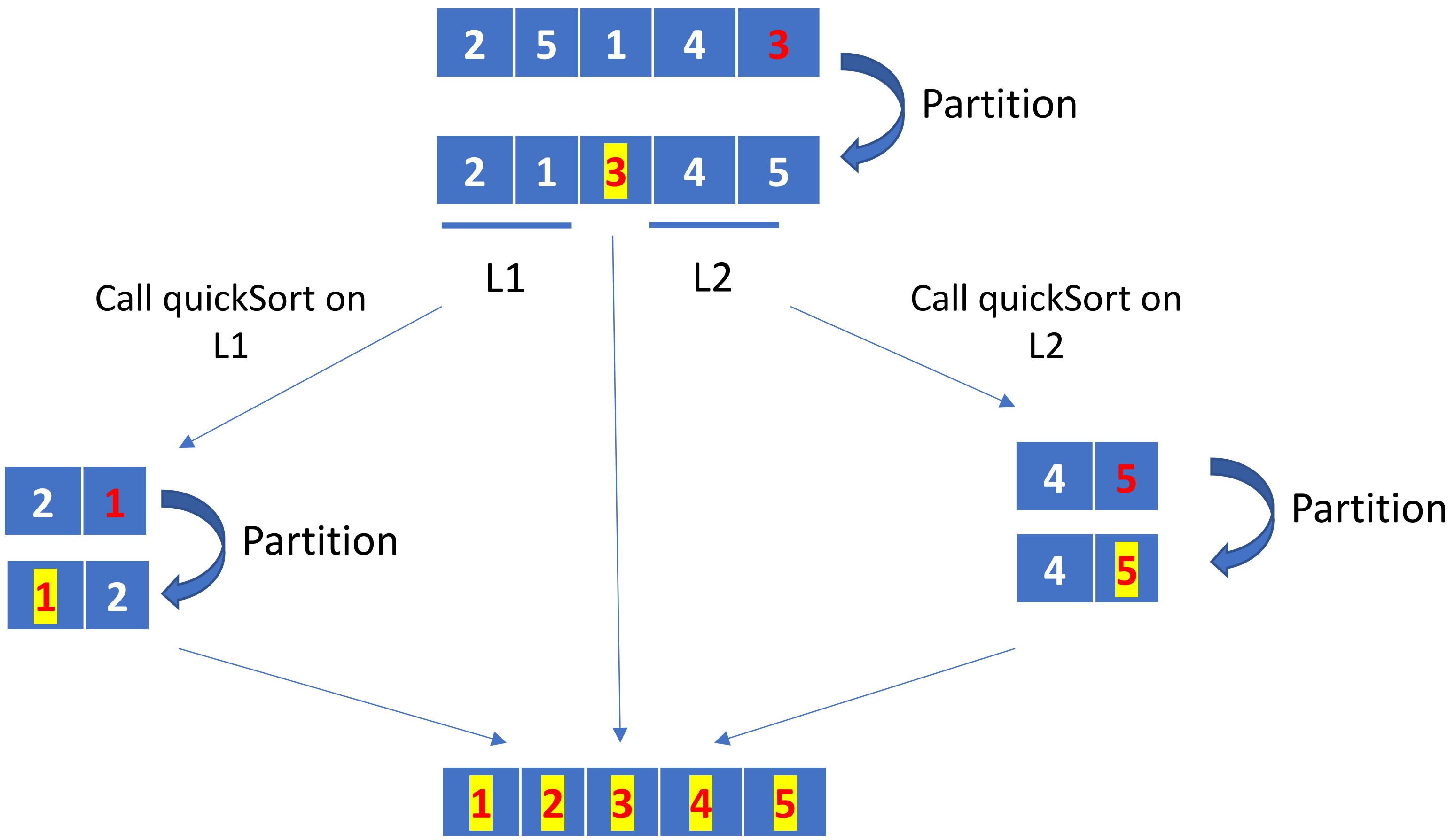
Pivot is in the correct location

Partition algorithm

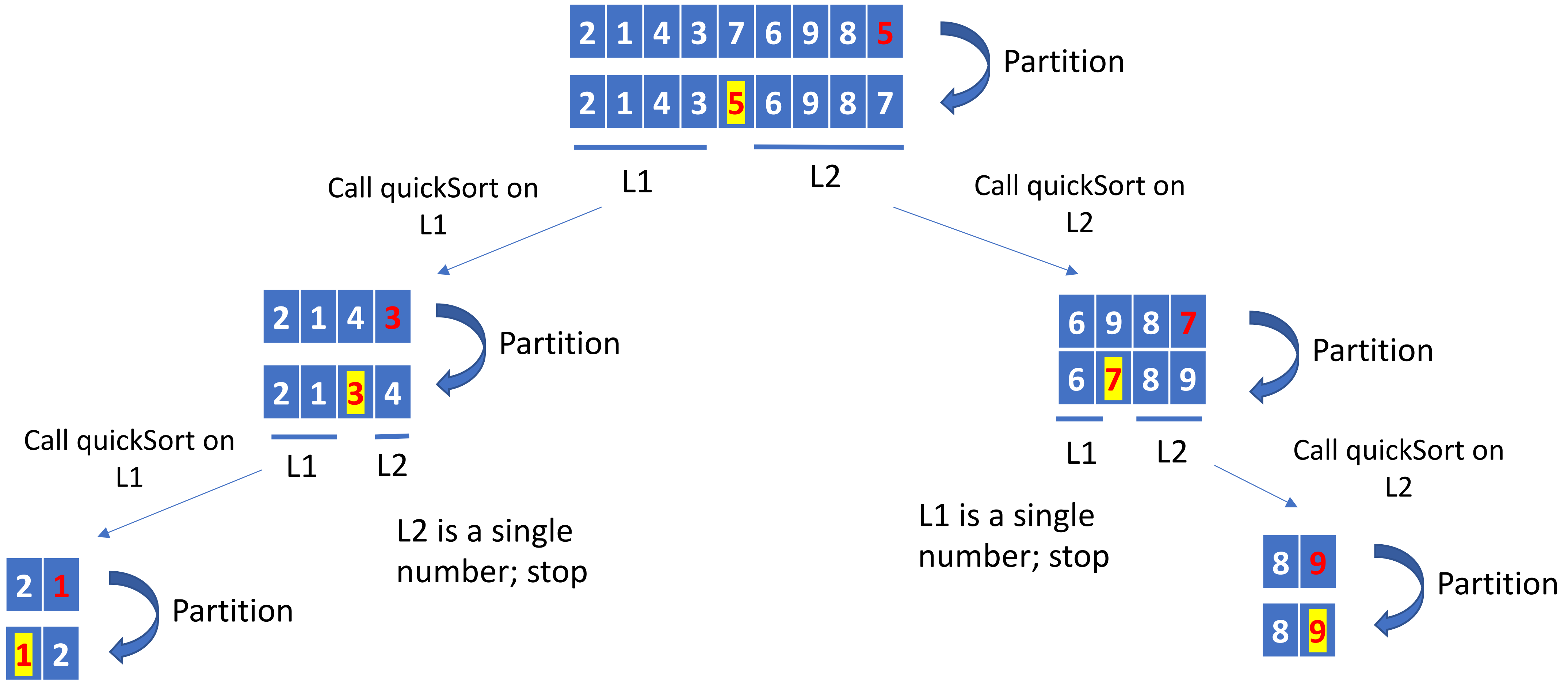
- Inputs:
  - *arr* (the list/array)
  - *low* (index of lower element),
  - *high* (index of last element, which will be the pivot)
- Set *left* = *low*
- Set *pivot* = *arr*[*high*]
- Set *right* = *high* – 1
- While *left* <= *right*:
  - Increase *left* by 1 until *arr*[*left*] > *pivot* (or left > right)
  - Decrease *right* by 1 until *arr*[*right*] <= *pivot* (or left > right)
  - If *left* < *right*, swap *arr*[*left*] and arr[*right*]
    - Increase *left* by 1
    - Decrease right by 1
- Swap arr[left] and arr[high]
- Return left



