Correlation functions and Sequential sources for $N \to N$, $\Delta \to N$ and the $\Delta \to \Delta$ form factors

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1 Useful definitions

We will consider the nucleon and Δ^+ interpolating fields (including Dirac index α and Lorentz index σ), such as

$$\begin{aligned} \chi^N_{\alpha L}(x) &= \epsilon^{abc}\left(d^{Ta}(x)C\gamma_5 u^b(x)\right)u^c_\alpha(x) \\ \chi^{\Delta^+}_{\sigma,\alpha L}(x) &= \epsilon^{abc}\left[2\left(d^{Ta}(x)C\gamma_\sigma u^b(x)\right)u^c_\alpha(x) + \left(u^{Ta}(x)C\gamma_\sigma u^b(x)\right)d^c_\alpha(x)\right] \end{aligned}$$

or some smeared version of these, denoted $\chi^N_{\alpha S}(x)$ or $\chi^{\Delta^+}_{\sigma,\alpha S}(x)$. In the following, the octet and decuplet baryons will be labelled by N and Δ, respectively. However, the results may be applied to any of the octet to decuplet transitions.

We will define some functions that are convenient for writing the correlation functions.

- `quarkContract13(a,b)`
  Epsilon contract 2 quark propagators and return a quark propagator. This is used for diquark constructions. Eventually, it could handle larger Nc. The numbers represent which spin index to sum over.

  The sources and targets must all be propagators but not necessarily of the same lattice type. Effectively, one can use this to construct an anti-quark from a di-quark contraction. In explicit index form, the operation `quarkContract13` does

  $$target^{k'k}_{\alpha\beta} = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'}_{\rho\alpha} * source2^{jj'}_{\rho\beta}$$

- `quarkContract14(a,b)`
  Epsilon contract 2 quark propagators and return a quark propagator.

  $$target^{k'k}_{\alpha\beta} = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'}_{\rho\alpha} * source2^{jj'}_{\beta\rho}$$

- `quarkContract23(a,b)`
  Epsilon contract 2 quark propagators and return a quark propagator.

  $$target^{k'k}_{\alpha\beta} = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'}_{\alpha\rho} * source2^{jj'}_{\rho\beta}$$

- `quarkContract24(a,b)`
  Epsilon contract 2 quark propagators and return a quark propagator.

  $$target^{k'k}_{\alpha\beta} = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'}_{\rho\alpha} * source2^{jj'}_{\beta\rho}$$

- `quarkContract12(a,b)`
  Epsilon contract 2 quark propagators and return a quark propagator.

  $$target^{k'k}_{\alpha\beta} = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'}_{\rho\rho} * source2^{jj'}_{\alpha\beta}$$

- `quarkContract34(a,b)`
  Epsilon contract 2 quark propagators and return a quark propagator.

$$target^{k'k}_{\alpha\beta} = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'}_{\alpha\beta} * source2^{jj'}_{\rho\rho}$$

- `colorContract(a,b,c)`
  Epsilon contract 3 color primitives and return a primitive scalar. The sources and targets must all be of the same primitive type (a matrix or vector) but not necessarily of the same lattice type. In explicit index form, the operation colorContract does

$$target = \epsilon^{ijk}\epsilon^{i'j'k'} * source1^{ii'} * source2^{jj'} * source3^{kk'}$$

  or

$$target = \epsilon^{ijk} * source1^{i} * source2^{j} * source3^{k}$$

We will define correlation functions for interpolating fields generically labelled as B. For example, for $N \to N$ we want

$$\begin{aligned}\langle B^{123}_{\gamma}(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle &= -\varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}\langle\left(\psi^{Ta}_{1,\alpha}(C\gamma_5)_{\alpha\beta}\psi^{b}_{2,\beta}\right)\psi^{c}_{3,\gamma}(x)\left(\bar{\psi}^{T\bar{a}}_{1,\bar{\alpha}}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}\bar{\psi}^{b}_{2,\bar{\beta}}\right)\bar{\psi}^{\bar{c}}_{3,\gamma}(0)\rangle \\ &= \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\beta}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\gamma}}\end{aligned}$$

$$\begin{aligned}\langle B^{123}_{\gamma}(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle &= -\varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}\langle\left(\psi^{Ta}_{1,\alpha}(C\gamma_5)_{\alpha\beta}\psi^{b}_{2,\beta}\right)\psi^{c}_{3,\gamma}(t)\left(\bar{\psi}^{T\bar{a}}_{1,\bar{\alpha}}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}\bar{\psi}^{b}_{3,\bar{\beta}}\right)\bar{\psi}^{\bar{c}}_{2,\gamma}(0)\rangle \\ &= -\varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{c}}_{\beta\bar{\gamma}}G^{(3)}(x,0)^{c\bar{b}}_{\gamma\bar{\beta}} \\ &= \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\gamma}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\beta}}\end{aligned}$$

and similarly we will consider $C\gamma_5 \to iC\gamma_k$ with labelling $B_{k,\gamma}$ for the decuplet baryons.

