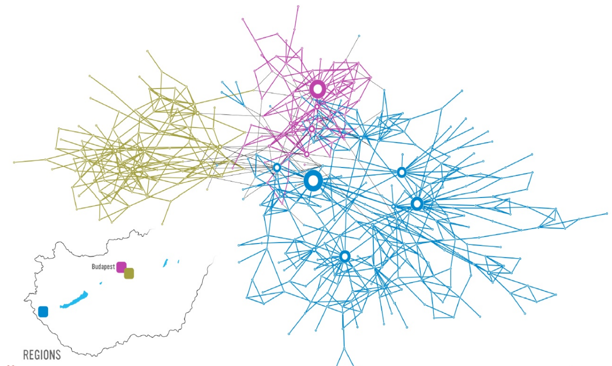


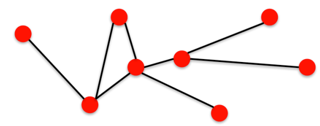
NETWORK SCIENCE

Network Science
Summer Research Institute 2018

IMAGE FROM ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE*. (2016) CAMBRIDGE UNIVERSITY PRESS.

WHAT IS A NETWORK?

A set of points joined in pairs by lines.



Network Science	Graph Theory
Network	Graph
Node	Vertex
Link	Edge
Often refers to real systems.	Mathematical representation of a network

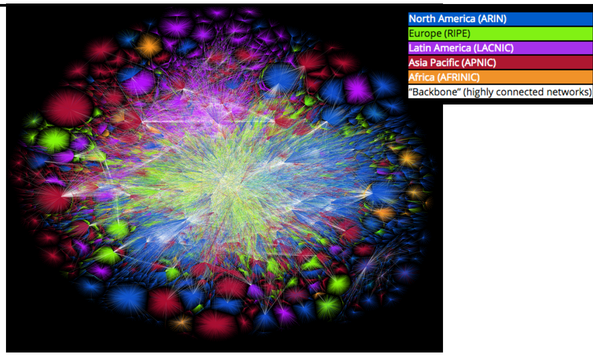
MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)
M.E.J. NEWMAN. *NETWORKS AN INTRODUCTION*. (2010) OXFORD UNIVERSITY PRESS

WHAT IS NETWORK SCIENCE?

The study of complex systems through a network which encodes the interactions between components.

NETWORK	NODES	LINKS	DIRECTED UNDIRECTED	N	L	⟨k⟩
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.33
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

TABLE FROM ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE*. (2016) CAMBRIDGE UNIVERSITY PRESS.



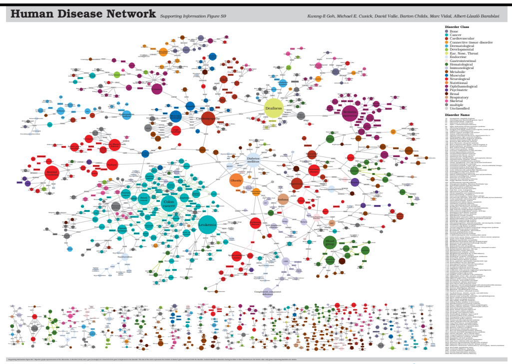
THE INTERNET

Visualization of the routing paths of the Internet. Barrett Lyon/The Opte Project, July 11, 2015

WWW.OPTE.ORG

FACEBOOK

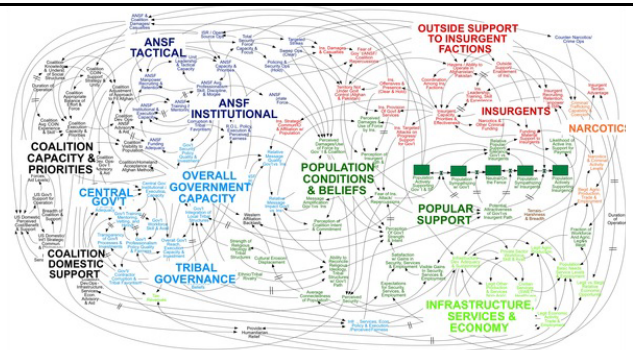
HTTPS://MEDIUM.COM/@JOHNROBB/FACEBOOK-THE-COMPLETE-SOCIAL-GRAPH-B58157EE6594



HUMAN DISEASE NETWORK

Diseases are connected if they have a common genetic origin.

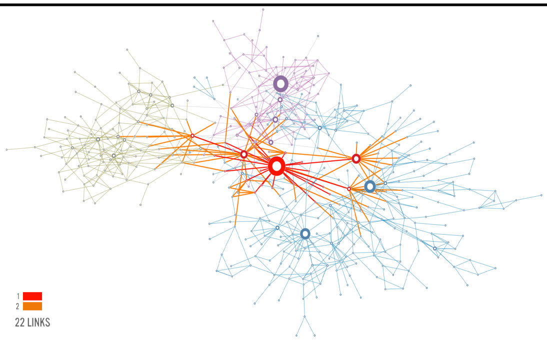
IMAGE FROM ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE* (2016) CAMBRIDGE UNIVERSITY PRESS.



MILITARY ENGAGEMENT

Designed during the Afghan war, 2012

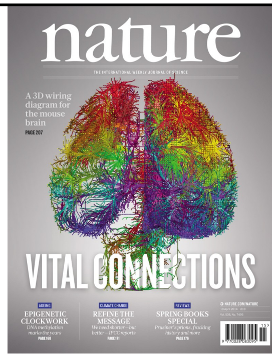
ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE* (2016) CAMBRIDGE UNIVERSITY PRESS.



COMPANY NETWORK

People are connected if one nominated the other as a source of information about organizational and professional issues.

ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE* (2016) CAMBRIDGE UNIVERSITY PRESS.

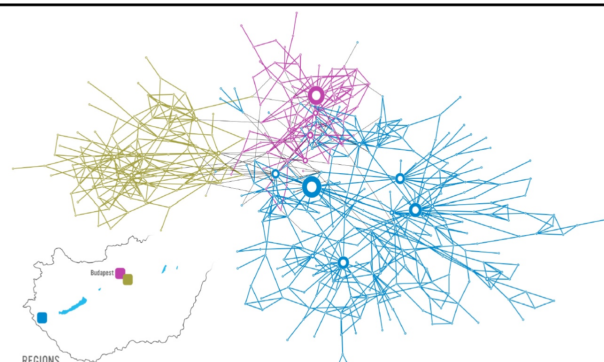


BRAIN NETWORK

April 10, 2014 Issue of *Nature*, neuronal connectivity in a mouse.

