of chaos of the dynamics) and n (number of neurons excited by the pattern). Anyway, these measures give clues for a general trend. The ability to discriminate between learned and non-learned patterns disappears while the system tries to learn too many things.

The last point is about dynamical coding. After learning, each learned pattern can be associated to a pattern of activity, made of N local limit cycles. For characterizing the attractor, we look at the mean output signal $m_{net}(t)=<x_i(t)>$. The question comes wether this limit cycle is specific of the pattern. It is difficult to give a formal answer to that question. We have observed that noisy versions of a learned pattern can induce strong changes in the characteristics of the attractor. This neighbour attractor can as well be a fixed point or a torus or a even strange attractor. Nevertheless, some general characteristics like gravity center or winding number remain in the same range than the original ones[2]. Moreover, the spatial repartition of neuronal activity remains very close from the original one. We compared on 5 network ($N=400$, $g=6$ and $\theta=0.3$) the vectors of mean outputs $(X_i)_{i=1..N}$ for regular and noisy versions of 1 learned pattern. The mean correlation between the two vectors is found to be 0,93 (noise=10%) and 0,87 (noise=20%). So, the coding of the input may both be seen in the topological characteristics of the attractor and in the spatial repartition of neuronal activity.

6   Conclusion

The learning scheme described in this article illustrates the rich dynamical behaviour of random recurrent neural networks. We have seen that the simulation of a learning rule, which uses the properties of individual neuronal dynamics, gives a good insight into the mechanism of learning and recognizing spatial patterns. Our network is moreover robust to noise addition. Some complementary simulations have to be made out in order to explore the question of network capacity. At last, the role of oscillations in neuronal computation could be explored in a more complex architecture including several clusters, and be relied to neurophysiological works[6].

References

[1] Cessac B: Increase in complexity in random neural networks. J de Phys I 1995; I.5: 409-432.

[2] Daucé E, Quoy M, Cessac B, Doyon B, Samuelides M: Self-Organization and Dynamics reduction in in recurrent networks: stimulus presentation and learning. Neural Networks. In press.

[3] Doyon B, Cessac B, Quoy M, Samuelides M: Chaos in neural networks with random connectivity. Int. J. of Bifurcation and Chaos 1993; 3-2: 279-291.

[4] Ginzburg I, Sompolinsky H: Theory of correlation in stochastic neural networks. Phys Rev E 1994; 50: 3171-3191.

[5] Herrmann M, Hertz J, Prugel-Bennett A: Analysis of synfire chains. Network: Computation in neural systems 1995; 6: 403-414.

[6] Gray, C M: Synchronous oscillations in neural systems: mechanisms an functions. J. Comput. Neurosciences 1994, 1: 11-38.

[7] Skarda C A, Freeman W J: How brain makes chaos in order to make sense of the world. Behav & Brain Sci 1987; 10 161-195.

step function which selects the input neurons, according to the value s. We have two time scales during the learning process : a fast one (given by t) for the iteration of the network dynamics (3) and a slow one (given by T) for the iteration of learning (4). In practice, one learning step is processed every 200 time steps.

The selected weights move almost continuously according to the sign of Δ_i, which tends to shift the neuron mean local field towards extreme (positive or negative) values. The initial change in mean output is amplified, and every neuron tends towards silence or saturation, which leads in both cases to a fall of the global dynamics. If the process is not stopped, it leads to a fixed point. In our experiments, we stop the process as soon as a limit cycle is reached. When the learning process ends, the network is able to *recognizes* the pattern, i.e. every new presentation of the pattern will lead to the same limit cycle reached at the end of the learning process (while the spontaneous dynamics remain chaotic).

Up to now, no analytical theory is available for the study of learning. The results below consequently come from simulated data, with networks of size $N=400$, $g=6$ and $\theta=0.3$. Our learning parameters are $\alpha=1$, $s=0.5$, $n=0.05N$. To learn a pool of P patterns, a cross training is carried out. One learning step is processed for each pattern of the pool, and this operation is reiterated until the network is reactive to every pattern of the pool. At each learning step, because of the selection on the input weights, only 5% of the weights are actually modified, of about $10^{-1}/N$ by weight. We verified that random modifications of the same order of magnitude do not lead to any change in the dynamics.

