Hyperon Masses

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GMO RLN

I. MOTIVATION

The Gell-Mann–Okubo (GMO) baryon mass relation is

$$\Delta_{\rm GMO} = M_\lambda + \frac{1}{3}M_\sigma - \frac{2}{3}M_N - \frac{2}{3}M_\Xi \,. \tag{1}$$

This quantity is interesting because it is a flavor-27 quantity, which means that the first non-vanishing correction to this relation in SU(3) baryon χPT arises from the one-loop sunset diagrams that are proportional to $m_q^{3/2}$ power, and thus non-analytic in the quark mass. One then anticipates that this quantity will exhibit a more rapid change quark mass dependence than the baryon masses themselves. While SU(3) baryon χPT is not a converging EFT in general, it may be that for this quantity, there is a convergence.

Both of these results from from DWF on asqtad ensembles, at a single lattice spacing of $a \approx 0.12$ fm and with $m_\pi \gtrsim 300$ MeV.

We have results on 30 ensembles with four lattice spacings, $a \approx \{0.06, 0.09, 0.12, 0.15\}$ fm and seven pion masses in the range $130 \lesssim m_\pi \lesssim 400$ MeV. Our aim is to explore how well SU(3) HBχPT describes the results, and, if the convergence pattern is healthy.

II. ANALYSIS NOTES

A. Fitting the correlators to extract GMO

We examined fitting the individual baryon correlation functions and then forming the GMO combination, but this yielded a result consistent with zero. In contrast, if we construct the GMO correlation function

$$C_{\rm GMO}(t) = \frac{C_\lambda(t) C_\sigma^{1/3}(t)}{C_N^{2/3}(t) C_\Xi^{2/3}(t)} \,, \tag{2}$$

and examine its effective mass, this yields values that are several sigma different from zero. We therefore need a more involved analysis of the correlation functions in order to extract precise values of GMO on each ensemble. Even if the two-point functions of each baryon are positive definite, we are not guaranteed that if we were to describe $C_{\rm GMO}(t)$ with a sum of exponentials, there overlap factors would all be positive.

However, we can probably fit this product correlation function in combination with the individual baryon correlation functions, in the following way, to obtain precise values of GMO. First, pull out the ground state overlap factors and energies from each single baryon correlation function, for which the correlator model is then

$$C_{\rm GMO}(t) = \frac{A_{\lambda,0} A_{\sigma,0}^{1/3}}{A_{N,0}^{2/3} A_{\Xi,0}^{2/3}} e^{-\Delta_{\rm GMO} t} \frac{\left[1 + \sum_n r_{\lambda,n} e^{-{\lambda,n} t}\right] \left[1 + \sum_n r_{\sigma,n} e^{-{\sigma,n} t}\right]^{1/3}}{\left[1 + \sum_n r_{N,n} e^{-{N,n} t}\right]^{2/3} \left[1 + \sum_n r_{\Xi,n} e^{-{\Xi,n} t}\right]^{2/3}} \,, \tag{3}$$

where $A_{B,0}$ are the ground state overlap factors for baryon B, $r_{B,n}$ are the ratio of the n^{th} excited state overlap factor to the ground state and $_{B,n}$ is the n^{th} excited state energy of baryon B. If this product correlation function is fit simultaneously with each individual baryon two-point function, then the only new parameter to be constrained is . We can go one step further and choose one of the ground state baryon masses to not be fit in addition to , but instead use the GMO formula, for example, one could replace M_Ξ in the fit by

$$M_\Xi \to \frac{3}{2}M_{\text{Ł}+\frac{1}{2}M_{\S}+M_N-\mathrm{GMO}}.(4)$$

XPT PRELIMINARIES

XPT, the low energy approximation of QCD, carries information about the spontaneous and explicit chiral symmetry breaking in QCD. The low energy structure of the theory depends on the size of the quark masses; Heavy quarks don't play a role since their DOFs are frozen at low energies. The overarching goal of XPT is to extract hadron properties, in terms of universal low-energy properties in an expansion about vanishing light quark masses, typically m_π. Flavor SU(3) is realized as a global symmetry of the hardon spectrum. In order to parameterize only the pion mass dependence of hadronic observables, SU(2) serves as a valid starting point, thereby sidestepping the potential convergence issues inherent in SU(3).

It is important to remark that homomorphisms of abstract groups to grpups of linear operators, symmetry groups to spaces of physical spaces in this context, form the basis for the theory of group representations.

