Global value chains and the transmission of exchange rate shocks to consumer prices *

Preliminary version

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Abstract

Following the 2008 financial crisis, inflation rates in advanced economies have been at odds with the prediction of a standard Phillips curve. This puzzle has triggered a debate on the global determinants of domestic prices. We contribute to this debate by investigating the impact of exchange rate shocks on consumer prices from 1995 to 2018. We focus on cost-push inflation through global value chains. We build on three sectoral world input-output datasets (WIOD and two versions of TiVA). We assume Cobb-Douglas production framework and work in a partial equilibrium setting. The construction of World Input-Output tables is data-demanding and WIOTs are typically released with a lag of several years. To address this gap, we use more up-to-date GDP and trade data, thus providing a tool for approximating the partial equilibrium impact of an exchange rate shock on consumer prices from 2015 onwards. Depending on countries, the absolute value of the elasticity of the household consumption expenditure (HCE) deflator to the exchange rate ranges from 5% to 35%, confirming the importance of global value chains in channeling external shocks to domestic inflation. Using data from WIOD on a sample of 43 countries, we find that the mean output-weighted elasticity of the HCE deflator to the exchange rate increased in absolute value from 7.5% in 2000 to 9.4% in 2008. After peaking in 2008, it declined to 8.8% in 2014. Extrapolations based on more up-to-date GDP and trade data suggest that the decline continued until 2016, before reversing in 2017 and 2018. Our findings are robust to using three different datasets.

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1 Introduction

Understanding inflation dynamics is key for monetary policy and policy makers. Since the Global Financial Crisis of 2008, inflation dynamics have triggered an intense debate in advanced economies. While inflation in the pre-crisis period followed a standard Phillips curve relationship, two consecutive puzzles have emerged. The "missing disinflation episode" emerged during the economic dowturn (2009-2011), when inflation remained much higher than suggested by the high level of economic slack. The "missing inflation puzzle" emerged from 2012 onwards, as inflation remained subdued despite the economic recovery (see Ball and Mazumder (2011); Coibion and Gorodnichenko (2015)). From a central bank perspective, the underlying question is whether the Phillips curve is a reliable tool to explain inflation in a globalised world. This triggered a very intense debate named "Phillips curve war" between supporters (among them Blanchard (2016), the European Central Bank (see Eser et al. (2020)) and Bank of France (see Berson et al. (2018)) and critics (led by Farmer and Nicolo (2018). All agree that external factors play a role in driving domestic inflation, either through more integration or more coordination. This paper contributes to this debate by quantifying the role of global value chains in inflation dynamics following an exchange rate shock in a Cobb-Douglas, partial equilibrium framework. Participation in global value chains strengthens cross-country linkages via trade in intermediate inputs. When value chains are global, fluctuations in exchange rates (we assume here an appreciation of a currency) affect household consumption deflator (HCE hereafter) through distinct channels: *i)* the prices of imported final goods sold directly to domestic consumers; *ii)* the prices of imported inputs feeding through domestic production; and *iii)* the price of exported inputs feeding through imported foreign production. Finally, *iv)*, changes in domestic and foreign production costs in turn pass through to the price of inputs for domestic and foreign goods and cause further production costs variations through input-output linkages.

Assuming a Cobb-Douglas production framework where firms have a simple cost-minimising behaviour, we compute the partial-equilibrium effects of an exchange rate shock on consumer prices. This exercise neglects all the much-studied determinants of the pass-through of exchange rate shocks into prices (see for example Berman et al. (2012)). For example, the pass-through depends on the intensity of competition in domestic markets: while an exchange rate appreciation lowers the price of imported inputs, a firm with limited competitive pressure may avail of greater profit margins rather than reduce prices in an effort to maintain its market share. The elasticity we compute thus overestimates the sensitivity of prices to exchange rate shocks. We also assume that all pricing occur using the currency of the producing country, despite the well-documented role of dominant-currency pricing (Gopinath et al. (2020)). Despite these shortcomings, this partial equilibrium approach is useful for identifying which countries and sectors are under pressure to adjust their prices when subject to exchange rate shocks.

We measure the partial equilibrium effect of exchange rate fluctuations on consumer prices, because the stabilisation of the consumer prices is a widespread policy objective for monetary authorities. We use world input-output tables covering twenty years of data (from 1995 to 2015). Most of the literature on global supply chains has focused on the role of input-output linkages in propagating shocks to production. By contrast, we examine the role of input-output linkages in propagating shocks to consumer prices. We analyse the impact of exchange rate shocks on

the main components of consumer prices (manufacturing goods, services, food and energy) and on the prices of imported and domestic final goods.

The construction of World Input-Output tables (WIOT hereafter) is data-demanding and WIOTs are typically released with a lag of several years. As a result, at the time of writing, the latest WIOT is five year old (2015). To address this gap, we use more up-to-date GDP and trade data to approximate the partial equilibrium impact of an exchange rate shock on the HCE deflator from 2016 onwards.

The absolute value of the elasticity of the HCE deflator to exchange rate shocks varies widely between countries, from 5% to 35%. In the euro area, the elasticity of the HCE delfator to changes in the value of the euro ranges from 6.5% to 18%. Such heterogeneity add to the challenges faced by the European Central Bank in stabilising prices in the monetary union. In a sample of 43 countries, the mean output-weighted elasticity of the HCE deflator to exchange rate shocks based on WIOD increased in absolute value from 7.5% in 2000 to 9.4% in 2008. It then declined to 8.8% in 2014. Extrapolations using more up-to-date GDP and trade data suggest that this decline continued to 2016 and was reversed in 2017 and 2018. Both recent versions of TIVA yield a similar evolution. Input-output mechanisms (i.e. excluding the change in the prices of imported final goods sold directly to domestic consumers) explain a large share of the elasticity, especially for large countries or for countries of the Eurozone subject to a shock on the Euro.

The paper is organised as follows. Section 2 reviews the related literature. Section 3 presents the methodology and the data sources. Section 4 computes the partial equilibrium impact of an exchange rate shock on HPE deflator up to 2015. Section 5 relies on up-to-date GDP and trade data to approximate the partial equilibrium impact of an exchange rate shock on HPE deflator from 2016 onwards.

2 Related literature

2.1 Trade in intermediate goods and the exchange rate pass-through

Our paper relates to several strands of literature. First, it relates to the literature that examines the link between global value chain (GVC) participation and the exchange rate pass-through (ERPT) to domestic prices. De Soyres et al. (2018) found evidence that an increase in production linkages, as proxied by trade in intermediate inputs, is strongly associated with higher inflation correlation. Georgiadis et al. (2019) estimate that the rise of GVC participation accounts for 50% of the decline in the ERPT to import prices observed since the mid-1990s.[1] Hagemejer et al. (2020) provide evidence that the decline in ERPT resulting from the enhanced participation in GVC may be nonlinear with respect to the country's position in the global value chain. They find that a growing backward GVC participation of the suppliers of imported intermediate inputs reduces the ERPT to producer prices. The ERPT for countries whose suppliers are strongly involved in GVC is significantly smaller than for economies whose suppliers do not participate in GVC.

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[1]The intuition is as follow: an exchange rate appreciation will increase the foreign-currency price of exports. However, when exports have high import contents, the appreciation will reduce the local-currency price of imported inputs, thus decreasing the foreign-currency price of exports. This will lead to a lower ERPT.

Our research also relates to the literature on the cross-border propagation of cost shocks. Auer et al. (2019) document that input-output linkages contribute substantially to synchronising producer price (PPI) inflation across countries. Antoun de Almeida (2016) shows that the cross-border sectors pairs which trade more intensively with each other in intermediate inputs display higher PPI inflation correlation, indicating price spillovers along the global supply chain.

Another strand of literature has focused on the link between the ERPT and the share of intermediate goods in imports. The empirical literature shows that the pass-through declines across the pricing chain. Ortega and Osbat (2020) find that the ERPT into import prices is high and fast, whereas the ERPT into final consumer prices is significantly smaller and relatively slower (see also Hahn (2003); Kunovac and Comunale (2017); Ben Cheikh and Rault (2017)).

From an input-output perspective, the ERPT to consumer prices depends on the direct and indirect import contents of consumption. Using WIOD and assuming a full exchange rate pass-through to import prices, Ortega and Osbat (2020) (box 1 prepared by Stefan Schaefer) find that the sum of the direct and indirect import contents in consumption was 16% in the HICP in 2014 in the euro area.

Another strand of literature investigates whether the ERPT is stable over time. Campa and Goldberg (2008) have documented a declining ERPT to import and consumer prices since the 1980s and 1990s. Özyurt (2016) also shows that the pass-through is partial in the euro area. The decline in the ERPT observed over the past two decades coincided with the increasing share of emerging economies in world trade and the accession of China to the WTO. By contrast, Leigh et al. (2017) find that both the ERPT and the price elasticity of trade volumes are stable over time. In the euro area, Ortega and Osbat (2020) show that the ERPT into total import prices has been broadly stable over time (at around 20%), while the ERPT to extra-euro area import prices has declined.

Our contribution to the literature is fourfold. While most of the literature focuses on producer prices, we focus on consumer prices, a variable of paramount interest for monetary policy. We compute the full partial equilibrium impact of exchange rate shocks through world input-output tables. By comparing the results obtained using three different databases, we make sure our results are robust to using different datasets. The construction of World Input-Output tables is data-demanding and WIOTs are typically released with a lag of several years. To address this gap, we use more up-to-date GDP and trade data, thus providing a tool for computing up-to-date estimates of the elasticity of consumer prices to exchange rate shocks.

2.2 The Input-Output model applied to a shock on production costs

The Leontief's production model (or Input-Output model, I-O thereafter) studies the impact of a demand shock in a closed economy (Leontief, 1951). The trade in value-added analysis reconciles international trade statistics with national I-O tables, and thus allows to extend Leontief's analysis to an international context. A number of studies (Hummels et al., 2001; Daudin et al., 2006, 2011; De Backer and Yamano, 2012; Johnson and Noguera, 2012; Koopman et al., 2014; Amador et al., 2015; Los et al., 2016) analyse the value added content of world trade. Some authors focus on Asia (Sato and Shrestha, 2014) or on the euro area (Cappariello and Felettigh, 2015).

Leontief's production model has a dual: the cost-push price model. In France, the Insee (Bourgeois and Briand, 2019) has developed the AVIONIC model based on French symmetric input-output tables. It measures the impact of an exogenous variation in the price of inputs on the price of production. Leontief's price model is broadly used in multi-sector, single-country macroeconomic models, for example, to measure the effect of changes in energy prices (Bournay and Piriou, 2015; Sharify, 2013). Implicitly, Bems and Johnson (2015) use it to focus on competitiveness. They compute real effective exchange rates weighted by the value-added trade structure to measure the impact of a demand change in value added on value added prices and final expenditure levels. Cochard et al. (2016) rely on an accounting approach to analyse the effects of costs on prices ("cost-push inflation") and shows that they are all the higher as countries are open.

The limitations of this approach are well known (Folloni and Miglierina, 1994). In particular, and although the division of global value chains largely takes place within multinational firms, it assumes a unique pricing system based on market prices and independent of firms' strategies. Still, this method provides a measure of the vulnerability of each sector to price or productivity shocks (Acemoglu et al., 2012; Carvalho, 2014). Hence, though unrealistic, this partial equilibrium approach using input-output tables is useful for identifying which countries and sectors are under pressure to adjust their prices when subject to exogenous cost shocks. For instance, it can show which euro area countries benefit most from an appreciation of the euro or whether adopting the euro has increased interdependence between member states.

3   The PIWIM model

Based on initial work from the OFCE (Observatoire Français des Conjonctures Économiques) Cochard et al. (2016), we have developed a model named «PIWIM» (Push-cost Inflation through World Input-output Matrices).

3.1   Defining a price shock in an I-O model

To identify which countries are most affected by a price shock through value-added and vertical international trade flows, we need a large structural matrix that integrates input flows between sectors, both within each country and between countries. This matrix traces the sectoral and geographical origin of inputs.

The standard I-O model relies on input-output tables registering transactions of goods and services (domestic or imported) at current prices. The I-O tables describe the sale and purchase relationships between producers and consumers within an economy. Each column describes, for each industry j, the intermediate consumption of goods and services from the various sectors. By extension, a "world" I-O table (WIOT) describes the sale and purchase relationships between producers and consumers in the whole world, differentiating between sectors in different countries. The WIOT has, on its diagonal, the country blocks with flows of domestic transactions of intermediate goods and services between industries. The "bilateral" blocks outside of the diagonal represent international flows of intermediate goods and services via bilateral sectoral exports and imports.

Traditionnaly, WIOTs are interpreted in a Leontieff framework of Leontieff production functions. That facilitates the study of the evolution of quantities in the economy. Here, we assume a Cobb-Douglas production function: the technical coefficients correspond to the share of each input in total costs. Hence, we can derive a price equation, following (De Soyres et al., 2018)). We define N as the product of the number of countries (I) and the number of sectors (J), $\mathcal{A}$ the matrix of technical input coefficient of dimension (N, N), and Y the gross output vector of dimension $(1, N)$.

In each sector of each (country,sector) there is a representative firm producing with domestic production factors (V) and domestic and imported intermediary inputs m according to a Cobb-Douglas technology:

$$Y_n = V_n^{\gamma_n} \times \prod_{n'=1}^{n'=N} m_{n,n'}{}^{a_{n,n'}} \text{ with } \gamma_n + \sum_{n'=1}^{n'=N} a_{n,n'} = 1$$

Where γ_n is the domestic production factors share, $a_{n,n'}$ the share of output from (country,sector) n' in the total production of (country,sector) n.

Assuming perfectly competitive firms and price at marginal cost, standard cost minimisation for each country leads to the following pricing system:

$$p_n = x_n \times w_n^{\gamma_k} \times \prod_{n'=1}^{n'=N} p_{n'}^{a_{n,n'}}, \forall n$$

With x the unit income of domestic production factors and x_n a constant depending only on parameters:

$$x_n = \gamma_k^{-\gamma_k} \times \prod_{n'=1}^{n'=N} a_{n,n'}^{-a_{n,n'}}$$

Taking logs, we have:

$$log(p_n) = log(x_n) + \gamma_n log(w_n) + \sum_{n'=1}^{n'=N} a_{n,n'}.log(p_{n'}), \forall n$$

Define P the vector of prices , Z a vector of $log(x_n) + \gamma_n.log(w_n)$, all of dimension $(1, N)$:

$$log(P) = Z + log(P)\mathcal{A} \tag{1}$$

Suppose an exogenous input price shock defined in log points (approximating percentages for small shocks). Define $\Delta^0 log(P)$ the shock vector of dimension $(1, N)$ computed as the difference between the original price vector $log(P^0)$ and the new vector $log(P^1)$. Then:

$$\Delta^0 log(P) = log(P^1) - log(P^0) = C,$$

with C the shock vector of dimension (1,N) that contains the direct effect of the shock on prices expressed in log points.