SAMPLE NETWORK SCIENCE APPLICATIONS

Network	Application
WWW	What web pages are most related to a search term?
Power Grid	What areas are vulnerable to power failures?
Protein Interactions	How do protein interactions impact human health?
Social / Company Networks	How does information spread?



NETWORK MEASURES

Network Science
Summer Research Institute 2018

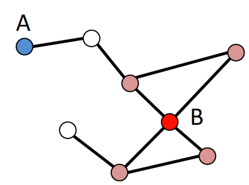
IMAGE FROM ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE*. (2016) CAMBRIDGE UNIVERSITY PRESS.

NODE DEGREE

The number of links connected to the node.

$k_A = 1$

$k_B = 4$

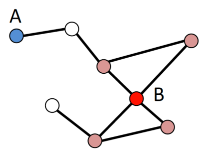


MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

AVERAGE DEGREE

$$\langle k \rangle = \frac{1}{N}\sum_{i=1}^{N} k_i = \frac{2L}{N}$$

N is the number of nodes
L is the number of links



$$\langle k \rangle = \frac{2 \cdot 9}{8} = \frac{9}{4}$$

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

AVERAGE DEGREE

NETWORK	NODES	LINKS	DIRECTED UNDIRECTED	N	L	$\langle k \rangle$
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.33
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

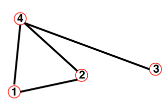
MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

ADJACENCY MATRIX

For a network with *n* nodes, we form an *nxn* matrix, *A*, such that

- $A_{ij} = 1$ if there is a link between node i and j
- $A_{ij} = 0$ if there is no link between node i and j

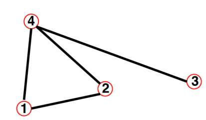
Example



$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

ADJACENCY MATRIX AND DEGREES



$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\boxed{k_i = \sum_{j=1}^{N} A_{ij}}$$

$$\boxed{k_j = \sum_{i=1}^{N} A_{ij}}$$

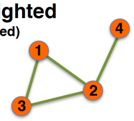
$$A_{ij} = A_{ji}$$
$$A_{ii} = 0$$

$$L = \frac{1}{2}\sum_{i=1}^{N} k_i = \frac{1}{2}\sum_{ij}^{N} A_{ij}$$

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

WEIGHTED GRAPHS

Unweighted
(undirected)

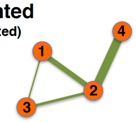


$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad <k> = \frac{2L}{N}$$

Weighted
(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} nonzero(A_{ij}) \qquad <k> = \frac{2L}{N}$$

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

CENTRALITY

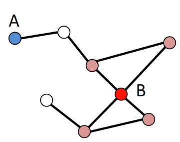
Which nodes are important based on their network?

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

DEGREE CENTRALITY — LOCAL MEASURE

Degree Centrality

The degree centrality of a node, *v*, is the degree of that node.

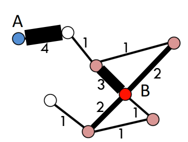


$k_A = 1$

$k_B = 4$

Weighted Degree Centrality

The weighted degree centrality of a node, *v*, is the sum of the weights of the incident edges.



$w_A = 4$

$w_B = 3+2+2+1 = 8$

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

THE PRINCESS BRIDE

	Albino	Ancient Booer	Assistant Brute	Buttercup	Die	Fezzik	Grandfather	Humperdinck	Impressive Clergyman	Inigo	King	Man in Black	Miracle Max	Mother	Queen	Rugen	The Kid	Valerie	Vizzini	Westley	Yellin
Albino	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Ancient Booer	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Assistant Brute	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1
Buttercup	0	1	0	0	0	1	3	6	1	3	1	3	0	0	1	1	2	0	3	9	1
Die	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
Fezzik	0	0	1	1	0	0	2	1	1	10	0	3	1	0	0	0	1	0	4	2	0
Grandfather	0	0	1	3	0	2	0	2	0	2	0	0	0	1	0	0	8	0	1	2	1
Humperdinck	0	0	0	6	0	1	2	0	2	1	0	1	0	0	0	4	1	0	0	5	3
Impressive Clergyman	0	0	0	1	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	1	1
Inigo	0	0	1	3	1	10	2	1	0	0	0	5	2	0	0	2	1	2	5	7	1
King	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Man in Black	0	0	0	3	0	3	0	1	0	5	0	0	0	0	0	0	0	0	2	0	0
Miracle Max	0	0	0	0	0	1	0	0	0	2	0	0	0	0	0	0	0	1	0	1	0
Mother	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Queen	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Rugen	0	0	0	1	1	0	0	4	0	2	0	0	0	0	0	0	0	0	0	1	0
The Kid	0	0	0	2	0	1	8	1	0	1	0	0	0	1	0	0	0	0	1	2	0
Valerie	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0
Vizzini	0	0	0	3	0	4	1	0	0	5	0	2	0	0	0	0	1	0	0	0	0
Westley	1	0	0	9	0	2	2	5	1	7	0	0	1	0	0	1	2	0	0	0	1
Yellin	0	0	1	1	0	0	1	3	1	1	0	0	0	0	0	0	0	0	0	1	0

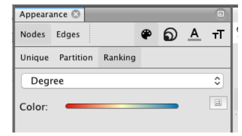
DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

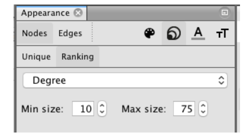
GEPHI & DEGREE CENTRALITY

1. Open the file PrincessBride.gexf
2. Under the Statistics panel click “Run” next to Average Degree.
   - This will give you a chart with the degree distribution (the number of vertices of each degree that appear in the graph).
   - Close this window.
3. Under the Statistics panel click “Run” next to Avg. Weighted Degree.
   - This will give the weighted degree distribution.
   - Close this window.
4. Click on the “Data Laboratory” button.
   - Here you can see the degree and weighted degree of each character.

