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# STDP-Induced Periodic Encoding of Static Patterns in Balanced Recurrent Neural Networks

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## Abstract

We present learning simulation results on a balanced recurrent neural network of spiking neurons, in a situation where the sensory context is dominated by the internal influence. Under our settings, the spontaneous responses are random-like and the different input patterns are difficult to discriminate. We present a simple implementation of the classical STDP learning rule : the "balanced STDP" where the potentiation and depression effects compensate. We show that under the application of this rule, the network level of activity is preserved. The only change is qualitative : starting from a near-random activity, the network learns to associate a periodic response in the presence of a particular static input. During the application of the STDP, the activity progressively regularizes and the internal period slightly shrinks. This strong periodic shaping of the response is obtained for relatively light weight changes. This particular response is moreover found to be specific to one particular pattern out of the initial set, displaying interesting selectivity properties.

## 1 Introduction

The brain is a massive recurrent neural network where most of the signal arriving at the neuron comes from the internal random-like self-feeding activity. A recurrent neural network can be seen as a non-autonomous dynamical system which undergoes perturbations from the external world. The interesting regimes lie in the domain where the incoming signal is in concurrence with the internal self-sustained signal. In that situation, the external sensory signal is not easy to discriminate from the rest of the activity. Neglecting some sensory signals may indeed be favorable most of the time. However, in a context of high competitiveness and survival, the critical issue is to identify and discriminate some sensory patterns of particular interest among a series of closely comparable patterns.

Among the possible mechanisms that may allow to manage such discrimination, *phase transition* mechanisms have been proposed as an efficient and biologically plausible implementation [1]. In a sensory context, some resonance or qualitative change may take place in the presence of some specific patterns, which become salient. This qualitative change in the dynamics could moreover help to produce specific motor responses.

We present in this paper such a synaptic mechanism that allows to extract an arbitrary sensory input pattern out of a set of sensory patterns in a context of strong and irregular self-feeding activity. The first section presents the model which is composed of a set of integrate-and-fire neurons under the influence of static input patterns. The learning rule is a simple implementation of the well-known Spike-Timing Dependent Plasticity (STDP) [2, 3]. The second section presents in detail the learning of a particular sensory pattern out of a set of arbitrary patterns. The selectivity of the response is then taken in consideration, and a general discussion follows.

## 2 Model

### 2.1 Neuron model

**Neuron update** Consider a set of neurons labelled by indexes  $i \in \{1, \dots, N\}$ . Taking into account the refractory period  $\tau_r$ , the neuron individual update is given by the following expression (which is a variant of the standard Integrate and Fire neuron model):

$$\text{if } \max_{u \in [t-\tau_r, t]} V_i(u) < \theta \text{ then } S_i(t) = \delta(V_i(t) - \theta) \text{ else } S_i(t) = 0 \quad (1)$$

$$\tau_m \frac{d\varepsilon_i}{dt} = -\varepsilon_i + S_i(t) \quad (2)$$

$$V_i(t) = \sum_j w_{ij} \varepsilon_j(t - \tau) - \theta \varepsilon_i(t) + I_i(t) \quad (3)$$

where the time  $t$  is expressed in ms, and  $V_i(t)$  is the membrane potential of the  $i^{th}$  neuron;  $S_i(t)$  is a spike (represented as an impulse with the use of the Dirac  $\delta$  function);  $\theta$  is the firing threshold;  $\tau_m$  is the membrane leak;  $\varepsilon_i$  is the trace of previous activation potentials, according to the neuron membrane leak;  $w_{ij}$  is the synaptic weight (from neuron  $j$  to the neuron  $i$ );  $\tau$  is the transmission delay;  $I_i(t)$  is the external input.