C directly affects the HCE deflator. In addition, firms face a change in their costs, which affects their prices according to equation 1. Hence, the price change is transmitted to country-specific industries that use shocked products as intermediate consumptions. The higher their reliance on shocked inputs, the higher the variation in their prices.

In a first step, the impact of the shock on each country-specific industry's output prices amounts to $\Delta^1 log(P) = C\mathcal{A}$.

In a second step, the shock is passed on to all country-specific industries that rely on those shocked inputs in their production processes. For the kth step, the increase in production prices amounts to $\Delta^k log(P) = C\mathcal{A}^k$.

As the technical coefficients are smaller than 1, the effect of the shock on input prices wears out as k increases. The overall effect of the shock is equal to the sum of the initial shock and all the changes in each step. Let us call S the total effect of the shock on prices (in log points), a vector $(1, N)$ composed of the elements s_{ij} measuring the total effect of the shock on the price of sector j in country i. We have:

$$S = C\left(I + \mathcal{A} + \mathcal{A}^2 + ... + \mathcal{A}^k + ...\right) = C(I - \mathcal{A})^{-1} \qquad (2)$$

with $(I - \mathcal{A})^{-1}$ the well-known of Leontief inverse matrix. If the shock is small enough, the elements of S can be understood as the elasticity of sectoral price to an input price shock. Using consumption shares to do a weighted average of these elasticities will generate the Household consumption expenditure (HCE) deflator elasticity to an input price shock [2].

3.2   Data

WIOTs are an extension of the national input-output tables. Input-output tables measure the relationships between the producers of goods and services (including imports) within an economy and the users of these goods and services (including exports). The national tables specify, in line, for each industry, the use of the product as intermediate or final use. In a national table, final use includes exports alongside domestic final uses, whereas exports are not a final use in world input-output tables. WIOTs show which foreign industry produces a good for a specific final use, and which foreign industry or final user uses the exports of a given country. For example, WIOTs show how much international trade is embedded in the consumption of a particular final product.

Aggregating national input-output tables into world input-output tables is challenging for many reasons. National input-output tables vary widely in terms of detail and scope, and are therefore not fully consistent. Furthermore, the availability of year-specific national input-output tables is limited, especially for developing economies.

This paper uses two multi-year WIOTs: the World Input Output Database (WIOD) and the Trade in Value Added Database (TiVA) from the OECD-ICIO.

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[2]This HCE deflator has different sectoral weights than the headline Consumer price inflation (CPI) as it also covers, for exemple, the rental equivalent of real estate expenditures

The World Input Output Database (WIOD) WIOD is hosted and updated by the University of Groningen (Netherlands) and benefits from the financial support of the European Commisssion. It contains time series of inter-country input-output tables from 2000 to 2014. It provides WIOTs that reconcile national input-output tables (or supply-use tables) with bilateral trade statistics. WIOD covers 43 countries (of which 28 members of the European Union) accounting for 85% of global GDP (see Table 1). It contains annual information for 56 industries. Therefore, for each year a full country-sector input-output matrix traces the importance of a supplying industry in one country for an industry in another country. The values in WIOTs are expressed in U.S. dollars at basic prices; market exchange rates were used for currency conversion (Timmer et al., 2015).

Europe	Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom
North America	Canada, United States
Latin America	Brazil, Mexico
Asia-Pacific	Australia, China, India, Indonesia, Japan, Korea, Taiwan
Other	Russia, Turkey

Table 1: **Economies included in the World Input-Output Database**

The Trade in Value Added (TiVA) database The TiVA database is compiled by the OECD and the WTO. It builds on the OECD harmonised individual country input-output tables to provide matrices of inter-industrial flows of goods and services in current prices (U.S. dollars). We use two versions of TiVA (see OECD (Decembre 2018) on the differences between the two datasets). The third revision, released in 2016, includes 64 economies (i.e. 35 OECD Countries, 28 non-OECD economies and the Rest of the world) and 34 industries, and covers the period 1995-2011. The fourth revision, released in 2018, includes 65 economies (see Table 2) and 36 sectors and covers the 2005-2015 period.

Europe	Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom
North America	Canada, United States
Latin America	Argentina, Brazil, Chile, Colombia, Costa Rica, Mexico (differentiating between three (Rev. 3) or two (Rev. 4) Mexico), Peru
Asia-Pacific	Australia, Cambodia, China (differentiating between four (Rev. 3) or three (Rev. 4) China), Hong Kong SAR, India, Indonesia, Japan, Khazakhstan (in Rev. 4), Korea, Malaysia, New Zealand, Philippines, Singapore, Taiwan, Thailand, Viet Nam
Other	Brunei, Israel, Morocco, Russia, Saudi Arabia, South Africa, Tunisia, Turkey

Table 2: **Economies included in TiVA revisions 3 (2016) and 4 (2018)**

Comparison of the WIOD and TiVA databases The WIOD and TiVA databases are constructed using similar assumptions (OECD and WTO (2011); Timmer et al. (2015); OECD

(Decembre 2018)). For example, both databases use the "import proportionality assumption" to allocate specific bilateral imports to using industries. National input-output statistics provide fow much of each product firms use for intermediary consumption and investment. However, the breakdown of these into domestic and imported product is not available. Hence, the construction of world input-output tables requires allocating imports to using industries. The import proportionality assumption assumes that the proportion of intermediates that an industry purchases from abroad is equal to the ratio of imports of intermediates to total domestic demand of intermediates in that product. This assumption can be misleading. Feenstra and Jensen (2012) find that shares of imported materials may differ substantially across U.S. industries. Based on Asian input-output tables, Puzzello (2012) finds that the proportionality assumption understates the use of foreign intermediate inputs. WIOD and TiVA databases, however, rely on the BEC (Broad Economic Categories) classification to map detailed six-digit products into intermediate use, final consumption and investment (Dietzenbacher et al., 2013).

One explicit difference between the databases is that WIOD assumes that the input mix is independant of the destination market. TiVA databases do not rely on this assumption for two countries (Mexico and China). They use different input-output tables for the exporting, import processeng and domestic-market oriented sectors of these countries (or at least two of those depending on the country and the version of the TiVA database). Yet (probably because our study does not focus on Mexico and China) the results we obtain from these different databases are very similar.

3.3 Nominal exchange rate shock

Implementing an exchange rate shock is more complex than implementing a production cost shock. The appreciation of the currency of country A leads to a fall in the national currency price of country A's imports, while the foreign-currency price of its exports increase. We are mainly interested is the impact of this shock on the consumer prices in country A. Yet, we also estimate the inflationary impact on countries that directly and indirectly, through third countries linkages, consume inputs from country A.

Suppose a world with two countries A and B, each having its own national currency, and a third currency for international transactions, the dollar. Following a 5% appreciation of the currency of country A against the two other currencies, the prices of country A expressed in dollars increase by 5% compared to those of country B expressed in dollars. Country B pays more for its imports of inputs, in dollars as well as in national currency, since its exchange rate against the dollar has not changed. Conversely, the prices of imported inputs in country A remain constant in dollars, since the prices of country B have not changed, and fall by half once expressed in country A's national currency. This has an impact on consumer prices (in national currency) in country A.

We assume that producers completely pass the exchange rate shock on their production prices. The change in the prices of imported goods is therefore transmitted to all domestic prices, both directly and through inter-industry linkages. These upward (downward) movements for country B (country A) affect all input prices in both countries.

The effects of the shock spread over multiple simultaneous production cycles. The overall impact of the shock in dollar terms is equal, for the shocked country A, to the rise in prices due to the exchange rate shock, minus direct and indirect decreases (via inter-industry linkages in the country), in national currency and then converted back into dollar terms, in the prices of inputs imported from B and disseminated to all branches. The overall impact on prices in dollar terms in country A is therefore lower than the initial exchange rate shock, as national currency prices are also affected. For country B, the final impact is equal to the cumulative direct and indirect effects of the higher prices of imported inputs.

In a global economy composed of I countries, each with J sectors, the appreciation of a country's currency i against all other currencies translates into a rise in country i's prices in dollars. The price of each sector will vary in percentage (approximated as log point assuming the shock is small enough) by: $c_\$^i$ for sectors in the shock-stricken country i and 0 in other countries. Hence, for each sector j in country i:

$$\Delta^0 log(p_{\$ij}) = log(p_{\$ij}^1) - log(p_{\$ij}^0) = c_{\$ij} = c_\$^i$$

And for each sector j in country $k (k \neq i)$,

$$\Delta^0 log(p_{\$kj}) = log(p_{\$kj}^1) - log(p_{\$kj}^0) = c_{\$kj} = 0$$

The appreciation affects producers through changes in relative prices between countries and, therefore, through changes in input prices traded between the shock-stricken country i and other countries.

Consider first the direct impact on other countries of the rise in imported input prices from shocked country i. For any sector l of a country k $(k \neq i)$, the increase in the producer price depends directly on the quantity of inputs imported from the shock-stricken country i, weighed by the variation in level of the price of inputs in dollars (i. e. the exchange rate shock). If $a_{kl,ij}$ is the share of inputs from the country i's sector j needed in the production of country's k sector l, we have :

$$\Delta^1 log(p_{\$kl}) = c_\$^i . a_{kl,i1} + \ldots + c_\$^i . a_{kl,ij} + \ldots + c_\$^i . a_{kl,iJ} = \sum_{j=1}^{J} c_\$^i . a_{kl,ij} = c_\$^i . \sum_{j=1}^{J} a_{kl,ij} \qquad (3)$$

For the shocked country, an appreciation of the currency has a disinflationary effect. In national currency, the prices of imported inputs fall in each sector by $c^i = -\frac{c_\$^i}{1+c_\$^i}$, or by $-9.09...\%$ with $c_\$^i = 10\%$.

We approximate these shocks by their log point equivalent[3]. This decline then spreads to all domestic-input using sectors. In sector j of the shocked country i, this fall amounts in national currency to:

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[3]We could have used the log point approximation earlier and write that $c^i = -c_\$^i$ with nearly the same results for small shocks.

$$\Delta^1 log(p_{ij}) = \sum_{l=1}^{l=J} c^i.a_{ij,1l} + \ldots + \sum_{l=1}^{l=J} c^i.a_{ij,kl} + \sum_{l=1}^{l=J} c^i.a_{ij,Il} = \left(-\frac{c_\$^i}{1+c_\$^i}\right).\sum_{\substack{k=1 \\ k \neq i}}^{k=I} \left[\sum_{l=1}^{l=J} a_{ij,kl}\right]$$

This shock can be converted into dollars:

$$\Delta^1 log(p_{\$ij}) = \left(1+c_\$^i\right).\left(-\frac{c_\$^i}{1+c_\$^i}\right) \sum_{\substack{k=1 \\ k \neq i}}^{k=I} \left[\sum_{l=1}^{l=J} a_{ij,kl}\right] \tag{4}$$

This yields the first step impact of the shock on all input prices of all countries.

To express this in matrix notation, we define two matrices that build on the world input-output matrix $\mathcal{A}$ defined in 3.1. These two matrices retain only the first-step effects of the exchange rate shock on the price of goods and services imported by the shocked country i and the first-step effects of the exchange rate shock on the price of goods and services imported by the rest of the world from country i. Compared to $\mathcal{A}$, we "close off" the links between a domestic input price shock and the price of goods as well as on the link between non-shocked input prices and the price of goods in a non-shocked country.

Let us first look at the shock from the perspective of countries that import inputs from country i.

Let $C_\$^i$ be the vector of log points changes in dollar prices following the appreciation of the currency of country i against all other currencies. Hence,

$$C_\$^i = \begin{pmatrix} 0 \ldots 0 \ldots c_{\$ij} \ldots c_{\$ik} \ldots 0 \ldots 0 \end{pmatrix}$$

with $c_{\$ij} = c_{\$ik} = c_\$^i$ for all sectors j and k in the shocked country i.

Building on Equation 3, the direct impact of the exchange rate shock on the other countries corresponds to the product of the shock vector $C_\$^i$ and a matrix $\mathcal{B}^i$. $\mathcal{B}^i$ builds on the large matrix $\mathcal{A}$ of technical coefficients, but only keeps the coefficients of each country's sectoral inputs imported from the shocked country i. The other coefficients are replaced by 0, including those of the block of country i concerning the domestic inputs of country i. The first-step impact of the appreciation of a currency against the dollar on the price of inputs in countries that are not shocked is equal to $C_\$^i \mathcal{B}^i$ with

$$C_\$^i \mathcal{B}^i = \begin{pmatrix} 0 \ldots c_\$^i \ldots 0 \end{pmatrix} \begin{pmatrix} 0 & \cdots & 0 \\ a_{11,ij} & 0 & a_{IJ,ij} \\ 0 & \cdots & 0 \end{pmatrix} \tag{5}$$

where each $a_{kl,ij}$ element of the line block represents the technical coefficient related to imports of inputs by sector l in country k (with $k \neq i$) from sector j of country i.

Let us now consider the shock from the perspective of the shocked country i.

Define C^i the vector of change in prices everywhere expressed in country i's currency.

$$C^i = \left(-\frac{c_\$^i}{1+c_\$^i}, \ldots 0 \ldots, -\frac{c_\$^i}{1+c_\$^i} \right)$$

From Equation 4, we can write the first-step impact for country i of the fall in input prices from the rest of the world. The first-step impact corresponds to the product of the shock vector C^i and a matrix $\tilde{\mathcal{B}}^i$. $\tilde{\mathcal{B}}^i$ builds on the large matrix $\mathcal{A}$ of which only the country blocks of the inputs imported by country i from other countries have been retained. The other coefficients are replaced by 0, including those of the block of country i concerning the domestic inputs of country i.

Notation table

- $a_{kl,ij}$: the share of inputs from the country i's sector j in the production of country's k sector l.

- $\mathcal{A}$: Matrix of world input-output coefficients.

- $\mathcal{B}^i$: $\mathcal{A}$ including only the coefficients of each country's sectoral inputs imported from country i (excluding domestic inputs in i).

- $\tilde{\mathcal{B}}^i$: $\mathcal{A}$ including only the country blocks of the inputs imported by country i from other countries (excluding domestic inputs in i).

- $c_\$^i$: exchange rate shock on the currency of country i in dollar. If it appreciates by 10%, $c_\$^i = 0.1$.

