"Multi-Level Models: What Are They and How Do They Work? "

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Why are MLM's important?

- Ordinary Least Squares (OLS) and all regression models under the General Linear Model (including logistic and poisson regression) assumes that all observations are independent of one another.
- Likewise, repeated measures (time1, time2, …) assume that all time points for a given individual are independent of one another (all pairs of time points), including serial dependency ("Durbin–Watson statistic").
- When data are "clustered," OLS may lead to inaccuracies in inference – standard errors may be too small – leads to overestimation of significance ("alpha-inflation").
- "Clusters" may include individuals within: dyads; twins; families; counseling groups; households; neighborhoods; schools; nursing units; hospitals (and repeated measures over time within an individual – the individual becomes the "cluster").

An example

- Subjects: 386 women across 40 women's groups with a focus on diet and weight control
- Group sizes range from 5-15
- Independent variable (measured for each individual) is "motivation."
- Outcome – weight loss
- The groups meet regularly and may have some level of cohesion – we may expect some correlation between the individuals with a given group.
- In fact, there is substantial "clustering" indicated by the fact that the groups differ substantially in average weight loss (9.75lbs-24.43lbs)

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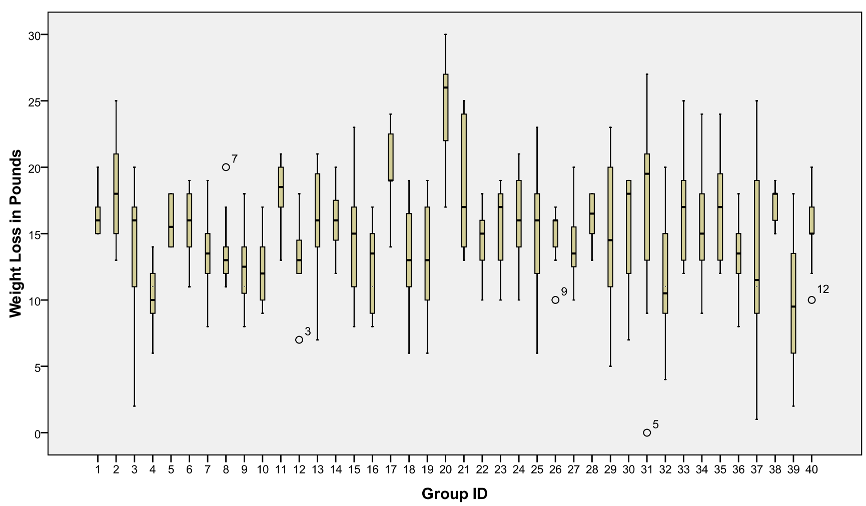
Average Weight Loss – 40 Groups



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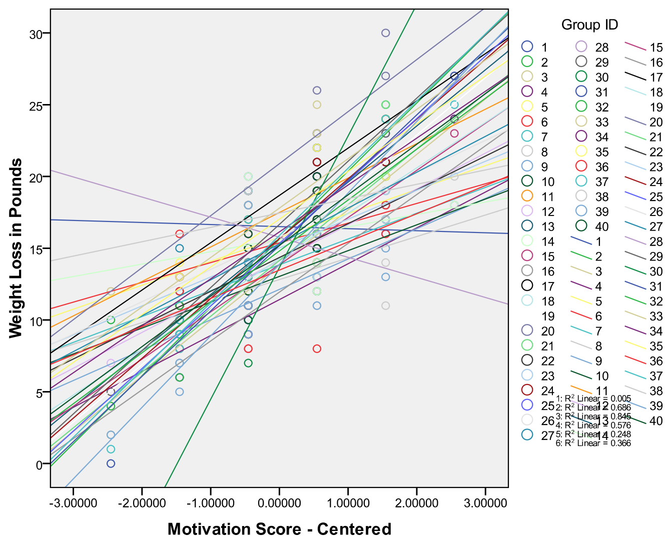
Weight Loss by Motivation (individual)



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Regression – Motivation(X) vs Wt Loss (Y) (no consideration of groups)

Model Summary[b]

