Measurement of the trade elasticity in frameworks with producer heterogeneity

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Technical note

Abstract

This technical note summarizes the main features of the two frameworks with efficiency heterogeneity used to microufound the gravity equation: the Ricardian and the Melitz-Chaney frameworks. It shows why bilateral trade data which do not include data on domestic prices do not allow estimating the efficiency heterogeneity parameter which corresponds to the trade elasticity in these frameworks. It recalls why any methodology which gives an estimate of the intersectoral efficiency heterogeneity parameter at the intersectoral level immediately informs on the intrasectoral efficiency heterogeneity parameter in the Melitz-Chaney framework under the assumption of a common efficiency distribution across sectors. Finally, it provides additional information on the direction of the estimation bias linked to zero trade flows in the procedure used to estimate the substitutability parameter of exporter-specific goods on bilateral trade data.

Keywords: trade elasticity, gravity equations, distance puzzle

Introduction

This is the extended technical note for the paper 'Heterogeneity and the Distance Puzzle' by Archanskaia E. and Daudin G. It goes over the main features of the two frameworks with efficiency heterogeneity used to microufound the gravity equation: the Ricardian and the Melitz-Chaney frameworks. It recalls the computation of the price index in each set-up. It shows how to get the result that bilateral trade data do not allow estimating the efficiency heterogeneity parameter which corresponds to the trade elasticity in these frameworks precisely because this data does not inform on the full distribution of prices in the destination. It also shows that the estimation procedure used in the paper to retrieve the Armington elasticity is valid across frameworks for computing the substitutability parameter of country-specific product bundles. Thus, across frameworks, the parameter computed in the paper gives the evolution of perceived substitutability of exporter-specific goods.

The note clarifies why the efficiency heterogeneity parameter estimated on aggregate data in the two frameworks with producer heterogeneity is actually one in the same. Indeed, in the seminal Eaton and Kortum model in perfect competition firms draw an efficiency characteristic from a Pareto distribution of productivity common across sectors, and this parameter is inherited at the intersectoral level as the parameter of the Fréchet distribution. Making the assumption that γ is not sector-specific in the Chaney framework entails that $\gamma = \theta$, i.e. the trade elasticity corresponds to the power law exponent of the intra- and intersectoral efficiency distribution. As shown by Imbs and Méjean (2011) the parameter estimated on aggregate data always corresponds to the implicit assumption of equality of sector-specific parameters. It follows that any methodology which allows computing the intersectoral efficiency dispersion in the Eaton and Kortum model would inform on the intrasectoral parameter in the Chaney (2008) model under the assumption that the productivity distribution is not sector specific.

Finally, the note formally derives the result that the estimated substitutability parameter of exporter-specific goods in the paper is overestimating the true underlying substitutability because of the prevalence of zero trade flows. The relative price is necessarily underestimated by more for small exporters.

1 Estimation of the trade elasticity in the Eaton and Kortum framework

This section recalls the main features of the Eaton and Kortum framework in perfect and monopolistic competition (see Eaton and Kortum (2002, 2010) for details). We recall the steps followed in Eaton and Kortum (2002, 2010) to derive the price index in order to show why the distribution of prices of effectively exported goods does not allow estimating the shape parameter of the intersectoral efficiency distribution under either perfect or monopolistic competition.

1.1 Set-up and notation in perfect competition

In each country i, there is a continuum of sectors k. Output in each sector is homogeneous, within and across producing countries. Output can be produced using one of the techniques for sector k available in country i. Production techniques vary in efficiency z. Efficiency is drawn from a Pareto distribution with parameter θ and efficiency lower bound at $\underline{z}$. This efficiency distribution is common across sectors, and the shape parameter of the Pareto is inherited at the intersectoral level as the shape parameter of the Fréchet distribution. It is assumed that each source accumulates its own stock of technology from which only its firms can make draws: it is through trade that countries acquire access to foreign technology stocks.

Techniques' arrival follows a Poisson process with parameter $\beta R\left(t\right)$, denoting research productivity and effort, respectively. Normalizing $\beta \underline{z}^{\theta} = 1$, and defining the stock of techniques available at t by $T\left(t\right) = \int\limits_{-\infty}^{t} R(v)\,\mathrm{d}v$, the number of techniques for producing output in sector k with efficiency $Z > z$ follows a Poisson distribution with parameter $\lambda = T(t)z^{-\theta}$. In perfect competition, only the best idea is operational within each country-sector, and its distribution is Fréchet. As the technology improvement process takes place independently within each sector k within the continuum of sectors, the distribution of the best-of ideas across sectors in each country is also Fréchet with parameter $T_i(t)z^{-\theta}$. The location parameter of the distribution $T_i(t)$ describes the country-specific stock of techniques (absolute advantage), while the variability parameter θ common to all countries measures the strength of comparative advantage (high θ means low efficiency variability).[1]

The cost of the bundle of inputs in country i, c_i, is the same across sectors. Sectors do not differ in input shares, inputs are mobile, production

[1]See Eaton and Kortum (2010).

is CRS. Factors of production can thus be shifted across sectors without bidding up factor prices. The unit cost of producing k in i is the realization of a random variable $W_i = c_i/Z_i$ where efficiency of country i in sector k is the realization of the random variable Z_i, with independent draws for each sector from the Fréchet distribution. With iceberg transport costs τ_{ij}, delivered price in destination j is the realization of a random variable $p_{ij} = c_i\tau_{ij}/Z_i$. The destination-specific price distribution parameter is defined by $\Phi_j = \sum T_s \left(c_s\tau_{sj}\right)^{-\theta}$.

1.2 Computation of the price index in perfect competition

This subsection shows that it is not possible to estimate the trade elasticity parameter in the Ricardian framework in perfect competition with data on observed cif prices of bilateral exports. For CES preferences, the ideal price index across the continuum of goods which j consumes is:

$$P_j\left(p\right)^{1-\sigma} = \int_0^\infty p^{1-\sigma}\,\mathrm{d}G_j\left(p\right)$$

In perfect competition, only the least cost goods delivered to destination j are effectively consumed. Thus, the destination specific price index is given by the $(1-\sigma)$ moment of the least cost distribution in destination j:

$$P_j\left(p\right)^{1-\sigma} = E\left[\left(W^{(1)}\right)^{1-\sigma}\right]$$

The properties of the distribution of the ordered costs help compute the relevant moment of the costs' distribution. Given truncation invariance of the Pareto, the conditional efficiency distribution for Z is:

$$\Pr\left[Z \geq z' | Z \geq z\right] \;\; = \;\; \left(z'/z\right)^{-\theta}$$

Using the definition $W = c/Z$, the conditional costs' distribution for $(w' < w)$ is:

$$\Pr\left[c/W \geq c/w' | c/W \geq c/w\right] \;\; = \;\; \Pr\left[W \leq w' | W \leq w\right] = \left(w'/w\right)^{\theta}$$

