Answers to Hubert Escaith

Guillaume Daudin[*]

December 7, 2018

1 Why not based on VA trade ?

Why don't we simply :
1. Compute the origin of the VA content of each good
2. Study how the price evolve following a shock on the price of VA in a country or another ?

Intuition:
That would not do because the price of, e.g. French VA does not change for everybody.

Doubt: is that enough an argument ? 1 sector, 2 countries

1.1 Evolution of VA price

$$A = \begin{pmatrix} a_{1,1} & a_{2,1} \\ a_{1,2} & a_{2,2} \end{pmatrix}$$

$$I - A = \begin{pmatrix} 1 - a_{1,1} & -a_{2,1} \\ -a_{1,2} & 1 - a_{2,2} \end{pmatrix}$$

$$(I - A)^{-1} = \frac{1}{(1 - a_{1,1})(1 - a_{2,2}) - a_{1,2}a_{2,1}} \begin{pmatrix} 1 - a_{2,2} & a_{2,1} \\ a_{1,2} & 1 - a_{1,1} \end{pmatrix} = z. \begin{pmatrix} 1 - a_{2,2} & a_{2,1} \\ a_{1,2} & 1 - a_{1,1} \end{pmatrix}$$

$$= \begin{pmatrix} u & v \\ w & x \end{pmatrix}$$

$$\text{French demand shares} = d = \begin{pmatrix} 1 - f \\ f \end{pmatrix}$$

$$(I - A)^{-1} d = \begin{pmatrix} u - uf + vf \\ w - wf + xf \end{pmatrix}$$

Donc, en cas de choc c pour le prix de la va dans le pays étranger (en monnaie française), on peut écrire un vecteur de choc : $C = (0, c)$. les prix varient tout d'abord de CA, puis CA^2, etc. Donc le vecteur de choc S (en monnaie française) est :

$$S = C + CA + CA^2... = C(I - A)^{-1} = \begin{pmatrix} cw & cx \end{pmatrix}$$

[*]PSL, Université Paris-Dauphine,Sciences Po, OFCE. E-mail: `guillaume.daudin@dauphine.psl.eu`

To measure the effect on French consumption prices, we do a weighted sum of these effects.

$$\bar{s} = c.\left[(1-f)\,w + xf\right] = c.\frac{(1-f)\,a_{1,2} + f\,(1-a_{1,1})}{(1-a_{1,1})\,(1-a_{2,2}) - a_{1,2}a_{2,1}} \tag{1}$$

If each nation's production only uses national inputs, we have a plausible :

$$\bar{s} = c.\frac{f}{1-a_{2,2}}$$

1.2 Exchange rate shock

Using the notations in the paper...

$$\hat{C}_{\$} = (-c, 0)$$
$$\mathcal{B} = \begin{pmatrix} 0 & a_{2,1} \\ 0 & 0 \end{pmatrix}$$
$$\tilde{\mathcal{B}} = \begin{pmatrix} 0 & 0 \\ a_{1,2} & 0 \end{pmatrix}$$

Hence

$$\begin{aligned} S &= (0, c) + \left[(0, -c.a_{2,1}) + (c.a_{1,2}, 0)\right] * \begin{pmatrix} u & v \\ w & x \end{pmatrix} \\ &= (0, c) + (c.a_{1,2}, -c.a_{2,1}) * \begin{pmatrix} u & v \\ w & x \end{pmatrix} \\ &= (0, c) + (u.c.a_{1,2} - w.c.a_{2,1}, v.c.a_{1,2} - x.c.a_{2,1}) \\ &= (u.c.a_{1,2} - w.c.a_{2,1}, c + v.c.a_{1,2} - x.c.a_{2,1}) \end{aligned}$$

and

$$\bar{s} = (u.c.a_{1,2} - w.c.a_{2,1}, c + v.c.a_{1,2} - x.c.a_{2,1}) \,.\, \begin{pmatrix} 1-f \\ f \end{pmatrix}$$
$$\bar{s} = c\left[f\,(1 + v.a_{1,2} - x.a_{2,1}) + (1-f)\,(u.a_{1,2} - w.a_{2,1})\right]$$

If each nation's production only uses national inputs, we have a plausible

$$\bar{s} = c.f$$

This seems to confirm that the exchange rate shock is not the same as the VA price shock.