					Change Statistics				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change
1	.738[a]	.545	.544	3.057	.545	459.569	1	384	.000

a. Predictors: (Constant), Motivation Score - Centered

b. Dependent Variable: Weight Loss in Pounds

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4295.678	1	4295.678	459.569	.000[a]
	Residual	3589.319	384	9.347		
	Total	7884.997	385			

a. Predictors: (Constant), Motivation Score - Centered

b. Dependent Variable: Weight Loss in Pounds

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	15.003	.156		96.409	.000	14.697	15.309
	Motivation Score - Centered	3.270	.153	.738	21.438	.000	2.970	3.570

a. Dependent Variable: Weight Loss in Pounds

Multilevel and Random Coefficients Models

Regression – 40 separate groups

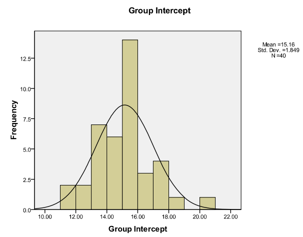
Coefficients[a]

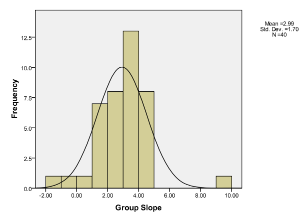
Group ID	Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound
1	1	(Constant)	16.507	.778		21.208	.000	14.712	18.301
		Motivation Score - Centered	-.143	.736	-.068	-.194	.851	-1.841	1.555
2	1	(Constant)	15.626	1.047		14.923	.000	13.150	18.102
		Motivation Score - Centered	4.750	1.215	.828	3.911	.006	1.878	7.622
3	1	(Constant)	13.820	.705		19.597	.000	12.194	15.446
		Motivation Score - Centered	3.862	.585	.919	6.600	.000	2.513	5.212
4	1	(Constant)	11.419	.846		13.496	.000	9.244	13.594
		Motivation Score - Centered	2.500	.960	.759	2.604	.048	.032	4.968
5	1	(Constant)	15.756	.730		21.594	.000	13.730	17.781
		Motivation Score - Centered	1.667	1.453	.498	1.147	.315	-2.367	5.701
6	1	(Constant)	15.373	.559		27.504	.000	14.156	16.591
		Motivation Score - Centered	1.375	.523	.605	2.630	.022	.236	2.514
7	1	(Constant)	13.845	.668		20.730	.000	12.390	15.300
		Motivation Score - Centered	1.800	.911	.496	1.977	.072	-.184	3.784
8	1	(Constant)	12.816	.766		16.741	.000	11.085	14.548
		Motivation Score - Centered	1.500	.649	.610	2.310	.046	.031	2.969
9	1	(Constant)	12.132	.799		15.177	.000	10.351	13.913
		Motivation Score - Centered	2.115	.912	.591	2.319	.043	.083	4.148
10	1	(Constant)	13.035	.517		25.233	.000	11.844	14.227
		Motivation Score - Centered	1.798	.469	.805	3.837	.005	.717	2.879
11	1	(Constant)	17.488	.968		18.060	.000	14.800	20.177
		Motivation Score - Centered	2.400	1.249	.693	1.922	.127	-1.068	5.868
12	1	(Constant)	14.133	.861		16.412	.000	11.920	16.347
		Motivation Score - Centered	2.500	.742	.833	3.371	.020	.594	4.406
13	1	(Constant)	15.692	1.010		15.535	.000	13.221	18.164
		Motivation Score - Centered	3.917	1.165	.808	3.363	.015	1.067	6.767
14	1	(Constant)	15.624	.591		26.428	.000	14.347	16.901
		Motivation Score - Centered	.866	.658	.343	1.316	.211	-.556	2.287
15	1	(Constant)	13.853	.873		15.875	.000	11.879	15.827
		Motivation Score - Centered	3.287	.749	.825	4.387	.002	1.592	4.983
16	1	(Constant)	11.941	1.014		11.779	.000	9.127	14.756
		Motivation Score - Centered	3.400	1.308	.793	2.600	.060	-.231	7.031
17	1	(Constant)	18.680	.733		25.485	.000	17.022	20.338