- c^i: exchange rate shock on the price of non-i goods expressed in the currency of i. $c^i = -\frac{c_\$^i}{1+c_\$^i}$. If $c_\$^i = 0.1$, $c^i = -0.0909...$

- C^i: vector of log point change in prices everywhere expressed in country i's currency: $C^i = \left(-\frac{c_\$^i}{1+c_\$^i}, \ldots 0 \ldots, -\frac{c_\$^i}{1+c_\$^i} \right) = (c_i, \ldots 0 \ldots, c_i)$

- $C_\$^i$: vector of log points changes in dollar prices following the appreciation of the currency of country i against all other currencies. $C_\$^i = \left(0 \ldots 0 \ldots c_{\$ij} \ldots c_{\$ik} \ldots 0 \ldots 0 \right)$ with $c_{\$ij} = c_{\$ik} = c_\$^i$

- $\hat{C}_\$^i$: vector of the log point change in dollar prices of goods and services from country i used as inputs in all other countries. $\hat{C}_\$^i = \left(0 \ldots \frac{c_\$^i}{1+c_\$^i}, \ldots 0 \right)$

- $p_{\$ij}$: price of goods produced by the sector j in country i in dollars (or any international reference currency)

The first step impact of the appreciation of the shocked country i on the price of its inputs corresponds, in national currency, to $C^i \tilde{\mathcal{B}}^i$ with:

$$C^i \tilde{\mathcal{B}}^i = \left(-\frac{c_{\$}^i}{1+c_{\$}^i}, \ldots 0 \ldots, -\frac{c_{\$}^i}{1+c_{\$}^i} \right) \begin{pmatrix} 0 & \ldots a_{ij,11} \ldots & 0 \\ 0 & 0 & 0 \\ 0 & \ldots a_{ij,IJ} \ldots & 0 \end{pmatrix} \tag{6}$$

where each $a_{ij,kl}$ element in the column block represents imports of inputs by sector j in country i from sector l of country k. We then convert this direct impact in dollars, by multiplying it by the new value of the national currency in dollars, $\left(1 + c_{\$}^i\right)$. The direct impact of the appreciation of the shocked country i on the price of its inputs corresponds, in dollars, to $\tilde{C}_{\$}^i \tilde{\mathcal{B}}^i$ with:

$$\tilde{C}_{\$}^i \tilde{\mathcal{B}}^i = \left(1 + c_{\$}^i\right) . C^i \tilde{\mathcal{B}}^i = \left(-c_{\$}^i \ldots 0 \ldots - c_{\$}^i \right) \begin{pmatrix} 0 & \ldots a_{ij,11} \ldots & 0 \\ 0 & 0 & 0 \\ 0 & \ldots a_{ij,IJ} \ldots & 0 \end{pmatrix} \tag{7}$$

The first-step effect on the world is therefore the sum of these vectors from equations 5 and 7, i. e. $C_{\$}^i.\mathcal{B}^i + \tilde{C}_{\$}^i\tilde{\mathcal{B}}^i$.

The input price shock then spreads to all sectors in all countries via the global intersectoral exchanges transcribed by the matrix of technical coefficients of the large matrix $\mathcal{A}$. This process will be simultaneously repeated several times, until the effects are completely exhausted. In the end, the total price effect of the exchange rate shock is equal to the sectoral shock itself, incremented by changes in input prices due to changes in imported input prices (both in the shocked country and in the non-shocked countries), and by all changes in prices during the production processes, i. e.:

$$\begin{aligned} S_{\$}^i = \Delta P_{\$}^i = C_{\$}^i \quad &+ \quad \left(C_{\$}^i.\mathcal{B}^i + \tilde{C}_{\$}^i\tilde{\mathcal{B}}^i \right) + \left(C_{\$}^i.\mathcal{B}^i + \tilde{C}_{\$}^i\tilde{\mathcal{B}}^i \right) \mathcal{A} + \left(C_{\$}^i.\mathcal{B}^i + \tilde{C}_{\$}^i\tilde{\mathcal{B}}^i \right) \mathcal{A}^2 \\ &+ \quad \ldots + \left(C_{\$}^i.\mathcal{B}^i + \tilde{C}_{\$}^i\tilde{\mathcal{B}}^i \right) \mathcal{A}^k + \ldots \end{aligned}$$

$$S_{\$}^i = C_{\$}^i + \left(C_{\$}^i.\mathcal{B}^i + \tilde{C}_{\$}^i\tilde{\mathcal{B}}^i \right) * (I - \mathcal{A})^{-1} \tag{8}$$

With $S_{\$}^i$ the total impact vector composed of the elements $s_{\$kj}^i$ showing the total impact of a shock on country i's exchange rate on the price of the country k's sector j in international currency in log points. Equation 8 gives the evolution of sectoral prices in international currency. Analysing this vector is the main objet of Cochard et al. (2016), which focuses on the evolution of price-competitiveness.

By contrast, this paper focuses on the effect of an exchange rate shock on the consumer prices. Hence, we are interested in the same impact expressed in national currency. To obtain the evolution of the sectoral prices of the shocked country in national currency, we remove the exchange rate shock in international currency, multiply the balance by the scalar of conversion equal to $\frac{1}{1+c_{\$}^i}$ and add the initial exchange rate shock in national currency.

$$S^i = C^i + \left(\frac{1}{1+c_\$^i}\right) * \left(S_\$^i - C_\$^i\right)$$

$$= C^i + \left(\frac{1}{1+c_\$^i}\right) * \left(C_\$^i.\mathcal{B}^i + \tilde{C}_\$^i\tilde{\mathcal{B}}^i\right) * (I - \mathcal{A})^{-1} \qquad (9)$$

$$= C^i + \left(\hat{C}_\$^i.\mathcal{B}^i + C^i\tilde{\mathcal{B}}^i\right) * (I - \mathcal{A})^{-1}$$

Where $\hat{C}_\$^i = \left(0 \ldots \frac{c_\$^i}{1+c_\$^i}, \ldots 0\right)$ is the increase in dollar prices of goods and services from country i used as inputs in all other countries.

S^i represents the overall impact of a exchange rate shock on prices in each sector of each country expressed in the currency of country i. It is expressed in log points. If the shock is small enough, its elements are the elasticity to an exchange rate shock of the prices consumers in country i are paying for each good. We return to this equation and its interpretation in section 5.

To convert this vector into the HCE deflator elasticity in country i, $\bar{s}^i$, we use the household consumption shares to compute a weighed average of the elements of S^i. Let HC^i be the vector of sectoral shares in country i's household consumption[4]:

$$HC^i = \begin{pmatrix} \frac{hc_{11}^i}{hc^i} \\ \ldots \\ \frac{hc_{kj}^i}{hc^i} \\ \ldots \\ \frac{hc_{IJ}^i}{hc^i} \end{pmatrix}$$

Where hc_{kj}^i corresponds to household consumption in country i of goods and services produced by sector j from country k and hc^i represents the total household consumption of country i.

$\bar{s}_i^{i,HC}$ provides the HCE deflator of country i elasticity to an exchange rate shock.

$$\bar{s}_i^{i,HC} = S^i.HC^i = \sum_{\substack{j=1\ldots J \\ k=1\ldots I}} s_{kj}^i.\frac{hc_{kj}^i}{hc^i} \qquad (10)$$

where s_{kj}^i is a coefficient of S^i for country i.

4 The impact of exchange rates fluctuations on consumer prices

We use the model presented in 3.3 to analyse the impact of an exchange rate shock on the household consumption expenditure (HCE) deflator. We assume that the shock is small enough to assimilate log and percentage points and hence consider $\bar{s}_i^{i,HC}$ as the HCE defator elasticity to an exchange rate shock. Following an appreciation of the national currency versus all other currencies, imported inputs and imported consumer goods become cheaper and domestic prices expressed in national currency decrease. Using WIOD, we find that the absolute value of the

---

[4]Consumption is at market prices, whereas the WIOTs are all at basic prices. This is not an issue as long as we assume that all taxes and subsidies on products are proportional, as is the main one, the value-added tax.

elasticity of the HCE deflator to an exchange rate shock is 0.055 (or 5.5%) for the US in 2011.

For euro-area members, we distinguish between the effect of the appreciation of the euro and the effect of the appreciation of a hypothetical national currency. For France, the elasticity of the HCE deflator to an appreciation of the euro is -0.076. The elastiticy of the HCE deflator to an appreciation of an hypothetical French national currency is -0.122.

Figure 1 shows that country elasticities are similar, accross our three databases both in 2011 and 2014.
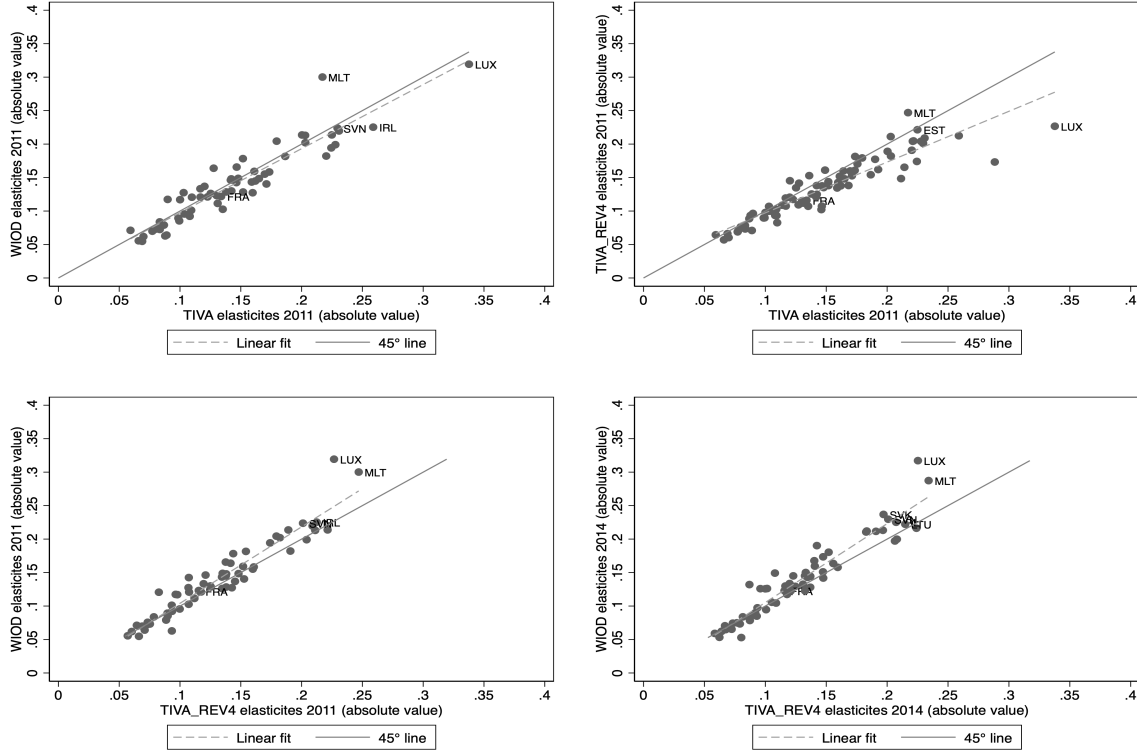
Figure 1: **Comparison of the HCE deflator elasticity to an exchange rate shock for WIOD, TIVA and TIVA rev. 4, 2011 and 2014**

Sources: WIOD, TIVA rev3 and TIVA rev4, authors' calculations

Using a sample of 43 countries, we plot the elasticity of the HCE deflator to the exchange rate over time. The anual evolution is the same regardless of the database (WIOD or TIVA) (see Figure 2). Using data from TIVA rev. 3 yields a slightly higher elasticity. Output-weighed results show a lower elasticity, reflecting the fact that large countries are relatively closed compared to small economies. Based on the third revision of TiVA database, we find that the output-weighed elasticity has increased by 25% between 1995 and 2008, reaching 10% in 2008. The elasticity has slightly declined afterwards.

Using data from WIOD, Figure 3 shows that, in absolute terms, the elasticity lies between 5% and 15%, but can be as high as 35%. In the euro area, the elasticity to changes in the value of the euro ranges from 6.5% to 18% depending on the member state. This heterogeneity adds to the challenges faced by the European Central Bank in stabilizing prices throughout a
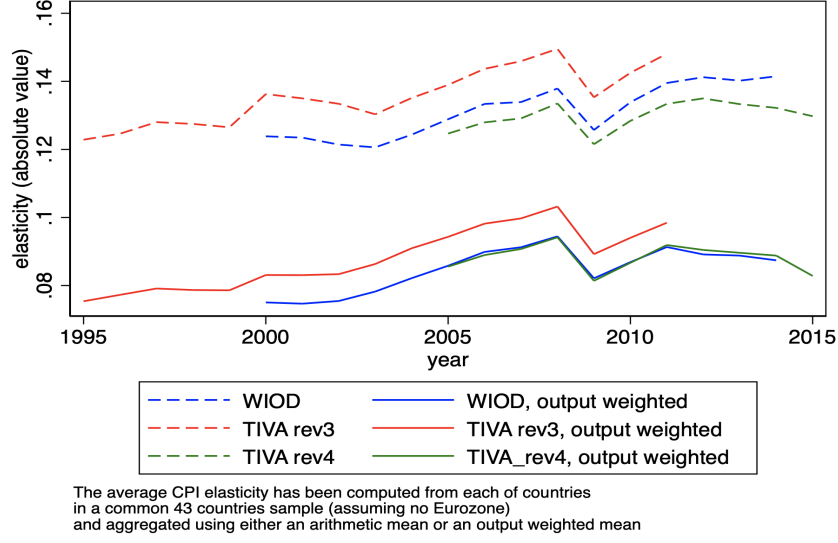
The average CPI elasticity has been computed from each of countries
in a common 43 countries sample (assuming no Eurozone)
and aggregated using either an arithmetic mean or an output weighted mean

Figure 2: **Comparison of the average HCE deflator elasticity to an exchange rate shock in the whole sample for WIOD and TIVA, 1995-2015**

Sources: WIOD, TIVA rev3, TIVA rev4 and authors' calculations

monetary union. Figure 4 shows that the value of the elasticity is closely related to the share of imported goods and services in household consumption.