With a suitable $T_{\alpha\beta}$ as some generic 4×4 matrix in Dirac spin space, and α, β are Dirac indices. We see that if we contract all the spin and color indices we arrive at the expressions

$$\begin{aligned}&T_{\bar{\gamma}\gamma}\langle B^{123}_{\gamma}(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle \\ &\quad = \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{tr}_S(\mathrm{quarkContract13}(G^{(1)}(x,0)(C\gamma_5), (C\gamma_5)G^{(2)}(x,0)))))\end{aligned}$$

$$\begin{aligned}&T_{\bar{\gamma}\gamma}\langle B^{123}_{\gamma}(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle \\ &\quad = \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{quarkContract13}(G^{(1)}(x,0)(C\gamma_5), (C\gamma_5)G^{(2)}(x,0))))\end{aligned}$$

where tr_C, tr_S and tr are traces in color, spin and all color-spin indices, resp.

2 Baryon 2-point correlation functions

2.1 $\Sigma^+\Sigma^+$, or PP:

For the case of the Σ^+/proton: $\Sigma^+_\gamma = B^{suu}_\gamma$ (for proton: $s \to d$). Note that $B^{123}_\gamma = -B^{213}_\gamma$.

$$\langle \Sigma^+_\gamma(x)\bar{\Sigma}^+_{\bar{\gamma}}(0)\rangle = \langle B^{123}_\gamma(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle_{\substack{1=s\\2,3=u}} + \langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{\substack{1=s\\2,3=u}}$$

The contractions needed here consist of the following:

$$\begin{aligned}
&T_{\bar{\gamma}\gamma}\langle B^{123}_\gamma(x)\bar{B}^{123}_{\bar{k},\bar{\gamma}}(0)\rangle \\
&\qquad = \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\beta}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\gamma}} \\
&\qquad = \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{tr}_S(\mathrm{qC13}(G^{(1)}(x,0)(C\gamma_5),(C\gamma_5)G^{(2)}(x,0)))))
\end{aligned}$$

$$\begin{aligned}
&T_{\bar{\gamma}\gamma}\langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle \\
&\qquad = \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\gamma}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\beta}} \\
&\qquad = \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{qC13}(G^{(1)}(x,0)(C\gamma_5),(C\gamma_5)G^{(2)}(x,0))))
\end{aligned}$$

Thus, the explicit final form for quarks $1,2,3$ being d, u and u, the correlation function is

$$\begin{aligned}
T\langle N(x)\bar{N}(0)\rangle &= \mathrm{tr}(T * \mathrm{tr}_C(U(x,0) * \mathrm{tr}_S(\mathrm{qC13}(D(x,0)(C\gamma_5),(C\gamma_5)U(x,0))))) \\
&+ \mathrm{tr}(T * \mathrm{tr}_C(U(x,0) * \mathrm{qC13}(D(x,0)(C\gamma_5),(C\gamma_5)U(x,0))))
\end{aligned}$$

2.2 $\Sigma^{*+}\Sigma^{*+}$, or $\Delta^+\Delta^+$:

Here Σ^{*+}/Δ^+: $\Sigma^{*+}_{k,\gamma} = 2B^{suu}_{k,\gamma} + B^{uus}_{k,\gamma}$ (for Δ^+: $s \to d$). Note that $B^{213}_{k,\gamma} = B^{123}_{k,\gamma}$.