		Motivation Score - Centered	3.296	.906	.771	3.637	.005	1.246	5.347
18	1	(Constant)	14.269	1.286		11.099	.000	10.964	17.574
		Motivation Score - Centered	3.167	1.239	.753	2.555	.051	-.019	6.353
19	1	(Constant)	14.890	.685		21.729	.000	13.382	16.399
		Motivation Score - Centered	4.000	.645	.882	6.201	.000	2.580	5.420
20	1	(Constant)	20.893	1.585		13.180	.000	16.819	24.968
		Motivation Score - Centered	3.625	1.192	.806	3.040	.029	.560	6.690
21	1	(Constant)	14.146	2.656		5.326	.013	5.694	22.598
		Motivation Score - Centered	3.885	1.731	.792	2.244	.111	-1.623	9.393
22	1	(Constant)	14.354	.583		24.611	.000	12.975	15.734
		Motivation Score - Centered	2.357	.701	.786	3.361	.012	.699	4.016
23	1	(Constant)	15.013	.623		24.081	.000	13.538	16.487
		Motivation Score - Centered	2.016	.491	.841	4.106	.005	.855	3.178
24	1	(Constant)	15.667	.780		20.083	.000	13.822	17.511
		Motivation Score - Centered	4.167	1.114	.816	3.739	.007	1.531	6.802
25	1	(Constant)	15.333	.878		17.468	.000	13.308	17.357
		Motivation Score - Centered	4.357	.956	.850	4.557	.002	2.152	6.562
26	1	(Constant)	15.770	.577		27.334	.000	14.406	17.134
		Motivation Score - Centered	2.250	.710	.768	3.169	.016	.571	3.929
27	1	(Constant)	15.300	.933		16.392	.000	13.220	17.380
		Motivation Score - Centered	2.500	1.271	.528	1.966	.078	-.333	5.333
28	1	(Constant)	15.765	1.186		13.295	.000	12.473	19.058
		Motivation Score - Centered	-1.400	2.522	-.267	-.555	.608	-8.402	5.602
29	1	(Constant)	16.661	1.054		15.808	.000	13.735	19.588
		Motivation Score - Centered	4.400	.770	.944	5.712	.005	2.261	6.539
30	1	(Constant)	13.656	.978		13.964	.001	10.544	16.768
		Motivation Score - Centered	9.167	1.912	.940	4.793	.017	3.080	15.253
31	1	(Constant)	15.185	1.026		14.808	.000	12.900	17.470
		Motivation Score - Centered	4.556	.705	.898	6.464	.000	2.986	6.127
32	1	(Constant)	14.811	.738		20.066	.000	13.203	16.420
		Motivation Score - Centered	3.548	.520	.892	6.826	.000	2.416	4.681
33	1	(Constant)	17.795	1.195		14.896	.000	14.971	20.620
		Motivation Score - Centered	3.441	1.264	.717	2.722	.030	.452	6.431
34	1	(Constant)	16.136	.586		27.537	.000	14.810	17.462
		Motivation Score - Centered	3.269	.510	.906	6.415	.000	2.116	4.422
35	1	(Constant)	17.026	.791		21.527	.000	15.237	18.815
		Motivation Score - Centered	3.328	.804	.810	4.137	.003	1.508	5.147
36	1	(Constant)	13.473	.778		17.312	.000	11.777	15.168
		Motivation Score - Centered	1.968	.957	.510	2.055	.062	-.118	4.054
37	1	(Constant)	15.887	.727		21.842	.000	14.303	17.472
		Motivation Score - Centered	4.685	.526	.932	8.910	.000	3.540	5.831
38	1	(Constant)	17.453	.666		26.203	.000	15.604	19.303
		Motivation Score - Centered	1.000	1.369	.343	.730	.506	-2.802	4.802
39	1	(Constant)	13.874	1.473		9.416	.000	10.269	17.480
		Motivation Score - Centered	4.979	1.236	.854	4.027	.007	1.954	8.004
40	1	(Constant)	15.198	.648		23.449	.000	13.772	16.625
		Motivation Score - Centered	3.525	1.264	.644	2.789	.018	.743	6.307