Quicksort example

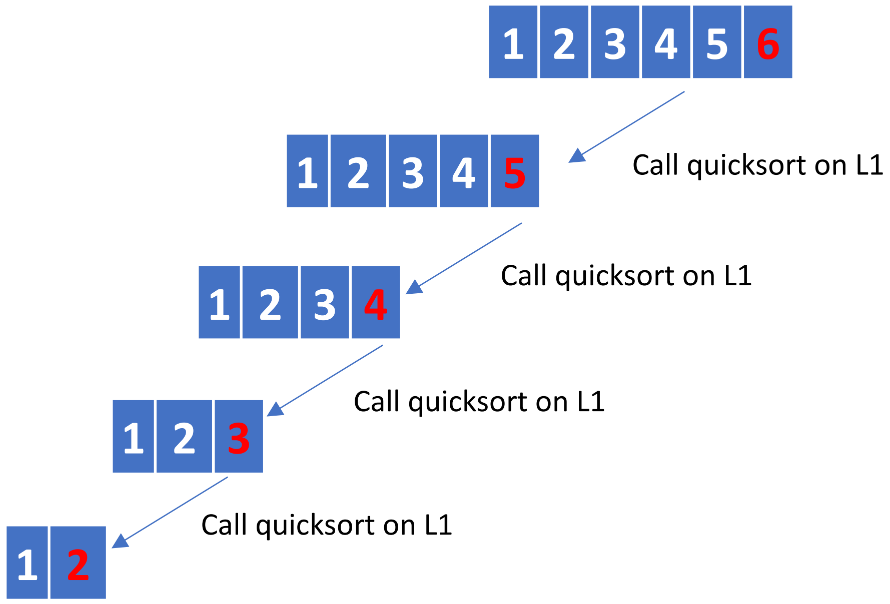


Quicksort example



Quicksort running times

- The *worst case* occurs if the original data is sorted, then the partition will keep the pivot in the last element and we will call quicksort on L1 = list containing all elements but the last one; L2 has no elements. The running time is $\theta(n^2)$
- Also see https://www.khanacademy.org/computing/computer-science/algorithms/quick-sort/a/analysis-of-quicksort



Partition looks at 6 elements

Partition looks at 5 elements

Partition looks at 4 elements

Partition looks at 3 elements

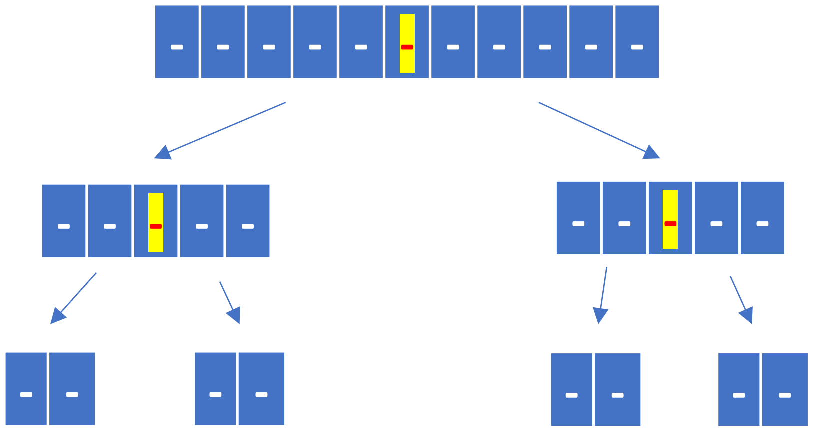
Partition looks at 2 elements

The total number of elements we look at is: 2 + 3 + …. + n

which is $\theta(n^2)$

Quicksort running times

- The *best case* occurs when the partitions are evenly balanced. In this case the number of sub-list pairs that get sorted is $\log n$. The running time is $\theta(n \log n)$.
- Also see https://www.khanacademy.org/computing/computer-science/algorithms/quick-sort/a/analysis-of-quicksort



Partition looks at 11 elements

Partition looks at 10 elements (~ 11 elements)

Partition looks at 8 elements (~ 11 elements)