This model displays the main properties of a spiking neuron model, in particular the resetting of the membrane potential after a spike is obtained with the subtracted term  $\theta \varepsilon_i(t)$  which corresponds to the value attained by the potential at the moment of the spike. The trace  $\varepsilon_i$  exponentially decays according to the leak factor. It may represent an instantaneous estimation of the firing rate of the neuron, and takes into account the whole history of the previous firings. Taking place in the right part of (3), the term  $w_{ij} \varepsilon_j$  corresponds to the postsynaptic potential at the synapse  $(i, j)$ . In the following simulation, we use a simple 1st order integration with resolution  $\delta_t$  between 0.5 and 1 ms.

**Neuron parameters** We use rather standard and plausible values for neuron parameters, i.e. the refractory period is  $\tau_r = 2$  ms, the membrane time constant is  $\tau_m = 10$  ms and the transmission delay is  $\tau = 10$  ms. The value of the firing threshold is fixed at 1. The resetting potential is 0. It can be noticed that the value of the neuron potential is not scaled to realistic values and thus considered as unitless. The temporal behavior of the neuron as a pulsing computing unit remains however consistent with the biology.

### 2.2 Weights setting

In modeling work, it is important to define the network parameters in the most generic way for the obtained behavior to be reproducible in various implementations. In particular, a change in the time resolution or in the number of neurons should not induce qualitative changes in the behavior of the system.

The networks we simulate belong to the category of random recurrent neural. In our model, the weights are independent and strongly heterogeneous. In particular, the strength of the total incoming weights varies from one neuron to another. This individual weights sum is defined by  $J_i = \sum_j w_{ij}$ . The realization of the weights sums  $J_i$ 's depends on the drawing of every incoming weight. If the individual weights obey to a distribution of mean  $\mu_w$  and standard deviation  $\sigma_w$  (where every weight is independent, i.e. there is no structure, neighbourhood or topology in the network), then we have  $\mu_J \simeq N\mu_w$  and  $\sigma_J \simeq \sqrt{N}\sigma_w^2$ .

The individual synaptic weights may thus obey to a distribution whose mean is  $\frac{\mu_J}{N}$  and whose standard deviation is  $\frac{\sigma_J}{\sqrt{N}}$ . In this paper, the individual synaptic weights are set according to a Gaussian draw  $\mathcal{N}\left(\frac{\mu_J}{N}, \frac{\sigma_J^2}{N}\right)$ . The extension to more elaborate distributions with sparse connectivity and explicit excitatory and inhibitory weights is however rather straightforward according to the  $(\mu_J, \sigma_J)$  constraint and does not qualitatively change the behavior of the network. In the same fashion, the simulations presented in the following take place on rather small neuronal networks composed of 200 neurons but could be extended with the same global parameters to much larger sizes.

The following parameters are chosen in order to allow the internal self-sustained activity to compete with the external stimulation. It can be noticed that a precise analysis of the spontaneous activity of comparable random networks of integrate-and-fire neurons is given in [4].

The weights sum mean is  $\mu_J = 0$  so that the excitatory influences compensate the inhibitory ones (balanced networks configuration). The value of the weights sum standard deviation is  $\sigma_J = 2.4 \times \tau_m$ . Those value correspond to a strong internal influence (which is typically the case when the weights sum overtakes the threshold - here  $\theta = 1$ ).

### 2.3 Network input

The inputs we send to the network are distributed among every neuron. We define a set of 10 static patterns  $(\mathbf{I}^{(p)})_{p=1..10}$  which are random vectors of size  $N$  and whose values are randomly set according to a random draw  $\mathcal{N}(0, \sigma_I^2)$  where  $\sigma_I = 2$  in the simulations. Each static pattern is presented to the network for a duration which overtakes the mixing time of the system in order to converge to a stationary response. For compatibility with eq.(3), we model in the following expression the contribution of the input to the membrane potential:

$$\text{if } S_i(t) = 0 \text{ (no spike at } t) \text{ then } \tau_m \frac{dI_i(t)}{dt} = -I_i(t) + I_i^{(p)} \text{ else } I_i(t) = 0 \quad (4)$$

so that the input contribution is resetted after each spike.