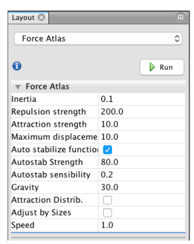
GEPHI & VISUALIZING DEGREE CENTRALITY

5. Go back to the “Overview.” Look at the options below the graph display.
6. You can thicken the edges to more clearly see the weights using the slider:
7. Turn Node labels on using
   - Change their size to match the size of the node using the menu with
8. Under “Appearance” you can rank the nodes by their degree or weighted degree, change the node size based on degree, change the colors of the nodes, etc.





GEPHI & LAYOUT

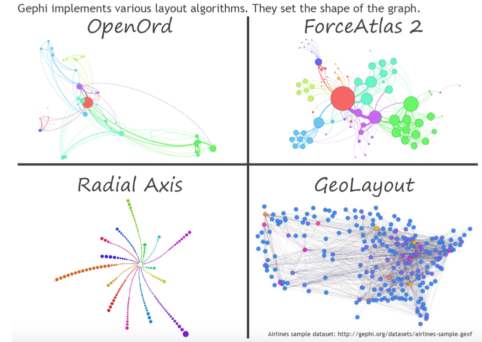


9. Go to the “Layout” tab and select “Force Atlas” from the drop down menu.
   - Idea is linked nodes attract each other and non-linked nodes repel each other.
   - Click “Run” to start the algorithm.
   - Set the “Repulsion strength” to 200,000 to expand the graph. Press enter.
   - Press “Stop” to stop the
10. Nodes may still overlap. Check off “Adjust by Sizes” and quickly run the algorithm again.
11. To stop labels from overlapping, run the “Label Adjust” layout.
12. Experiment with different layouts. Look at the Gephi tutorial for Layouts to help you understand the parameters. Focus on ForceAtlas, ForceAtlas2, Fruchterman-Reingold, and OpenOrd https://gephi.org/users/tutorial-layouts/.
13. You can preview your graph by going to the the “Preview.” Click “Refresh” to see what the graph will look like. You can turn labels on/off, change the edges from curved to straight, etc.
14. Then you can export the graph as an svg or pdf. Alternatively, you can use the “screen shot” button at any time to get a high resolution png.

GEPHI LAYOUTS

Various layouts exist

Gephi implements various layout algorithms. They set the shape of the graph.



HTTPS://GEPHI.ORG/TUTORIALS/GEPHI-TUTORIAL-LAYOUTS.PDF

GEPHI LAYOUTS

In general, select one according to the feature of the topology you want to highlight:

emphasis DIVISIONS *OpenOrd*	*emphasis* COMPLEMENTARITIES *ForceAtlas, Yifan Hu, Frushterman-Reingold*
emphasis RANKING *Circular, Radial Axis*	*emphasis* GEOGRAPHIC REPARTITION *GeoLayout*

Graphic Adjustements

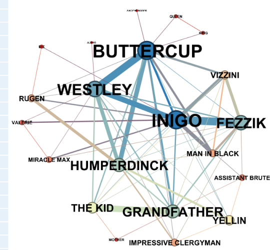
- Label Adjust
- Noverlap
- Expansion
- Contraction

HTTPS://GEPHI.ORG/TUTORIALS/GEPHI-TUTORIAL-LAYOUTS.PDF

DEGREE VS. WEIGHTED DEGREE

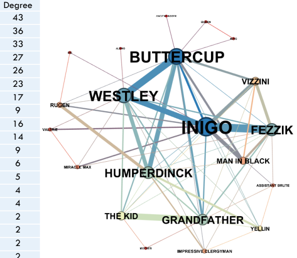
Degree

	Degree
INIGO	14
BUTTERCUP	14
WESTLEY	12
FEZZIK	11
HUMPERDINCK	10
GRANDFATHER	10
THE KID	8
YELLIN	7
VIZZINI	6
MAN IN BLACK	5
RUGEN	5
IMPRESSIVE CLERGYMAN	5
MIRACLE MAX	4
ASSISTANT BRUTE	4
VALERIE	3
DIE	2
KING	2
MOTHER	2
QUEEN	2
ALBINO	1
ANCIENT BOOER	1



Weighted Degree

	Weighted Degree
INIGO	43
BUTTERCUP	36
WESTLEY	33
FEZZIK	27
HUMPERDINCK	26
GRANDFATHER	23
THE KID	17
YELLIN	9
VIZZINI	16
MAN IN BLACK	14
RUGEN	9
IMPRESSIVE CLERGYMAN	6
MIRACLE MAX	5
ASSISTANT BRUTE	4
VALERIE	4
DIE	2
KING	2
MOTHER	2
QUEEN	2
ALBINO	1
ANCIENT BOOER	1



DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

EIGENVECTOR CENTRALITY

A node is important if it is connected to important nodes.

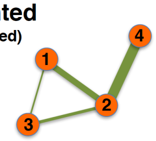
"Weighted degree centrality with a feedback loop: A [node] gets a boost for being connected to important [nodes]." (Beveridge, 2016)

The eigenvector centrality, x_i, of node i comes from solving the linear system of equations

$$x_i = \sum_{j \in V} A_{ij} x_j$$

where j is a neighbor of i.

Weighted
(undirected)



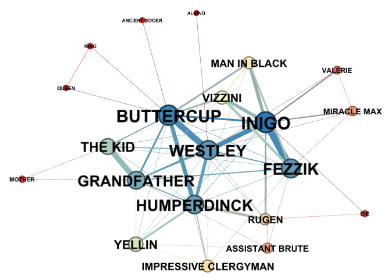
$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)
A. BEVERIDGE, J. SHAN. NETWORK OF THRONES. *MATH HORIZONS*, **APRIL** (2016) 18-22.

GEPHI & EIGENVECTOR CENTRALITY

1. Under the Statistics panel click "Run" next to Eigenvector Centrality.
   - This will give you a chart with the eigenvector centrality distribution (the number of vertices of each degree that appear in the graph).
   - Close this window.
2. Click on the "Data Laboratory" button.
   - Here you can see the eigenvector centrality of each character.
3. Under "Appearance" you can rank the nodes by their eigenvector centrality.