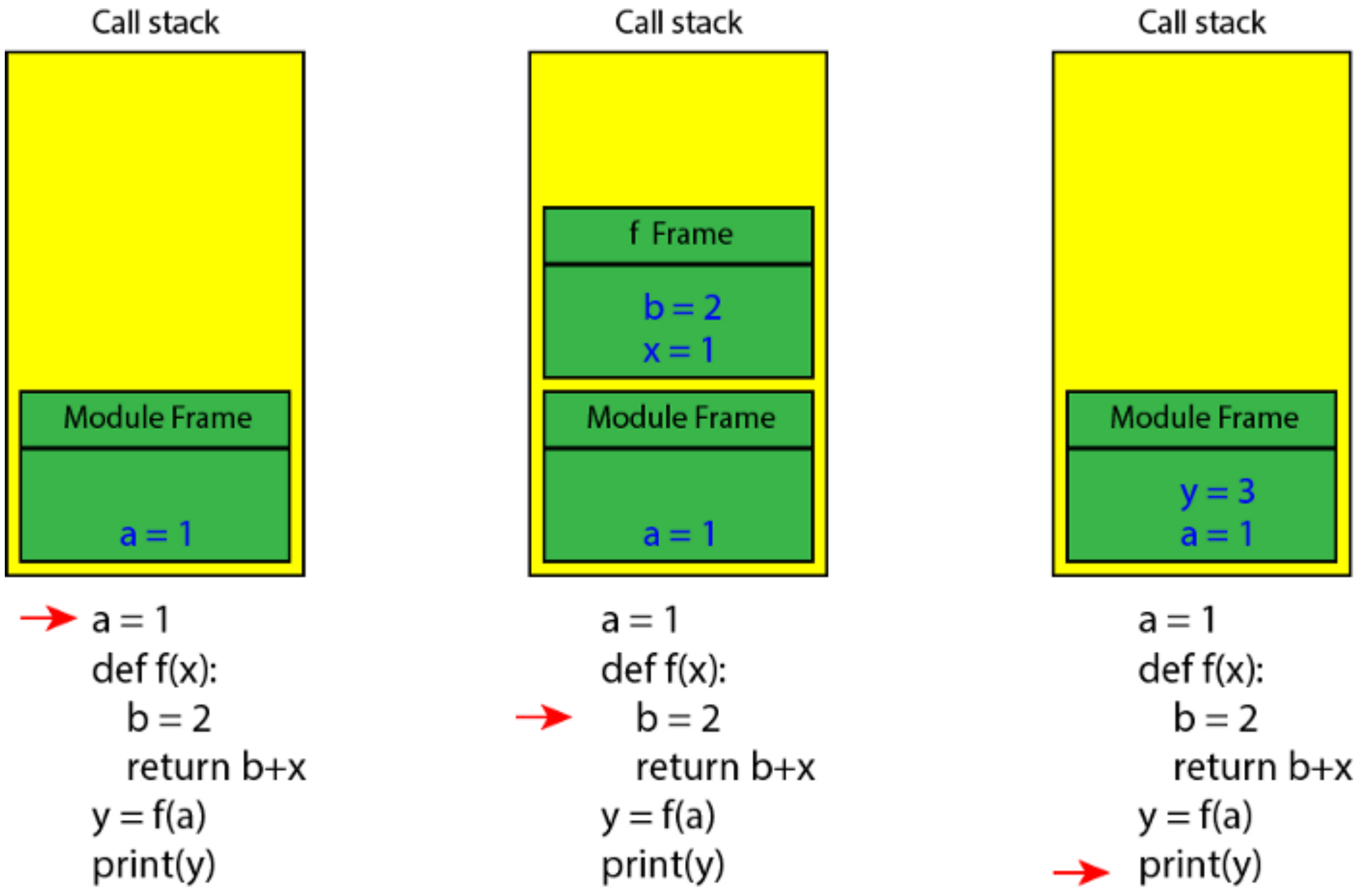
The total number of elements we look at is approximately 11 x $\lfloor \log n \rfloor$ which is $\theta(n \log n)$

Selection sort and Quicksort algorithms

	Selection sort		Quicksort	
	Time	Additional Space	Time	Additional Space
Best	$\theta(n^2)$	$\theta(1)$	$\theta(n \log n)$	$\theta(\log n)$
Worst	$\theta(n^2)$	$\theta(1)$	$\theta(n^2)$	$\theta(n)$
Average	$\theta(n^2)$	$\theta(1)$	$\theta(n \log n)$	$\theta(\log n)$

- Which algorithm is the *best*?

When a function is called, information is stored in the *call stack*



In quicksort, recursive function calls are stored on the stack,

- *n* times in the worst case
- roughly *log n* times in the best/average case.