Figure 5 contrasts the contribution of domestic versus imported goods to the HCE deflator elasticity to an exchange rate shock. We define

$$\bar{s}_i^{i,HC} = \bar{s}_{i,imp}^{i,HC} + \bar{s}_{i,dom}^{i,HC} = S^i . HC^{i,dom} + S^i . HC^{i,imp} \tag{11}$$

Where:

$$
\begin{aligned}
HC^i &= HC^{i,dom} + HC^{i,imp} \\
&= \begin{pmatrix} 0 \\ \dots \\ \frac{hc_{ij}^i}{hc^i} \\ \dots \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{hc_{11}^i}{hc^i} \\ \dots \\ 0 \\ \dots \\ \frac{hc_{I,J}^i}{hc^i} \end{pmatrix}
\end{aligned} \tag{12}
$$

For example,

$$\bar{s}_{i,imp}^{i,HC} = \sum_{\substack{j=1\dots J \\ k=1\dots I \\ k \neq i}} s_{kj}^i . \frac{hc_{kj}^i}{hc^i} \tag{13}$$

Figure 5 shows that changes in the prices of imported final consumer goods contribute more to the total effect than changes in the prices of domestic goods. Although imported final consumer goods account for a smaller share of total consumption than domestic goods, they are the most impacted by the initial exchange rate shock. Imported final consumer goods also explain the differences in price elasticities observed between open and less open economies.
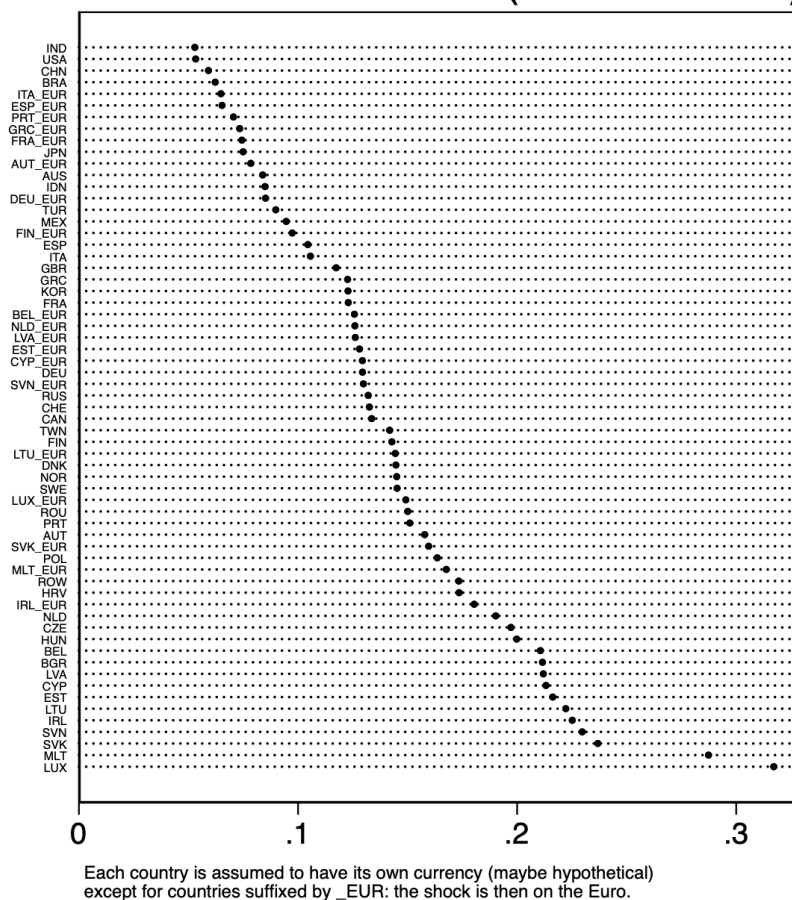
16

Figure 3: **Distribution of the HCE deflator elasticity to a 100% exchange rate appreciation (WIOD) - 2014.**

Sources: WIOD and authors' calculations

Figure 6 analyses the impact of global inflationary shocks on the main components of the HCE deflator (manufacturing goods, services, food and energy). Non-energy industrial goods make the bulk of the total impact. However, services also play a significant role, especially in advanced economies. Although services are mainly produced domestically and do not rely much on imported inputs, they make up a substantial share of total consumption. Similarly, domestic core inflation (all products except food and energy) accounts for a significant share of the total impact (Figure 7), reflecting the weight of domestic services and non-energy industrial goods in total consumption.

The 45° line serves as a comparison to the regression line
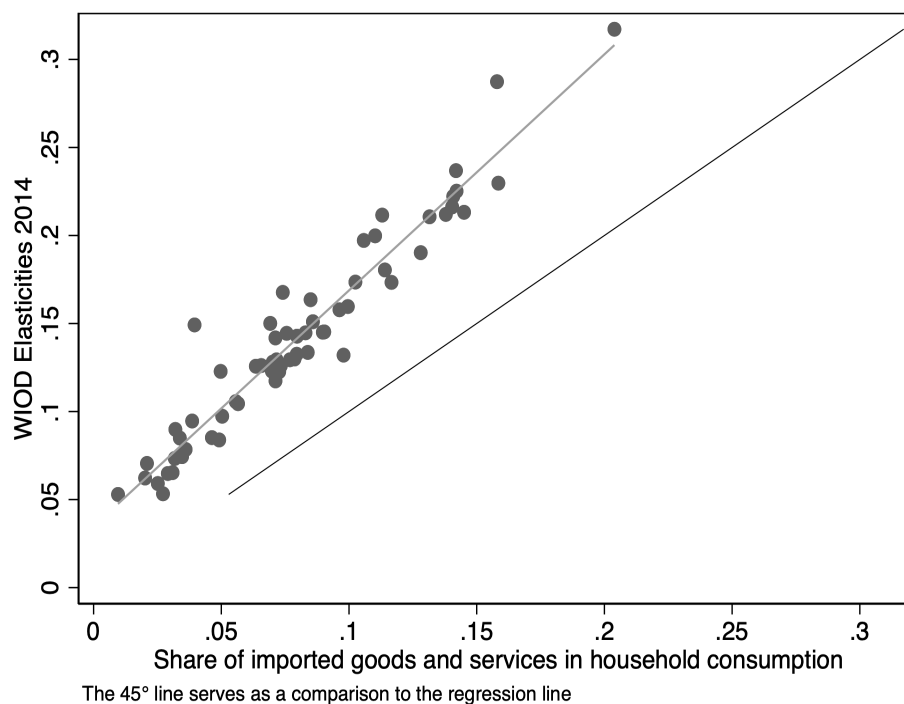
Figure 4: **HCE deflator elasticity to an exchange rate shock and the share of imported consumption in total consumption (WIOD)**

Sources: WIOD and authors' calculations

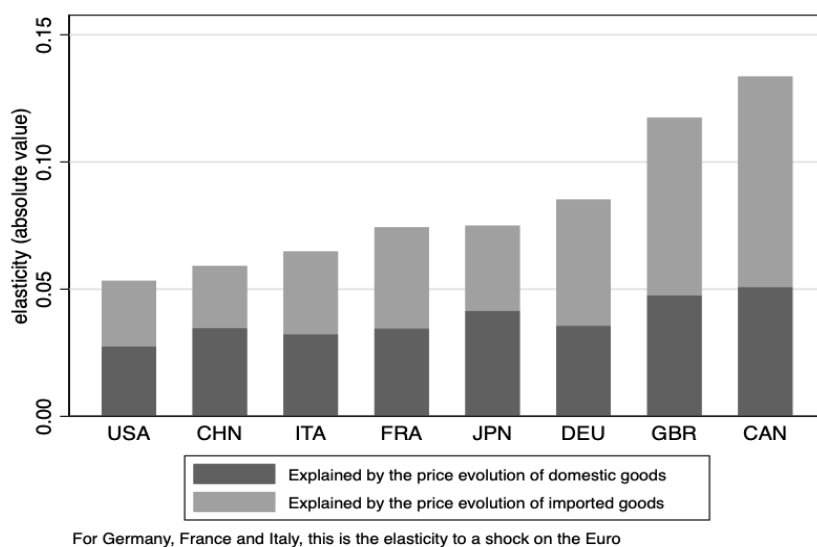For Germany, France and Italy, this is the elasticity to a shock on the Euro

.

Figure 5: **Contribution of imported and domestic final goods and services to the HCE deflator elasticity to an exchange rate shock**
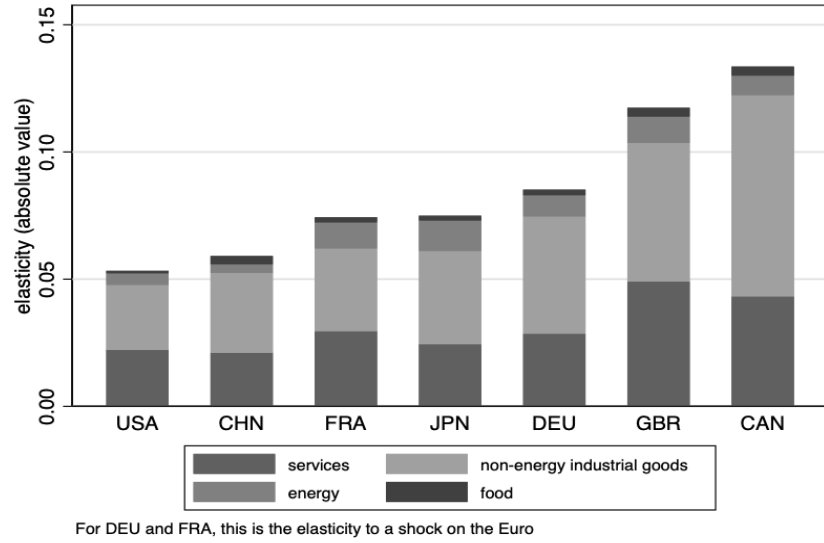
Sources: WIOD and authors' calculations

18

Figure 6: **Contribution of different products to the HCE deflator elasticity to an exchange rate shock**
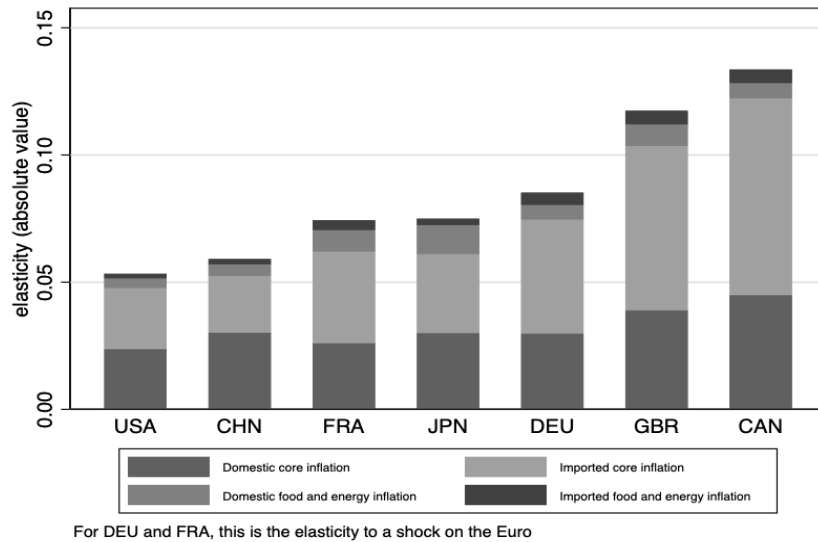
Sources: WIOD and authors' calculations

Figure 7: **Contribution of domestic and imported components to the HCE deflator elasticity**

Sources: WIOD and authors's calculations

5 Can we extrapolate the HCE deflator elasticity?

5.1 Doing without the world input-output matrices

World input-output matrices are not available for the most recent years: the latest years covered by WIOD and TiVA rev4 are, respectively, 2014 and 2015. In addition, using WIOTs involves cumbersome computations. Given these difficulties, we look for a simpler way to compute the

elasticity of the HCE deflator to the exchange rate. We break down $\overline{s}_i{}^{i,HC}$ into different elements classified by ease of use and computation. Let us start from equation 9. We have:

$$
\begin{aligned}
S^i &= && C^i && + \left(\hat{C}^i_\$.\mathcal{B}^i + C^i\tilde{\mathcal{B}}^i \right) * (I - \mathcal{A})^{-1} \\
S^i &= && \underbrace{C^i}_{\substack{\text{(E1) direct effect through} \\ \text{imported consumption goods}}} && + \underbrace{C^i\tilde{\mathcal{B}}^i}_{\substack{\text{(E2) effect on} \\ \textit{domestic}\ \text{consumption goods} \\ \text{through } \textit{imported}\ \text{inputs}}} && + \underbrace{\hat{C}^i_\$.\mathcal{B}^i}_{\substack{\text{(E3) effect on} \\ \textit{imported}\ \text{consumption goods} \\ \text{through } \textit{domestic}\ \text{inputs}}}
\end{aligned}
$$

$$
+ \underbrace{\left(\hat{C}^i_\$.\mathcal{B}^i + C^i\tilde{\mathcal{B}}^i \right) * (I - \mathcal{A})^{-1} * \mathcal{A}}_{\text{(E4) residual}}
$$

(14)

C^i and $\hat{C}^i_\$$ have a large number of zeros. So, we can write, defining $HC^{i,dom}$ and $HC^{i,imp}$ as the domestic and imported shares of HC^i and adjusting the dimension of $E1$, $E2$ and $E3$.

$$
\begin{aligned}
\overline{s}_i^{i,HC} &= S^i.HC^i = E1.HC^i + E2.HC^i + E3.HC^i + E4.HC^i \\
&= E1.HC^{i,imp} + E2.HC^{i,dom} + E3.HC^{i,imp} + E4.HC^i
\end{aligned}
$$

(15)

When the domestic currency appreciates, $E1.HC^{i,imp}$ (for short $E1.HC$), $E2.HC^{i,dom}$ (for short $E2.HC$) reduce the consumer prices of the country i whereas $E3.HC^{i,imp}$ (for short $E3.HC$) increases them.

This decomposition differs from equation 11. Equation 11 focuses on the contribution of domestic versus imported goods to the HCE deflator elasticity to an exchange rate shock. By contrast, equation 15 highlights the transmission channels of the shock.

Figure 8 plots the shares of $E1.HC$, $E2.HC$, $E3.HC$ and $E4.HC$ (shortening $E4.HC^i$) in $\overline{s}_i^{i,HC}$. $E1.HC$ dominates. While $E3.HC$ is negligible, $E4.HC$ accounts for 10% to 30% of $\overline{s}_i^{i,HC}$ for most countries except China. On the whole, as shown by Figure 9, input-output mechanisms (i.e. everything but $E1.HC$) explain a large share of the elasticity, especially for large countries or countries of the Eurozone subject to a shock on the Euro. This share has increased untill 2013-2014, implying an increasing need of data from WIOTs to perform our computations (see Figure 10).

$E1.HC$ and $E2.HC$ can be computed with national input-output matrices whereas world input-output matrices are needed for computing $E3.HC$ and $E4.HC$. Although world input-output matrices are not available for the most recent years, $E4.HC$ can be inferred from easier-to-compute elements of $\overline{s}_i^{i,HC}$.