Replacing z by its value in the Poisson distribution of techniques' efficiency, the number of techniques for production of output in a given sector k is distributed Poisson with parameter $T(t)c^{-\theta}w^{\theta}$ (using $z = c/w$). The probability of no technique allowing production with cost less than w arriving in a unit interval is given by $\Pr[X=0] = \frac{\left(T(t)(c/w)^{-\theta}\right)^0 \exp\left\{-T(t)(c/w)^{-\theta}\right\}}{0!} = \exp\left\{-T(t)(c/w)^{-\theta}\right\}$. Then the probability of a lower cost draw arriving is given by $1 - \Pr[X=0]$. The distribution of the lowest cost is then Weibull with parameter $T(t)c^{-\theta}w^{\theta}$:

$$F(w) = \Pr[W \leq w] = 1 - \exp\left\{-T(t)c^{-\theta}w^{\theta}\right\}$$

Similarly, the second-least cost probability distribution is given by the probability that at least 2 techniques had arrived which cost of production was lower than w:

$$F_2(w) = \Pr[W^{(2)} \leq w] = 1 - \sum_{v=0}^{1} \frac{\left[T(t)c^{-\theta}w^{\theta}\right]^v \exp\left\{-T(t)c^{-\theta}w^{\theta}\right\}}{v!}$$

This is just $1 - \Pr[X=0] - \Pr[X=1]$. Indeed, for the second-lowest cost to be lower than w, we need to subtract the probability that no cost draw was lower than w as well as the probability that only one draw were lower than w. More generally, ordered costs $W^{(\alpha)}$, where α denotes the rank of the cost in the pool of available techniques, are random variables which have a gamma distribution given by:[2]

$$F_\alpha(w) = \Pr[W^{(\alpha)} \leq w] = 1 - \sum_{v=0}^{\alpha-1} \frac{\left[T(t)c^{-\theta}w^{\theta}\right]^v \exp\left\{-T(t)c^{-\theta}w^{\theta}\right\}}{v!}$$

Defining $\Phi = T(t)c^{-\theta}$ and $\lambda = \theta w^{\theta-1}\Phi$, the ordered costs' pdf is given by:

$$F'_\alpha(w) = \lambda \exp\left\{-\Phi w^{\theta}\right\} - \exp\left\{-\Phi w^{\theta}\right\} \left[\sum_{v=1}^{\alpha-1} \frac{\left(\Phi w^{\theta}\right)^{v-1} v\lambda}{v!} - \frac{\lambda\left(\Phi w^{\theta}\right)^{v}}{v!}\right]$$

where the summation in squared brackets simplifies to $\left[\lambda - \frac{\lambda\left(\Phi w^{\theta}\right)^{\alpha-1}}{(\alpha-1)!}\right]$. The pdf is:

$$f_\alpha(w) = \frac{\lambda\left(\Phi w^{\theta}\right)^{\alpha-1} \exp\left\{-\Phi w^{\theta}\right\}}{(\alpha-1)!} = \frac{\theta\Phi^{\alpha} w^{\theta\alpha-1} \exp\left\{-\Phi w^{\theta}\right\}}{\Gamma(\alpha)}$$

In perfect competition, the pdf for prices and costs for surviving producers are both given by the least cost pdf. For $\alpha = 1$:

$$f_1(w) = \theta\Phi w^{\theta-1} \exp\left\{-\Phi w^{\theta}\right\}$$

[2]The gamma distribution describes the waiting time in a Poisson process until the α change is observed. And since in this case draws are made from a Pareto distribution, the parameter θ accounts for draws' variability.

The $(1-\sigma)$ moment of the least cost distribution is:

$$E\left[\left(W^{(1)}\right)^{1-\sigma}\right] = \int_0^\infty w^{1-\sigma} f_1(w)\,\mathrm{d}w$$

Computing this moment for j:[3]

$$E\left[\left(W_j^{(1)}\right)^{1-\sigma}\right] = \int_0^\infty w^{1-\sigma}\theta\Phi_j w^{\theta-1}\exp\left\{-\Phi_j w^\theta\right\}\,\mathrm{d}w$$

Define $v = \Phi_j w^\theta$, therefore $dv = \Phi_j \theta w^{\theta-1} dw$ and $(v/\Phi)^{(1-\sigma)/\theta} = w^{1-\sigma}$. Changing the variable of integration and rearranging:

$$E\left[\left(W_j^{(1)}\right)^{1-\sigma}\right] = \Phi_j^{-(1-\sigma)/\theta}\int_0^\infty v^{(1-\sigma)/\theta}\exp\left\{-v\right\}\,\mathrm{d}v$$

By the definition of the gamma function with parameter: $\gamma = 1 + \frac{1-\sigma}{\theta}$, the integral is equal to $\Gamma\left[\frac{\theta+1-\sigma}{\theta}\right]$. The price index in j which is well defined under parameter restrictions $1 \leq \sigma < \theta + 1$ is:

$$P_j^{1-\sigma} = \Phi_j^{-(1-\sigma)/\theta}\Gamma\left[\frac{\theta+1-\sigma}{\theta}\right]$$

Solving for Φ_j:

$$\Phi_j = P_j^{-\theta}\left(\Gamma\left[\frac{\theta+1-\sigma}{\theta}\right]\right)^{\frac{\theta}{1-\sigma}}$$

Since the distribution of potential costs is invariant to trade costs, the $(1-\sigma)$ moment for the least-costs distribution over potential costs for goods produced in i and delivered to j is:

$$P_{ij}^{1-\sigma} = \Phi_i^{-(1-\sigma)/\theta}\Gamma\left[\frac{\theta+1-\sigma}{\theta}\right]$$

where $\Phi_i = T_i(c_i\tau_{ij})^{-\theta}$. Solving for Φ_i:

$$\Phi_i = P_{ij}^{-\theta}\left(\Gamma\left[\frac{\theta+1-\sigma}{\theta}\right]\right)^{\frac{\theta}{1-\sigma}}$$

[3] Lemma 2 in Eaton and Kortum (2010).

Replacing Φ_i and Φ_j by their values in the trade share equation, we get an expression for the trade share in terms of relative prices:

$$\frac{X_{ij}}{X_j} = \left(\frac{P_{ij}}{P_j}\right)^{-\theta} \tag{1}$$

The problem is that while the distribution of prices in destination j is observed, the potential distribution of costs from i to j is not. This is because all not-lowest cost producers are eliminated through trade selection. As shown by Eaton and Kortum (2002), observed imports from i to j adjusted for the trade share inherit the price distribution in the destination:

$$\begin{aligned}
\pi_{ij} G_j(p) &= \int_0^p \exp\left[-\sum_{s=1}^{N} T_s \left(c_s \tau_{sj}\right)^{-\theta} p^{\theta}\right] \theta T_i \left(c_i \tau_{ij}\right)^{-\theta} p^{\theta-1} \,\mathrm{d}p \\
&= \frac{T_i \left(c_i \tau_{ij}\right)^{-\theta}}{-\Phi_j} \int_0^p \exp\left[-\Phi_j p^{\theta}\right] \left[-\Phi_j\right] \theta p^{\theta-1} \,\mathrm{d}p \\
&= \pi_{ij} \left[1 - \exp\left\{-\Phi_j p^{\theta}\right\}\right]
\end{aligned}$$

To estimate θ using information on observed prices, it is therefore necessary to devise an estimation procedure which uses destination price indices. Three remarks are in order before we move on to monopolistic competition.

- The correct weights for computing source and destination price indices are destination-specific since for products exported to j, i inherits j's pdf. The destination-specific price index should be computed over all goods in j, with j-weights.