EIGEN VECTOR CENTRALITY



	Eigenvector Centrality
INIGO	1
BUTTERCUP	0.96795
WESTLEY	0.891104
FEZZIK	0.890407
HUMPERDINCK	0.866192
GRANDFATHER	0.834149
THE KID	0.726221
YELLIN	0.637546
VIZZINI	0.575288
MAN IN BLACK	0.502557
IMPRESSIVE CLERGYMAN	0.497184
RUGEN	0.45704
ASSISTANT BRUTE	0.392665
MIRACLE MAX	0.357085
VALERIE	0.263957
MOTHER	0.18194
DIE	0.171062
KING	0.130344
QUEEN	0.130344
ANCIENT BOOER	0.113701
ALBINO	0.104436

DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

COMPARISON

	Degree Centrality	Weighted Degree Centrality	Eigenvector Centrality
INIGO	1	1	1
BUTTERCUP	1	2	2
WESTLEY	3	3	3
FEZZIK	4	4	4
HUMPERDINCK	5	5	5
GRANDFATHER	5	6	6
THE KID	7	7	7
YELLIN	8	10	8
VIZZINI	9	8	9
MAN IN BLACK	10	9	10
RUGEN	10	10	12
IMPRESSIVE CLERGYMAN	10	12	11
MIRACLE MAX	13	13	14
ASSISTANT BRUTE	13	14	13
VALERIE	15	14	15
KING	16	16	18
QUEEN	16	16	19
DIE	16	16	17
MOTHER	16	16	16
ANCIENT BOOER	20	20	20
ALBINO	21	21	21

DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

PAGERANK

PageRank is the idea behind the Google search engine.

"Each vertex has an inherent importance $\beta \geq 0$, along with an importance acquired from its neighbors." (Beveridge, 2016)

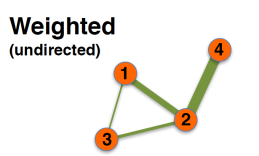
A neighbors importance is divided evenly among its neighbors.

The PageRank, y_i, of vertex i is given by

$$y_i = \alpha \sum_{j \in V} \frac{A_{ij}}{k_j} y_j + \beta$$

where $\alpha + \beta = 1, \alpha, \beta \geq 0$ and j is a neighbor of i.

Researchers usually use $\beta = 0.15$.



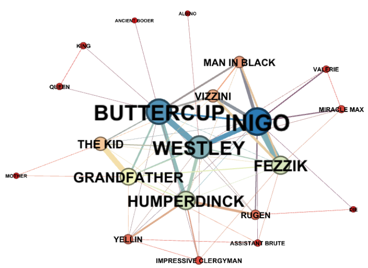
$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

A. BEVERIDGE, J. SHAN. NETWORK OF THRONES. *MATH HORIZONS*, APRIL (2016) 18-22.

GEPHI & PAGERANK

1. Under the Statistics panel click "Run" next to PageRank.
   - You will need to choose, p, which is like α
2. Click on the "Data Laboratory" button.
   - Here you can see the PageRank of each character.
3. Under "Appearance" you can rank the nodes by their PageRank.

PAGERANK



	PageRank
BUTTERCUP	0.108767
INIGO	0.100573
WESTLEY	0.090182
FEZZIK	0.077843
GRANDFATHER	0.072204
HUMPERDINCK	0.070926
THE KID	0.059063
YELLIN	0.051667
VIZZINI	0.044783
RUGEN	0.040841
IMPRESSIVE CLERGYMAN	0.038447
MAN IN BLACK	0.038235
MIRACLE MAX	0.033206
ASSISTANT BRUTE	0.031673
VALERIE	0.026687
KING	0.023938
QUEEN	0.023938
DIE	0.020194
MOTHER	0.019558
ANCIENT BOOER	0.013748
ALBINO	0.013527

DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

COMPARISON

	Degree Centrality	Weighted Degree Centrality	Eigenvector Centrality	PageRank
INIGO	1	1	1	2
BUTTERCUP	1	2	2	1
WESTLEY	3	3	3	3
FEZZIK	4	4	4	4
HUMPERDINCK	5	5	5	6
GRANDFATHER	5	6	6	5
THE KID	7	7	7	7
YELLIN	8	10	8	8
VIZZINI	9	8	9	9
MAN IN BLACK	10	9	10	12
RUGEN	10	10	12	10
IMPRESSIVE CLERGYMAN	10	12	11	11
MIRACLE MAX	13	13	14	13
ASSISTANT BRUTE	13	14	13	14
VALERIE	15	14	15	15
KING	16	16	18	16
QUEEN	16	16	19	17
DIE	16	16	17	18
MOTHER	16	16	16	19
ANCIENT BOOER	20	20	20	20
ALBINO	21	21	21	21

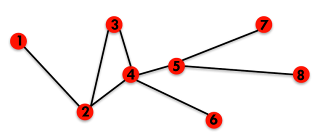
DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

PATHS AND DISTANCE

A *path* is a sequence of nodes in which each nodes is adjacent to the next one.

The *distance (shortest path)* between two nodes is the number of edges in the shortest path connecting them.

The distance from node i to node j is denoted d_{ij}.



M.E.J. NEWMAN. (2010). *NETWORKS: AN INTRODUCTION.* OXFORD UNIVERSITY PRESS.

CLOSENESS CENTRALITY– GLOBAL MEASURE

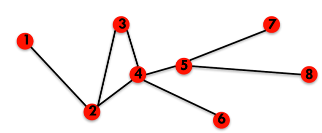
A nodes' average distance to all other nodes is given by

$$\ell_i = \frac{1}{n} \sum_{j \in V} d_{ij}$$

where *n* is the number of nodes.

The *closeness centrality* of node i is

$$C_i = \frac{1}{\ell_i} = \frac{n}{\sum_{j \in V} d_{ij}}$$

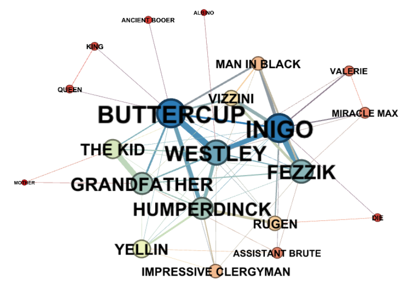


M.E.J. NEWMAN. (2010). *NETWORKS: AN INTRODUCTION.* OXFORD UNIVERSITY PRESS.

GEPHI & CLOSENESS CENTRALITY

1. Under the Statistics panel click “Run” next to Avg. Path Length.
   - This will give several centrality measures, including closeness centrality.
2. Click on the “Data Laboratory” button.
   - Here you can see the closeness centrality of each character.
3. Under “Appearance” you can rank the nodes by their closeness centrality.