$$\begin{aligned}
\langle \Sigma^{*+}_{k,\gamma}(x)\bar{\Sigma}^{*+}_{\bar{k},\bar{\gamma}}(0)\rangle &= 4\langle B^{123}_{k,\gamma}(x)\bar{B}^{123}_{\bar{k},\bar{\gamma}}(0)\rangle_{\substack{1=s\\2,3=u}} + 2\langle B^{123}_{k,\gamma}(x)\bar{B}^{123}_{\bar{k},\bar{\gamma}}(0)\rangle_{\substack{1,2=u\\3=s}} \\
&+ 4\langle B^{123}_{k,\gamma}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle_{\substack{1=s\\2,3=u}} + 4\langle B^{123}_{k,\gamma}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle_{\substack{1,3=u\\2=s}} \\
&+ 4\langle B^{123}_{k,\gamma}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle_{\substack{1,2=u\\3=s}}
\end{aligned}$$

2.3 $\Delta^+\Delta^+$:

This correlator is the same as the $\Sigma^{*+}\Sigma^{*+}$. In the case the $s \to d$, we have Δ^+: $\Delta^+_{k,\gamma} = 2B^{duu}_{k,\gamma} + B^{uud}_{k,\gamma}$. Note that $B^{213}_{k,\gamma} = B^{123}_{k,\gamma}$. For degenerate u and d quarks, we have (up to a factor of 6)

$$\langle \Delta^+_{k,\gamma}(x)\bar{\Delta}^+_{\bar{k},\bar{\gamma}}(0)\rangle = \langle B^{123}_{k,\gamma}(x)\bar{B}^{123}_{\bar{k},\bar{\gamma}}(0)\rangle + 2\langle B^{123}_{k,\gamma}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle$$

2.4 $\Lambda\Lambda$:

Here $\Lambda_\gamma = 2B^{uds}_\gamma + B^{sdu}_\gamma + B^{usd}_\gamma$ where $B^{213}_\gamma = -B^{123}_\gamma$.

$$\begin{aligned}
\langle \Lambda_\gamma(x)\bar{\Lambda}_{\bar{\gamma}}(0)\rangle &= 4\langle B^{123}_\gamma(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle_{1=u,2=d,3=s} \\
&+ \langle B^{123}_\gamma(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle_{1=s,2=d,3=u} + \langle B^{123}_\gamma(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle_{1=s,2=u,3=d} \\
&+ 2\langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=d,2=u,3=s} + 2\langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=u,2=d,3=s} \\
&+ 2\langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=d,2=s,3=u} + 2\langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=u,2=s,3=d} \\
&- \langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=s,2=d,3=u} - \langle B^{123}_\gamma(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=s,2=u,3=d}
\end{aligned}$$

2.5 $\Delta^+ P$:

Here $P_\gamma = B_\gamma^{duu}$ where $B_\gamma^{123} = -B_\gamma^{213}$. $\Delta^+_{k,\gamma} = 2B^{duu}_{k,\gamma} + B^{uud}_{k,\gamma}$ where $B^{213}_{k,\gamma} = B^{123}_{k,\gamma}$.

$$\begin{aligned}
\langle P_\gamma(x)\bar{\Delta}^+_{\bar{k},\bar{\gamma}}(0)\rangle &= 2\langle B_\gamma^{du_1u_2}(x)\bar{B}^{du_1u_2}_{\bar{k},\bar{\gamma}}(0)\rangle + 2\langle B_\gamma^{du_1u_2}(x)\bar{B}^{du_2u_1}_{\bar{k},\bar{\gamma}}(0)\rangle \\
&+ \langle B_\gamma^{du_1u_2}(x)\bar{B}^{u_1u_2d}_{\bar{k},\bar{\gamma}}(0)\rangle + \langle B_\gamma^{du_1u_2}(x)\bar{B}^{u_2u_1d}_{\bar{k},\bar{\gamma}}(0)\rangle \\
&= 2\langle B_\gamma^{123}(x)\bar{B}^{123}_{\bar{k},\bar{\gamma}}(0)\rangle_{1=d,2=u_1,3=u_2} + 2\langle B_\gamma^{123}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle_{1=d,2=u_1,3=u_2} \\
&- \langle B_\gamma^{123}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle_{1=u_1,2=u_2,3=d} - \langle B_\gamma^{123}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle_{1=u_1,2=d,3=u_2}
\end{aligned}$$