- Price imputation for goods that i does not export to j using prices observed in j cannot be justified since it is only in the precise range that i effectively exports to j that i has j's price distribution. Nothing can be inferred about non-observed prices since out of the exported product range i has a price distribution which differs from j's. We cannot infer P_{ij} using prices observed in j, nor can we infer P_{ij} using prices observed in i.

- The data we use does not allow estimating the efficiency heterogeneity parameter in the Ricardian framework because observed prices of exports from i to j do not correspond to the least-cost distribution in source i across the goods' continuum given i technology parameters. Instead they mimick the least cost distribution in j given technology parameters in all sources and each source bilateral costs with j.

1.3 Set-up and notation in monopolistic competition

In monopolistic competition each firm's production technique is its private property and each firm's output is a specific variety within some sector k. The number of varieties within each sector is infinite but countable. Each variety is indexed by α, the order of appearance in in the ordered costs' distribution among production techniques available in sector k. Consumption choices at the intrasectoral level are described by a CES aggregator:

$$Q(k) = \left[\sum_{\alpha=1}^{\infty} Q^{(\alpha)}(k)^{\frac{\sigma'-1}{\sigma'}}\right]^{\frac{\sigma'}{\sigma'-1}}$$

with $\sigma' \geq \sigma > 1$ which are respectively the intra- and intersectoral substitution elasticities, and σ' finite. The share of expenditure on some intrasectoral variety is then a function of the relative price of the variety to the sectoral price index:

$$\frac{X^{(\alpha)}(k)}{X(k)} = \left[\frac{P^{(\alpha)}(k)}{P(k)}\right]^{1-\sigma'}$$

where the sectoral price index is:

$$P(k) = \left[\sum_{\alpha=1}^{\infty} P^{(\alpha)}(k)^{1-\sigma'}\right]^{\frac{1}{1-\sigma'}}$$

Since intersectoral consumption choices are represented by a CES aggregator, the expenditure on a sectoral good is given by:

$$\frac{X(k)}{X} = \left[\frac{P(k)}{P}\right]^{1-\sigma}$$

where the overall price index is:

$$P = \left[\int_0^1 P(k)^{1-\sigma}\,\mathrm{d}k\right]^{\frac{1}{1-\sigma}}$$

Eaton and Kortum (2010) show that under appropriate parameter restrictions, two set-ups are possible for monopolistic competition to get a well-defined price index.[4] First, the set-up in which there are no overhead costs

[4] See Theorem 2 in Eaton and Kortum (2010).

and therefore all varieties are available : parameter restrictions are $1 < \sigma < \theta + 1 < \sigma'$ (in perfect competition: $\sigma' \to \infty$). Second, the set-up in which there are overhead costs which restrict available varieties to the subset $\alpha = 1, ..., A(k)$ where $A(k) = \max\left\{\alpha : W^{(\alpha)}(k) \leq \overline{w}\right\}$. Parameter restrictions are $1 < \sigma < \theta + 1$ and $\sigma' = \sigma$ with an upper bound on costs $\overline{w}$.[5] In this note, we recall the main steps of deriving the price index with overhead costs and a bounded number of available varieties (see Eaton and Kortum (2010) for details) because this set-up is directly comparable to Chaney (2008) where bilateral fixed costs of entry into foreign markets are incorporated into the model. We show that in this set-up the efficiency heterogeneity parameter θ cannot be estimated with our data while the substitutability parameter σ can be estimated.

1.4 Price index computation with monopolistic competition in intrasectoral varieties

Overhead costs restrict the number of varieties available in each sector to the subset $\alpha = 1, ..., A(k)$ where $A(k) = \max\left\{\alpha : W^{(\alpha)}(k) \leq \overline{w}\right\}$. Further, $\sigma' = \sigma$ by assumption. In this set-up it is indifferent whether the price index is computed as the $(1 - \sigma)$ moment of expected sectoral prices or as the $(1 - \sigma)$ moment of goods' prices across the goods' continuum.

As shown by Eaton and Kortum (2010), the aggregate price index is computed in two steps: first as a function of Φ and of the cost cut-off $\overline{w}$, then by deriving the expression for $\overline{w}$ as a function of Φ, the ratio of market size X to fixed entry costs E defined in terms of labor, and model parameters σ and θ.

The sectoral price index is now given by the restricted set of varieties:

$$P(k)^{1-\sigma} = \sum_{\alpha=1}^{A(k)} \left(P^{(\alpha)}(k)\right)^{1-\sigma}$$

Forming the expectation across all sectors and using the result from profit

[5] The true parameter restriction in Eaton and Kortum (2010) is $\sigma' \geq \sigma$ but in practice they work with $\sigma' = \sigma$.

maximization that price is a fixed mark-up over costs (see below):

$$\begin{aligned} E\left[P^{1-\sigma}(k)\right] &= E\left[\sum_{\alpha=1}^{A(k)} P^{(\alpha)}(k)^{1-\sigma}\right] \\ E\left[\sum_{\alpha=1}^{A(k)} \left(MW^{(\alpha)}(k)\right)^{1-\sigma}\right] &= M^{1-\sigma} E\left[\sum_{\alpha=1}^{A(k)} W^{(\alpha)}(k)^{1-\sigma}\right] \end{aligned}$$

The conditional distribution of costs is given by $Pr[W^{\alpha} \leq w | W^{\alpha} \leq \overline{w}] = \left(\frac{w}{\overline{w}}\right)^{\theta}$, with density $\theta w^{\theta-1}\overline{w}^{-\theta}$, and the number of costs allowing production with unit cost less than $\overline{w}$ is given by $\Phi\overline{w}^{\theta}$.[6] This allows rewriting the expectation formed over the sum of ordered costs in the range $[1, ..., A(k)]$ as the product of the expected number of cost draws below $\overline{w}$ and the expected cost of each such draw:

$$\begin{aligned} E\left[P(k)^{1-\sigma}\right] &= M^{1-\sigma}\Phi\overline{w}^{\theta} \int_0^{\overline{w}} w^{1-\sigma}\theta w^{\theta-1}\overline{w}^{-\theta}\,\mathrm{d}w \\ &= M^{1-\sigma}\Phi\theta \int_0^{\overline{w}} w^{\theta-\sigma}\,\mathrm{d}w \\ &= M^{1-\sigma}\Phi\frac{\theta}{\theta-\sigma+1}\overline{w}^{\theta-\sigma+1} \end{aligned} \tag{2}$$

The zero profit condition is used to express $\overline{w}$ as a function of Φ and X/E. Variable profit of some variety is $\left\{\Pi^v(w) = (p-w)X^{(\alpha)}(k)/p\right\}$. Profit maximization gives $p = Mw$ where $\{M = \sigma/(\sigma-1)\}$. Replacing in the expression for variable profit:

$$\Pi^v(w) = \frac{(M-1)w}{Mw}X^{(\alpha)}(k) = \frac{X^{(\alpha)}(k)}{\sigma}$$

Using the demand equation for the variety $\left\{X^{(\alpha)}(k) = \left(\frac{P^{(\alpha)}(k)}{P}\right)^{(1-\sigma)} X\right\}$ and expressing price in terms of the marginal cost:

$$\Pi^v(w) = \frac{X}{\sigma}\left(\frac{Mw}{P}\right)^{(1-\sigma)}$$

[6]For an individual good, the number of costs which deliver cost less than some upper bound w is distributed Poisson with parameter Φw^{θ}. Across the goods' continuum Φw^{θ} is the measure of goods which can be produced with cost less than w.