CLOSENESS CENTRALITY



	Closeness Centrality
BUTTERCUP	0.769231
INIGO	0.769231
WESTLEY	0.714286
FEZZIK	0.689655
GRANDFATHER	0.666667
HUMPERDINCK	0.666667
THE KID	0.625
YELLIN	0.606061
VIZZINI	0.571429
RUGEN	0.555556
IMPRESSIVE CLERGYMAN	0.540541
MAN IN BLACK	0.540541
MIRACLE MAX	0.5
ASSISTANT BRUTE	0.5
VALERIE	0.487805
KING	0.454545
QUEEN	0.454545
DIE	0.454545
ANCIENT BOOER	0.444444
ALBINO	0.425532
MOTHER	0.416667

DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

COMPARISON

	Degree Centrality	Weighted Degree Centrality	Eigenvector Centrality	PageRank	Closeness Centrality
INIGO	1	1	1	2	1
BUTTERCUP	1	2	2	1	1
WESTLEY	3	3	3	3	3
FEZZIK	4	4	4	4	4
HUMPERDINCK	5	5	5	6	6
GRANDFATHER	5	6	6	5	5
THE KID	7	7	7	7	7
YELLIN	8	10	8	8	8
VIZZINI	9	8	9	9	10
MAN IN BLACK	10	9	10	12	12
RUGEN	10	10	12	10	10
IMPRESSIVE CLERGYMAN	10	12	11	11	11
MIRACLE MAX	13	13	14	13	13
ASSISTANT BRUTE	13	14	13	14	14
VALERIE	15	14	15	15	15
KING	16	16	18	16	16
QUEEN	16	16	19	17	17
DIE	16	16	17	18	18
MOTHER	16	16	16	19	21
ANCIENT BOOER	20	20	20	20	19
ALBINO	21	21	21	21	20

DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

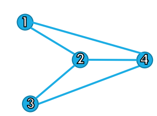
BETWEENNESS CENTRALITY – GLOBAL

The *betweenness centrality* "measures how frequently that [node] lies on short paths between other pairs of [nodes]." High betweenness means the node is a “broker of information.” (Beveridge, 2016)

The betweenness, z_i, of node i is

$$z_i = \sum_{j,k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}}$$

where σ_{jk} is the number of shortest paths from j to k and $\sigma_{jk}(i)$ is the number of those paths that go through node i.



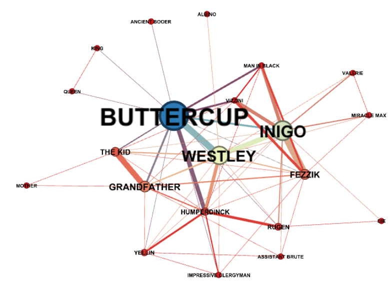
Pairs of Nodes	σ_{jk}	$\sigma_{jk}(2)$	$\frac{\sigma_{jk}(2)}{\sigma_{jk}}$
1, 2	1	1	1
1, 3	2	1	0.5
1, 4	1	0	0
2, 3	1	1	1
2, 4	1	1	1
3, 4	1	0	0
		z_2	3.5

A. BEVERIDGE, J. SHAN. NETWORK OF THRONES. *MATH HORIZONS*, **APRIL** (2016) 18-22.

GEPHI & BETWEENNESS CENTRALITY

1. Under the Statistics panel click "Run" next to Avg. Path Length.
   - This will give several centrality measures, including betweenness centrality.
2. Click on the "Data Laboratory" button.
   - Here you can see the betweenness centrality of each character.
3. Under "Appearance" you can rank the nodes by their betweenness centrality.

BETWEENNESS CENTRALITY



	Betweenness Centrality
BUTTERCUP	60.048413
INIGO	36.30119
WESTLEY	32.548413
GRANDFATHER	13.715079
FEZZIK	11.580556
THE KID	8.511111
HUMPERDINCK	5.148413
RUGEN	3.875
YELLIN	2.886111
VIZZINI	0.6
MIRACLE MAX	0.333333
MAN IN BLACK	0.166667
IMPRESSIVE CLERGYMAN	0.142857
ASSISTANT BRUTE	0.142857
VALERIE	0
KING	0
QUEEN	0
DIE	0
ANCIENT BOOER	0
ALBINO	0
MOTHER	0

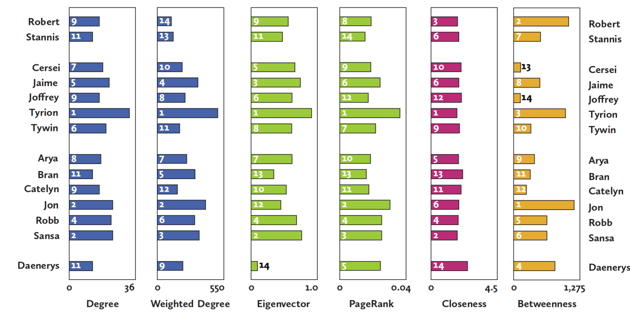
DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

COMPARISON

	Degree Centrality	Weighted Degree Centrality	Eigenvector Centrality	PageRank	Closeness Centrality	Betweenness Centrality
INIGO	1	1	1	2	1	2
BUTTERCUP	1	2	2	1	1	1
WESTLEY	3	3	3	3	3	3
FEZZIK	4	4	4	4	4	5
HUMPERDINCK	5	5	5	6	6	7
GRANDFATHER	5	6	6	5	5	4
THE KID	7	7	7	7	7	6
YELLIN	8	10	8	8	8	9
VIZZINI	9	8	9	9	10	10
MAN IN BLACK	10	9	10	12	12	12
RUGEN	10	10	12	10	10	8
IMPRESSIVE CLERGYMAN	10	12	11	11	11	13
MIRACLE MAX	13	13	14	13	13	11
ASSISTANT BRUTE	13	14	13	14	14	14
VALERIE	15	14	15	15	15	15
KING	16	16	18	16	16	16
QUEEN	16	16	19	17	17	17
DIE	16	16	17	18	18	18
MOTHER	16	16	16	19	21	21
ANCIENT BOOER	20	20	20	20	19	19
ALBINO	21	21	21	21	20	20

DATA FROM KAMINSKI, J., SCHOBER, M., ALBALADEJO, R., ZASTUPAILO, O., HIDALGO, C. (2012). "MOVIEGALAXIES - SOCIAL NETWORKS IN MOVIES", HTTP://MOVIEGALAXIES.COM, AUGUST 2012, MAY 9, 2018.