a. Dependent Variable: Weight Loss in Pounds

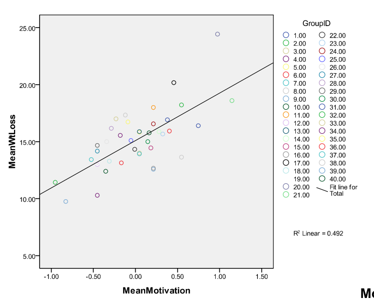
Statistics

		Group Intercept	Group Slope
N	Valid	40	40
	Missing	0	0
Mean		15.1574	2.9920
Std. Deviation		1.84908	1.70027





Regression of Group Averages (avg motivation & avg weight loss)



WARNING: "ECOLOGICAL FALLACY" – cannot make generalizations from the group down to the individual level.

This approach only tells us the relationship between the group mean motivation and the group's average weight loss – NOT about individuals performance.

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics R Square Change	F Change	df1	df2	Sig. F Change
1	.701[a]	.492	.479	1.92098	.492	36.791	1	38	.000

a. Predictors: (Constant), MeanMotivation

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound
1	(Constant)	15.095	.304		49.669	.000	14.480	15.711
	MeanMotivation	4.161	.686	.701	6.066	.000	2.772	5.550

a. Dependent Variable: MeanWtLoss

Regression – control for groups 39 (g-1) dummy variables

Model Summary[c]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.545[a]	.297	.218	4.003	.297	3.746	39	346	.000
2	.839[b]	.704	.669	2.602	.407	473.697	1	345	.000

a. Predictors: (Constant), gr39, gr30, gr21, gr38, gr29, gr28, gr16, gr11, gr5, gr20, gr18, gr12, gr4, gr13, gr33, gr26, gr24, gr23, gr2, gr22, gr25, gr10, gr3, gr1, gr35, gr34, gr17, gr15, gr8, gr31, gr27, gr9, gr19, gr6, gr7, gr32, gr36, gr37, gr14

b. Predictors: (Constant), gr39, gr30, gr21, gr38, gr29, gr28, gr16, gr11, gr5, gr20, gr18, gr12, gr4, gr13, gr33, gr26, gr24, gr23, gr2, gr22, gr25, gr10, gr3, gr1, gr35, gr34, gr17, gr15, gr8, gr31, gr27, gr9, gr19, gr6, gr7, gr32, gr36, gr37, gr14, Motivation Score - Centered

Coefficients[a]

Model		Unstandardized Coefficients: B	Std. Error	Standardized Coefficients: Beta	t	Sig.	95.0% Confidence Interval for B: Lower Bound	Upper Bound
1	(Constant)	15.264	.722		21.138	.000	13.844	16.684
	gr1	-1.192	1.098	-.042	-1.086	.278	-3.352	.967
	gr2	1.254	1.130	.042	1.110	.268	-.969	3.476
	gr3	-1.409	1.095	-.050	-1.287	.199	-3.563	.744
	gr4	-3.564	1.223	-.105	-2.914	.004	-5.970	-1.159
	gr35	1.743	1.067	.064	1.634	.103	-.355	3.841
	gr36	-1.598	1.003	-.066	-1.593	.112	-3.572	.375
	gr37	-.199	1.007	-.008	-.197	.844	-2.180	1.782
	gr38	2.444	1.285	.067	1.902	.058	-.084	4.971
	gr39	-2.931	1.178	-.092	-2.488	.013	-5.247	-.614
	Motivation Score - Centered	3.119	.143	.704	21.765	.000	2.837	3.400

a. Dependent Variable: Weight Loss in Pounds

ANCOVA – same as (g-1) dummy vars

Tests of Between-Subjects Effects

Dependent Variable:Weight Loss in Pounds

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
Corrected Model	5548.748[a]	40	138.719	20.485	.000	.704	819.398	1.000
Intercept	79914.173	1	79914.173	11801.135	.000	.972	11801.135	1.000
motivatc	3207.753	1	3207.753	473.697	.000	.579	473.697	1.000
group	1253.070	39	32.130	4.745	.000	.349	185.044	1.000
Error	2336.249	345	6.772					
Total	94765.000	386						
Corrected Total	7884.997	385						

a. R Squared = .704 (Adjusted R Squared = .669)

b. Computed using alpha = .05

Parameter Estimates

Dependent Variable:Weight Loss in Pounds

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval: Lower Bound	Upper Bound	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	15.264	.722	21.138	.000	13.844	16.684	.564	21.138	1.000
motivatc	3.119	.143	21.765	.000	2.837	3.400	.579	21.765	1.000
[group=1]	-1.192	1.098	-1.086	.278	-3.352	.967	.003	1.086	.191
[group=2]	1.254	1.130	1.110	.268	-.969	3.476	.004	1.110	.198
[group=3]	-1.409	1.095	-1.287	.199	-3.563	.744	.005	1.287	.250

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How to estimate the "degree" of clustering? Intraclass Correlation (ICC)

- The ICC measures the proportion of the total variance of an outcome variable that is accounted for by the clustering of the cases.
- OLS regression (and GLM) assumes that there is no amount of variance accounted for by these "clusters"/groups; ICC=0.
- ICC varies from 0 to 1 (normally). [NOTE: ICC can be negative, but it is rare – this would mean that individuals within a given "cluster" were more different from one another within the group than with individuals outside the group.]
- The ICC may be initially estimated from a one-way ("Fixed Effects") ANOVA on the groups or "clusters."
- Even for ICC's as small as 0.01, for n=25 subjects per group, the actual alpha=0.11 (which is more than 2 times the nominal alpha=0.05). "Alpha-inflation" increases as ICC and number of subjects per group increases.