In order to have an insight into specificity of learning, we add a gaussian noise to previously learned patterns, and measure the specific reactivity to different classes of patterns made with a noise of 5%, 10%, 15% and 20% (% based on signal/noise ratio). The results are in Table 1.

Table 1: Specific reactivity to learned patterns with an additive gaussian noise, on one network, $N=400$, $g=6$ and $\theta=0.3$ (measured on 100 noisy versions on the basis of a pool of 5 learned patterns)

noise	5%	10%	15%	20%
reactivity	88%	87%	78%	76%

We can see that the specific reactivity remains high even for relatively loud noise (20%), which shows the robustness of our learning scheme.

The evolution of (non-specific) reactivity with P can help for an estimation of the effective capacity. We made a measure on random networks with $N=400$, $g=6$ and $\theta=0.3$. Ten networks were to learn 1 random pattern, ten others were to learn 2 random patterns, etc... until $P=10$. For each value of P, on each network, we measured reactivity after learning by presenting random non-learned patterns. We observed that reactivity does not increase significantly until $P=8$ (it remains lower than 5%), and then strongly increases for $P=9$ (reactivity=30%) and $P=10$ (reactivity=70%). So the capacity of the network seems to be overrun for $P=9$. However, a complete measure of the capacity should take into account others parameters, especially g (which determines the degree

4   Reactivity to stimulations

We present to the network a spatial binary signal. We are interested in the dynamical response of the network to this information. The input set (pattern) $I=(I_i)_{i=1..N}$ is a vector of N binary values. A subset of size $n<<N$, is equal to a, while the remainder is null. In our simulations, we take $a=1$.

$$(3) \begin{cases} u_i(t+1) = \sum_{j=1}^{N} J_{ij}x_j(t) - \theta + I_i \\ x_i(t+1) = f_g(u_i(t+1)) \end{cases}$$

When the pattern is present, the characteristics of the dynamical system are modified. We call it forced dynamics (3). The mean output of the directly excited neurons increases significantly, and, by reaction, dynamics as a whole are reorganized.

Starting from chaotic spontaneous dynamics, we call *reactivity* the proportion of patterns which lead the system towards non-chaotic dynamics. This reactivity has to be evaluated before any learning. For instance, with $N=400$, $g=6$ and $\theta=0.3$, on the basis of 10 network simulations with chaotic spontaneous dynamics, the reactivity (measured on samples of 100 random patterns) is 3%. A low reactivity means that chaos is stable in the spontaneous dynamics, so that the forced dynamics may remain chaotic for almost every pattern presentation.

Hebbian learning will be applied on the forced dynamics. The aim is to induce a specific reactivity to several random patterns arbitrarily chosen.

5   Learning

Let us specify the context of learning. In our simulations, on the basis of a chaotic spontaneous dynamics, one will say there is recognition if the presentation of a learned pattern leads to non-chaotic dynamics, of type $T2$ torus, limit cycle or fixed point (the dynamics are non-chaotic if largest Lyapunov exponent is < 0). This may be viewed as an interpretation of Freeman's paradigm, in which chaos would be related to an "I don't know" state, and cycle dynamics to a recognition state [7].

Our learning rule is derived from Hebb's rule. In a classical Hebbian process, the synaptic weights are reinforced when the pre and post-synaptic signals are correlated. Here, our learning process takes place when something new happens in the local fields of the neurons. When the neuronal activities change after pattern presentation, the synaptic modifications will rely on correlated changes in pre- and post-synaptic activity. Globally, the network learns novelty.