1.3 Residual issue

Starting from 4.2 in the paper

$$E1 = C(i.e.C^i) = (0,c)$$
$$E2 = C.\tilde{\mathcal{B}} = (0,c)\,.\begin{pmatrix} 0 & 0 \\ a_{1,2} & 0 \end{pmatrix} = (c.a_{1,2}, 0)$$

$$E1.HC = (0,c)\,.\begin{pmatrix} 1-f \\ f \end{pmatrix} = f.c$$
$$E2.HC = (c.a_{1,2}, 0)\,.\begin{pmatrix} 1-f \\ f \end{pmatrix} = c.a_{1,2}.\,(1-f)$$
$$\bar{s} - E1.HC - E2.HC =$$
$$c\left[f\left(1 + v.a_{1,2} - x.a_{2,1}\right) + (1-f)\left(u.a_{1,2} - w.a_{2,1}\right)\right] - c\left(f + a_{1,2}.\,(1-f)\right)$$
$$= c\left[a_{1,2}\left((1-f)(1+u) + vf\right) + a_{2,1}\left((1-f)\,w - x\right)\right]$$

Easy : we can normalize the shock c to 1.

$$residual = a_{1,2}\left((1-f)(1+u) + vf\right) + a_{2,1}\left((1-f)\,w - x\right)$$

How can we continue to show that this thing does not depend on the openness/size of the economy?
Idea : Hypothesis that (but it does not help)

$$a_{1,2} = a_{2,1}$$

and (does not help)

$$a_{1,1} = a_{2,2}$$

? More interesting

$$\frac{a_{1,1}}{a_{1,2}} = \frac{a_{2,2}}{a_{2,1}} = \frac{1-f}{f}$$

and

$$a_{1,1} + a_{1,2} = a$$

So...

$$a_{1,1} = (1-f)a$$
$$a_{1,2} = fa$$

Then

$$\begin{aligned}
residual &= fa\left(\left(1-f\right)\left(1+u\right)+vf\right)+a_{2,1}\left(\left(1-f\right)w-x\right)\\
&= z.\left[fa\left(\left(1-f\right)\left(1+\left(1-a_{2,2}\right)\right)+a_{2,1}f\right)+a_{2,1}\left(\left(1-f\right)af-(1-f)a\right)\right]\\
&= z\left[fa\left(\left(1-f\right)\left(2-a_{2,2}\right)\right)+a_{2,1}f\right)-a_{2,1}a\left(1-f\right)^2\right]\\
&= \frac{fa\left(\left(1-f\right)\left(2-a_{2,2}\right)\right)+a_{2,1}f\right)-a_{2,1}a\left(1-f\right)^2}{\left(1-a_{1,1}\right)\left(1-a_{2,2}\right)-a_{1,2}a_{2,1}}\\
&= \frac{fa\left(\left(1-f\right)\left(2-a_{2,2}\right)\right)+a_{2,1}f\right)-a_{2,1}a\left(1-f\right)^2}{\left(1-a(1-f)\right)\left(1-a_{2,2}\right)-afa_{2,1}}
\end{aligned}$$

Pour dériver, je développe:

$$residual = \frac{f^2a\left(-2+a_{2,2}\right)+fa\left(2-a_{2,2}+2a_{2,1}\right)-aa_{2,1}}{fa\left(1-a_{2,2}-a_{2,1}\right)+1-a+a_{2,2}(1+a)}$$

Le signe de cette dérivée est celui de :

$$\begin{gathered}
\left(2fa\left(-2+a_{2,2}\right)+a\left(2-a_{2,2}+2a_{2,1}\right)\right)\left(fa\left(1-a_{2,2}-a_{2,1}\right)+1-a+a_{2,2}(1+a)\right)\\
-a\left(1-a_{2,2}-a_{2,1}\right)\left(f^2a\left(-2+a_{2,2}\right)+fa\left(2-a_{2,2}+2a_{2,1}\right)-aa_{2,1}\right)
\end{gathered}$$

Et alors là... Aucune idée du signe de cette expression...

$$\begin{gathered}
\left(fa\left(1-a_{2,2}-a_{2,1}\right)+1-a+a_{2,2}(1+a)\right)>0\\
a\left(1-a_{2,2}-a_{2,1}\right)>0\\
\left(2fa\left(-2+a_{2,2}\right)+a\left(2-a_{2,2}+2a_{2,1}\right)\right)=a\left[\left(1-a_{2,2}\right)\left(1-2f\right)\right]+2a2,1>0
\end{gathered}$$