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Intraclass Correlation

ANOVA

One-Way Fixed Effects

Weight Loss in Pounds

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2340.996	39	60.026	3.746	.000
Within Groups	5544.002	346	16.023		
Total	7884.997	385			

$$ICC_{ANOVA} = \frac{\hat{\tau}}{\hat{\tau} + \hat{\sigma}^2} = \frac{MS_{treatment} - MS_{error}}{MS_{treatment} + (n-1)MS_{error}}$$

$$\text{when groups sizes unequal use } \tilde{n} = M_n - \left[\frac{sd^2(n_j)}{(gM_n)}\right] = 9.63$$

$$\frac{60.0225 - 16.0231}{[60.0225 + ((9.63-1)16.0231)]} = 0.22$$

Where M_n mean number of cases per group; $sd^2(n_j)$ is the variance of the number of cases per group; g=number of groups

Estimated ICC=0.22 indicates substantial clustering **
[for ICC=0.20, n=10, actual alpha=0.28 not 0.05 (Kreft, deLeeuw 1998)]

"Fixed vs. Random Coefficients"

OLS regression – only the outcomes and errors/residuals are considered to be random (i.e. samples from a larger population)

$$\underline{y_i} = B_0 + B_1 x_i + \underline{e_i}$$

Random

Fixed

$$\underline{y_{ij}} = \underline{B_{0j}} + \underline{B_{1j}} x_{ij} + \underline{r_{ij}}$$

"Random Coefficient" regression – now the intercepts and slopes are also considered to be RANDOM (i.e. samples from a larger population)

"Multi-Level / Hierarchical Linear Models"

i=individual j=group

Level 1: Individual Level; "micro-level" $\underline{y_{ij}} = \underline{B_{0j}} + \underline{B_{1j}} x_{ij} + \underline{r_{ij}}$

Level 2: Group Level; "macro-level" $\underline{B_{0j}} = \gamma_{00} + \underline{\mu_{0j}}$ & $\underline{B_{1j}} = \gamma_{10} + \underline{\mu_{1j}}$

Combine Level 1 and Level 2 $\underline{y_{ij}} = \gamma_{00} + \gamma_{10} x_{ij} + \underbrace{\left(\underline{\mu_{0j}} + \underline{\mu_{1j}} x_{ij} + \underline{r_{ij}}\right)}$

"Additional New Parameters" – Variance Components – help to capture the impact of the group structure on the relationship of the predictors on the dependent variable.

$$Variance(\text{level 1 residual}) = Var(r_{ij}) = \sigma^2$$

$$Variance(\text{intercepts}) = Var(\mu_{0j}) = \tau_{00}$$

$$Variance(\text{slopes}) = Var(\mu_{1j} x_{ij}) = \tau_{11}$$

$$Covariance(\text{between slopes \& intercepts}) = \tau_{01}$$

Unconditional Cell Means Model

Calculate ICC from the RC model using only the group membership (no level 1 predictor) – equivalent to a "One-way Random Effects ANOVA"

$$y_{ij} = B_{0j} + r_{ij} \quad = \quad y_{ij} = \gamma_{00} + \left(\mu_{0j} + r_{ij}\right)$$

```
MIXED pounds BY group
  /CRITERIA=CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)
  /FIXED=| SSTYPE(3)
  /METHOD=REML
  /PRINT=CPS CORB COVB DESCRIPTIVES G  LMATRIX R SOLUTION TESTCOV
  /RANDOM=INTERCEPT | SUBJECT(group) COVTYPE(UN)
  /EMMEANS=TABLES(OVERALL).
```

Estimates of Covariance Parameters[a]

	Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
							Lower Bound	Upper Bound
$\hat{\sigma}^2$	Residual		16.069431	1.225085	13.117	.000	13.839088	18.659222
$\hat{\tau}_{00}$	Intercept [subject = group]	Variance	4.906446	1.560373	3.144	.002	2.630653	9.151041

a. Dependent Variable: pounds.

$$\mathrm{ICC}_{\mathrm{RC}} = \frac{\hat{\tau}_{00}}{\hat{\tau}_{00} + \hat{\sigma}^2} = \frac{4.906}{(4.906 + 16.069)} = 0.234$$

[for ICC=0.20, n=10, actual alpha=0.28 not 0.05 (Kreft, deLeeuw 1998)]

Unconditional Cell Means Model

$$y_{ij} = B_{0j} + r_{ij} \quad = \quad y_{ij} = \gamma_{00} + \left(\mu_{0j} + r_{ij}\right)$$

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		16.069431	1.225085	13.117	.000	13.839088	18.659222
Intercept [subject = group]	Variance	4.906446	1.560373	3.144	.002	2.630653	9.151041

a. Dependent Variable: pounds.