QCD vacuum breaks chiral symmetry down to $U(3)_v$, the diagonal subgroup of SU(3). Eight massless goldstone bosons appear, each coupled via F_0 to conserved axial-vector current, thereby producing a pole in the 2-pt function. Eight parameter XPT is a simply connected lie group. Physics of the goldstone bosons, the pion fields, describe a low energy EFT called XPT; The gauge principle generates interactions between matter fields through exchange of massless gauge bosons.

One expands the physical vacuum state of QCD to obtain xpt via the treatment of the baryon fields as heavy static fermions. In the chiral limit, where the quark masses are taken to zero, the momentum transfer between baryons by pion exchange, $p^\mu = m_B v^\mu + k^\mu$, is $\lll m_B$, thus the baryon velocity is conserved.

The mass splittings in SU(3) within a multiplet possess non-trivial chiral transformation properties, thus are treated as perturbations. They can be treated as perturbations since their value is much less than the average baryon mass.

SCATTERED NOTES

h_{abc} is invariant under cyclic permutations i.e. $h_{abc} = h_{bca} = h_{cab}$

lagrangian containing only light flavor quarks in chiral limit, where $m_u, m_d, m_s \to 0$, starting pt. for low-energy QCD.

global symms in the above lagrangian appear manifest in chirality matrix γ_5

a chiral (field) var is one under which parity is tranformed into neither orig. var nor its neg.

trans: $x \to -x$ if x is a vector

Replacement of the fermion field by a boson field leads to a free field theory

using this field theory, compute correlation functions of **fermion bilinears** explicity. in the language of renormalization groups, the model contains a line of fixed points parameterized by the coupling constant g.

$$J^\mu_a = \frac{\partial \delta \mathcal{L}}{\partial \partial_\mu \epsilon_a} \partial_\mu J^\mu_a = \frac{\partial \delta \mathcal{L}}{\partial\ \epsilon_a}$$

light quark mass dependence of hardron masses det. by XPT analytic terms depended on lECS of the chiral Lagrangian

lattice quark masses may be too large for SU(3) xpt to be valid, perturbative xpt behavior occurs only for $m_q < m_s$ WHY SU3 FLAVOR SYMMETRY EVIDENT IN BARYON PHENOMENOLOGY

$1/n_c$

$1/n_c$ expansion constrains strucutre of baryon xpt

chiral corrections to the chiral lagrangian ?? have to respect spin-flavor structure of $1/n_c$ expansion

mass relations(function of m_q) project baryon masses onto diff spin-flavor channels

sometimes perturbative qcd is called asymptotic freedom

operator expansion for mass splittings of octet and decuplet uses quark operators as operator basis, can also use skyrme operator basis. what is diff??

can use lo xpt to relate quark masses to pion masses

in isospin limit of SU(3), only 2 independent quark masses, 3 independent meson masses. one can always convert from a quark mass expansion to a meson mass expansion via the gell-mann-okubo relation

Extrapolation is insensitive to hyperon couplings

Concerned with the convergence of the EFT.

In the $S = 1$ spectrum, have virtual baryons. $m_\pi < m_\Delta$ in order to decay. When $m_\pi > m_\Delta$, spin-1/2 is actually stable as there is no imaginary mass. Is this the correct implication

NUCLEON SIGMA TERM

The sum of sigma terms determines the coupling strength of nucleons to WIMPs via spin-independent channel. Sigma terms represent the contributions of explicit chiral symmetry breaking to the nucleon mass. This value is renomalization scheme and scale independent due to it being a scalar and determined via other fit parameters, namely the nucleon mass and ϵ_π.

Need to employ baryon xpt to extrapolate lattice data for M_N with heavy quarks to the physical and chiral limits, while including finite volume effects. Perform SU(2) baryon xpt to compute extrapolations of sigma terms and mass of nucleon with the light quark mass as expansion parameter.

A nucleon (proton or neutron) couples to the external higgs field via g_q where q is the quark falvor. Elementary fermions are massive, $m_f = g_f \frac{\partial m_f}{\partial g_f}$ where g_f is the fermion-higgs coupling.