We try to infer $\overline{s}_i^{i,HC}$ from $E1.HC$ and $E2.HC$. Figure 11 depicts the relationship between $\overline{s}_i^{i,HC}$ and $E1.HC + E2.HC$ according to equation 16. The high R^2 (0.98) suggests that $E1.HC + E2.HC$ is a good predictor of $\overline{s}_i^{i,HC}$.

$$
\overline{s}_i^{i,HC} = \alpha + \beta \left(E1.HC^{i,imp} + E2.HC^{i,dom} \right) + \varepsilon_i
$$

(16)

We check whether the relationship is constant over time by estimating yearly cross-sections of equation 16. With the exception of 2009, the relationship is broadly stable (see Figures 12 and 13).

We obtain similar results with TiVA (see Appendix C). Our results suggest that we can approximate the HCE deflator elasticity for the most recent years, using the share of imported
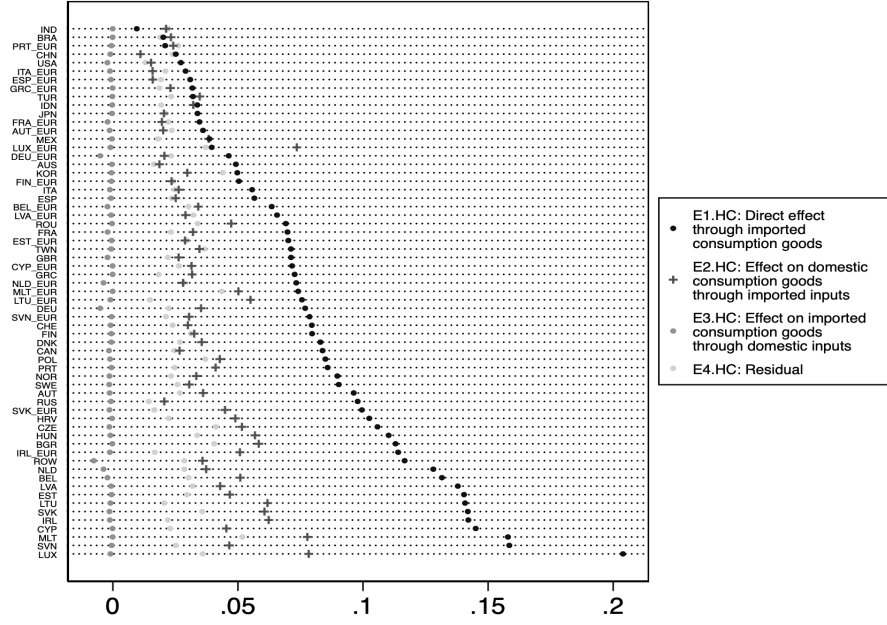
Figure 8: **Decomposition of $\overline{s}_i^{i,HC}$ into E1.HC, E2.HC, E3.HC and E4.HC**
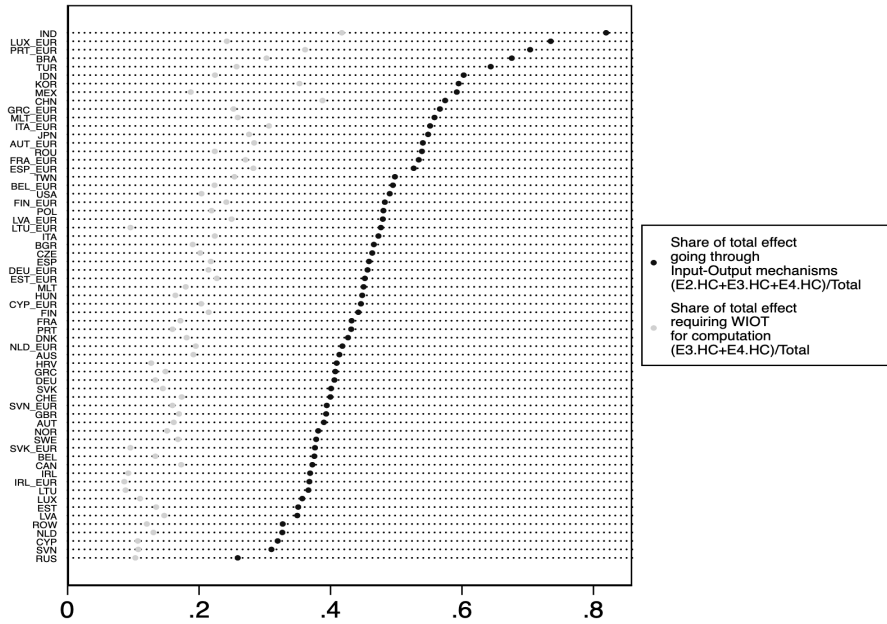
Sources: WIOD and authors' calculatons

Figure 9: **Decomposition of $\overline{s}_i^{i,HC}$**

Sources: WIOD and authors' calculations

goods in household consumption and the share of imported inputs in household consumption of domestic goods. $E1.HC + E2.HC$ is a good predictor of the total effects. Yet, they cannot be
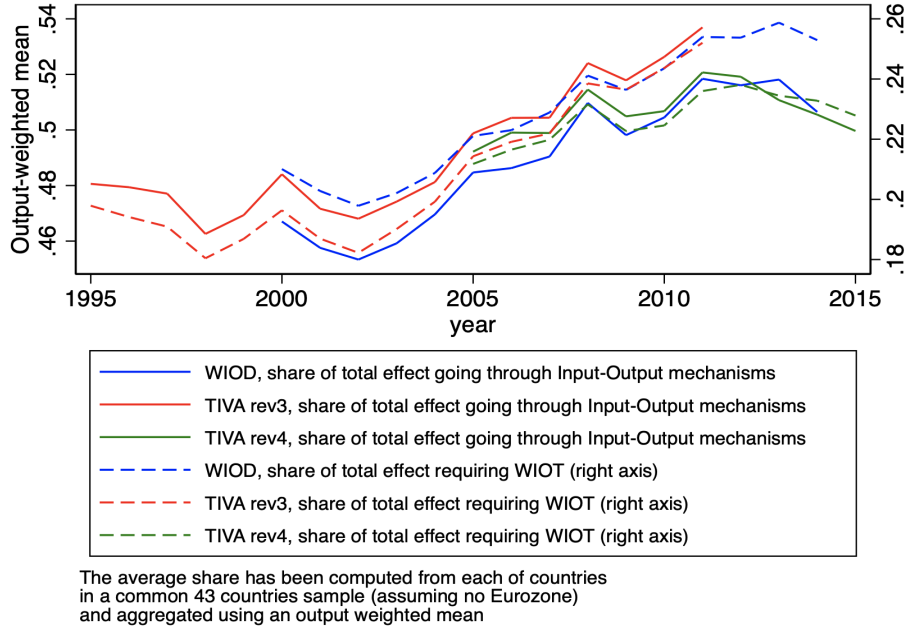
Figure 10: **Decomposition of $\overline{s}_i^{i,HC}$ through time**

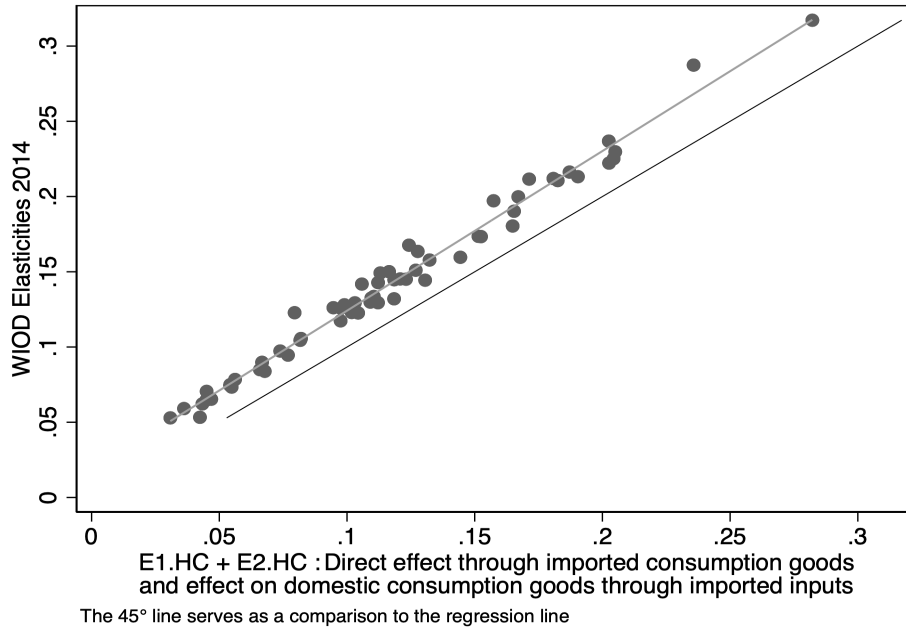Sources: WIOD, TIVA rev3, TIVA rev4 and authors' calculations

Figure 11: **Comparison of $\overline{s}_i^{i,HC}$ and $E1.HC^{i,imp} + E2.HC^{i,dom}$**

Sources: WIOD and authors' calculations

extrapolated in a multiplicative way, as the other effects ($E3.HC + E4.HC$) add to them rather than amplifying them. They are of similar size for small open economies and large closed ones. It might be that the small economy counterbalances its small size with its large openness rate
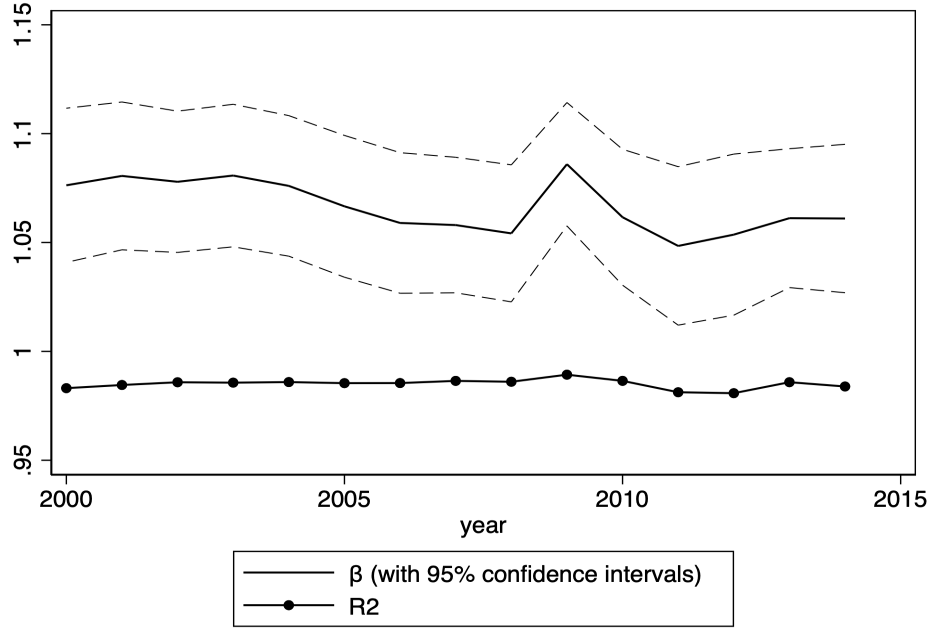
Figure 12: **Evolution of β (the coefficent of E1.HC+E2.HC) and R^2 over time (WIOD)**
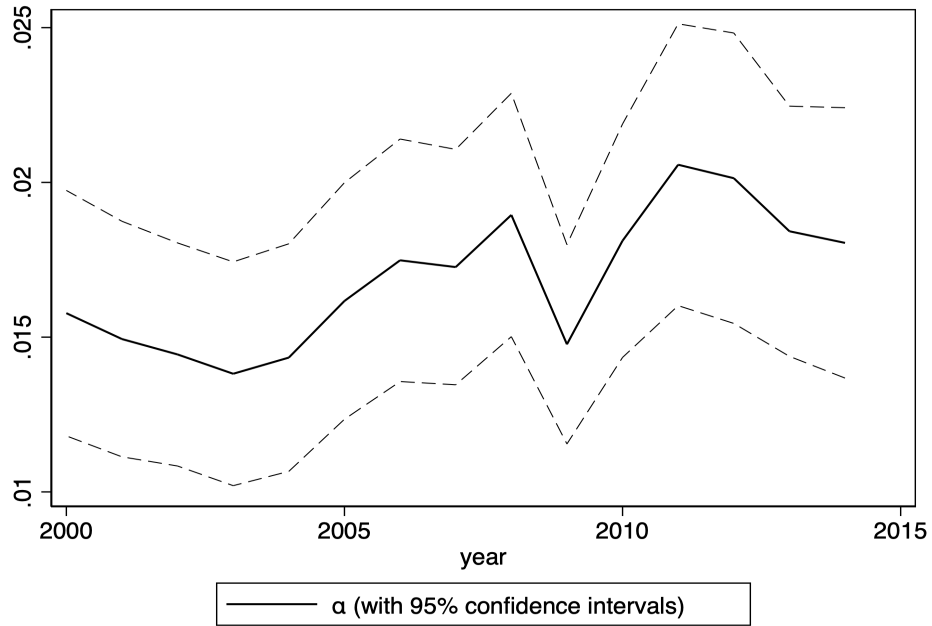
Sources: WIOD and authors' calculations

Figure 13: **Evolution of α (the constant) over time (WIOD)**

Sources: WIOD and authors' calculations

and vice versa.[5]

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[5]Although this functional form might seem counterintuitive (one might expect that the elasticity is an affine function of openness as summarized by $E1.HC$), the analytical examination of the two-country, one-sector case shows that it is plausible (see Appendix D).

5.2 Doing without TiVA and WIOD, but keeping Eurostat

However, even these data ($E1.HC$ and $E2.HC$) are not up-to-date for a large number of countries. The share of imports in household final consumption and in intermediate consumption for the production of domestic household final consumption are not routinely computed by national statistical institutes. We have to use a proxy. It is easy to identify consumption and intermediary goods imports using UN Comtrade data and the BEC classification. While the World Bank provides regular estimates for household consumption, it does not provide an estimate for intermediate consumptions. Eurostat provides estimates for intermediate consumptions in the case of European countries. Combining these three data sources, we compute the share of imported consumption goods in household consumption and the share of imported inputs in all inputs.

