Proxy for the import contents

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2 countries: Home and Foreign. Foreign variables are starred.

Vectors of production: Q, Q^*. Denote $\mathcal{Q} = \begin{bmatrix} Q \\ Q^* \end{bmatrix}$ the vector of production in both countries.

Direct domestic requirement matrices are A, A^*. Import requirement matrices are B, B^*. The world input-output table is $\mathcal{A} = \begin{pmatrix} A & B^* \\ B & A^* \end{pmatrix}$.

Sectoral vectors of domestic consumption are C for domestically produced consumption and C^* for imported consumption, with total consumption normalized to 1. Denote $\mathcal{C} = \begin{bmatrix} C \\ C^* \end{bmatrix}$.

We have $\mathcal{Q} = \mathcal{A}\mathcal{Q} + \mathcal{C}$ so $\mathcal{Q} = \mathcal{L}\mathcal{C}$, with $\mathcal{L} = (I - \mathcal{A})^{-1}$ the world Leontiev inverse (I is the identity matrix).

Denote the submatrices of $\mathcal{L}$ as: $\mathcal{L} = \begin{pmatrix} L & M^* \\ M & L^* \end{pmatrix}$.

Then we have $\begin{bmatrix} Q \\ Q^* \end{bmatrix} = \begin{pmatrix} L & M^* \\ M & L^* \end{pmatrix} \begin{bmatrix} C \\ C^* \end{bmatrix}$. Sectoral domestic production associated to a unit of domestic consumption is given by $Q = LC + M^*C^*$.

If I get it right, the vector of import content of domestic consumption should be

$$\underbrace{C^*}_{\text{direct}} + B\Big(\underbrace{LC}_{\substack{\text{indirect due to}\\ \text{domestic production}}} + \underbrace{M^*C^*}_{\substack{\text{indirect due to}\\ \text{domestic production of}\\ \text{inputs for Foreign}}} \Big).$$

The import content itself is the scalar product of $\mathbf{1}$ (a vector of ones) and the vector of import content: $\mathbf{1} \cdot C^* + \mathbf{1} \cdot B(LC + M^*C^*)$.

Next, denote $\tilde{L} = (I - A)^{-1}$ the inverse Leontiev of domestic production alone (ignoring the world input-output linkages). This is what we would use with traditional matrices from one country only. We can decompose the import content in the following way:

$$\underbrace{\mathbf{1} \cdot C^*}_{\text{direct}} + \mathbf{1} \cdot B\Big(\underbrace{\tilde{L}C}_{\substack{\text{indirect due to}\\ \text{domestic I/O linkages}}} + \underbrace{(L - \tilde{L})C}_{\substack{\text{global value chain}\\ \text{of domestic production}}} + \underbrace{M^*C^*}_{\substack{\text{indirect due to}\\ \text{domestic production of}\\ \text{inputs for Foreign}}} \Big). \tag{1}$$

The first two terms can be computed with I/O matrices from 1 country only. The last two terms require the full world I/O matrix and reflect global value chains. I think this expression generalizes easily to the case of many countries (the square matrices B and M^* should simply become rectangular).

Can we make a crude approximation of this formula using only simple ratios? Denote $\delta = \mathbf{1} \cdot C^*$ the share of direct imports in consumption, $\beta = \mathbf{1} \cdot B$ the share of imported inputs in total domestic production. Let α the share of domestic inputs in total domestic production and μ the total share of inputs in total production: we have $\alpha + \beta = \mu$ and μ is typically $\approx 1/2$. Finally, define $\tilde{\beta} = \beta/(1 - \mu)$ the share of imports in total inputs used by the domestic economy.

Assuming homogeneous sectors, we get a very crude approximation of the second term:[1] $\beta(1-\delta)/(1-\alpha) = \beta(1-\delta)/(1-\mu+\beta) \approx \beta(1-\delta)/(1-\mu) \approx \tilde{\beta}(1-\delta)$ to the first order in β. My guess is that the total import content should be highly correlated with

$$\delta + (1-\delta)\tilde{\beta}.$$

If you run a regression of the import content on δ and $(1-\delta)\tilde{\beta}$, you should get a decent fit. The coefficient of δ should be slightly higher than 1, the difference with 1 potentially reflecting global value chain effect captured by the last term of (1). The coefficient of $(1-\delta)\tilde{\beta}$ might also be different from 1, perhaps capturing the third term of (1).

[1] The matrix B becomes β, $\tilde{L}$ becomes $1/(1-\alpha)$, C becomes $1-\delta$.