The rule is the following one:

$$(4) \quad J_{ij}(T+1) = J_{ij}(T) + \frac{\alpha}{N}\Delta_i(T) \cdot (X_j(T) - s) \cdot \Theta(\Delta_j(T) - s)$$

The quantity Δ_i measures the change in neuron i, after the pattern presentation, by making the difference of the mean outputs after and before presentation. Parameter α gives the intensity of learning. X_j is the mean output of input neuron j . $\Theta()$ is the unit

An analytical approach has been developped using Mean Field Theory (MFT), which statistically describes the network behavior at the thermodynamic limit ($N{\rightarrow}\infty$). A small number of parameters (θ and g for the model presented here) are enough to give the evolution laws of $u_i(t)$ and $x_i(t)$. Depending on θ and g, the stationary dynamical mode is either a fixed point, or the Gaussian process described below [1]:

$$(2)\begin{cases} U(0) \ = \ U \\ U(t) \ = \ U + B(t) \end{cases}$$

where U is a Gaussian random variable and $B(t)$ a centered white noise. The MFT description of a network's dynamic is very accurate, but ignores some finite size particularities, like quasi-periodicity routes, seen as a succession of dynamical states between fixed point and stochastic dynamical activity.

3   Neuronal activity

The vector of mean local fields $U=(\overline{u_i(t)})_{i=1..N}$ has approximatively a Gaussian distribution. The transfer function being a sigmoid with values on [0,1], the neurons whose mean local field is very negative have an output close to zero. Their local field signal $u_i(t)$ is flattened through the sigmoïd. Those neurons are almost silent. In the same way, the neurons with very positive mean local field saturate and behave like additional thresholds. In both cases, silent and saturated neurons do not favour the propagation of dynamics through the network. Dynamics are carried by the other ones that we will call dynamic neurons. Those neurons amplify the signal $u_i(t)$ and propagate it through the network.

We define the *amplification rate* of a neuron i as the ratio var($x_i(t)$)/var($u_i(t)$), roughly equal to $f_g{}'(U_i)^2$. If the amplification is >1, the neuron is said dynamic. Elsewhere, if the amplification is <1, the neuron is either silent or saturated, depending on its mean local field. In a typical network with size $N=900$, sigmoid gain $g=6$, threshold $\theta=0.3$, the proportions of silent, dynamic and saturated neurons are respectively 51%, 40% and 9% (mean on 5 networks). Figure 2, whose samples are taken out of a network with chaotic dynamics, presents these three types of neuronal activity. Note there is no correlation between the three signals.
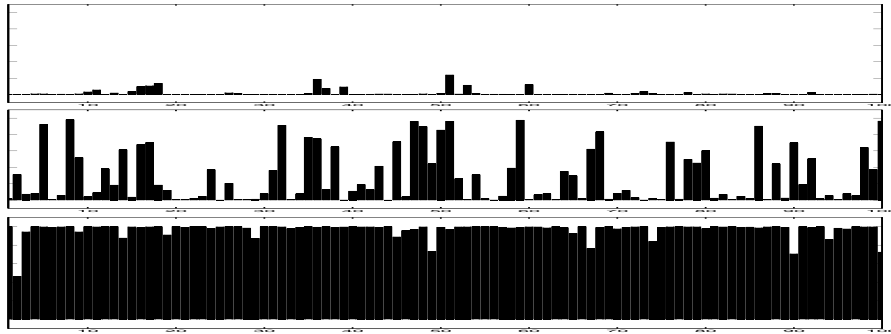

Figure 2*: Output of 3 neurons out of the same network, on 100 time steps, N=100, g=6, θ=0.3. 1*[st]*: almost silent activity. 2*[nd]*: dynamic activity. 3*[rd]*: saturation.

the non-homogeneity of neuronal activity. In section 4 we precise the characteristics of the pattern-forced dynamics. In section 5 we present the learning process and measure its efficacy. We then discuss the problems of capacity and dynamical coding. We conclude in section 6.

2   A discrete-time dynamical system

Our model comprises N neurons, connected by synaptic weights J_{ij} (weigh of the signal of neuron j towards neuron i). The network is fully connected ($J_{ij} \neq 0$ for $i \neq j$) and has no memory of its former state ($J_{ii}=0$). The weights J_{ij} are independent samples of a centered normal law: $\mathcal{P}_{Jij}=\mathcal{N}(0, \frac{1}{N-1})$, $i \neq j$. Thanks to the scaling law, the variance of the sum of the input weights of a neuron, is constant $\forall N$.

