Hypothesis Testing: Coin example, with background explanation

We will carry out a hypothesis test to test whether a coin is fair.

The null and alternative hypotheses are as follows:

H0: p = 0.50
H1: p != 0.50

where p = probability of heads on a single coin toss

After carrying out an experiment, a coin is flipped 100 times, and lands heads up 62 times. Is there evidence the coin is biased?

From the Central Limit Theorem, if p is the probability of success, and we expect at least 15 successes and failures, then a sample proportion obtained from $n \geq 30$ independent samples has the following probability distribution:

$$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

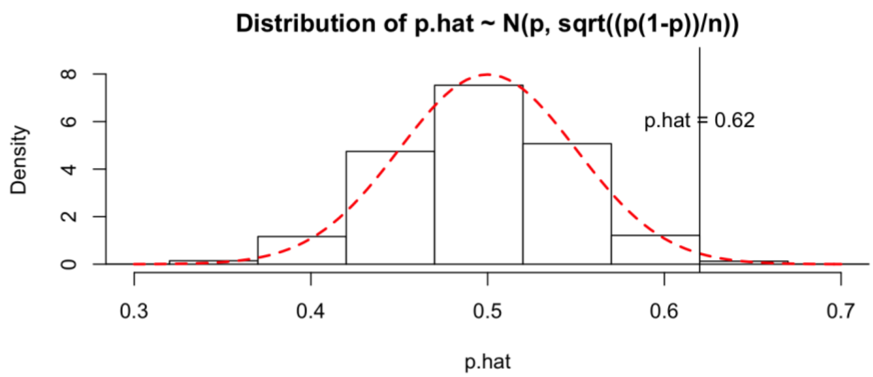
To carry out a hypothesis test, calculate a test statistic

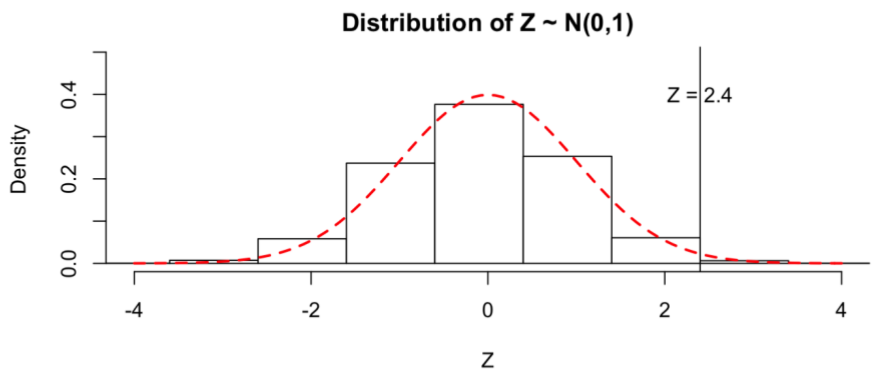
$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

and $Z \sim N(0,1)$ if p_0 is the true probability of success according to H_0.

If the observed $\hat{p}$ (or observed test statistic) is unlikely under H_0, then we will reject H_0 (our assumptions are incorrect).

If we assume H_0 is true (that p = 0.50), how unlikely is our observed sample proportion of 0.62?





Context behind the *p-value*: our observed proportion (0.62) differs from the expected value (which is 0.50) by 0.12. How likely is it, under H_0, that out observed proportion would differ by at least 0.12? What is the probability that the sample proportion differs from the expected proportion by at least the observed amount?

Z-scale interpretation:

Our Z test statistic, calculated using the formula above, is 2.4. Remember Z is the number of standard deviations from the mean. So a question equivalent to the above one is: How likely is it that our observed proportion is at least 2.4 standard deviations from the mean?

In either case, we look at the area under the curve in *both* tails, and find that this p-value is 0.0164.

It is very *unlikely* that we our observation is at least 2.4 standard deviations from the mean – in fact, this only happens about 1.6% of the time. It is equally unlikely that our observed sample proportion would differ from 0.50 by at least a value of 0.12.

Therefore, we reject H_0 and accept the H_1. There is sufficient evidence that this coin is biased (and in this case, biased towards heads).

Conclusions regarding H_0.

- If p-value < 0.05 (or the specified significance level), then reject H_0 and accept H_1. There is sufficient evidence in favor of the alternative hypothesis.

- If p-value is NOT < 0.05, we fail to reject H_0. We say that there is not sufficient evidence to reject H_0.

In both cases, you must state the conclusion in the context of the problem. For example if you are testing that a coin is biased, and your p-value is 0.23, you would conclude that there is NOT sufficient evidence to conclude that the coin is biased.

Note: You can never conclude that H_0 is true! Hypothesis testing is only designed to test for evidence against H_0.

Why?

For example, you flip a coin 30 times and get 20 heads. What do you conclude?