### 2.4 Spontaneous activity

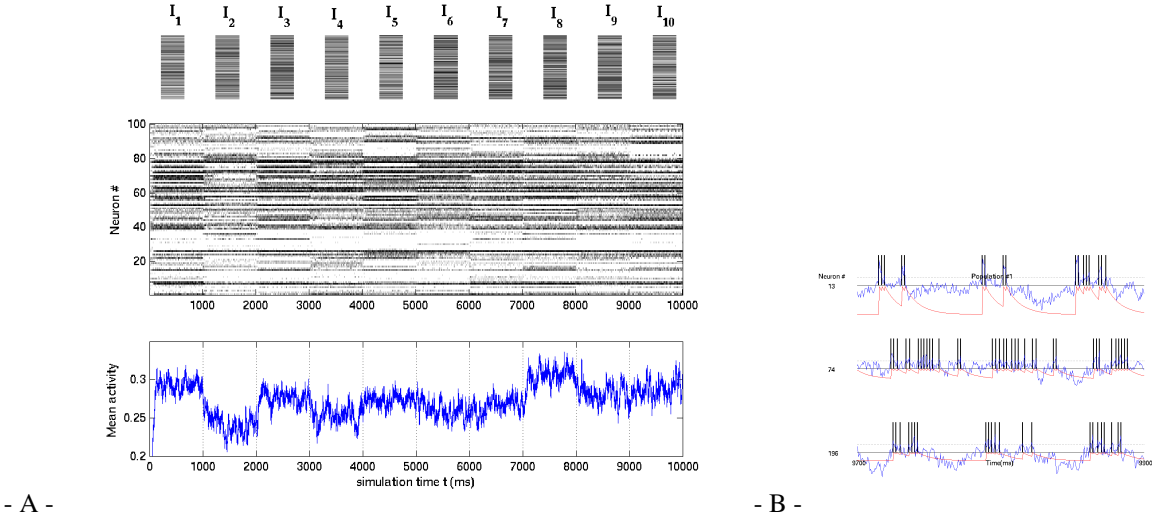


Figure 1: **Spontaneous dynamics.** - **A** - The 10 different input patterns obey to a random draw. They are shown on the upper part of the figure. The activity diagram of 100 neurons (out of 200) is given in the middle, under the successive influence of those patterns (every pattern lasts for 1 s). The lower part gives the mean activity of the whole population. - **B** - Three individual neuronal activities are extracted from the activity diagram between  $t = 9700$  and  $t = 9900$ . The neuron potential is in blue, the input contribution in red, and the spikes appear as vertical bars.

It is well-known that recurrent neuronal networks where the inhibitory influence compensates (or overtakes) the excitatory influence may develop a sustained irregular (chaotic) activity [5]. Depending on the respective excitatory or inhibitory influences, this activity may be purely irregular, or on the contrary display regular or irregular synchronous firing patterns [6].

The consequence of weights sum heterogeneity is less questioned in the literature, as the weights sum are often taken homogeneous. The consequence of the weights sum heterogeneity is the variability of individual neurons levels of activity: in a given population, the local unbalance implies that some neurons are repeatedly excited and saturate and other are repeatedly inhibited and fade

away. This heterogeneity in the activity pattern is however tunable with the inputs. A change in the input may partly participate in the remapping of the activity.

We present in figure 1A the activity diagram of 100 neurons of the network under the influence of 10 different static input signals. Every static input pattern lasts for 1 s, leading to a reorganization of the network activity. The global activity is irregular. During the presentation of a given pattern, the firing rate is different for every neuron, depending both on the internal and on the external influences. When a new pattern is presented, some changes occur in the dynamics. Some neurons become silent, other increase their firing rate, so that a particular activity pattern is obtained for every input pattern.

It can be noticed however that most of the neurons remain unsensitive to the input influence, remaining either silent or saturated during the whole simulation. In our particular parameter setting, the internal self-sustained activity dominates the external influence.

The dynamics globally remains very irregular and its temporal characteristics are not easy to extract. The first characteristics is the *asynchrony* which directly results from the balance between the excitatory and inhibitory influences. Despite the asynchrony, some repeatable patterns can be observed with a zoom on the diagram (see figure 1B). Some sequences of activation over 50-60 ms seem to be present intermittently, but do not last in a stable way.