For entry to be profitable, the fixed entry cost has to be covered by variable profit: $\left\{E \leq \frac{X}{\sigma}\left(\frac{Mw}{P}\right)^{(1-\sigma)}\right\}$. Rearranging to solve for the cut-off cost $\overline{w}$ gives:

$$\overline{w} = \left(\frac{E\sigma}{X}\right)^{1/(1-\sigma)} \frac{P}{M}$$

Replacing $\overline{w}$ by its value in the expression for the price index:

$$P^{1-\sigma} = M^{1-\sigma} \frac{\Phi\theta}{\theta-(\sigma-1)} \left(\frac{E\sigma}{X}\right)^{\frac{\theta-(\sigma-1)}{1-\sigma}} P^{\theta-(\sigma-1)} M^{-(1-\sigma)-\theta}$$

Rearranging and simplifying:

$$P = \Phi^{-1/\theta} M \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-1/\theta} \left(\frac{X}{E\sigma}\right)^{\frac{-[\theta-(\sigma-1)]}{(\sigma-1)\theta}} \qquad (3)$$

In monopolistic competition, just as in perfect competition, the destination-specific price index $P_j = \left[\int_0^1 P_j(k)^{1-\sigma}\,\mathrm{d}k\right]^{1/(1-\sigma)}$ which is well defined under parameter restrictions $1 \leq \sigma = \sigma' < \theta + 1$ and upper bound on available varieties $\overline{w}$ can be used to estimate the underlying heterogeneity parameter θ. Using (3), the destination parameter $\Phi_j = \sum_{s=1}^{N} T_s(c_s\tau_{sj})^{-\theta}$ can be written as a function of the ideal price index across the surviving firms from all sources present in this destination:

$$\Phi_j = P_j^{-\theta} M^{\theta} \left(\frac{\theta-(\sigma-1)}{\theta}\right) \left(\frac{E_j\sigma}{X_j}\right)^{\frac{\theta-(\sigma-1)}{(\sigma-1)}}$$

Fixed entry costs are defined in terms of destination market labor: $E_j = c_j L_j$ where c_j is the labor cost in j and L_j is the number of mobilized labor units. This definition of entry costs means that we do not first ask whether the firm is present in the domestic market i and as a second step whether it is able to export to j, but rather we ask whether the firm from i can get in any market j (including the domestic i market) given its cost draw from the i-specific productivity distribution and the fixed entry cost in j which has to be paid by any firm wishing to operate in j. The measure of active sellers in j is defined across all firms active in j: $H_j = \Phi_j \overline{w_j}^{\theta}$. This is just the number of expected cost draws below the cost cut-off. Using the definition of cut-off cost:

$$H_j = \Phi_j \left[\left(\frac{\theta - (\sigma - 1)}{\theta} \frac{X}{E\sigma} \right)^{1/\theta} \Phi^{-1/\theta} \right]^{\theta} \quad = \quad \frac{\theta - (\sigma - 1)}{\theta\sigma} \frac{X}{E}$$

Under the assumption that there is no upper bound on productivity draws and that entry costs are market-specific and common to all firms, a subset of producers from each source i survives in all sectors k in each destination j. Trade-driven selection does not impede observing source-specific costs' distribution across the goods' continuum. This source-specific distribution is observed in the sectoral bilateral trade data. The price index for goods delivered from i to j is defined by:

$$P_{ij} \quad = \quad \left[\int_0^1 P_{ij}(k)^{1-\sigma} \, \mathrm{d}k \right]^{1/(1-\sigma)}$$

Using (2) this price index is:

$$P_{ij} \quad = \quad M \left(\frac{\Phi_{ij}\theta}{\theta - \sigma + 1} \right)^{1/1-\sigma} \overline{w_j}^{\frac{\theta - \sigma + 1}{(1-\sigma)}} \tag{4}$$

where $\Phi_{ij} = T_i(c_i\tau_{ij})^{-\theta}$ is the source-specific Poisson parameter for cost draws inclusive of bilateral trade costs while the cost cut-off is destination-specific. What matters for a firm from i to get into market j is its ability to overcome the fixed entry cost in j given its cost draw from the i-specific efficiency distribution.[7]

Using (2), the cif price of i exports relatively to the overall price index in j is:

$$\frac{P_{ij}}{P_j} \quad = \quad \frac{M \left(\frac{\Phi_{ij}\theta}{\theta-\sigma+1} \right)^{1/1-\sigma} \overline{w_j}^{\frac{\theta-\sigma+1}{(1-\sigma)}}}{M \left(\frac{\Phi_j\theta}{\theta-\sigma+1} \right)^{1/1-\sigma} \overline{w_j}^{\frac{\theta-\sigma+1}{(1-\sigma)}}}$$

Simplifying and solving for the Φ-ratio:

$$\frac{\Phi_{ij}}{\Phi_j} = \left(\frac{P_{ij}}{P_j} \right)^{(1-\sigma)}$$

[7] In the next section it will be shown that allowing for explicit fixed costs of exporting on top of entry costs in the domestic market will induce a selection margin among firms able to survive in i. But also in that model the fixed costs of exporting are paid in destination-specific labor units. This additional selection margin will not change the conclusion that from observed prices of exports from i to j only the substitutability parameter σ can be estimated.

Replacing the Φ-ratio in the right hand side of the trade share equation, we get back the CES demand equation:

$$\frac{X_{ij}}{X_j} = \left(\frac{P_{ij}}{P_j}\right)^{(1-\sigma)} \tag{5}$$

Thus, observed cif prices of exports can be used to estimate the substitutability parameter σ, but not the underlying heterogeneity parameter θ.

Several remarks are in order.

- In monopolistic competition with an upper bound on costs, the characteristics of our data only allow estimating the substitutability parameter σ, just as in perfect competition. In perfect competition, the cost distribution in i could not be deduced from observed prices of i products sold in j because the surviving lowest-cost firms replicated the price distribution in j for the first-best cost across all possible sources exporting to j. In monopolistic competition this price distribution is observed, but the cost cut-off is defined in terms of the destination-specific Poisson parameter for cost draws. This means that the Φ parameter which carries the $-1/\theta$ exponent is the destination-specific parameter which cancels in the ratio P_{ij}/P_j. This leaves a market share equation which underlines the tight link between the relative Φ's of the source and destination and their realized price distributions: i's market share in j is equal to the relative realized price of its goods in j to the power $(1-\sigma)$ or alternatively to the ratio Φ_{ij}/Φ_j.[8] To estimate θ using observed prices it is necessary to devise an estimation procedure which works with destination-specific price indices.

- Destination-specific sectoral weights should be used in any type of aggregation across observed sectoral relative prices since market shares and composite good prices for any source are determined by the overall destination-specific price distribution.

- Under model assumptions some trade would be observed in every sector k between all pairs ij. There being no zero trade flows, the question of whether prices should be imputed would not arise, and both trade shares and composite good relative prices would be computed over observed trade values and product prices.[9]

[8] In perfect competition the CES aggregator at the intersectoral level could be used to estimate σ using the fact that i's market share in j is given by the relative price of products in which i is lowest cost supplier to the price index over lowest cost draws across the goods' continuum in j.

