

Analysis of Algorithms: Sorting algorithms (Bubble sort and Quicksort)

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Course Notes: <https://gdancik.github.io>

What do we mean by Sorting?

- One of the most common operations in computer science is to sort data numerically or alphabetically
- We have seen previously that sorted data can be searched much more efficiently than unsorted data. Why?
- In addition, for presentation purposes, elements such as names, states, ages, GPAs, etc, are often displayed in sorted order (numeric data may be sorted from low to high or high to low; when we say that numeric data is sorted we will mean low to high)

| | | | | | |
|----|----|----|---|----|----|
| 11 | 21 | 18 | 3 | 15 | 19 |
|----|----|----|---|----|----|

- The list above in sorted order is: 3, 11, 15, 18, 19, and 21

Bubble sort

- Find the maximum element in the list (all n elements)
 - Swap this maximum element with the last element in the list
- Find the maximum element in the list (first $n - 1$ elements)
 - Swap this maximum element with the second to last element in the list
- Find the maximum element in the list (first $n - 2$ elements)
 - Swap this maximum element with the third to last element in the list
- This process repeats until we are down to the first element. This is the minimum element, which is now the first element in the list

| | | | | | |
|----|----|----|---|----|----|
| 11 | 21 | 18 | 3 | 15 | 19 |
|----|----|----|---|----|----|

Bubble sort (example)

- We search all $n = 6$ elements for the maximum, and swap this maximum element with the last one in the list (the 6th one)



The max is 21 → swap this with the last element



Bubble sort (example)

- We search the first $n-1 = 5$ elements for the maximum, and swap this maximum element with the 5th one (or the second to last one)

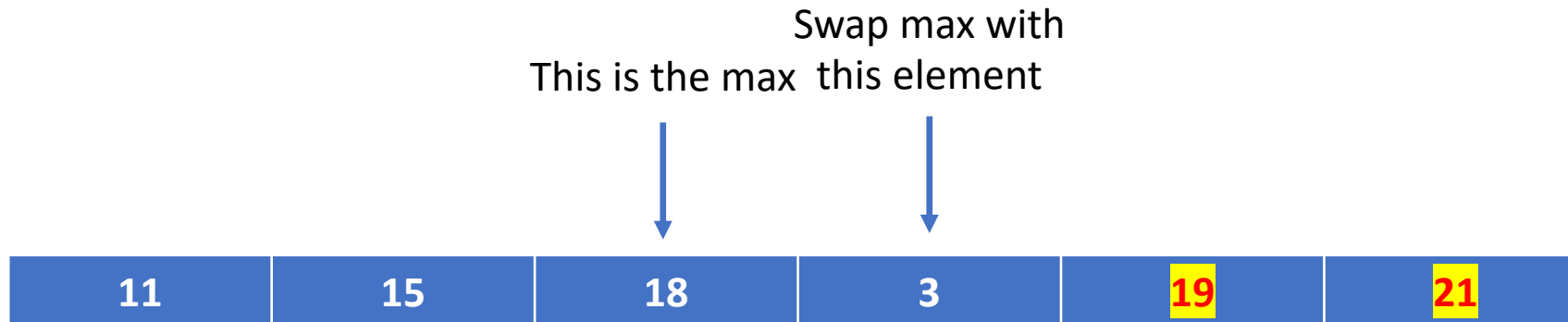


The max is 19 → swap this with the 5th element



Bubble sort (example)

- We search the first $n-2 = 4$ elements for the maximum, and swap this maximum element with the 4th one

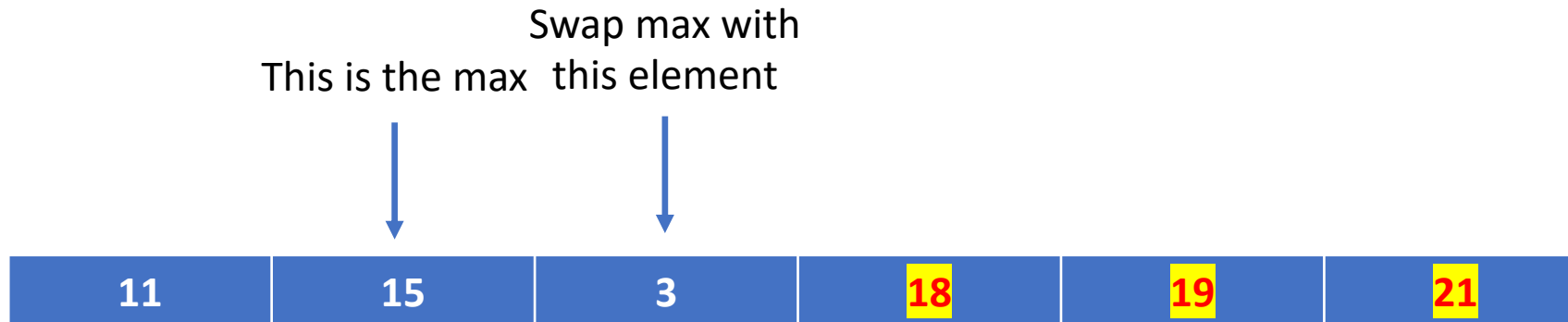


The max is 18 → swap this with the 4th element



Bubble sort (example)