### 3 Weights adaptation

Learning generically refers to the slow process by which a neural network modifies its own structure under the influence of an environmental context. The neuron synapses are modulated by the characteristics of the signals arriving at the synaptic interface (local mechanism). The sum of the local changes is expected to produce an improvement of the network response. In the basic Hebbian rule statement [7], a synapse is reinforced when the arrival of a pre-synaptic signal repeatedly coincides with post-synaptic spike emission. The emission of a post-synaptic spike *after* pre-synaptic firing is thus facilitated.

The biological relevance of the Hebbian rule has long been conjectural since the first observation of a potentiation mechanism based on the co-activation of pre-synaptic and post-synaptic neurons [8]. The lasting potentiation of the synapse is commonly called "Long Term Potentiation" (LTP), and the reverse "Long Term Depression" (LTD). More recent observations have shown that the timing of spike arrivals may be of critical importance in the mechanism of synaptic potentiation [2, 3]. This mechanism has been called Spike-Timing Dependent Plasticity (STDP). The STDP can be seen as a coincidence detection mechanism whose precision may be of the order of few milliseconds. The main effect of a STDP rule is to potentiate the sequential co-activation : the EPSP<sup>1</sup> that anticipates the arrival of a spike on the post-synaptic neurons lead to a synaptic potentiation. The EPSP taking place several milliseconds *after* spike emission leads to a synaptic depression.

Several implementations of the STDP update mechanism are proposed in the litterature. Those implementations mainly classify into one-to-one and all-to-all implementations [9]. In the one-to-one implementation, the weight update only depends on the most recent occurrence of pre-synaptic/post-synaptic events. In an all-to-all scheme, the weights update depend on every previous synaptic event (knowing that the influence of the less recent events fades with time). The following scheme belongs to the second category:

$$\frac{dw_{ij}(t)}{dt} = \tau_m \frac{\alpha}{N} [S_i(t)\varepsilon_j(t - \tau) - \varepsilon_i(t)S_j(t - \tau)] \quad (5)$$

Remind here that the  $S_i(t)$ 's are discrete impulse events, so that the weights update takes place (in positive or negative) by steps at discrete moments. This update scheme divides in two terms. The first term corresponds to the synapse potentiation. A significant weight increment takes place when a post-synaptic spike occurs with a strong pre-synaptic trace (recent pre-synaptic excitations). The second term corresponds to the synapse depression. The weight undergoes significant decrement when a pre-synaptic pulse arrives at the synapse with a strong pre-synaptic trace (i.e. after post-synaptic excitation). There results a global facilitation of pre-post sequences of excitation, and a depression of post-pre sequences of excitation (see Figure 2).

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<sup>1</sup>Excitatory Post Synaptic Potential

It can be noticed that with this setting, the rule is symmetrical i.e. the potentiation compensates the depression. This rule can thus be considered as *balanced*. It can also be noticed that the storing of a trace is not very expensive computationally speaking, and corresponds to one of the most simple realizations on the STDP learning rule.

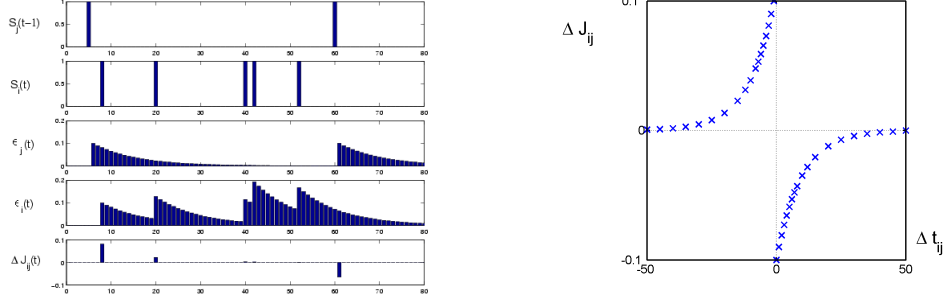


Figure 2: The left figure gives the time evolution of the traces  $\epsilon_i$  and  $\epsilon_j$  according to random pre-synaptic and post-synaptic spike trains. The resolution is  $\delta t = 1$  ms. The resulting weight change  $\Delta w_{ij}$  is on the lower plot. The right figure gives the value of the weight change according to the temporal difference between a pre-synaptic/post-synaptic pair of spikes in the particular case  $\tau = 1$  ms.