[9] In the data zero trade flows are observed. The model must be modified to generate

1.5 Zero trade flows and the overestimation bias

Zero trade flows are a prevalent feature of the data. This means that while we observe sectoral market shares of the source in all sectors, we do not have the full set of information on the realized price distribution of the source across the continuum of sectors. This has to be explicitly taken into account in the estimation procedure. In this subsection we show that the Eaton and Kortum model provides two important keys for understanding the relationship between observed price indices and the underlying realized price distributions across the continuum of goods.

First, unobserved prices are necessarily above the maximum price observed in the destination in any sector. This is because the cost cut-off $\overline{w}$ defines the maximum landed price which makes entry profitable in the destination across the continuum of goods. Second, unobserved prices are source-specific because they are drawn from the source efficiency distribution: the relevant parameter is $\Phi_{ij}\overline{w_j}^{\theta}$ which gives the expected number of draws in i below the cost cut-off in the destination. These two features allow characterizing the relationship between the price index constructed from observed prices for any source i exporting to j and the underlying true price index across the goods' continuum. This characterization combined with the implications of the variance in the number of zeros across exporters to a given destination leads to the conclusion that the true underlying substitutability parameter is lower than the parameter estimated using observed prices.

Use the expression for the price index in (4) to show that an increase in the cost cut-off leads to a decrease in the ideal price index due to the variety effect: as the cut-off increases, the number of zero trade flows decreases.

$$\begin{aligned}\frac{\partial P_{ij}}{\partial \overline{w}} &= M\left(\frac{\Phi_{ij}\theta}{\theta-\sigma+1}\right)^{1/1-\sigma}\frac{\theta-\sigma+1}{1-\sigma}\overline{w_j}^{\frac{\theta}{(1-\sigma)}} \\ &= \frac{1}{1-\sigma}M(\Phi_{ij}\theta)^{1/(1-\sigma)}(\theta-\sigma+1)^{-\sigma/(1-\sigma)}\overline{w_j}^{\theta/(1-\sigma)} \qquad (6)\end{aligned}$$

where $\Phi_{ij} = T_i(c_i\tau_{ij})^{-\theta}$ is the source-specific Poisson parameter for cost draws inclusive of bilateral trade costs while the cost cut-off $\overline{w_j}$ is destination-specific. The first component is negative given $\sigma > 1$ while all other components are positive. The ideal price index decreases when the cost cut-off increases which means that as the cut-off increases, more firms from i are able to enter j market. Because we work with the goods' continuum, each additional firm corresponds to the elimination of a sectoral zero trade flow.

zero trade flows. One way to do this is to assume an upper bound on productivity draws. Another way to do this which we follow in the paper is to assume that zero trade flows correspond to mismeasured very small trade flows.

Use the expression for the average price of goods from i to show that eliminating the variety effect we verify that the price index corrected for the variety effect increases in the cost cut-off. Equation (2) gives the expression for the expected price of goods exported by i across the goods' continuum. The measure of active sellers from i in j for a given cost cut-off is given by $H_{ij} = \Phi_{ij}\overline{w_j}^{\theta}$. The expected average price is:

$$\begin{aligned} \frac{1}{H_{ij}} E\left[P(k)_{ij}^{1-\sigma}\right] &= \frac{\Phi_{ij}\overline{w_j}^{\theta-\sigma+1}}{\Phi_{ij}\overline{w_j}^{\theta}} M^{1-\sigma}\left(\frac{\theta}{\theta-\sigma+1}\right) \\ &= (M\overline{w_j})^{1-\sigma}\left(\frac{\theta}{\theta-\sigma+1}\right) \end{aligned}$$

The price index corrected for the variety effect $P_{ij}^{COR} = \left\{H_{ij}^{-1}E\left[P(k)_{ij}^{1-\sigma}\right]\right\}^{1/(1-\sigma)}$ is increasing in $\overline{w_j}$. Marginal entrants from i which reduce the number of sectoral zeros are high cost relatively to firms from i already present in the market.

$$P_{ij}^{COR} = M\left(\frac{\theta}{\theta-\sigma+1}\right)^{1/(1-\sigma)}\overline{w_j}$$

Consider the total differential of the price index P_{ij} assuming σ, θ invariant:

$$dP_{ij} = \frac{\partial P_{ij}}{\partial \Phi_{ij}} d\Phi_{ij} + \frac{\partial P_{ij}}{\partial \overline{w_j}} d\overline{w_j}$$

Consider an increase in the cost cut-off with Φ_{ij} constant. For any exporter i to j:

$$dP_{ij} = \left\{\frac{1}{1-\sigma} M(\Phi_{ij}\theta)^{1/(1-\sigma)}(\theta-\sigma+1)^{-\sigma/(1-\sigma)}\overline{w_j}^{\theta/(1-\sigma)}\right\} d\overline{w_j}$$

Consider two exporters i and s to some destination j. For a given small change in the cost cut-off, the relative change in their price indices is given by:

$$\frac{dP_{ij}}{dP_{sj}} = \frac{\partial P_{ij}/\partial \overline{w_j}}{\partial P_{sj}/\partial \overline{w_j}} = \left(\frac{\Phi_{ij}}{\Phi_{sj}}\right)^{1/1-\sigma}$$

Given $\sigma > 1$, $\frac{dP_{ij}}{dP_{sj}} < 1$ if and only if $\frac{\Phi_{ij}}{\Phi_{sj}} > 1$, eg if $\Phi_{ij} > \Phi_{sj}$. We conclude that for a given increase in the cost cut-off, the number of zero trade flows is reduced quicker for the exporter with a higher Φ[10] **and** his price index in j increases by less than the price index of the source with a smaller Φ.

[10] This comes from the definition of the mass of entrants $H_{ij} = \Phi_{ij}\overline{w}^{\theta}$. Differentiating with respect to the cost cut-off, $dH_{ij}/dH_{sj} = \Phi_{ij}/\Phi_{sj}$.

Consider the relative market share of i and s in j.

$$\frac{X_{ij}}{X_{sj}} = \frac{\Phi_{ij}}{\Phi_{sj}}$$

It is straightforward that the source with the higher Φ has higher market share in the destination.

Combining all of the previous results:

- the relatively high Φ source will have a higher market share and a lower price index in destination
- for a given cost cut-off, the relatively high Φ source will have a smaller share of sectoral zero trade flows
- for a given increase in the cost cut-off, the relatively high Φ source will have a higher number of additional entrants (eg a stronger reduction in sectoral zero trade flows) while its price index will increase by less.

