Study of Exotic Hadrons with Lattice QCD and Distillation

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Bonn

October 2024

I hereby declare that this thesis was formulated by myself and that no sources or tools other than those cited were used.

Bonn, ... Date

.... Signature

1. Gutachter: Prof. Dr. Stefan Krieg
2. Gutachterin: Prof. Dr. Anne Jones

Acknowledgements

I would like to thank ...

You should probably use \chapter* for acknowledgements at the beginning of a thesis and \chapter for the end.

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Hadron Spectroscopy on the Lattice

2.1 Gauge Field Smearing

In order to achieve overlap with states of interest in the continuum, namely the low-lying states, one must being with smearing of the quark fields via some smoothing function. A brief exposition of *Jacobi smearing* will follow.

$$\tilde{\psi}_{a\alpha}(x) = \mathcal{S}_{ab}(x,y)\psi_{b\alpha}(y) \tag{2.1}$$

Here, x, y are lattice sites, a, b are color indices, α a spin component.

This construction is derived from the "parent" representation of a gauge-invariant, spatially symmetric operation, which serves as a means of improving the projection onto low-lying states in correlation functions:

$$\tilde{\Psi}(\vec{x},t) = \sum_{\vec{y}} L(\vec{x},\vec{y})\psi(\vec{y},t) \tag{2.2}$$

Define the jacobi smearing operator as

$$\nabla \tag{2.3}$$

2.2 Distillation Smearing

The information we need to extract is

$$V^{\dagger}M^{-1}V \to \tau$$

where τ is the perambulator matrix on a single time slice. $V(t)$ is a matrix with $4 \times N_v$ columns constructed from eigenvectors of the covariant 3d Laplace operator. It is important to note that $V(t)$ does not act on Dirac components. Thus, $V(t)$ is a block identity in Dirac space and each block contains the first N_v eigenvectors $v_i(t)$. A given column $V^{(i,\alpha)}(t)$ has entries

$$V^{(i,\alpha)}(t)_{\vec{x},t',\beta} = v_i(t)_{\vec{x}}\delta_{tt'}\delta_{\alpha\beta}$$

Propagators transform with tensor product structure

$$\text{Lattice} \otimes \text{Matrix(Nc)} \otimes \text{Matrix(Ns)} \otimes \text{Complex}$$

check this and format with a figure from the TALK AT NRW FAIR We can work out these dimensions for ourselves; A distilled propagator stored on disk has dimensions $2*8*2*4*4*10*10*16$ with the dictionary

$$\text{complex} * \text{snk} * \text{src} * N \times N_\sigma * \text{tslice}$$

At this point, we need to actually perform contractions to obtain the correlator

$$C_M^{(2)}(t', t) = Tr[\Phi^B(t')\tau(t', t)\Phi^A(t)\tau(t, t')]$$

where

$$\Phi^A_{\alpha\beta}(t) = V^\dagger(t)[\Gamma^A(t)]_{\alpha\beta}V(t) \equiv V^\dagger(t)\mathcal{D}^A(t)V(t)S^A_{\alpha\beta}$$

and

$$\tau_{\alpha\beta}(t', t) = V^\dagger(t')M^{-1}_{\alpha\beta}(t', t)V(t)$$

is the perambulator, defined by the lattice representation of the Dirac operator, M. See [https://arxiv.org/abs/0905.2160v

CHAPTER 3

Computational Setup

Distillation is costly initially both in storage and component construction. For the di-meson system we are investigating, the contraction cost is not the dominant contribution. We will use the MultiGrid (MG) solver from `QUDA`, `Chroma` with `Superbblas` support, the `PRIMME` eigensolver, and `Numpy Einsum` for contractions. The amount of computation and storage scales with the lattice size N and the rank of the distillation basis, n. The optimal rank of the distillation basis is determined experimentally, but it is proportional to the spatial volume of the lattice.

3.1 Cost and Storage of Distillation

Computation	Operations cost	Memory footprint
Distillation basis[a]	N^3Tn^3D	N^3nT
Meson elementals[b]	N^3Tn^3	$N^3n + n^3$
Perambulators[c]	N^3Tn	N^3Tn
Contractions[d]	n^4T	n^3T

[a] Generate colorvector matrix elements
[b] Contract two matrices → tensor
[c] Projection of the inverse Dirac operator → square matrices
[d] Contract together matrix elements and perambulators

once a suitable set of perambulators compute, **reuse** to correlate a collection of interpolators

In order to perform spectroscopy calculations for a given ensemble, there exists a sequential dependency chain:

- ✓ **Ensemble generation:** $N_f = 2 + 1$ quark flavors, a tree level Symanzik improved gluon action and 6-stout dynamical smeared Wilson fermions

- ✓ **HPC Tasks:** Generation of distillation basis, perambulators, meson elementals using `Chroma` with `superbblas` support on the Jureca cluster at JSC

- Construct **Di-meson distilled operators** using Hadspec method of subduction coefficents and helicity operators

✓ Perform **contractions** of multi-hadron operators → 2pt correlators

- Construct correlation function coming from the **GEVP** in the right irreducible representation

- **Compute spectrum** and energy shifts w.r.t to the DD^* threshold for a heavy quark mass close to the charm quark mass.

- **Lüscher analysis** to obtain finite volume energies from Scattering amplitudes

- **Search for Poles** AKA when an attractive potential is not deep enough to hold a bound state

3.2 Two-Point Correlation function

APPENDIX A

Useful information

In the appendix you usually include extra information that should be documented in your thesis, but not interrupt the flow.

The LaTeX WikiBook [**latexwiki**] is a useful source of information on LaTeX.

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