## 4 Simulation

We address in this paper the question of the periodic encoding of static patterns in recurrent networks. The idea is to learn to associate a static input to a particular firing pattern in a self-organized way. It is important to notice that no straightforward correspondance can be established between the particular static pattern and the resulting activity. The treatment that takes place inside the network is not signal processing, but only the selection and transformation of a static input in a dynamic pattern.

### 4.1 Learning process

In the considered simulation, the learning process lasts between  $t = 10$  s and  $t = 13$  s, with the  $10^{th}$  static pattern. The value of the learning parameter is  $\alpha = 0.03$ . We present in figure 3 some aspects of the activity evolution during the learning process.

Figure 3A presents a sliding autocorrelogram of the mean activation signal. At each time step  $t$ , the signal  $s(t, \dots, t + 250)$  is correlated with  $s(t + \tau, \dots, t + 250 + \tau)$  for  $\tau$  in  $0..250$  ms. The red zones denotes a high correlation while the deep blue zones denote low correlation values. Such a sliding autocorrelogram offers a synthetic view of the ongoing transformation taking place in the dynamics. The method is comparable to a sliding Fourier transform, but the number of periodic repetitions appears here in a more straightful way.

Starting from a rather unstructured activity, a periodic wave is found to progressively emerge and stabilize in the mean activation signal, which appear in the form of a series of red stripes in the diagram. This initial periodicity (of the order of 60 ms) is also visible in some individual activities (see figure 3B) at the beginning of the learning process. For increasing times, the activity becomes more regular and most of all the series of stripes shrinks from left to right, corresponding to a clear decrease of the internal period. At the end of the process ( $t = 13$  s), the internal period is of the order of 40 ms. The tightening of the internal period also appears in the individual activities (see figure 3C)

Moreover, if we compare the individual activities at the beginning and at the end of the process (see figures 3B and 3C), we first see that the firing rate of the individual neurons remains unchanged. This point is due to the balance of the STDP rule: the synapses potentiations and the depressions compensate, and the mean amplitude of the neuron input is preserved. Moreover, despite the change in periodicity, the internal structure of the activity is also preserved. Despite the period evolution, the