GAME OF THRONES – A STORM OF SWORDS



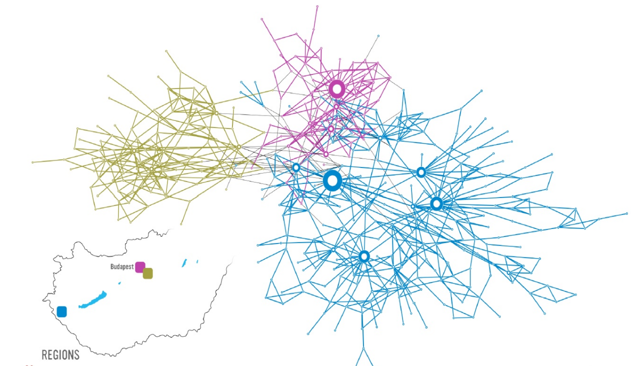
A. BEVERIDGE, J. SHAN. NETWORK OF THRONES. *MATH HORIZONS*, **APRIL** (2016) 18-22.

EXERCISE

For each of the following, come up with an example network and a particular node in that network that has:

1. High closeness centrality, but low degree centrality.
2. High degree centrality, but low closeness centrality.
3. High betweenness centrality, but low closeness centrality.
4. High closeness centrality, but low betweenness centrality.
5. High degree centrality, but low betweenness centrality.
6. High betweenness centrality, but low degree centrality.

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)



COMMUNITIES

Network Science
Summer Research Institute 2018

IMAGE FROM ALBERT-LASZLO BARABASI'S *NETWORK SCIENCE.* (2016) CAMBRIDGE UNIVERSITY PRESS.

COMMUNITIES IN BELGIUM MOBILE PHONE NETWORK

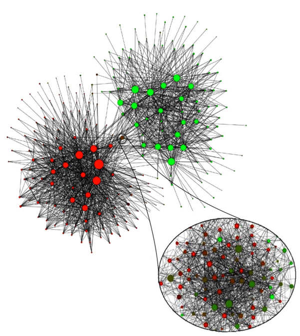
Communities extracted from call patterns.

Nodes – communities, size of each node reflects the number of individuals in the community (>100)

Links – calls between communities

Colors reflect the language spoken in each community – red: French, green: Dutch

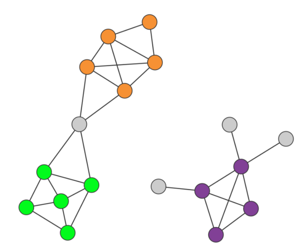
A community of communities connect the two main clusters. This community has less language separation – Brussels the country's capital.



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

HYPOTHESES FOR COMMUNITIES

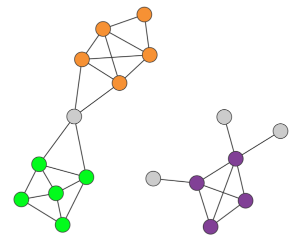
1. A network's community structure is uniquely encoded in its wiring diagram.
2. A community corresponds to a connected subgraph.
   - All members of a community are connected by a path that stays in the community.
3. Communities correspond to locally dense neighborhoods of a network.
   - Nodes in a community have a higher probability of linking to each other than to nodes not in the community.



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

CLIQUES AS COMMUNITIES?

- A clique is a complete subgraph of k nodes.
- Triangles are frequent; larger cliques are rare.
- Communities do not necessarily correspond to complete subgraphs.



STRONG AND WEAK COMMUNITIES

Consider a connected subgraph, C, with N_C nodes.

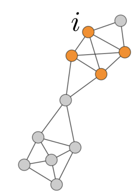
Internal Degree, k_i^{int}, is the number of links incident with node *i*, that connect to other nodes in C.

External Degree, k_i^{ext}, is the number of links incident with node *i*, that connect to nodes not in C.

If $k_i^{ext} = 0$, then all neighbors of *i* belong to C and C is a good community for *i*.

If $k_i^{int} = 0$, then all neighbors of *i* belong to other communities and C is not a good community for *i*.

$k_i^{int} = 3$
$k_i^{ext} = 1$

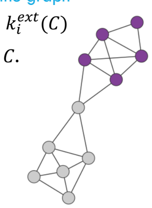


STRONG AND WEAK COMMUNITIES

Strong Community: Each node of C has more links within the community than with the rest of the graph

$k_i^{int}(C) > k_i^{ext}(C)$

For all $i \in C$.

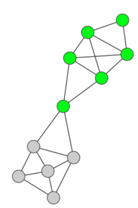


Weak Community: The total internal degree of C is greater than the total external degree.

$\sum_{i \in C} k_i^{int}(C) > \sum_{i \in C} k_i^{ext}(C)$

This is a relaxation of the strong community. It allows some vertices to violate $k_i^{int}(C) > k_i^{ext}(C)$

Every strong community is a weak community.



GRAPH PARTITIONING

How many ways are there to partition a network into two communities?

<u>Graph Bisection</u>:

Divide a network into two equal non-overlapping subgraphs such that the number of links between the nodes in the two groups is minimized.

Two subgroups of sizes n_1 and n_2, total number of combinations $\frac{N!}{n_1! n_2!}$.

When $n_1 = n_2 = N/2$, this is approximately $\frac{2^{N+1}}{\sqrt{N}}$.

When N=10, this would give 256 partitions (1 ms).

When N=100, this would give 10^{26} partitions (10^{21} years).

GRAPH PARTITION – HISTORY

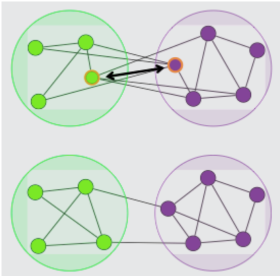
Partition the full wiring diagram of an integrated circuit into smaller subgraphs, so that they minimize the number of connections between them.

2.5 billion transistors

GRAPH PARTITION – HISTORY

Kerninghan-Lin Algorithm for Graph Bisection

- Partition a network into two groups of predefined size. This partition is called a cut.
- Inspect each pair of nodes, one from each group. Identify the pair that results in the largest reduction of cut size (links between the two groups) if we swap them.
- Swap them.
- If no pair reduces the cut size, swap the pair that increases it the least.
- The process is repeated until each node is moved once.