For the nucleon: $M_N f_{qN} = \sigma_{qN} := g_q \frac{\partial M_n}{\partial g_q} = m_q \frac{\partial M_N}{\partial m_q}$
$\frac{\partial M_n}{\partial g_q} = \langle N \,|\, \bar{q}q \,|\, N \rangle$
The overarching aim is to determine M_π dependence on M_N and compute $\sigma_{\pi N}$ via the feynman-hellman theorem. **Result:** $M_N(M\pi)$

Derivation of the perturbative nucleon mass

XPT EXPRESSIONS FOR DELTA AND NUCLEON

In order to compute the difference between the average nucleon and average delta masses (eqn. 35), we need to compute eqns. 17 and 27 (nucleon, delta mass extrapolations, respectively) and the several orders(lo,nlo,n2lo), therein? (18-22 for nucleon and 28-30 for delta)? https://arxiv.org/abs/hep-lat/0501018 gives xpt expressions and mass expansions about m_π^2
https://arxiv.org/abs/hep-lat/0405007v2 where lecs are defined

simultaneous fits for Delta

Poisson summation formula

Poisson summation formula $\Rightarrow$ functional equation for **jacobi theta function** $\Rightarrow$ that of the **Riemann Zeta function**
$\theta(s) = \sum_{n=-\infty}^{\infty} e^{-\pi n^2 s}$ where $s \in \mathbb{R}$, however, one can also take $s \in \mathbb{C}$ and apply the Poisson summation formula to the Schwarz function $f(x) = e^{-\pi s x^2}, \hat{f}(p) = \frac{1}{\sqrt{s}} e^{-\pi \frac{p^2}{s}}$ which is just the functional equatiion of the theta function.

The more general theta function, the Jacobi theta function, is a function of two complex variables $\vartheta(z,\tau) = \sum_{n=-\infty}^{\infty} \exp(\pi i n^2 \tau + 2\pi i n z)\,.$

Why do we need finite volume corrections?

At finite temperature, the spectral density of the correlator differs from the zero temperature case so that it may not be dominated by the lowest intermediate state with quantum numbers of the currents at large t. Loop integrals get replaced with loop sums since LQCD calculations are carried out on a lattice with finite volume L^3, thus, the calculated hadronic masses will have some dependence on L. Due to the finite nature of the box with periodic boundary conditions, the spectrum of the hamiltonian operator is then discrete; The corresponding energy values $M_i(L)$ within the spectrum carry a L dependence. The afroementioned pointlike stable particles are accompanied by a cloud of virtual particles; This cloud, when "squeezed" by the box, causes the energy to deviate from the infinite volume mass. The probability for a single quark to separate from its partner(s), since of course quarks are confined and always coupled to a quark(s) via gluon(s), rapidly goes to zero as the size of L grows. Thus, the leading finite size effect on the hadron masses as L increases is due to the "squeezing" of the virtual pion cloud around the particles.

Meson pole, $p^2 = -m^2$ in the euclidean propagator has **unit residue**

The correlation functions of ϕ can be expanded in a series of feynman diagrams with momentum space propagators $\tilde{\Delta}(p;m) = (m^2 + p^2)^{-1}1$. The feynman rules for finite L correlation functions are the same as in infinite volume except that the space-like components of loop and external momenta are restricted to discrete values $\mathbf{p} = \frac{2\pi}{L}\mathbf{n}, n \in \mathbb{Z}^{\not\Vdash}$

$\mathbb{Z}^{\mathbf{3}}$ Gauge fields on a graph

It is possible to define gauge fields on an abstract graph $\mathcal{G}$, as on regular lattices.

In order to study the L dependence of feynman diagrams as $L \to \infty$, one works in position space rather than momentum space. The infinite volume propagator is $\Delta(x;m) = \int e^{ipx}(m^2+p^2)^{-1}\frac{d^4p}{(2\pi)^4}$

bound states give rise to poles in the analytically continued forward elastic meson scattering amplitude

MESON XPT

Pion Mass at one-loop

Using the power counting scheme $\epsilon_\pi = \frac{m_\pi}{\Lambda_\chi}, \Lambda_\chi = 4\pi F$, we compute the correction to the pion mass at one loop order. Note that our convention is $F = f\sqrt{2}$. $\phi = \sum_{i=1}^{3}\phi_i\tau_i = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{bmatrix}, \chi = 2B\hat{m}1$