- We search the first $n - 3 = 3$ elements for the maximum, and swap this maximum element with the 3rd one

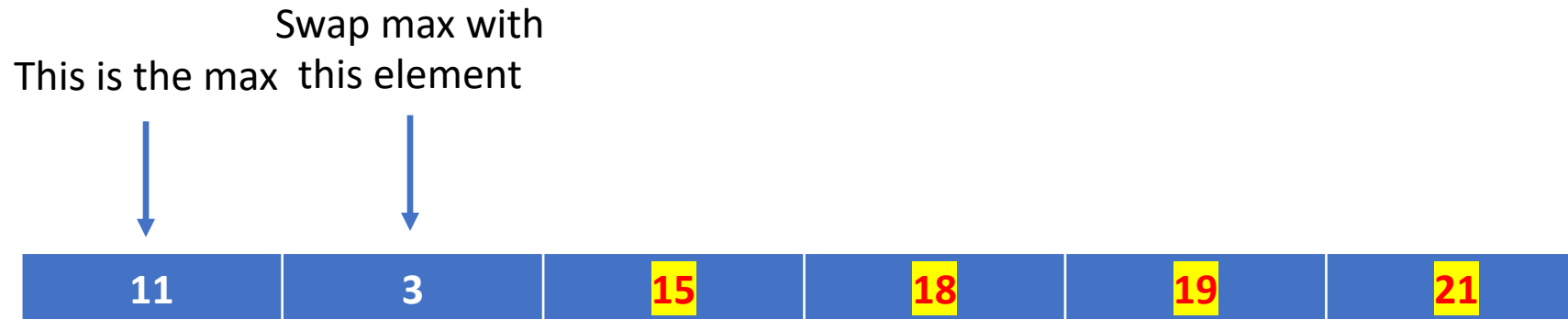


The max is 15 → swap this with the 3rd element



Bubble sort (example)

- We search the first $n - 4 = 2$ elements for the maximum, and swap the maximum element with the 2nd one



The max is 11 → swap this with the 2nd element



Bubble sort (example)

- Once we have only 1 element left (we are finding the max of just the 1st element), then we are done. The list is now sorted.



Bubble sort algorithm

- $end = n - 1$
- while $end > 0$:
 - Set max_index to the index of the maximum element between $values[0]$ through $values[end]$
 - Swap $values[max_index]$ and $values[end]$
 - Set $end = end - 1$

Bubble sort algorithm

- $end = n - 1$
- while $end > 0$:
 - Set max_index to the index of the maximum element between $values[0]$ through $values[end]$
 - Swap $values[max_index]$ and $values[end]$
 - Set $end = end - 1$
 - Set max_index to 0
 - Set i to 0
 - While $i \leq end$:
 - If $values[i] > values[max_index]$:
 - set max_index to i
 - set $i = i + 1$

Bubble sort algorithm

- $end = n - 1$

- while $end > 0$:

Executed $n - 1$ times (while loop)

- Set max_index to 0

- Set i to 0

- While $i \leq end$:

- If $values[i] > values[max_index]$:

- set max_index to i

- Set $i = i + 1$

Executed up to $n - 1$ times each time (while loop)

- Swap $values[max_index]$ and $values[end]$

- Set $end = end - 1$

This suggests an order of magnitude of n^2

Bubble sort algorithm

- $end = n - 1$
- while $end > 0$:
 - Set max_index to 0
 - Set i to 0
 - While $i \leq end$:
 - If $values[i] > values[max_index]$:
 - set max_index to i
 - Set $i = i + 1$
 - Swap $values[max_index]$ and $values[end]$
 - Set $end = end - 1$

Assume that $n = 4$

| end | # iterations of inner while loop |
|-----|----------------------------------|
| 3 | 4 |
| 2 | 3 |
| 1 | 2 |
| 0 | - |

In general, for a list of size n , the total number of inner loop iterations is:

$$2 + 3 + 4 + \dots + n$$

This is $n(n+1)/2 - 1$, which has an order of magnitude of n^2 .

Quicksort

- STAY TUNED!