The dynamics equation is the following one:

$$(1)\begin{cases} u_i(t+1) = \sum_{j=1}^{N} J_{ij} x_j(t) - \theta \\ x_i(t+1) = f_g(u_i(t+1)) \end{cases}$$

The transfer function f_g is a sigmoid whose values are on [0,1] and whose gain is g - $f_g(u)=(1+\tanh(g.u))/2$ -. Each neuron calculates its output $x_i(t)$ from its local field $u_i(t)$. The threshold $\theta \geq 0$ is equal for every neuron. The slope of the transfer function $g/2$ represents the sensibility to local field variations.

The system develops a rich dynamical behaviour by tuning g. For low values of g, the system converges to a fixed point. A continuous increase of g leads the system from fixed point to chaos following a quasi-periodicity route [3]. The evolution of the stationary dynamics attractors with g are presented on figure 1.
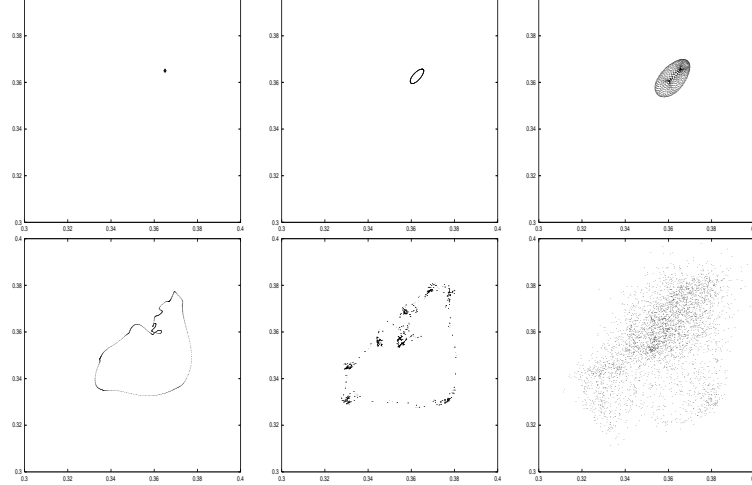

Figure 1: Representation in the ($t,t+1$) space of the mean output signal with -first line- g=4 (fixed point), g=4.42 (cycle), g=4.5 ($T2$ torus), -second line- g=5 (frequency locking), g=5.5 (frontier of chaos) and g=6 (chaos). The chaoticity of a dynamics is determined with the largest Lyapunov exponent. N=100, θ=0.3.

Novelty Learning in a Discrete Time Chaotic Network

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Abstract

Although extraordinarily complexes, the mental processes can be seen as products of the neuronal dynamical system. In this context, biological observations make it possible to emit the conjecture that recognition of a form or a stimulus leads to a reduction of neuronal dynamics. This paper proposes a generic model for the study of such dynamics by learning random stimuli. We implement a Hebb-like learning rule, which reinforces the innovation in a network stimulated by a random input. The network learns to react specifically to one or more learned inputs. An estimation of the networks reactivity after learning brings encouraging results in terms of capacity. Then the question of dynamical coding is evoked in terms of limit cycles associated to specific patterns.

1   Introduction

Neurobiology shows every day the extreme richness of the processes developed by the brain for data processing. The modeling of neural networks as non-linear dynamical systems can clarify some aspects of neuronal computation[4][5]. The model we present here is an asymmetrical discrete-time recurrent network, well designed for the development of complex dynamics. A statistical analysis of this model has been previously published [1].

The way of processing information with chaos is our main concern. Freeman's paradigm will be our biological background. On the basis of recordings on the olfactory bulb of rabbits and cats, Freeman associates the recognition of a known odor to a specific limit cycle attractor, the waking rest activity being chaotic[7].

In section 2 we present the basic properties of the model. In section 3 we point out