The contractions needed here consist of the following:

$$\begin{aligned}
&T_{\bar{\gamma}\gamma}\langle B_\gamma^{123}(x)\bar{B}^{123}_{\bar{k},\bar{\gamma}}(0)\rangle \\
&\qquad= \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_k)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\beta}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\gamma}} \\
&\qquad= \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{tr}_S(\mathrm{qC13}(G^{(1)}(x,0)(C\gamma_k),(C\gamma_5)G^{(2)}(x,0)))))
\end{aligned}$$

$$\begin{aligned}
&T_{\bar{\gamma}\gamma}\langle B_\gamma^{123}(x)\bar{B}^{132}_{\bar{k},\bar{\gamma}}(0)\rangle \\
&\qquad= \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_5)_{\alpha\beta}(C\gamma_k)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\gamma}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\beta}} \\
&\qquad= \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{qC13}(G^{(1)}(x,0)(C\gamma_k),(C\gamma_5)G^{(2)}(x,0))))
\end{aligned}$$

2.6 $P\Delta^+$:

Here $P_\gamma = B_\gamma^{duu}$ where $B_\gamma^{123} = -B_\gamma^{213}$. $\Delta^+_{k,\gamma} = 2B^{duu}_{k,\gamma} + B^{uud}_{k,\gamma}$ where $B^{213}_{k,\gamma} = B^{123}_{k,\gamma}$.

$$\begin{aligned}
\langle \Delta^+_{k,\gamma}(x)\bar{P}_{\bar{\gamma}}(0)\rangle &= 2\langle B_{k,\gamma}^{du_1u_2}(x)\bar{B}^{du_1u_2}_{\bar{\gamma}}(0)\rangle + 2\langle B_{k,\gamma}^{du_1u_2}(x)\bar{B}^{du_2u_1}_{\bar{\gamma}}(0)\rangle \\
&+ \langle B_{k,\gamma}^{u_1u_2d}(x)\bar{B}^{du_1u_2}_{\bar{\gamma}}(0)\rangle + \langle B_{k,\gamma}^{u_1u_2d}(x)\bar{B}^{du_2u_1}_{\bar{\gamma}}(0)\rangle \\
&= 2\langle B_{k,\gamma}^{123}(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle_{1=d,2=u_1,3=u_2} + 2\langle B_{k,\gamma}^{123}(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=d,2=u_1,3=u_2} \\
&- \langle B_{k,\gamma}^{123}(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=u_1,2=u_2,3=d} - \langle B_{k,\gamma}^{123}(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle_{1=u_2,2=u_1,3=d}
\end{aligned}$$

The contractions needed here consist of the following:

$$\begin{aligned}
&T_{\bar{\gamma}\gamma}\langle B_{k,\gamma}^{123}(x)\bar{B}^{123}_{\bar{\gamma}}(0)\rangle \\
&\qquad= \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_k)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\beta}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\gamma}} \\
&\qquad= \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{tr}_S(\mathrm{qC13}(G^{(1)}(x,0)(C\gamma_5),(C\gamma_k)G^{(2)}(x,0)))))
\end{aligned}$$

$$\begin{aligned}
&T_{\bar{\gamma}\gamma}\langle B_{k,\gamma}^{123}(x)\bar{B}^{132}_{\bar{\gamma}}(0)\rangle \\
&\qquad= \varepsilon^{abc}\varepsilon^{\bar{a}\bar{b}\bar{c}}(C\gamma_k)_{\alpha\beta}(C\gamma_5)_{\bar{\alpha}\bar{\beta}}G^{(1)}(x,0)^{a\bar{a}}_{\alpha\bar{\alpha}}G^{(2)}(x,0)^{b\bar{b}}_{\beta\bar{\gamma}}G^{(3)}(x,0)^{c\bar{c}}_{\gamma\bar{\beta}} \\
&\qquad= \mathrm{tr}(T * \mathrm{tr}_C(G^{(3)}(x,0) * \mathrm{qC13}(G^{(1)}(x,0)(C\gamma_5),(C\gamma_k)G^{(2)}(x,0))))
\end{aligned}$$

3 Baryon 3-point functions

We now want to construct baryon 3-point functions with an electromagnetic current

$$J(x) = \sum_f e_f \bar{q}_f(x)\mathcal{O}(x,y)q_f(y)$$

where we sum over all flavors f, and $\mathcal{O}$ is composed of some gamma matrices and some covariant displacement function – possibly only a delta function. The 3-point functions take the form

$$\Gamma^{(3)} = \langle B(x_2)J(x_1)\bar{B}(0)\rangle$$

for some interpolating field $B(x)$. We see that the current is inserted on each line in the 2-point function.