Starting from the SU(3) matrix $U(x) = exp(i\frac{\phi(x)}{F_0})$, the most general effective Lagrangian Is $L_{eff} = \frac{F_0^2}{4}Tr(\partial_\mu U\partial^\mu U^\dagger)$. The lagrangian L_2 in the $SU(2)_L \times SU(2)_R$ is $L_2 = \frac{F^2}{8}Tr(\partial_\mu U\partial^\mu U^\dagger) + \frac{F^2}{8}2B\hat{M}Tr(U + U^\dagger)$. Again, we are working in the chiral limit where $m_u = m_d = 0$ with $m_s = phys$. When working in SU(3) sector, $m_s = 0$. It is also worth noting that in order to obtain the mass of the Goldstone boson, $M_\pi^2 = 2B_0\hat{m}$, we have to work in the isospin-symmetric limit $m_u = m_d = \hat{m}$. To expand this lagrangian in powers of ϕ, we begin by substituting in the value of U, the SU(3) matrix. We obtain

HEAVY BARYON XPT

Baryon Chiral Perturbation Theory by Jenkins and Manohar

The starting point of any EFT is the most general effective Lagrangian that respects the symmetries of the underlying theory, that of which is being approximated by a low-energy description. The “base case” is when one chooses the energy to be sufficiently small such that only pions are treated as explicit DOF. Therefore, we need some pion transformation properties:

1. According to Noether’s theorem, there are six conserved currents that generate the lie algebra of the chiral group, $SU(2)_L \times SU(2)_R$ defined by

$$[Q_I^i, Q_I^j] = i\epsilon^{ijk}Q_I^k, [Q_L^i, Q_R^j] = 0$$

Requirements for usage: Pion momentum is small and baryons are nearly on-shell, the mass of the heavy baryon is irrelevant in this EFT.

The octet of the $\frac{1}{2}$ baryons, represented as a traceless 3×3 matrix B with elements as complex, four-component Dirac fields, transforms as an octet under the adjoint representation of $SU(3)_V$. In order to construct the effective Lagrangian, we start with the group $G = SU(2)_L \times SU(2)_R$.

We want to show that the leading order correction to the nucleon masses are:

$$\delta M_p = +2\alpha_N \frac{2B\delta}{4\pi F_\pi} - 4\sigma_N \frac{M_\pi^2}{4\pi F_\pi} + NNLO+$$
$$\delta M_n = -2\alpha_N \frac{2B\delta}{4\pi F_\pi} - 4\sigma_N \frac{M_\pi^2}{4\pi F_\pi} + NNLO+$$

We begin by constructing the πN lagrangian adhering to the transformation rules of the nucleon and quark fields.

The covariant derivative is expressed as

LO self-energy correction of nucleon masses

Here, we will compute the leading order correction to the nucleon mass using heavy baryon chiral perturbation theory. First, some notation. We define the spurion field $\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$ where $u^2 = U$. The nucleon operator is constructed as $\bar{N}N = \bar{N}^i N_i = [\bar{p}p + \bar{n}N]$. We begin with computing the mass shift from the operator proportional to $\bar{N}N tr(\chi_+)$.

$$\begin{aligned} tr(\chi_+) &= 2tr(\chi) + \ldots \\ &= 2tr(2Bm_Q) \\ &= 4(2B\hat{m}) + \ldots \end{aligned}$$

From Eq (48) of 0904.2404 as is, the nucleon operators (b1, b5, b6, b8), and compute their correction to the nucleon self-energies

Convert Eq (48) of above, to our updated normalization (from new notes), and repeat

Take LO Lagrangian from notes (1.10) and expand up to two pion fields, and use these operators to compute (what you will see) are N2LO corrections to the nucleon mass

FITTING STRATEGIES

Model Averaging

To construct a model average, one must perform an extrapolation which is insensitive to the hyperon couplings. The output of this model average provides information on the convergence of the EFT used to perform the fit. We will use the extrapolated values of the hyperon masses to test mass relations from the $\frac{1}{N_c}$ expansion.

Performing a model average involves the computation of the covariance matrix between two sets of statistics computed on a set of models, namely, a simultaneous fit of two masses, for instance, of M_N, M_Δ.

III. MEETING 9/8/21

Suggestion by Andre to load the lam chi data later in the fitting routine
put in physical point as a data point in order to anchor the result
seeking a global fit with simultaneous fit of F_π with $\frac{M_N}{\Lambda_\chi}$
$\frac{\partial F}{\partial \epsilon_\pi} = y * (y^2 - x^2) - (3/2) * x^2 * y * log(x^2) - y^3 * log(4 * (y/x)^2)$