We mimick equation 16 by equation 17. We estimate successive cross-sections of equation 17 to check whether the proxy is satisfactory.

$$\overline{s}_i^{i,HC} = \alpha \quad +\beta_1 \frac{\text{imported consumption goods}_i}{\text{household consumption}_i} \\ +\beta_2 \left[\frac{\text{imported intermediate goods}_i}{\text{intermediate consumption}_i} * \frac{\text{domestic consumption goods}_i}{\text{household consumption}_i} \right] + \varepsilon_i \tag{17}$$

In the same way, Figure 14 mimics Figure 12. The results are less encouraging: the R^2 is smaller and declining over time, and the estimated coefficient is not constant.
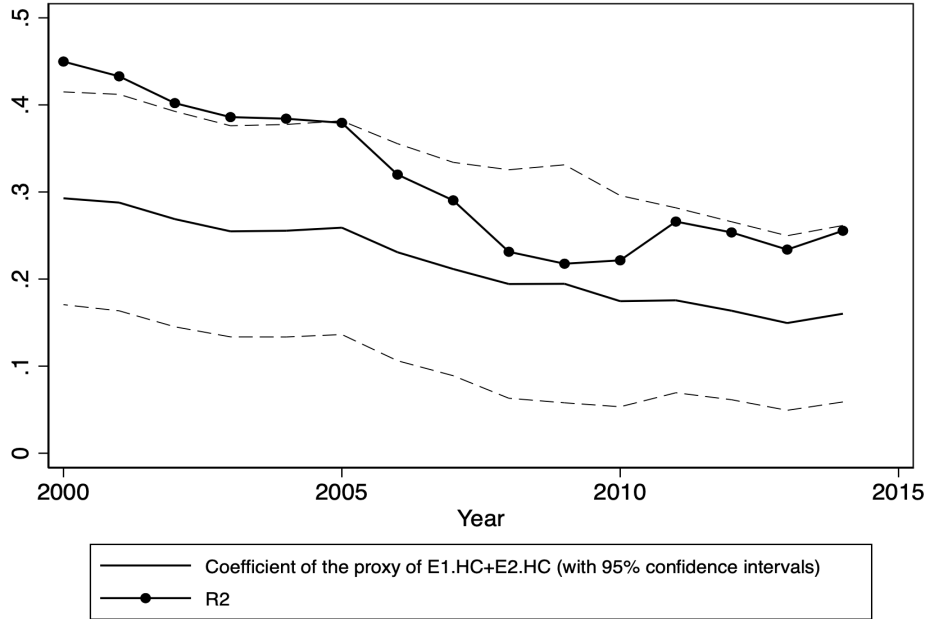

Figure 14: **Evolution of β and $R2$ (WIOD) using Eurostat data to approximate E1.HC + E2.HC (limited number of countries)**

Sources: WIOD and authors' calculations

Yet, estimating successive cross-sections of equation 17 is a demanding test to establish a link between the elasticity computed by PIWIM based on WIOD data and more up-to-date data assembled from various sources. It does not allow to exploit country-specific information on the

determinant of the elasticity. A less demanding test is to run a panel with country fixed-effects, assuming that β is constant over time but that it explains only within-country variations. To take into account year-specific shocks, we add two year-specific variables : the GDP-weighted mean of each variable of interest (see equation 18).

$$
\begin{aligned}
\bar{s}_{i,t}^{i,t,HC} = \alpha \quad & + \beta_1 \frac{\text{imported consumption goods}_{i,t}}{\text{household consumption}_{i,t}} \\
& + \beta_2 \left[\frac{\text{imported intermediate goods}_{i,t}}{\text{intermediate consumption}_{i,t}} * \frac{\text{domestic consumption goods}_{i,t}}{\text{houshold consumption}_{i,t}} \right] \\
& + \beta_3 \frac{\text{Total imported consumption goods}_t}{\text{Total household consumption}_t} \\
& + \beta_4 \left[\frac{\text{Total imported intermediate goods}_t}{\text{Total intermediate consumption}_t} * \frac{\text{Total domestic consumption goods}_t}{\text{Total household consumption}_t} \right] \\
& + fe_i + \varepsilon_{i,t}
\end{aligned}
\tag{18}
$$

We run the panel regressions for the period 2000 to 2008. We then estimate the out-of-sample elasticity for each country i for 2014. The outcome is close to the elasticity computed with WIOD for 2014 despite a small downward bias (see Figure 15). Hence, we could use this approach to estimate the HCE deflator elasticity to the exchange rate from 2015 onwards.
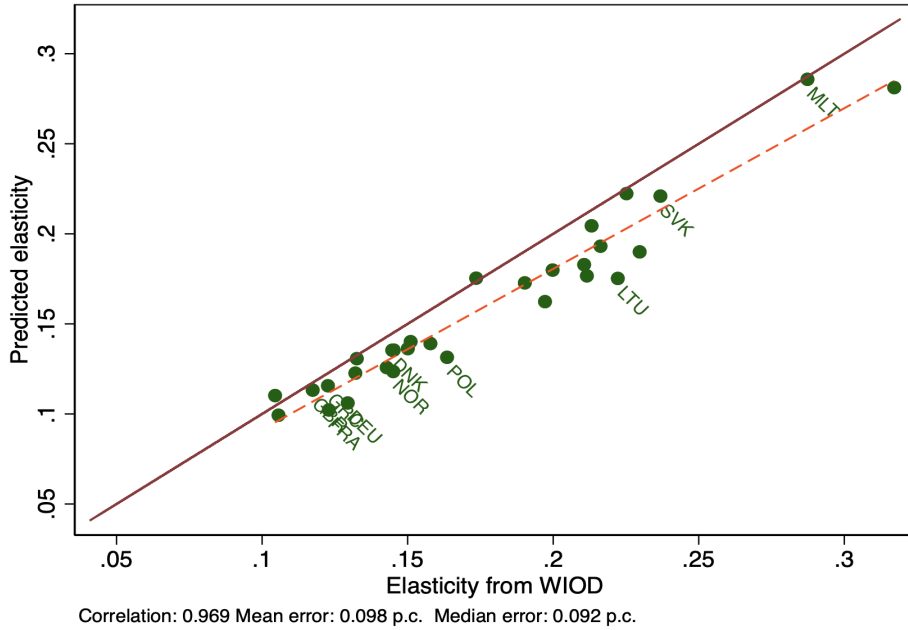

Figure 15: **Comparing the HCE deflator elasticity in 2014 (WIOD) and the prediction from a panel regression on the 2000-2008 period with fixed effects using Eurostat data.**

5.3   Doing with only World Bank and Comtrade data

Data on intermediate consumption and household consumption are not available for all countries. As a result, our regressions only include a limited number of observations. To expand our panel, we use an even simpler proxy for E1.HC+E2.HC that requires only trade data from Comtrade and GDP data from the World Bank, both available until 2018. As a result, we can include

many more countries in the new panel (see equation 19).

$$\bar{s}_{i,t}^{i,t,HC} = \alpha \quad +\beta_1 \frac{\text{imported consumption goods}_{i,t}}{\text{GDP}_{i,t}}$$
$$+\beta_2 \frac{\text{imported intermediate goods}_{i,t}}{\text{GDP}_{i,t}}$$
$$+\beta_3 \frac{\text{Total imported consumption goods}_t}{\text{Total GDP}_t} \qquad (19)$$
$$+\beta_4 \frac{\text{Total imported intermediate goods}_t}{\text{Total GDP}_t}$$
$$+fe_i + \varepsilon_{i,t}$$

The out-of-sample prediction remains satisfactory, although the mean and median errors are larger (see Figure 16). Our findings are robust to using other databases (revision 3 and revision 4 of TIVA).[6]
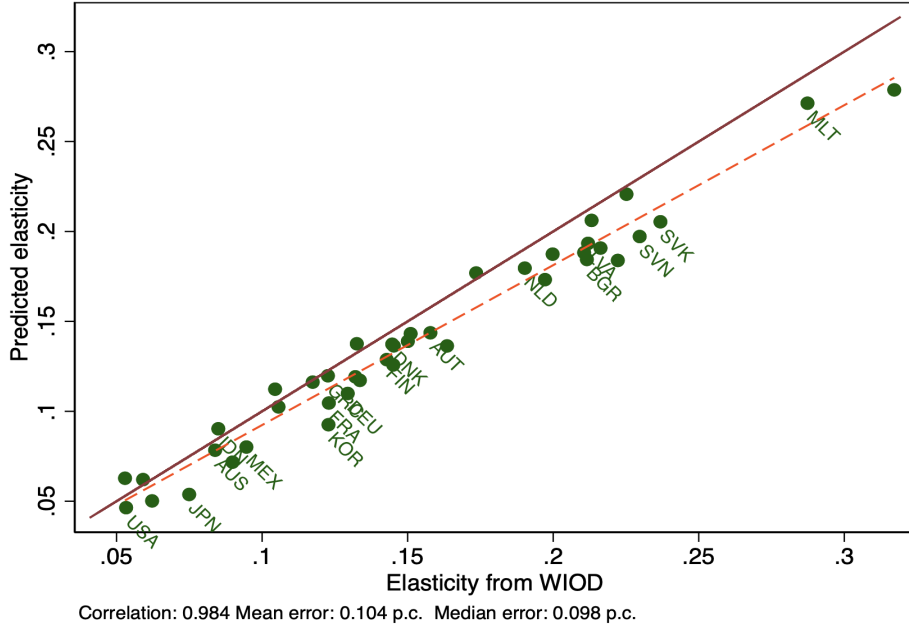

Correlation: 0.984 Mean error: 0.104 p.c. Median error: 0.098 p.c.

Figure 16: **Comparing the HCE deflator elasticity in 2014 (WIOD) and the prediction from a panel regression on the 2000-2008 period with fixed effects using only World Bank and Comtrade data.**

Sources: WIOD, World Bank, Comtrade and authors' calculations

Using these equations, we can predict the HCE deflator elasticity from 2016 onwards to make up for the lack of WIOTs. Figure 17 shows the predictions. The in-sample predictions seem rather robust, giving us confidence in the quality of the out-of-sample predictions.

6 Conclusion

In this paper, we investigate the role of GVCs for inflation dynamics within a unified framework of input-output databases (WIOT and TIVA) from 1995 to 2018. Our main results are threefold. First, we confirm the importance of Global Value Chains in explaining inflation dynamics. The Houshold consumption expenditure (HCE) deflator elasticity to a shock on the domestic currency

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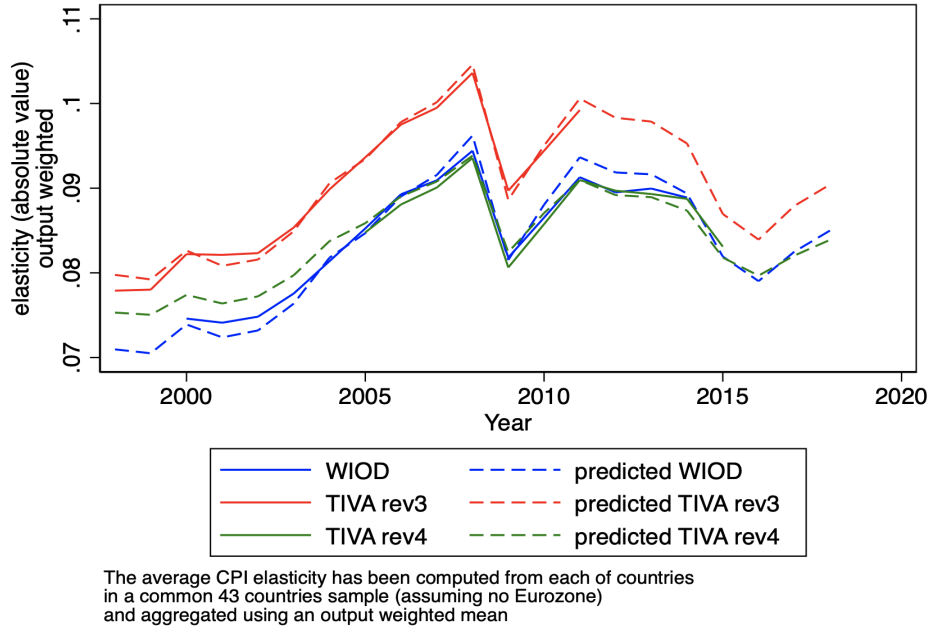[6]Results are available upon request

Figure 17: **Comparing the output-weighed HCE deflator elasticity to its prediction using only World Bank and Comtrade data.**

Sources: WIOD, World Bank, Comtrade and authors' calculations

ranges from 5% to 35%, depending mainly on the openness of countries. Within the euro area, the range of elasticities is large, adding to the challenges faced by the European Central Bank in stabilising prices throughout the monetary union. Input-output mechanisms explain a large share of the elasticity, especially for large countries or for countries of the Eurozone subject to a shock on the Euro. Our results are robust to using different databases (WIOD, TiVA 2016 and TiVA 2018).

Second, we show that the direct impact (through imported final goods) and domestic Input-Output linkages (i.e. domestic final goods produced using foreign inputs) account for most of the propagation of an exchange rate shock to domestic prices. First-round effects explain three-quarters of the propagation of exchange rate shocks to domestic prices. By contrast, we find a limited role for the second-round effects, i.e. the additional transmission of lower domestic input prices to other sectors of the domestic economy and other countries occurring during subsequent production cycles. We analyse the HCE deflator main components elasticity to an exchange rate shocks. Domestic core inflation (defined as inflation excluding food and energy) accounts for a significant share of the total elasticity, mainly reflecting the weight of domestic services and non-energy industrial goods in total consumption.

Third, we provide a tool to make up for the lack of timely WIOTs. The construction of World Input-Output tables is data-demanding and WIOTs are typically released with a lag of several years. To address this gap, we use more up-to-date GDP and trade data, which can be easily updated and used to make up for the lack of WIOTs; We thus provide a tool for approximating the HCE deflator elasticty to an exchange rate shock from 2015 onwards.