We want expressions for the individual u, d, etc. quark contributions within the 3-point function. These can be formed directly from the 2-point function using a generalized propagator of the form

$$\widetilde{U}(x_2,x_1,0) = U(x_2,x_1)\Gamma U(x_1,0) = \gamma_5 U^\dagger(x_1,x_2)\gamma_5\Gamma U(x_1,0)\ .$$

for the illustrative case of the simple u-quark Γ-matrix inserted current. The 3-point function is constructed from the 2-point function after inserting into each line, resp.

3.1 Illustrative case NJN: $\bar{u}\Gamma u$ insertion

Consider the case of the proton: $P_\gamma = B_\gamma^{duu}$ and neutron $N_\gamma = B_\gamma^{udd}$. Note that $B_\gamma^{123} = -B_\gamma^{213}$. The u quark contribution to the proton 3-point function amounts to summing $\widetilde{U}$ quark contributions in the 2-point function. If one writes the 2-point correlator as a function of the quarks

$$\begin{aligned}\Gamma^{(2)}(D(x,0),&U_1(x,0),U_2(x,0)) = T\langle P(x)\bar{P}(0)\rangle\\ &= T\langle B^{123}(x)\bar{B}^{123}(0)\rangle_{\substack{1=d\\2,3=u}} + T\langle B^{123}(x)\bar{B}^{132}(0)\rangle_{\substack{1=d\\2,3=u}}\\ &= \mathrm{tr}(T * \mathrm{tr}_C(U_2(x,0) * \mathrm{tr}_S(\mathrm{qC13}(D(x,0)(C\gamma_5),(C\gamma_5)U_1(x,0)))))\\ &+ \mathrm{tr}(T * \mathrm{tr}_C(U_2(x,0) * \mathrm{qC13}(D(x,0)(C\gamma_5),(C\gamma_5)U_1(x,0))))\quad ,\end{aligned}$$

then the $\bar{u}\mathcal{O}u$-insertion in the 3-point function is

$$\Gamma_u^{(3)}(D,U_1,U_2) = \Gamma^{(2)}(D(x_2,0),\widetilde{U}(x_2,x_1,0),U_2(x_2,0)) + \Gamma^{(2)}(D(x_2,0),U_1(x_2,0),\widetilde{U}(x_2,x_1,0))$$

where $\widetilde{U}(x_2,x_1,0) = U(x_2,x_1)\mathcal{O}(x_1,y)U(y,0)$. Thus, in explicit form the u-quark contribution to the proton 3-point function is

$$\begin{aligned}T\langle P(x_2)\ &\bar{u}(x_1)\mathcal{O}(x_1,y)u(y)\ \bar{P}(0)\rangle\\ &= \mathrm{tr}(T * \mathrm{tr}_C(\widetilde{U}(x_2,x_1,0) * \mathrm{tr}_S(\mathrm{qC13}(D(x_2,0)(C\gamma_5),(C\gamma_5)U(x_2,0)))))\\ &+ \mathrm{tr}(T * \mathrm{tr}_C(\widetilde{U}(x_2,x_1,0) * \mathrm{qC13}(D(x_2,0)(C\gamma_5),(C\gamma_5)U(x_2,0))))\\ &+ \mathrm{tr}(T * \mathrm{tr}_C(U(x_2,0) * \mathrm{tr}_S(\mathrm{qC13}(D(x_2,0)(C\gamma_5),(C\gamma_5)\widetilde{U}(x_2,x_1,0)))))\\ &+ \mathrm{tr}(T * \mathrm{tr}_C(U(x_2,0) * \mathrm{qC13}(D(x_2,0)(C\gamma_5),(C\gamma_5)\widetilde{U}(x_2,x_1,0))))\end{aligned}$$

The rest of the form factors follow in such a simple fashion. We tabulate them for convenience.

3.2 pJp

Proton 2-point function is

$$\Gamma^{(2)}(D,U,U) = T\langle B^{123}(x)\bar{B}^{123}(0)\rangle_{\substack{1=d \\ 2,3=u}} + T\langle B^{123}(x)\bar{B}^{132}(0)\rangle_{\substack{1=d \\ 2,3=u}}$$

$\bar{u}\mathcal{O}u$-insertion:

$$\Gamma^{(3)}_u(D,U,U) = \Gamma^{(2)}(D(x_2,0),\widetilde{U}(x_2,x_1,0),U(x_2,0)) + \Gamma^{(2)}(D(x_2,0),U(x_2,0),\widetilde{U}(x_2,x_1,0))$$