$\hat{\sigma}^2$ = 16.069 is significant (pval<0.001) – 16.069 units of within class variance in pounds lost that might be significantly accounted for by motivation predictors.

$\hat{\tau}_{00}$ = 4.906 is also significant (pval=0.002) – 4.906 units of between class variance in pounds lost (reflects differences between groups in mean pounds lost); and since this is significantly > 0, indicates that there is random variation among the intercepts of the various groups – WE SHOULD NOT IGNORE CLUSTERING.

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Random Coefficient Model

$$\underline{y_{ij}} = \gamma_{00} + \gamma_{10}x_{ij} + \left(\underline{\mu_{0j}} + \underline{\mu_{1j}}x_{ij} + \underline{r_{ij}}\right)$$

Mixed Level 1 & 2

```
MIXED pounds BY group WITH motivatc
  /CRITERIA=CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)
  /FIXED=motivatc | SSTYPE(3)
  /METHOD=REML
  /PRINT=CPS CORB COVB DESCRIPTIVES G  LMATRIX R SOLUTION TESTCOV
  /RANDOM=INTERCEPT motivatc | SUBJECT(group) COVTYPE(UN)
  /EMMEANS=TABLES(OVERALL).
```

$$\underbrace{\underline{y_{ij}} = \underline{B_{0j}} + \underline{B_{1j}}x_{ij} + \underline{r_{ij}}}_{\text{Level 1}} \qquad \underbrace{\underline{B_{0j}} = \gamma_{00} + \underline{\mu_{0j}} \qquad \underline{B_{1j}} = \gamma_{10} + \underline{\mu_{1j}}}_{\text{Level 2}}$$

Model Dimension[b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	motivatc	1		1	
Random Effects	Intercept + motivatc[a]	2	Unstructured	3	group
Residual				1	
Total		4		6	

$\hat{\gamma}_{00}$ $\hat{\gamma}_{10}$ $\hat{\tau}_{00}$ $\hat{\tau}_{01}$ $\hat{\tau}_{11}$ $\hat{\sigma}^2$

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax may yield results that differ from those produced by prior versions. If you are using version 11 syntax, please consult the current syntax reference guide for more information.

b. Dependent Variable: pounds.

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Consider the Fixed Effects Portion of the Model

$$\underline{y_{ij}} = \underbrace{\gamma_{00} + \gamma_{10}x_{ij}}_{\text{Fixed}} + \underbrace{\left(\underline{\mu_{0j}} + \underline{\mu_{1j}}x_{ij} + \underline{r_{ij}}\right)}_{\text{Random}}$$

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept ($\hat{\gamma}_{00}$)	15.137701	.279667	35.497	54.128	.000	14.570231	15.705171
motivatc ($\hat{\gamma}_{10}$)	3.118162	.210667	37.075	14.801	.000	2.691339	3.544985

a. Dependent Variable: pounds.

Weight Loss (Y_{ij}) = 15.137701 lbs $\hat{\gamma}_{00}$
+ 3.118 units_motivation/lbs *(motivation_ctr (x_{ij})) $\hat{\gamma}_{10}$

Both the intercept and slope are significant (both pval<0.001).

So, for every unit increase in motivation, 3.118 more lbs are lost; with an average weight loss per group of 15.14 lbs.

Multilevel and Random Coefficients Models

Consider the Random Effect Portion of the Model

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual ($\hat{\sigma}^2$)		5.933224	.475980	12.465	.000	5.069965	6.943470
Intercept + motivatc [subject = group]	UN (1,1) ($\hat{\tau}_{00}$)	2.397058	.741332	3.233	.001	1.307464	4.394681
	UN (2,1) ($\hat{\tau}_{01}$)	.585041	.384745	1.521	.128	-.169046	1.339127
	UN (2,2) ($\hat{\tau}_{11}$)	.932841	.376054	2.481	.013	.423316	2.055655

a. Dependent Variable: pounds.

$\hat{\tau}_{11}$ variance of the slopes across groups is significant (again do not ignore clustering)

Cannot test this via ANCOVA nor (g-1) dummy variables in regression.

$\hat{\tau}_{01}$ the covariance between slopes and intercepts was positive indicating the higher the average pounds lost in a group (intercept), the stronger the positive relationship of motivation on weight loss; but this was not significant.