NUMBER OF COMMUNITIES

Community Detection

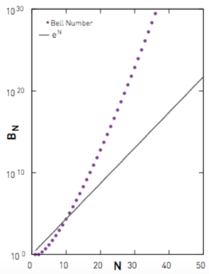
The number and size of communities are unknown at the beginning.

Partition

Division of a network into groups of nodes, so that each node belongs to to one group.

Bell Number: number of possible partitions of N nodes

$B_N = \frac{1}{e}\sum_{j=0}^{N}\frac{j^N}{j!}$



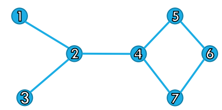
HIERARCHICAL CLUSTERING

1. Determine how similar nodes are using the adjacency matrix.
2. Hierarchical clustering iteratively identifies groups of nodes with high similarity, following one of two strategies:
   a) Agglomerative Algorithms: Merge nodes and communities with high similarity.
   b) Divisive Algorithms: Split communities by removing links that connect nodes with low similarity.
3. Hierarchical Tree or dendrogram used to visiualize the history of the merging or splitting process the algorithm follows. Horizontal cuts of this tree offer various community partitions.

GIRVAN-NEWMAN ALGORITHM - DIVISIVE

1. Define a centrality measure for the edges.
   - Link betweenness – the number of shortest paths between all node pairs that run along a link.
2. Compute the centrality of each link. Remove the link with the largest centrality; in case of a tie, choose randomly.
3. Recalculate the centrality of each link.
4. Repeat until all links are removed.

Progress can be represented using a tree or *dendrogram*.

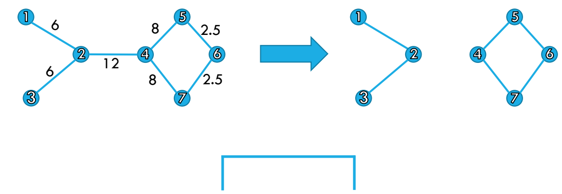


MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)
M.E.J. NEWMAN. (2010). *NETWORKS: AN INTRODUCTION.* OXFORD UNIVERSITY PRESS.

GIRVAN-NEWMAN ALGORITHM - DIVISIVE

1. Define a centrality measure for the edges.
2. Compute the centrality of each link. Remove the link with the largest centrality; in case of a tie, choose randomly.
3. Recalculate the centrality of each link.
4. Repeat until all links are removed.

Progress can be represented using a tree or *dendrogram*.

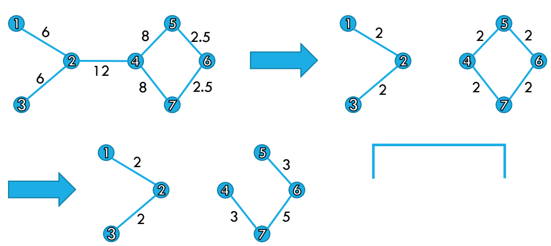


MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)
M.E.J. NEWMAN. (2010). *NETWORKS: AN INTRODUCTION.* OXFORD UNIVERSITY PRESS.

GIRVAN-NEWMAN ALGORITHM - DIVISIVE

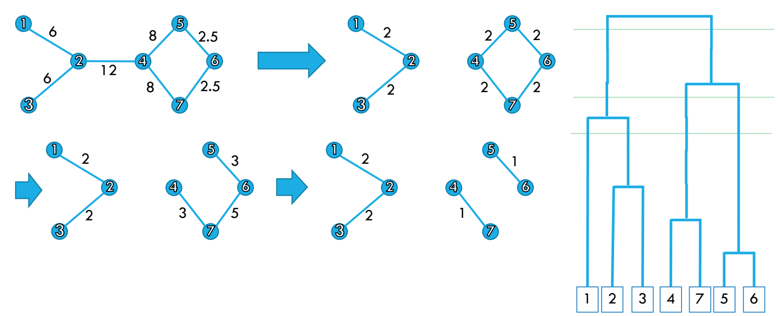
1. Define a centrality measure for the edges.
2. Compute the centrality of each link. Remove the link with the largest centrality; in case of a tie, choose randomly.
3. Recalculate the centrality of each link.
4. Repeat until all links are removed.

Progress can be represented using a tree or *dendrogram*.

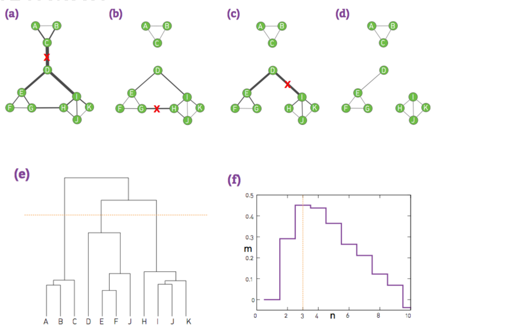


MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)
M.E.J. NEWMAN. (2010). *NETWORKS: AN INTRODUCTION.* OXFORD UNIVERSITY PRESS.

GIRVAN-NEWMAN ALGORITHM - DIVISIVE

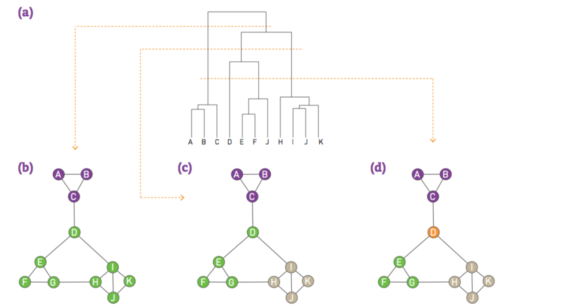


GIRVAN-NEWMAN



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

WHERE TO CUT?

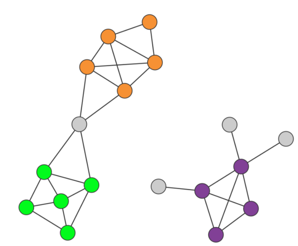


MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

GIRVAN-NEWMAN ALGORITHM IN R

HYPOTHESES FOR COMMUNITIES

1. A network's community structure is uniquely encoded in its wiring diagram.
2. A community corresponds to a connected subgraph.
   - All members of a community are connected by a path that stays in the community.
3. Communities correspond to locally dense neighborhoods of a network.
   - Nodes in a community have a higher probability of linking to each other than to nodes not in the community.