It follows that for a given pair of exporters with $X_{ij}/X_{sj} > 1$, the observed relative price index P_{ij}/P_{sj} is higher than the true underlying realized price distribution in the two sources across the continuum of sectors: $P_{ij}/P_{sj} >> P^R_{ij}/P^R_{sj}$. The observed price index is underestimated by more for small exporters. Generalizing to N exporters: for a given variance in market shares, the variance in unobserved true realized price distributions is higher than the variance in observed prices because of the strong negative correlation in the number of lacking prices and market shares. Another way to see that the price index is underestimated by more for small exporters is to note that unobserved prices in any sector for any source are higher than the maximum observed price in the destination (implication 1 of the model). As the small exporter has a greater share of unobserved prices, its true underlying price index is necessarily higher than the true underlying price index of a big exporter.[11]

The substitutability parameter estimated on observed price indices and market shares ($\widetilde{\sigma}$) is overestimated relatively to the true underlying substitutability parameter (σ). This is because the variation in the number of unobserved prices across exporters entails that a given percentage change in

[11] We verify that in the data smaller market share is associated with a greater number of sectoral zero trade flows, as predicted by the model. We find that overtime, the strength of the negative correlation between the number of zeros and market shares is progressively reduced. In terms of the model, this means that the variance in Φ's is reduced overtime. This is consistent with the finding by Levchenko and Zhang (2011) that there is a reduction in the strength of comparative advantage across countries over the period 1962-2009.

market share is linked to a higher underlying (relatively to observed) percentage change in prices.

$$
\begin{aligned}
(\widetilde{\sigma}-1) &= \ln\left[\frac{X_{ij}/X_{sj}}{P_{sj}/P_{ij}}\right] \\
&\geq \\
(\sigma-1) &= \ln\left[\frac{X_{ij}/X_{sj}}{P^R_{sj}/P^R_{ij}}\right]
\end{aligned}
\tag{7}
$$

2 Estimation of the trade elasticity in the Melitz-Chaney framework

This section goes over the main features of the Melitz-Chaney microfoundation of the gravity structure of trade. First, it presents the Chaney (2008) model and the computation of the price index. It shows that it is not possible to estimate the degree of efficiency variability across firms using data on observed cif prices of bilateral exports. Only the substitutability parameter σ can be estimated with these data.

The section recalls that if bilateral fixed costs of trade depend on distance, the distance elasticity estimated in gravity equations in the context of the Chaney framework would be inversely dependent on σ. Our finding that the substitutability parameter has increased in 1962-2009 would only deepen the distance puzzle. If however it is assumed that fixed costs are not distance-dependent, then the trade elasticity in the Chaney framework would correspond to the efficiency heterogeneity parameter γ.

To estimate γ on aggregate trade trade it is necessary to make the assumption of a common efficiency variability parameter across sectors so that $\gamma_k = \gamma$.[12] This section concludes that given the structural proximity of this model to Eaton and Kortum (2010), γ can be estimated using the methodology suggested in 1 to estimate θ.[13] This methodology is based on destination-specific price indices.

Each source i produces a homogeneous good indexed $k = 0$ with CRS technology. This good is freely traded. This pins down relative wages: the price of this good is normalized to 1, and the wage in country i, c_i, is the

[12] See Imbs and Méjean (2011).

[13] This is because we seek to estimate the trade elasticity parameter for aggregate trade. Crozet and Koenig (2010) show how the panel dimension of firm-level exports' data can be used to identify sector-specific γ_k, σ_k, and ρ_k. Non-trade data such as firm-level sales or employment has been used to estimate Pareto distribution parameters at the sectoral or economy-wide level in Luttmer (2007); Di Giovanni et al. (2011); Chatterjee and Rossi-Hansberg (2012).

number of units of the numeraire good produced with one unit of labor. Thus, labor efficiency may vary across countries.

There is a finite number of sectors $k = 1, ..., K$ which each consist of a continuum of differentiated varieties. A firm, characterized by efficiency $z_{k,\alpha}$ drawn from a productivity distribution assumed common across countries but sector-specific, produces some variety $\alpha(k)$, paying a constant firm-specific marginal cost $w_i(k) = c_i/z^{(\alpha)}(k)$.

To enter any market, the firm pays per unit pair- and sector-specific variable trade costs $\tau_{ij}(k)$ as well as sector-specific fixed costs which are assumed to be pair-specific $f_{ij}(k)$.

Consumer preferences are assumed well-represented by a Cobb-Douglas utility function at the intersectoral level and CES at the intrasectoral level. The substitutability parameter σ_k is sector-specific. Varieties within any sector are indexed by α.

$$U = q_0^{\mu_0} \prod_{k=1}^{K} \left[\int_0^1 q_k(\alpha)^{\frac{\sigma_k - 1}{\sigma_k}} \, \mathrm{d}\alpha \right]^{\mu_k \frac{\sigma_k}{\sigma_k - 1}}$$

Firms draw their productivity from a Pareto distribution with support $[1, \infty)$, and it is assumed that the shape parameter γ_k of the Pareto is common across countries but sector-specific.[14]

$$\Pr(Z \leq z) = 1 - z^{-\gamma_k}$$

As product differentiation is assumed costless, each firm optimally chooses to produce a unique variety. Varieties within each sector can be ranked by the efficiency $z^{(\alpha)}(k)$ of the producing firm. Given CES demand, the price of any variety is a constant mark-up charged over production and variable trade costs:

$$p_{ij}^{(\alpha)}(k) = \frac{\sigma_k}{\sigma_k - 1} \left[c_i \tau_{ij}(k) / z^{(\alpha)}(k) \right]$$

Given CD intersectoral demand, expenditure in country j on goods in sector k is given by $\mu_k Y_j$. Using the properties of the CES aggregator at the intrasectoral level, demand for the firm-specific variety is:

$$x_{ij}^{(\alpha)}(k) = \left[\frac{p_{ij}^{(\alpha)}(k)}{P_j(k)} \right]^{1-\sigma_k} \mu_k Y_j$$

[14]Parameter restrictions are $\sigma_k < \gamma_k + 1$.

where $P_j(k)$ is the ideal price index in country j in sector k. Define the firm-specific production cost inclusive of variable trade costs $v_{ij}^{(\alpha)}(k) = c_i \tau_{ij}(k)/z^{(\alpha)}(k)$. Using the zero profit condition for firms in sector k in source i exporting to destination j, we can express the cut-off cost $\overline{v_{ij}(k)}$ as a function of the sectoral ideal price index $P_j(k)$ and the fixed bilateral trade cost $f_{ij}(k)$. Net profits of a firm are given by:

$$\pi_{ij}^{(\alpha)}(k) = \left[p_{ij}^{(\alpha)}(k) - v_{ij}^{(\alpha)}(k)\right] q_{ij}^{(\alpha)}(k) - f_{ij}(k)$$

where $q_{ij}^{(\alpha)}(k) = x_{ij}^{(\alpha)}(k)/p_{ij}^{(\alpha)}(k)$. Defining the mark-up $M = \sigma_k/(\sigma_k - 1)$, replacing $q_{ij}^{(\alpha)}(k)$ and $p_{ij}^{(\alpha)}(k)$ by their values, and setting $\pi_{ij}^{(\alpha)}(k) = 0$ for the cut-off firm:

$$f_{ij}(k) = \left(\frac{M\overline{v_{ij}(k)}}{P_j(k)}\right)^{1-\sigma_k} \frac{\mu_k Y_j}{\sigma_k}$$

Therefore, the cut-off cost for entering market j for firm from i in sector k is:

$$\overline{v_{ij}(k)} = \left[\frac{\mu_k Y_j}{f_{ij}(k)\sigma_k}\right]^{1/(\sigma_k-1)} \frac{P_j(k)}{M}$$

The sectoral ideal price index $P_j(k)$ is given by the summation across all countries of the firms with a cost below the cut-off:

$$P_j(k) = \left[\sum_{i=1}^{N} c_i L_i \int_0^{\overline{v_{ij}(k)}} \left[M v_{ij}(k)\right]^{1-\sigma_k} \mathrm{d}v_{ij}(k)\right]^{1/(1-\sigma_k)}$$

where $c_i L_i$ is total labor income in source i which is proportional to the potential mass of entrants in any sector k in i.