$\hat{\sigma}^2$ the addition of motivation as a predictor reduced the within group variance in pounds lost by 63% (from 16.069 to 5.933)

$\hat{\tau}_{00}$ the addition of motivation as a predictor also reduced the between group differences in avg. pounds lost (intercepts) by 51% (4.906 to 2.397)

SO, overall there were fluctuations between the groups in terms of mean pounds lost – these fluctuations were systematically related to the mean motivation level of individuals within these groups.

MLM with Predictor at Level 2 (group variable) "treatment" = diet program vs control

$$\underline{y_{ij}} = \underbrace{\gamma_{00} + \gamma_{10}x_{ij} + \gamma_{01}W_j + \gamma_{11}W_j x_{ij}}_{\text{Fixed}} + \underbrace{\left(\underline{\mu_{0j}} + \underline{\mu_{1j}}x_{ij} + \underline{r_{ij}}\right)}_{\text{Random}}$$

```
MIXED pounds BY group WITH motivatc treatc
  /CRITERIA=CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)
  /FIXED=motivatc treatc motivatc*treatc | SSTYPE(3)
  /METHOD=REML
  /PRINT=CPS CORB COVB DESCRIPTIVES G  LMATRIX R SOLUTION TESTCOV
  /RANDOM=INTERCEPT motivatc | SUBJECT(group) COVTYPE(UN)
  /EMMEANS=TABLES(OVERALL).
```

Estimates of Fixed Effects[a]

	Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
$\hat{\gamma}_{00}$	Intercept	15.165612	.259287	33.414	58.490	.000	14.638337	15.692886
$\hat{\gamma}_{10}$	motivatc	3.130055	.184638	32.155	16.952	.000	2.754031	3.506078
$\hat{\gamma}_{01}$	treatc	1.527592	.528671	33.305	2.889	.007	.452377	2.602808
$\hat{\gamma}_{11}$	motivatc * treatc	1.245004	.376767	32.758	3.304	.002	.478251	2.011757

Estimates of Covariance Parameters[a]

	Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
$\hat{\sigma}^2$	Residual		5.933076	.475382	12.481	.000	5.070820	6.941953
$\hat{\tau}_{00}$	Intercept + motivatc [subject = group]	UN (1,1)	1.967013	.656943	2.994	.003	1.022170	3.785225
$\hat{\tau}_{01}$		UN (2,1)	.144647	.314042	.461	.645	-.470865	.760159
$\hat{\tau}_{11}$		UN (2,2)	.556332	.301327	1.846	.065	.192440	1.608318

a. Dependent Variable: pounds.

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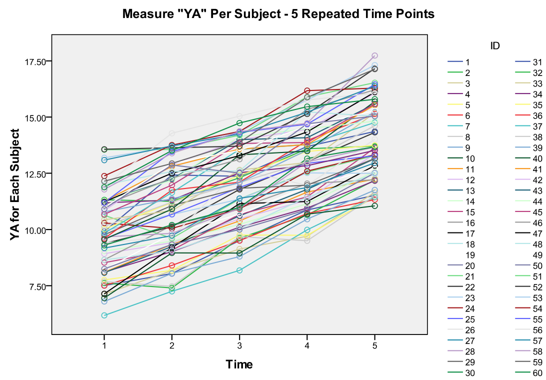
Repeated Measures

Investigate the following hypothetical data set. YA is the outcome measures over 5 time points for 60 individuals



Multilevel and Random Coefficients Models

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YA-Model 1
(random intercept – cell means model)

$$\underline{y_{ij}} = \underline{B_{0j}} + \underline{r_{ij}} \quad = \quad \underline{y_{ij}} = \gamma_{00} + \left(\underline{\mu_{0j}} + \underline{r_{ij}}\right)$$

```
MIXED YA BY ID
  /CRITERIA=CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)
  /FIXED=| SSTYPE(3)
  /METHOD=REML
  /PRINT=CPS CORB COVB DESCRIPTIVES G  LMATRIX R SOLUTION TESTCOV
  /RANDOM=INTERCEPT | SUBJECT(ID) COVTYPE(UN)
  /EMMEANS=TABLES(OVERALL).
```

Estimates of Covariance Parameters[a]

	Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
$\hat{\sigma}^2$	Residual		3.023271	.275986	10.954	.000	2.527978	3.615603
$\hat{\tau}_{00}$	Intercept [subject = ID]	Variance	2.515843	.577176	4.359	.000	1.604738	3.944238

a. Dependent Variable: YA.