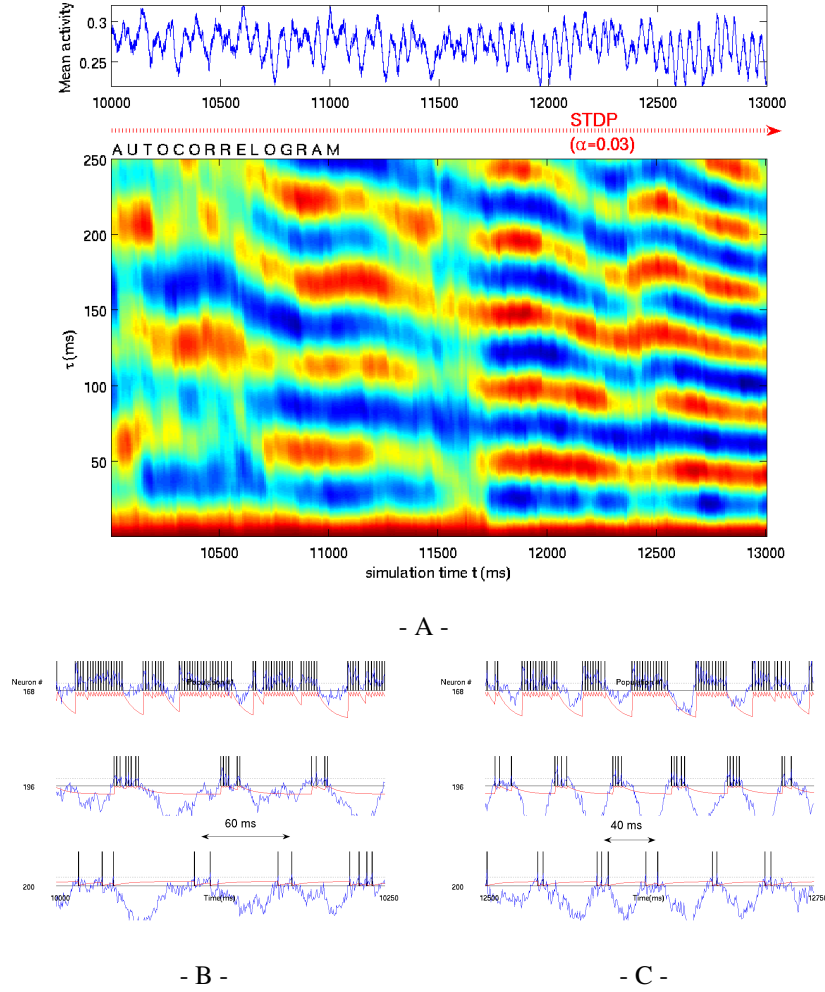


Figure 3: - **A** - Sliding auto-correlogram evolution during the learning process. The time is from left to right. The upper figure gives the mean activation. The lower figure is a sliding auto-correlogram corresponding to the upper trace. - **B** - Individual neuronal activities at the beginning of the learning process. - **C** - Individual neuronal activities at the end of the learning process.

neurons #168 and #200 remain in phase while the neurons #196 and #200 remain in antiphase. The rule thus stabilizes and consolidate a repeatable path of activation which was intermittently present in the initial activity. This consolidation property is to be compared with the polychronous sets consolidation presented in [10].

This simulation thus gives a clear insight in the regularization mechanism realized by our STDP rule. Starting from a mostly unstructured activity, the rule helps the system to lock on a particular sequence of activation, and thus consolidates this particular (pre-post) path with an increase of the corresponding weights. For instance, knowing the transmission delay is  $\tau = 10$  ms, the link from the neuron #200 to the neuron #196 is potentiated ( $\Delta w_{196,200} = 0.227$ ), corresponding to a facilitation of the  $200 \rightarrow 196$  transmission. On the contrary, the neurons #200 and #168 are in phase, and given the 10 ms delay, the pre-synaptic spike arrives after the post-synaptic bursting activity, and the link is depressed ( $\Delta w_{168,200} = -0.234$ ). Note that the amplitude of the weights change is small in comparison to the initial absolute value of the weights. Due to balance of the rule, there is no significant deviation in the positive or negative directions, i.e.  $E(\Delta w_{ij})_{i,j \in \{1, \dots, N\}} \simeq 0$  where  $\Delta w_{ij}$  is the total weight change. We find a change rate  $\frac{\sigma(\Delta w_{ij})_{i,j \in \{1, \dots, N\}}}{\sigma(w_{ij}^0)_{i,j \in \{1, \dots, N\}}} \simeq 0.08$  where  $w_{ij}^0$  is the

initial weight and the  $\sigma$  function gives an estimate of the standard deviation. The weights change is thus in the order of 8%.

## 4.2 Selectivity of the response

From a physicist point of view, the qualitative change obtained under the influence of a particular pattern is comparable to a phase transition from a disordered regime to a periodic one. A comparable effect of Hebbian learning has been observed in the literature in firing rate recurrent neural networks models [11].