Using the assumption that the productivity distribution is sector but not country-specific, defining the cut-off productivity draw $\overline{z_{ij}(k)}$ as a function of $\overline{v_{ij}(k)}$, change the variable of integration to z to get:

$$P_j(k) = \left[\sum_{i=1}^{N} c_i L_i \int_{\overline{z_{ij}(k)}}^{\infty} \left[M c_i \tau_{ij}(k)\right]^{1-\sigma_k} \gamma_k z^{\sigma_k - 1} z^{-\gamma_k - 1} \mathrm{d}z\right]^{1/(1-\sigma_k)}$$

Rearranging and solving for the integral:

$$P_j(k) \quad = \quad M\left(\frac{-\gamma_k}{\sigma_k-(\gamma_k+1)}\right)^{1/(1-\sigma_k)}\left[\sum_{i=1}^{N} c_i L_i \left(c_i\tau_{ij}(k)\right)^{1-\sigma_k}\left(\overline{z_{ij}(k)}\right)^{\sigma_k-(\gamma_k+1)}\right]^{1/(1-\sigma_k)}$$

Working with the summation where we replace $\overline{z_{ij}(k)}$ by its value:

$$\sum_{i=1}^{N} c_i L_i \left(c_i\tau_{ij}(k)\right)^{1-\sigma_k}\left(c_i\tau_{ij}(k)/\overline{v_{ij}(k)}\right)^{\sigma_k-1-\gamma_k} = \sum_{i=1}^{N} c_i L_i \left(c_i\tau_{ij}(k)\right)^{-\gamma_k}\left(\overline{v_{ij}(k)}\right)^{\gamma_k-(\sigma_k-1)}$$

Defining $T_i = c_i L_i$, replacing $\overline{v_{ij}(k)}$ by its value, we get:

$$\sum_{i=1}^{N} T_i \left(c_i\tau_{ij}(k)\right)^{-\gamma_k}\left\{\left[\frac{\mu_k Y_j}{f_{ij}(k)\sigma_k}\right]^{1/(\sigma_k-1)}\frac{P_j(k)}{M}\right\}^{\gamma_k-(\sigma_k-1)}$$

Rearranging gives:

$$\left[\frac{\mu_k Y_j}{\sigma_k}\right]^{\frac{\gamma_k-(\sigma_k-1)}{(\sigma_k-1)}}\left[\frac{P_j(k)}{M}\right]^{\gamma_k-(\sigma_k-1)}\sum_{i=1}^{N} T_i \left(c_i\tau_{ij}(k)\right)^{-\gamma_k}\left[f_{ij}(k)\right]^{1-\frac{\gamma_k}{(\sigma_k-1)}}$$

Define $\Psi_j(k) = \sum_{i=1}^{N} T_i \left(c_i\tau_{ij}(k)\right)^{-\gamma_k}\left[f_{ij}(k)\right]^{1-\frac{\gamma_k}{(\sigma_k-1)}}$. Replacing in the expression for the sectoral price index:

$$P_j(k)^{1-\sigma_k} \quad = \quad M^{1-\sigma_k}\frac{\gamma_k}{\gamma_k-(\sigma_k-1)}\left[\frac{\mu_k Y_j}{\sigma_k}\right]^{\frac{\gamma_k-(\sigma_k-1)}{(\sigma_k-1)}}\left[\frac{P_j(k)}{M}\right]^{\gamma_k-(\sigma_k-1)}\Psi_j(k)$$

Solving for the sectoral price index in j:

$$P_j(k)^{-\gamma_k} \quad = \quad M^{-\gamma_k}\frac{\gamma_k}{\gamma_k-(\sigma_k-1)}\left[\frac{\mu_k Y_j}{\sigma_k}\right]^{\frac{\gamma_k-(\sigma_k-1)}{(\sigma_k-1)}}\Psi_j(k)$$

Using the definition of exports for an individual firm, and the proportionality assumption on the mass of potential entrants in any country i, derive the expression for total bilateral sectoral exports.

$$X_{ij}(k) \quad = \quad c_i L_i \int_{\overline{z_{ij}(k)}}^{\infty} x_{ij}^{(\alpha)}(k)\gamma_k z^{-(\gamma_k+1)}\,\mathrm{d}z$$

Using $T_i = c_i L_i$ and replacing $x_{ij}^{(\alpha)}(k)$ by its value:

$$X_{ij}(k) = T_i \mu_k Y_j \int_{\overline{z_{ij}(k)}}^{\infty} \left(\frac{p_{ij}^{(\alpha)}(k)}{P_j(k)} \right)^{1-\sigma_k} \gamma_k z^{-(\gamma_k+1)} \, \mathrm{d}z$$

Replacing $p_{ij}^{(\alpha)}(k)$ by its value:

$$X_{ij}(k) = T_i \mu_k Y_j \gamma \left(\frac{M c_i \tau_{ij}(k)}{P_j(k)} \right)^{1-\sigma_k} \int_{\overline{z_{ij}(k)}}^{\infty} z^{\sigma_k - 1} z^{-(\gamma_k+1)} \, \mathrm{d}z$$

Solving for the integral:

$$X_{ij}(k) = T_i \frac{\overline{z_{ij}(k)}^{\sigma_k - 1 - \gamma_k}}{\gamma_k - (\sigma_k - 1)} \mu_k Y_j \gamma_k \left(\frac{M c_i \tau_{ij}(k)}{P_j(k)} \right)^{1-\sigma_k}$$

Using the definition of the cut-off productivity,

$$\overline{z_{ij}(k)} = M c_i \tau_{ij}(k) P_j(k)^{-1} (\mu_k Y_j / \sigma_k)^{1/(1-\sigma_k)} f_{ij}(k)^{-1/(1-\sigma_k)}$$

total bilateral exports are given by:

$$X_{ij}(k) = T_i \left(\frac{M c_i \tau_{ij}(k)}{P_j(k)} \right)^{\sigma_k - 1 - \gamma_k} \left(\frac{\mu_k Y_j}{\sigma_k} \right)^{\frac{\sigma_k - 1 - \gamma_k}{1-\sigma_k}} f_{ij}(k)^{\frac{\gamma_k - (\sigma_k - 1)}{1-\sigma_k}} \frac{\gamma_k}{\gamma_k - (\sigma_k - 1)} \mu_k Y_j \left(\frac{M c_i \tau_{ij}(k)}{P_j(k)} \right)^{1-\sigma_k}$$

Simplifying:

$$X_{ij}(k) = T_i \frac{\gamma_k}{\gamma_k - (\sigma_k - 1)} \left[\frac{M c_i \tau_{ij}(k)}{P_j(k)} \right]^{-\gamma_k} [\mu_k Y_j]^{-\gamma_k/(1-\sigma_k)} [f_{ij}(k) \sigma_k]^{\frac{\gamma_k - (\sigma_k - 1)}{(1-\sigma_k)}}$$