$$\mathrm{ICC}_{RC} = \frac{\hat{\tau}_{00}}{\hat{\tau}_{00} + \hat{\sigma}^2} = \frac{2.516}{(2.516 + 3.023)} = 0.454$$

DO NOT IGNORE CLUSTERING

YA-Model 3
(random intercept and slope & fixed time)

$$\underline{y_{ij}} = \gamma_{00} + \gamma_{10} x_{ij} + \left(\underline{\mu_{0j}} + \underline{\mu_{1j}} x_{ij} + \underline{r_{ij}}\right)$$

i=time; j=individual

```
MIXED YA BY ID WITH TIME
  /CRITERIA=CIN(95) MXITER(100) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE) PCONVERGE(0.000001, ABSOLUTE)
  /FIXED=TIME | SSTYPE(3)
  /METHOD=REML
  /PRINT=CPS CORB COVB DESCRIPTIVES G  LMATRIX R SOLUTION TESTCOV
  /RANDOM=INTERCEPT TIME | SUBJECT(ID) COVTYPE(UN)
  /EMMEANS=TABLES(OVERALL).
```

Estimates of Fixed Effects[a]

	Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
$\hat{\gamma}_{00}$	Intercept	8.790029	.242639	59	36.227	.000	8.304510	9.275547
$\hat{\gamma}_{10}$	TIME	1.053117	.023202	59.000	45.389	.000	1.006689	1.099544

Estimates of Covariance Parameters[a]

	Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
$\hat{\sigma}^2$	Residual		.228306	.024066	9.487	.000	.185692	.280700
$\hat{\tau}_{00}$	Intercept + TIME [subject = ID]	UN (1,1)	3.281270	.650907	5.041	.000	2.224275	4.840556
$\hat{\tau}_{01}$		UN (2,1)	-.048610	.047100	-1.032	.302	-.140924	.043704
$\hat{\tau}_{11}$		UN (2,2)	.009470	.006415	1.476	.140	.002510	.035727

a. Dependent Variable: YA.

Notes on the MLM approach to RM-ANOVA for YA – Model 3

- **Model 3 added "time" as a fixed predictor to the model to determine whether there was a significant "linear" effect of time.**
- **There is a significant effect of adding time to the model ($\hat{\sigma}^2$ residual error dropped from 3.02 down to 0.228), which is still significant.**
- **Adding time to the model also accounted for all the significant variance in the slopes of YA on time, e.g. differences between slopes were no longer significant $\hat{\tau}_{11}$ (pval = 0.140). And the covariance (between slopes and intercepts) is also not significant.**
- **Individual mean differences (intercepts) $\hat{\tau}_{00}$ (pval<0.001) remain a significant source of variance.**

Summary

- If you suspect group (or cluster) effects, i.e. subjects within your groups may exhibit some level of "cohesion" or "alikeness," leading you to think they are more alike within groups than between groups, the ICC should be calculated and evaluated.
- If ICC is close to 0, run both OLS regression and a Random Coefficient ("mixed") model and compare. Evaluate the "random variance components."
- Try comparing RM-ANOVA with the "mixed" multi-level model approach where "time" is level 1 and the individual is level 2. Evaluate the impact of adding time to the model.

References

- Cohen, Jacob; Cohen, Patricia; West, Stephen; Aiken, Leona. "Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences" 3rd edition, Lawrence Erlbaum Associates Inc., 2003.
  - [All data/examples in this lecture came from this book –
    - mostly Chapter 14 (Randon Coefficient Regression and Multilevel Models) and
    - Section 15.4 (Multilevel Regression of Individual Changes Over Time)]
- Raudenbush; Stephen W.; Bryk, Anthony S. "Hierarchical Linear Models: Applications and Data Analysis Methods" 2nd edition, SAGE publications, 2002.

*

- Kreft, Ita; de Leeuw, Jan. "Introducing Multilevel Modeling," SAGE publications, 1998.
- Tabachnick, Barbara G.; Fidell, Linda S. "Using Multivariate Statistics," 5th edition, Pearson Education Inc., 2007. [Chapter 15 focuses on Multilevel Linear Modeling.]

VIII. Statistical Resources and Contact Info

SON S:\Shared\Statistics_MKHiggins\website2\index.htm

[updates in process]

Working to include tip sheets (for SPSS, SAS, and other software), lectures (PPTs and handouts), datasets, other resources and references

Statistics At Nursing Website: [website being updated] http://www.nursing.emory.edu/pulse/statistics/

And Blackboard Site (in development) for "Organization: Statistics at School of Nursing"

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