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Appendix A   WIOD Sectors

A01	Crop and animal production, hunting and related service activities
A02	Forestry and logging
A03	Fishing and aquaculture
B	Mining and quarrying
C10-C12	Manufacture of food products, beverages and tobacco products
C13-C15	Manufacture of textiles, wearing apparel and leather products
C16	Manufacture of wood and of products of wood and cork, except furniture; articles of straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c.
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31-C32	Manufacture of furniture; other manufacturing
C33	Repair and installation of machinery and equipment
D35	Electricity, gas, steam and air conditioning supply
E36	Water collection, treatment and supply
E37-E39	Sewerage and other waste management services
F	Construction
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
G46	Wholesale trade, except of motor vehicles and motorcycles
G47	Retail trade, except of motor vehicles and motorcycles
H49	Land transport and transport via pipelines
H50	Water transport
H51	Air transport
H52	Warehousing and support activities for transportation
H53	Postal and courier activities
I	Accommodation and food service activities
J58	Publishing activities
J59-J60	Motion picture, video and television programme production; programming and broadcasting activities
J61	Telecommunications
J62-J63	Computer programming, consultancy; information service activities
K64	Financial service activities, except insurance and pension funding
K65	Insurance, reinsurance and pension funding, except compulsory social security
K66	Activities auxiliary to financial services and insurance activities
L68	Real estate activities
M69-M70	Legal and accounting activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M73	Advertising and market research
M74-M75	Other professional, scientific and technical activities; veterinary activities
N	Administrative and support service activities
O84	Public administration and defence; compulsory social security
P85	Education
Q	Human health and social work activities
R-S	Other service activities
T	Activities of households as employers; producing activities of households for own use
U	Activities of extraterritorial organizations and bodies

Table 3: **Industries in WIOD**

Appendix B   Comparison of S and S^i in the two-country, one-sector case

In this appendix, we use the two-country and one-good case to illustrate the difference between a price shock (section 3.1) and an exchange rate shock (section 3.3)

B.1   Effect of a price shock based of VA contents

Using the notations of the paper, we have in the two-country and one good case:

$$\mathcal{A} = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$$I - \mathcal{A} = \begin{pmatrix} 1 - a_{1,1} & -a_{1,2} \\ -a_{2,1} & 1 - a_{2,2} \end{pmatrix}$$

$$(I - \mathcal{A})^{-1} = \frac{1}{(1 - a_{1,1})(1 - a_{2,2}) - a_{2,1}a_{1,2}} \begin{pmatrix} 1 - a_{2,2} & a_{1,2} \\ a_{2,1} & 1 - a_{1,1} \end{pmatrix} = z. \begin{pmatrix} 1 - a_{2,2} & a_{1,2} \\ a_{2,1} & 1 - a_{1,1} \end{pmatrix}$$

$$= \begin{pmatrix} u & v \\ w & x \end{pmatrix}$$

$$\text{Country 1 demand shares} = d = \begin{pmatrix} 1 - f \\ f \end{pmatrix}$$

$$(I - \mathcal{A})^{-1} d = \begin{pmatrix} u - uf + vf \\ w - wf + xf \end{pmatrix}$$

When a shock c occurs on the prices of country 2 (the currency does not matter here), we have the following initial shock vector : $C = (0, c)$. In the first instance, this has an impact on prices $C\mathcal{A}$, and then $C\mathcal{A}^2$, etc. Hence the total effect of the shock S is:

$$S = C + C\mathcal{A} + C\mathcal{A}^2 \ldots = C(I - \mathcal{A})^{-1} = \begin{pmatrix} cw & cx \end{pmatrix}$$

To measure the effect on French consumption price index, we compute a weighted sum of these effects.

$$\bar{s} = c.\left[(1 - f)w + xf\right] = c.\frac{(1 - f)a_{2,1} + f(1 - a_{1,1})}{(1 - a_{1,1})(1 - a_{2,2}) - a_{2,1}a_{1,2}} \tag{20}$$

If each nation's production only uses national inputs, we have:

$$\bar{s} = c.\frac{f}{1 - a_{2,2}}$$

B.2   Exchange rate shock

Using the notations in the paper, we have:

$$C = \left(0, \frac{-c_\$}{1 + c_\$}\right) = (0, -c)$$

$$C_\$ = (c_\$, 0)$$

$$\tilde{C}_\$ = (0, -c_\$)$$

$$\hat{C}_\$ = \left(\frac{c_\$}{1 + c_\$}, 0\right) = (c, 0)$$

$$\mathcal{B} = \begin{pmatrix} 0 & a_{1,2} \\ 0 & 0 \end{pmatrix}$$

$$\tilde{\mathcal{B}} = \begin{pmatrix} 0 & 0 \\ a_{2,1} & 0 \end{pmatrix}$$

Hence

$$S = (0, c) + [(0, -c.a_{1,2}) + (c.a_{2,1}, 0)] * \begin{pmatrix} u & v \\ w & x \end{pmatrix}$$

$$= (0, c) + (c.a_{2,1}, -c.a_{1,2}) * \begin{pmatrix} u & v \\ w & x \end{pmatrix}$$

$$= (0, c) + (u.c.a_{2,1} - w.c.a_{1,2}, v.c.a_{2,1} - x.c.a_{1,2})$$

$$= (u.c.a_{2,1} - w.c.a_{1,2}, c + v.c.a_{2,1} - x.c.a_{1,2})$$

and

$$\bar{s} = (u.c.a_{2,1} - w.c.a_{1,2}, c + v.c.a_{2,1} - x.c.a_{1,2}) . \begin{pmatrix} 1 - f \\ f \end{pmatrix}$$

$$\bar{s} = c \left[f \left(1 + v.a_{2,1} - x.a_{1,2}\right) + (1 - f) \left(u.a_{2,1} - w.a_{1,2}\right)\right]$$

If each nation's production only uses national inputs, we have a plausible

$$\bar{s} = c.f$$

This confirms that an exchange rate shock differs from a price shock.

Appendix C   Comparision of $\bar{s}_i^{i,HC}$ and $E1.HC^{i,imp} + E2.HC^{i,dom}$ in the TIVA rev. 3 and TIVA rev. 4 databases

Figures 18, 19 and 20 for TIVA rev. 3 and Figures 21, 22 and 23 for TIVA rev. 4 show that we get a good prediction of the partial equilibrium effects of an exchange rate shock on consumption prices by using simply the share of imported final consumption goods and services and the share of imported intermediate goods in domestic final consumption.
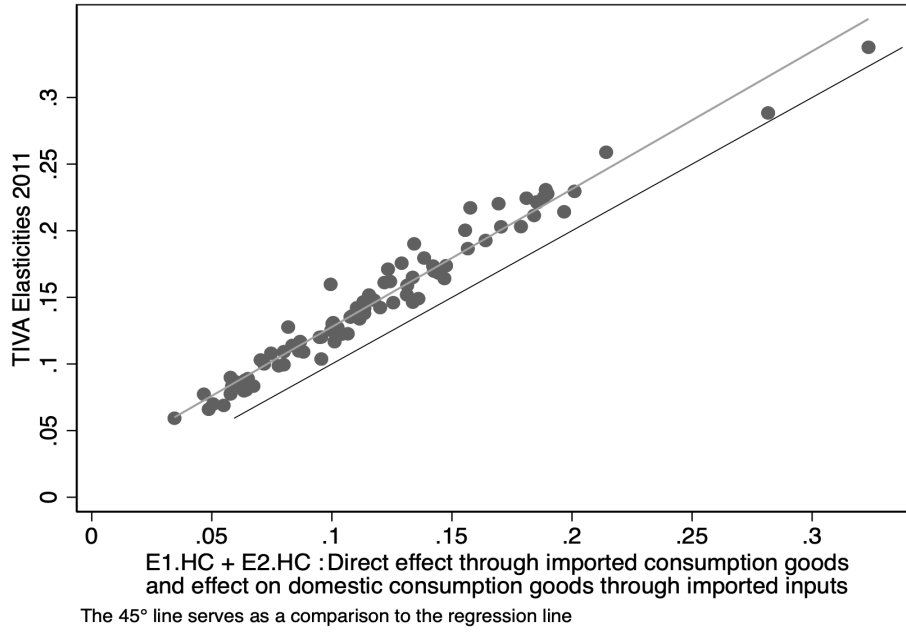

Figure 18: **Comparing $\bar{s}_i^{i,HC}$ and $E1.HC^{i,imp} + E2.HC^{i,dom}$ (TIVA rev. 3)**
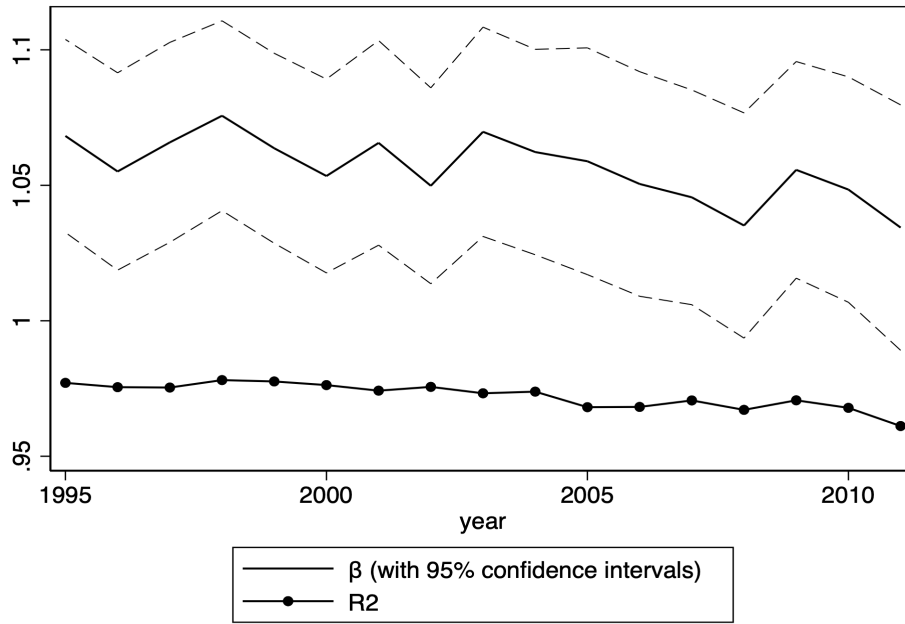
Sources: TIVA rev3 and authors' calculations

Figure 19: **Evolution of β and $R2$ (TIVA rev. 3)**
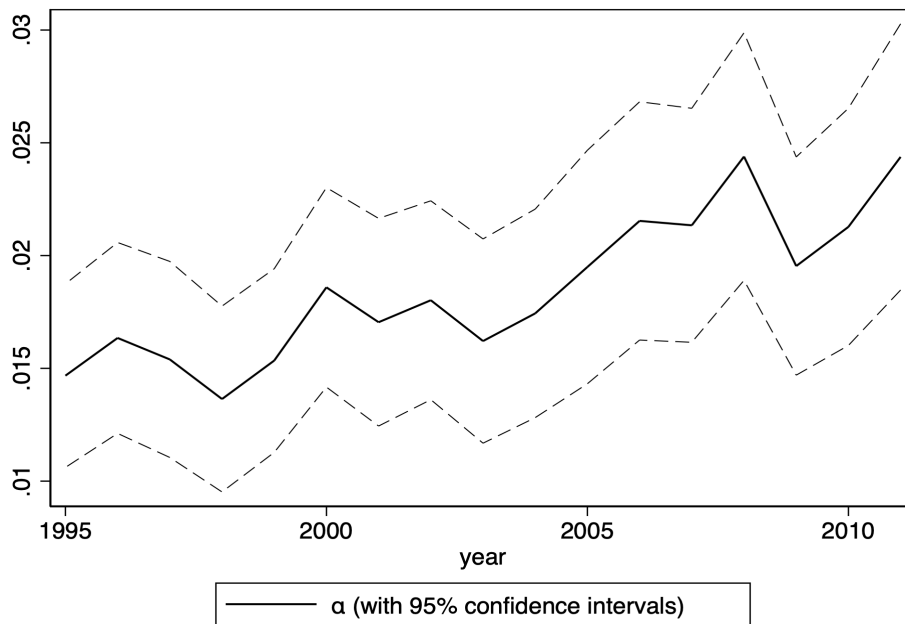
Sources: TIVA rev3 and authors' calculations

Figure 20: **Evolution of α (TIVA rev. 3)**

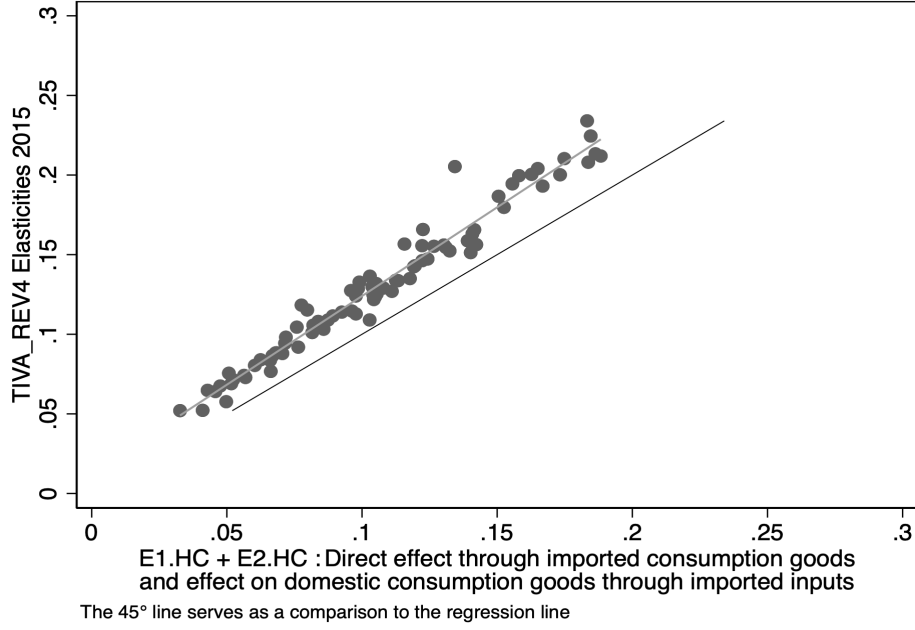Sources: TIVA rev3 and authors' calculations

Figure 21: **Comparison between** $\bar{s}_i^{i,HC}$ **and** $E1.HC^{i,imp} + E2.HC^{i,dom}$ **(TIVA rev. 4)**
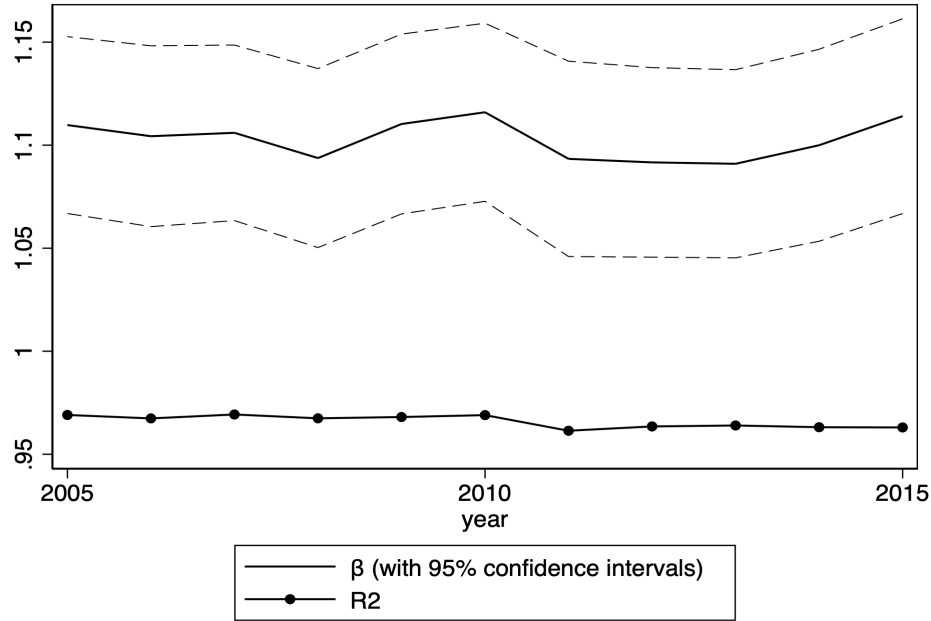
Sources: TIVA rev4 and authors' calculations