Replacing $P_j(k)^{-\gamma_k}$ by its value and simplifying:

$$X_{ij}(k) = T_i \mu_k Y_j \Psi_j(k)^{-1} [c_i \tau_{ij}(k)]^{-\gamma_k} [f_{ij}(k)]^{\frac{\gamma_k - (\sigma_k - 1)}{(1-\sigma_k)}}$$

Define $\Psi_{ij}(k) = T_i [c_i \tau_{ij}(k)]^{-\gamma_k} [f_{ij}(k)]^{\frac{\gamma_k - (\sigma_k - 1)}{(1-\sigma_k)}}$. The market share of country i in sector k in j is:

$$\frac{X_{ij}(k)}{\mu_k Y_j} = \frac{\Psi_{ij}(k)}{\Psi_j(k)}$$

Having obtained a market share equation at the sectoral level in terms of the Ψ-ratio, we want to express this ratio in terms of observed price distributions in our data. Having solved for the sectoral price index in the destination $P_j(k)$ as a function of $\Psi_j(k)$, we just need to solve for the ideal price index $P_{ij}(k)$ across varieties exported by i to j in sector k:

$$P_{ij}(k)^{1-\sigma_k} = c_i L_i \int_{\overline{z_{ij}(k)}}^{\infty} p_{ij}^{(\alpha)}(k)^{1-\sigma_k}\,\mathrm{d}z$$

Replacing $p_{ij}^{(\alpha)}(k)$ by its value and solving for the integral:

$$P_{ij}(k)^{1-\sigma_k} = \frac{\gamma_k}{\gamma_k-(\sigma_k-1)} T_i \left(M c_i \tau_{ij}(k)\right)^{1-\sigma_k} \overline{z_{ij}(k)}^{\sigma_k-\gamma_k-1}$$

Replacing the cut-off productivity by its value:

$$P_{ij}(k)^{1-\sigma_k} = \frac{\gamma_k}{\gamma_k-(\sigma_k-1)} T_i \left[M c_i \tau_{ij}(k)\right]^{-\gamma_k} P_j(k)^{\gamma_k-(\sigma_k-1)} \left[\frac{\mu_k Y_j}{f_{ij}(k)\sigma_k}\right]^{(\sigma_k-\gamma_k-1)/(1-\sigma_k)}$$

Replacing $P_j(k)^{\gamma_k}$ by its value, rearranging, and simplifying:

$$\left(\frac{P_{ij}(k)}{P_j(k)}\right)^{1-\sigma_k} = \frac{\Psi_{ij}(k)}{Psi_j(k)}$$

Thus, the observed price distribution from i to j can be used in the sectoral market share equation to get the substitutability parameter σ_k, but not the efficiency variability parameter γ_k.

Consider a reformulation of the utility function of consumers as Cobb-Douglas between the homogeneous good and the bundle constituted by the differentiated goods, and assume a two-tier CES function for the differentiated goods' bundle:

$$U = q_0^{1-\mu} \left[\sum_{k=1}^{K} Q(k)^{\sigma-1/\sigma}\right]^{\mu(\sigma/\sigma-1)}$$

where sectoral consumption across all varieties α is $Q(k) = \left[\int_0^1 q(\alpha)^{\frac{\sigma_k-1}{\sigma_k}}\,\mathrm{d}\alpha\right]^{\frac{\sigma_k}{\sigma_k-1}}$.

The parameter we seek to estimate is the trade elasticity at the level of aggregate trade in differentiated goods. This amounts to assuming away sector-specific dimensions: $\gamma_k = \gamma$ and $\sigma_k = \sigma$. This allows redefining the bundle constituted by differentiated goods as a continuum of differentiated varieties.

Suppressing sectoral subscripts, we get the following expressions for aggregate bilateral exports, the price index in the destination, and the Ψ-parameters:

$$X_{ij} = \frac{T_i \left[c_i \tau_{ij}\right]^{-\gamma} \left[f_{ij}\right]^{\frac{\gamma-(\sigma-1)}{(1-\sigma)}}}{\sum_{s=1}^{N} T_s \left(c_s \tau_{sj}\right)^{-\gamma} \left[f_{sj}\right]^{\frac{\gamma-(\sigma-1)}{(1-\sigma)}}} \mu Y_j$$

$$P_j = M \left(\frac{\gamma}{\gamma-(\sigma-1)}\right)^{-1/\gamma} \left[\frac{\mu Y_j}{\sigma}\right]^{\frac{\gamma-(\sigma-1)}{\gamma(1-\sigma)}} \Psi_j^{-1/\gamma}$$

$$\Psi_j = \sum_{s=1}^{N} T_s \left(c_s \tau_{sj}\right)^{-\gamma} \left[f_{sj}\right]^{\frac{\gamma-(\sigma-1)}{(1-\sigma)}}$$

$$\Psi_{ij} = T_i \left[c_i \tau_{ij}\right]^{-\gamma} \left[f_{ij}\right]^{\frac{\gamma-(\sigma-1)}{(1-\sigma)}}$$

These expressions are structurally similar to the ones obtained in the monopolistic set-up of the Eaton and Kortum framework with a bounded number of available varieties because firm entry in foreign markets is determined by destination-specific characteristics. The difference resides in that Ψ_j in the Chaney framework includes a pair-specific bilateral fixed cost while in the Eaton and Kortum framework this fixed cost is invariant by source and therefore not in Φ_j.

In the Chaney framework, the functional form of the bilateral fixed entry cost f_{ij} is not defined while in the Eaton and Kortum framework the overhead cost is defined in terms of the number and cost of labor units specific to the destination: $f_{ij} = f_j = c_j F_j$ where F_j is the number of workers. Chaney (2008) provides some empirical evidence of distance-dependent fixed costs of trade. A simple way of making these fixed costs source- and distance-dependent would be to write: $f_{ij} = c_j F_{ij} = c_j dist^{\rho_1}$. The functional form for variable trade costs would still be defined as $\tau_{ij} = dist^{\rho_0}$. Bilateral exports are then given by:

$$X_{ij} = \frac{T_i {c_i}^{-\gamma} {dist_{ij}}^{-(\gamma\rho_0+\rho_1\gamma/(\sigma-1)-\rho_1)}}{\sum_{s=1}^{N} T_s {c_s}^{-\gamma} {dist_{sj}}^{-(\gamma\rho_0+\rho_1\gamma/(\sigma-1)-\rho_1)}} \mu Y_j$$

In this case, the distance elasticity estimated in gravity equations is a combination of σ, γ, and ρ parameters. Given that the substitutability parameter dampens the sensitivity of trade flows to trade barriers, our finding of an increasing σ only deepens the distance puzzle. Given our results for σ, an increase in the distance coefficient would have to be explained by changes in γ, ρ_0, and ρ_1, with at least one of these parameters strictly increasing in 1962-2009.

If however it is assumed that, rather counter-intuitively, the number of labor units required to export to some destination depends on the distance between the source and the destination, the model would become equivalent to the Eaton and Kortum framework in monopolistic competition with a bounded number of available varieties due to an overhead cost defined as $f_{ij} = f_j = c_j F_j$. The distance coefficient would be defined as $\gamma\rho$. In this case, our results on the evolution of the substitutability would neither deepen, nor explain the distance puzzle.

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