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

MODULARITY

Add a random hypothesis:

Randomly wired networks are not expected to have a community structure.

Imagine a partition into n_C communities $\{C_C, c = 1, n_C\}$

Modularity $M(C_c) = \frac{1}{2L}\sum_{i,j=1}^{N}(A_{ij} - P_{ij})\delta(C_i - C_j)$

Original data

Expected connections, a model

Relative to a specific partition

Modularity is a measure associated to a partition.

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

MODULARITY

Maximal Modularity Hypothesis

The partition with the maximum modularity *M* for a given network offers the optimal community structure.

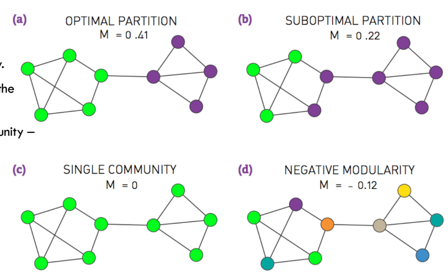
Goal: Find the partition into communities that maximizes *M*.

MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

MODULARITY

a) Optimal Partition – maximizes modularity
b) Suboptimal Partition – positive modularity.
c) Single Community, assigning all nodes to the same community – modularity 0
d) Assigning each node to a different community – negative modularity

Modularity is size dependent.



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

MODULARITY BASED COMMUNITY DETECTION

Greedy Algorithm – iteratively join nodes if the move increases the new partitions modularity.

1. Assign each node to a community of its own. That is, start with N communities.
2. Inspect each pair of communities connected by at least one link and compute the modularity variation, ΔM, obtained if we merge these two communities.
3. Identify the community pairs for which ΔM is the largest and merge them. Modularity of a particular partition is always calculated from the full topology of the network.
4. Repeat step 2 until all nodes are merged into a single community.
5. Record for each step and select the partition for which the modularity is maximal.

There are other algorithms that are better.

Limits to Modularity – cannot detect communities smaller than $\sqrt{2L}$ where L is the number of links.

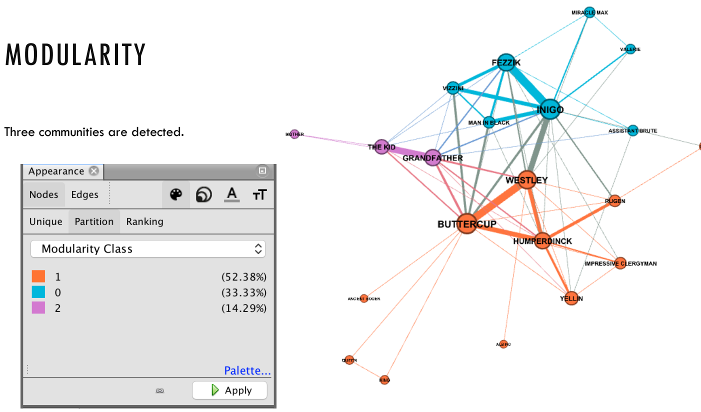
MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

GEPHI & MODULARITY

1. Under the Statistics panel click “Run” next to Modularity.
   - Choose “randomize,” choose whether or not to include edge weights. Adjust the resolution as desired.
2. Click on the “Data Laboratory” button.
   - Here you can see the modularity class of each character.
3. Under “Appearance” you can partition the nodes by their modularity class.

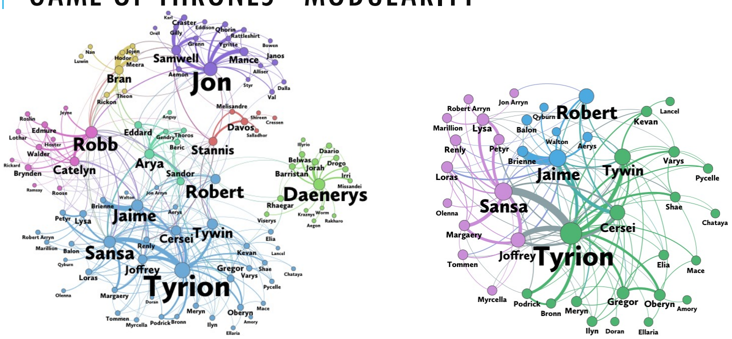
MODULARITY

Three communities are detected.

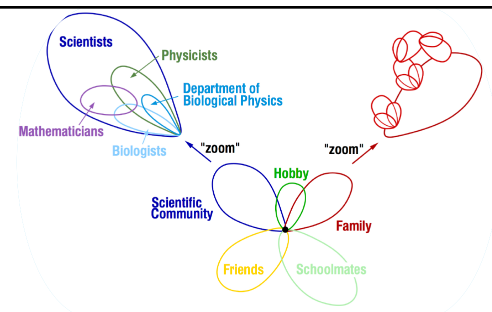


MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)

GAME OF THRONES - MODULARITY



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)



OVERLAPPING COMMUNITIES

Schematic representation of the communities surrounding T. Vicsek who introduced the concept of overlapping communities.

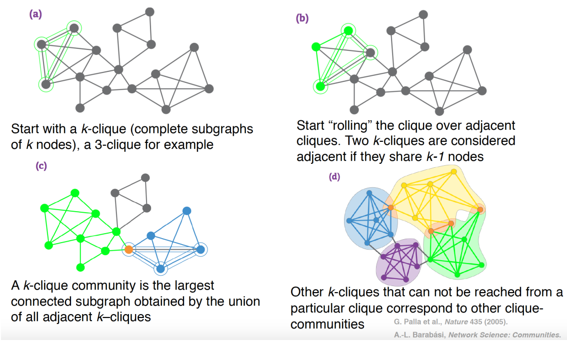
MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)
G. PALLA ET AL., *NATURE* 435 (2005)

CLIQUE PERCOLATION (CFINDER)

Views a community as the union of overlapping cliques.

The Cfinder package that implements Clique Percolation Method can be downloaded at:

www.cfinder.org



MODIFIED FROM SLIDES FOR ALBERT-LASZLO BARABASI'S COMPLEX NETWORKS COURSE (HTTPS://WWW.BARABASILAB.COM/COURSE)