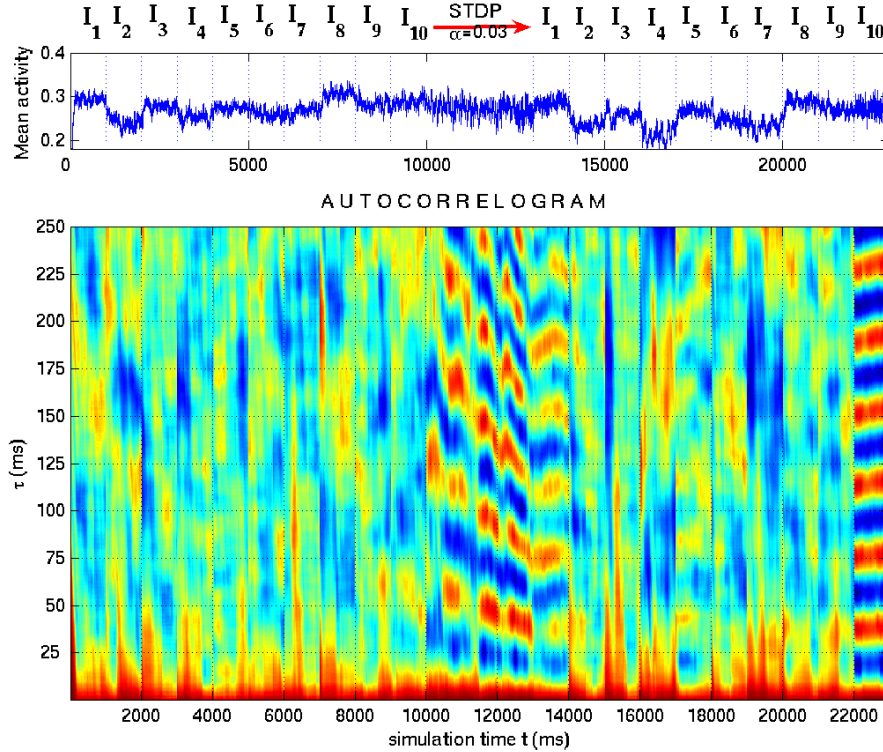


Figure 4: **Sliding Autocorrelation diagram : whole simulation.** The time is from left to right. The upper figure gives the mean activation. The lower figure is a sliding autocorrelogram corresponding to the upper trace. Patterns 1 to 10 are successively presented between  $t=0$  and  $t=10$  s. Then pattern #10 is learned during 3 s. The full sequence of 10 patterns is then presented from  $t=13$  to  $t=23$  s.

The main issue here is the selectivity capability of the network after learning. Is the network response specific to the learned pattern or does it respond in a similar way to other non-specific patterns? We thus tested the network response to the 10 arbitrary patterns before and after learning. A sliding autocorrelogram of the full 23 s simulation is presented on figure 4. The initial response to the 10 input patterns is clearly non-periodic, as mentioned previously. The locking on a periodic pattern and its progressive tightening of the periodic stripes appears from  $t=10$  to  $t=13$  s. After learning, the network periodic response is clearly specific to the learned pattern, while the other patterns only induce a non-periodic response.

The response to the other patterns has however been modified, and a weak periodic autocorrelation is for instance perceptible in the response to the pattern #1. More generally, the strength of the learning parameter  $\alpha$  and the duration of the learning process influences the selectivity of the response. In that simulation, the weight changes remain rather small, but a too long continuation of the learning process would lead the system to a global and nonspecific periodic response. A study of the loading capacity in such a system is also to be done, in order to estimate to the best how many different patterns can be learned in the same way.

## 5 Discussion

Our implementation of the STDP learning rule has shown a massive regularization effect with light weight changes, which is not to our knowledge present in the literature (see [10] or [12] for instance). The result we present, which use one of the simplest weights, neuron and learning rule implementation, are easily generalisable to other models, and combinable with other weight adaptation rules (short term depression for instance).

From a biological modeling point of view, this study emphasizes the role of qualitative transition rather than mean activity changes, and may be considered as a complementary approach to classical feed-forward input selectivity mechanisms. It is highly probable that several extraction mechanisms combine in the real brain to produce appropriate responses.

Some questions remain at stake at the present stage, in particular for an estimation of the loading capacity in multi-patterns learning. This study would also benefit a comparison with real data, in particular in the olfactory [13] or motor [14] systems.

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