Figure 22: **Evolution of** β **and** $R2$ **(TIVA rev. 4)**

Sources: TIVA rev4 and authors' calculations

Appendix D Study of the decomposition of the shock in the two-country, one-sector case

D.1 The issue

As a reminder from the paper, where $\bar{s}_i^{i,HC}$ is the effect of an exchange rate shock on consumption prices :

36

$$
\begin{aligned}
\bar{s}_i^{i,HC} &= S^i.HC^i = E1.HC^i + E2.HC^i + E3.HC^i + E4.HC^i \\
&= E1.HC^{i,imp} + E2.HC^{i,dom} + E3.HC^{i,imp} + E4.HC^i
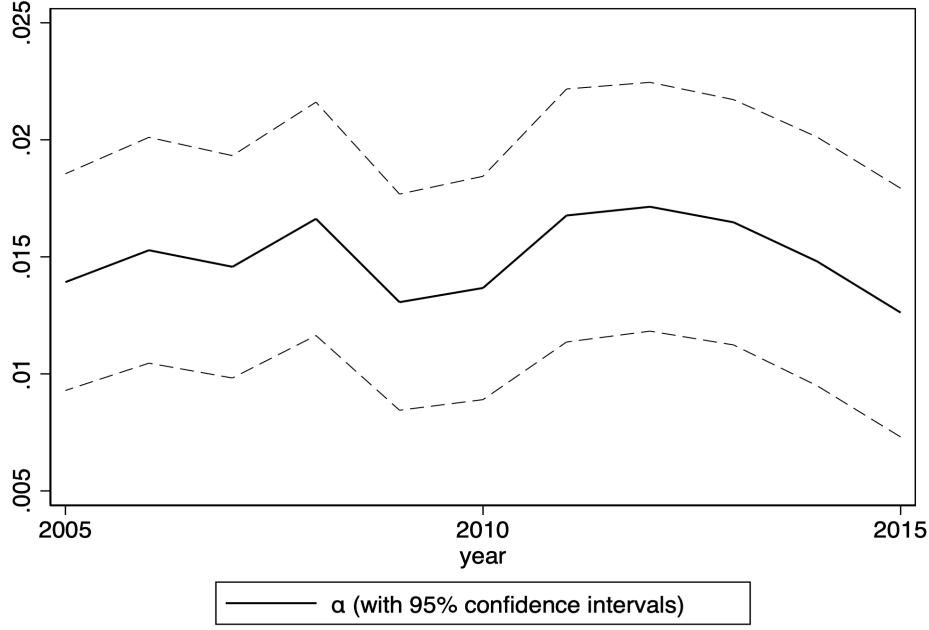\end{aligned}
\tag{21}
$$

Figure 23: **Time evolution of α (TIVA rev. 4)**

Sources: TIVA rev4 and authors' calculations

and

$$
\begin{aligned}
S^i &= & C^i & & + \left(\hat{C}^i_\$.\mathcal{B}^i + C^i \tilde{\mathcal{B}}^i \right) * (I - \mathcal{A})^{-1} \\
S^i &= & \underbrace{C^i}_{\substack{\text{(E1) direct effect through} \\ \text{imported consumption goods}}} & + & \underbrace{C^i \tilde{\mathcal{B}}^i}_{\substack{\text{(E2) effect on} \\ \textit{domestic} \text{ consumption goods} \\ \text{through } \textit{imported} \text{ inputs}}} & + & \underbrace{\hat{C}^i_\$.\mathcal{B}^i}_{\substack{\text{(E3) effect on} \\ \textit{imported} \text{ consumption goods} \\ \text{through } \textit{domestic} \text{ inputs}}}
\end{aligned}
$$

$$
+ \underbrace{\left(\hat{C}^i_\$.\mathcal{B}^i + C^i \tilde{\mathcal{B}}^i \right) * (I - \mathcal{A})^{-1} * \mathcal{A}}_{\text{(E4) residual}}
$$

$$(22)$$

When the shock corresponds to an appreciation of the domestic currency, $E1$ and $E2$ reduce country i's consumer prices whereas $E3$ increases them. $E1$ and $E2$ are easy to compute with national input-output matrices, whereas world input-output matrices are needed for computing $E3$ and $E4$.

Unexpectedly, $E3 + E4$ seems to be constant, whatever the openness rate of the economy (see Figure 11).

Let us focus on the two-country, one-sector economy :

$$E1 = C = (0, -c)$$

$$E2 = C.\tilde{\mathcal{B}}^i = (0, -c).\begin{pmatrix} 0 & 0 \\ a_{2,1} & 0 \end{pmatrix} = (-c.a_{2,1}, 0)$$

$$E3 = (c, 0).\begin{pmatrix} 0 & a_{1,2} \\ 0 & 0 \end{pmatrix} = (0, c.a_{1,2})$$

$$E1.HC = (0, -c).\begin{pmatrix} 1 - f \\ f \end{pmatrix} = -f.c$$

$$E2.HC = (-c.a_{2,1}, 0).\begin{pmatrix} 1 - f \\ f \end{pmatrix} = -c.a_{2,1}.(1 - f)$$

$$E3.HC = (0, c.a_{1,2}).\begin{pmatrix} 1 - f \\ f \end{pmatrix} = f.c.a_{1,2}$$

We do not lose any generality by normalising the shock c to 1.

And, developed from SAGE :

$$\bar{s} - E1.HC - E2.HC = \left(-\frac{a_{12}a_{21}^2 - a_{11}a_{21}a_{22} + (a_{11} - a_{12})a_{21} - \left(a_{12}a_{21}^2 - a_{11}a_{21}a_{22} - (a_{11} - 1)a_{12} + (a_{11} - 2a_{12})a_{21}\right)f}{a_{12}a_{21} - (a_{11} - 1)a_{22} + a_{11} - 1} \right)$$

$$(23)$$

We assume that: $\frac{a_{1,1}}{a_{2,1}} = \frac{1-f}{f}$ and $a_{1,1} + a_{2,1} = a$.

So $a_{1,1} = (1 - f)a$ and $a_{2,1} = fa$.

Then:

$$\bar{s} - E1.HC - E2.HC = \left(-\frac{\left(a^2 a_{12} + a^2 a_{22} - a^2\right)f^3 - \left(2 a^2 a_{22} - 2 a^2 + \left(a^2 + a\right)a_{12}\right)f^2 + \left(a^2 a_{22} - a^2 + a_{12}\right)f}{(a-1)a_{22} - (aa_{12} + aa_{22} - a)f - a + 1} \right)$$

$$(24)$$

According to SAGE, the derivative of this according to f is:

$$\left(\begin{array}{c} -\dfrac{\left(\left(a^2 a_{12} + a^2 a_{22} - a^2\right)f^3 - \left(2 a^2 a_{22} - 2 a^2 + \left(a^2 + a\right)a_{12}\right)f^2 + \left(a^2 a_{22} - a^2 + a_{12}\right)f\right)(aa_{12} + aa_{22} - a)}{\left((a-1)a_{22} - (aa_{12} + aa_{22} - a)f - a + 1\right)^2} \\ -\dfrac{a^2 a_{22} + 3\left(a^2 a_{12} + a^2 a_{22} - a^2\right)f^2 - a^2 - 2\left(2 a^2 a_{22} - 2 a^2 + \left(a^2 + a\right)a_{12}\right)f + a_{12}}{(a-1)a_{22} - (aa_{12} + aa_{22} - a)f - a + 1} \end{array} \right)$$

The sign of this expression is difficult to study. We hence move to a numerical application.

D.2 Numerical application

Based on WIOD 2014, we compute the ratio between value added and production. The computation with the WIOD data is : egen total=rowtotal(vAUS1-vROW) and then (161-74)/161=54%.

To simplify, we assume that the ratio is equal to 0.5.

$$a_{1,1} + a_{1,2} = a_{2,1} + a_{2,2} = 50\%$$

$$\frac{a_{1,2}}{a_{1,1} + a_{1,2}} = f$$

$$a_{2,1} = 0.48$$

$$a_{2,2} = 0.02$$

In that case:

$$\bar{s} - E1.HC - E2.HC = \frac{-0.125\,f^3 + 0.245\,f^2 - 0.11\,f}{-0.25\,f - 0.26}$$
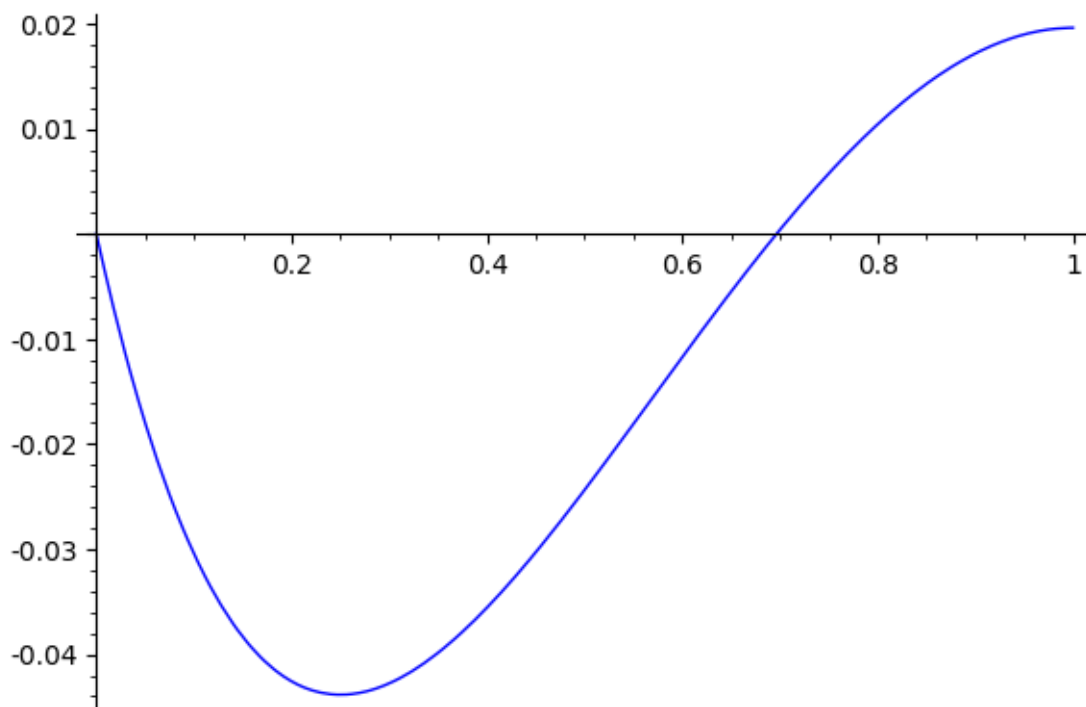
Which yields Figures 24 and 25.

Figure 24: $\bar{s} - E1.HC - E2.HC$ as a function of the openness rate

Actual openness rates in the sample vary between 0.15 and 0.5. In that zone, the relationship between the openness rate and the residual is not monotonous (see Figure 24).

Figure 25 confirms that, in that numerical exercise, the total effect is dominated by the direct effect through imported consumption goods and, to a lesser extent, the effect on domestic consumption goods through imported inputs. The other effects are approximately additive if the openness rate is betweem 0.15 and 0.5.
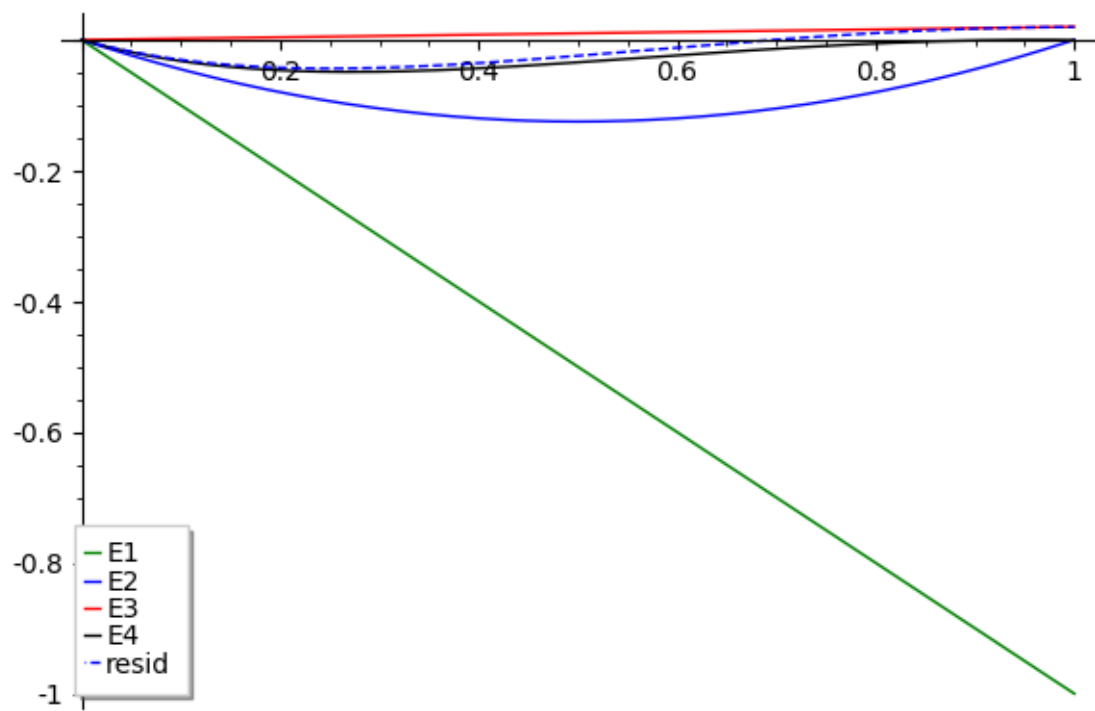
Figure 25: E1.HC, E2.HC, E3.HC, E4.HC and the "residual" $(\bar{s}-E1.HC-E2.HC)$ as a function of the openness rate