$\bar{d}\mathcal{O}d$-insertion:

$$\Gamma^{(3)}_d(D,U,U) = \Gamma^{(2)}(\widetilde{D}(x_2,x_1,0),U(x_2,0),U(x_2,0))$$

3.3 nJn

Neutron 2-point function is

$$\Gamma^{(2)}(U,D,D) = T\langle B^{123}(x)\bar{B}^{123}(0)\rangle_{\substack{1=u \\ 2,3=d}} + T\langle B^{123}(x)\bar{B}^{132}(0)\rangle_{\substack{1=u \\ 2,3=d}}$$

$\bar{u}\mathcal{O}u$-insertion:

$$\Gamma^{(3)}_u(U,D,D) = \Gamma^{(2)}(\widetilde{U}(x_2,x_1,0),D(x_2,0),D(x_2,0))$$

$\bar{d}\mathcal{O}d$-insertion:

$$\Gamma^{(3)}_d(U,D,D) = \Gamma^{(2)}(U(x_2,0),\widetilde{D}(x_2,x_1,0),D(x_2,0)) + \Gamma^{(2)}(U(x_2,0),D(x_2,0),\widetilde{D}(x_2,x_1,0))$$

3.4 $\Delta^+ P$:

Here $P_\gamma = B^{duu}_\gamma$ where $B^{123}_\gamma = -B^{213}_\gamma$. $\Delta^+_{k,\gamma} = 2B^{duu}_{k,\gamma} + B^{uud}_{k,\gamma}$ where $B^{213}_{k,\gamma} = B^{123}_{k,\gamma}$. After contracting over the project T, the 2-point correlation function is

$$\begin{aligned}
\Gamma^{(2)}(D,U,U) &= T\langle P(x)\bar{\Delta}^{+}_{\bar{k}}(0)\rangle \\
&= 2\langle TB^{123}(x)\bar{B}^{123}_{\bar{k}}(0)\rangle_{\substack{1=d \\ 2,3=u}} + 2\langle TB^{123}(x)\bar{B}^{132}_{\bar{k}}(0)\rangle_{\substack{1=d \\ 2,3=u}} \\
&\quad - \langle TB^{123}(x)\bar{B}^{132}_{\bar{k}}(0)\rangle_{\substack{1,2=u \\ 3=d}} - \langle TB^{123}(x)\bar{B}^{132}_{\bar{k}}(0)\rangle_{\substack{1,3=u \\ 2=d}}
\end{aligned}$$

$\bar{u}\mathcal{O}u$-insertion:

$$\Gamma^{(3)}_u(D,U,U) = \Gamma^{(2)}(D(x_2,0),\widetilde{U}(x_2,x_1,0),U(x_2,0)) + \Gamma^{(2)}(D(x_2,0),U(x_2,0),\widetilde{U}(x_2,x_1,0))$$

$\bar{d}\mathcal{O}d$-insertion:

$$\Gamma^{(3)}_d(D,U,U) = \Gamma^{(2)}(\widetilde{D}(x_2,x_1,0),U(x_2,0),U(x_2,0))$$

4 Sequential Sources

NEED SOME INTRO HERE ON THE SOURCE TRICK - FIDDLING WITH APPLYING DIRAC OP AT x_1

As a final step, `Chroma` takes the sequential source from below and does $\gamma_5 S^\dagger \gamma_5$.

4.1 NJN

Proton 2-point function is

$$\Gamma^{(2)}(D,U,U) = T\langle B^{123}(x)\bar{B}^{123}(0)\rangle_{\substack{1=d\\2,3=u}} + T\langle B^{123}(x)\bar{B}^{132}(0)\rangle_{\substack{1=d\\2,3=u}}$$

Sequential source for u-quark:

$$\begin{aligned} X &= \mathrm{qC24}(U(C\gamma_5),(C\gamma_5)D) \\ S_u(x_2) &= T * X + \mathrm{tr}_S(X) * T - \mathrm{qC13}((C\gamma_5)U(C\gamma_5),UT) - \mathrm{transposeSpin}(\mathrm{qC12}(UT,(C\gamma_5)U(C\gamma_5))) \end{aligned}$$

Sequential source for d-quark:

$$S_d(x_2) = -\mathrm{qC14}(TU(C\gamma_5),(C\gamma_5)U) - \mathrm{transposeSpin}(\mathrm{qC12}(UT,(C\gamma_5)U(C\gamma_5)))$$

4.2 $\Delta^+ P$: