# Baryon chiral perturbation theory using a heavy fermion lagrangian

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Baryon chiral perturbation theory is developed using an effective lagrangian in which the baryons appear as heavy static fields. The chiral logarithmic corrections to the axial current for semileptonic hyperon decay and for the analysis of the strangeness content of the proton are computed as examples. The corrections are as big as the lowest order values, which implies that $F$ and $D$ cannot be reliably extracted from hyperon semileptonic decays.

Low-momentum processes involving hadrons can be related using chiral symmetry. These relations are best derived by constructing an effective low-energy lagrangian which implements the chiral symmetry and its breaking [1]. The effective lagrangian for the strong interactions contains an $\mathrm{SU}(3)_{\mathrm{L}}\times\mathrm{SU}(3)_{\mathrm{R}}$ chiral symmetry which is spontaneously broken to the diagonal SU(3) subgroup. A pseudoscalar octet of pseudo-Goldstone bosons, the pion fields, appears in the effective lagrangian as a manifestation of this breakdown. This effective lagrangian has a power series expansion in derivatives and the chiral symmetry breaking quarks mass matrix. Higher dimension operators are suppressed by inverse powers of the chiral symmetry breaking scale $\Lambda_\chi\sim 1$ GeV. Thus a term with two additional derivatives will be suppressed by $M_\pi^2/\Lambda_\chi^2$, where $M_\pi$ is the $\pi$, $K$ or $\eta$ mass. Baryon fields also can be incorporated into the effective theory in a chirally consistent manner. Unfortunately, higher derivative operators involving baryon fields are not suppressed since $m_B/\Lambda_\chi\sim \mathrm{O}(1)$, where $m_B$ is the baryon mass. Since higher dimension operators with derivatives acting on baryon fields cannot be neglected, a systematic calculation using this approach is problematic. In principle, all the terms which are powers of $(m_B/\Lambda_\chi)$ can be summed, but it is difficult to see how to perform this summation in practice.

In this paper, we develop a consistent derivative expansion for baryons in a chiral effective field theory by treating the baryon fields as heavy static fermions. The static field limit for heavy fermions has recently been used with great success to study hadrons containing a heavy quark [2–5]. An essential observation of that work is that hadronic momentum transfer due to gluon exchange can be much smaller than, and independent of, the heavy quark mass for many processes. This same result can be used here, because in the chiral limit, the momentum transferred between baryons by pion exchange is small compared to the baryon mass, so the baryon velocity is effectively conserved. We will use the formalism developed by Georgi for the study of heavy quarks [3]. The baryon momentum can be written as

$$p^\mu = m_B v^\mu + k^\mu\,, \tag{1}$$

where $m_B$ is the baryon mass, and $v{\cdot}k \ll m_B$ is proportional to the amount by which the baryon is off-shell. The effective theory can be written in terms of baryon fields $B_v$ with a definite velocity $v^\mu$ which are related to the original baryon fields $B$ by

$$B_v(x) = \exp(\mathrm{i} m_B \not{v} v_\mu x^\mu)\; B(x)\;. \tag{2}$$

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(One can have a field with definite position and velocity because $[v^\mu, x^\nu] = i\hbar g^{\mu\nu}/m_B \to 0$ in the heavy fermion limit.) These new baryon fields obey a modified Dirac equation

$$\not\partial B_v = 0 \tag{3}$$

which no longer has the mass term. Derivatives on $B_v$ produce powers of $k$ rather than $p$ so that higher derivative terms in the effective theory are suppressed by powers of $k/\Lambda_\chi$ rather than $p/\Lambda_\chi$, and are small. Thus we now have a systematic expansion in powers of derivatives.

We could write down the effective baryon theory by taking the usual baryon chiral lagrangian, and rewriting it in terms of the fields $B_v$ using the formulae in ref. [3]. However, since the effective theory is the most general lagrangian consistent with broken chiral symmetry, it is simpler to directly write down the most general possible lagrangian involving $B_v$. The fields in the effective theory are the pion field,

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} (1/\sqrt{2})\pi^0 + (1/\sqrt{6})\eta & \pi^+ & K^+ \\ \pi^- & -(1/\sqrt{2})\pi^0 + (1/\sqrt{6})\eta & K^0 \\ K^- & \bar{K}^0 & -(2/\sqrt{6})\eta \end{pmatrix}, \tag{4}$$

the $\xi$ and $\Sigma$ fields (with $f \approx 93$ MeV)

$$\xi = \exp(i\pi/f), \quad \Sigma = \xi^2 = \exp(2i\pi/f), \tag{5}$$

and the baryon field

$$B_v = \begin{pmatrix} (1/\sqrt{2})\Sigma_v^0 + (1/\sqrt{6})\Lambda_v & \Sigma_v^+ & p_v \\ \Sigma_v^- & -(1/\sqrt{2})\Sigma_v^0 + (1/\sqrt{6})\Lambda_v & n_v \\ \Xi_v^- & \Xi_v^0 & -(2/\sqrt{6})\Lambda_v \end{pmatrix}. \tag{6}$$

Under a $SU(3)_L \times SU(3)_R$ transformation these fields transform as

$$\Sigma \to L\Sigma R^\dagger, \quad B_v \to UB_vU^\dagger, \quad \xi \to L\xi U^\dagger = U\xi R^\dagger, \tag{7,8}$$

where $U$ is defined implicitly by eq. (8). The effective theory is most conveniently written in a basis in which $B \to UBU^\dagger$, rather than one in which $B_L \to LBL^\dagger$ and $B_R \to RBR^\dagger$, because the Dirac spinor for a heavy fermion at rest is not a chiral eigenstate. Thus removal of the mass term is most conveniently done in a basis in which $B_L$ and $B_R$ both transform in the same way. Only the average mass of the baryon multiplet arising from spontaneous chiral symmetry breaking can be removed. The SU(3) mass splittings within a multiplet have non-trivial chiral transformation properties, and are treated as perturbations. This procedure is consistent because the mass splittings are much smaller than the average mass.

The Dirac structure of the effective theory simplifies considerably in the heavy baryon limit. To see this, it is useful to introduce the velocity projection operator

$$P_v = \tfrac{1}{2}(1 + \not v), \quad B_v = P_v B_v, \tag{9}$$

and the spin operators $S_v^\mu$ which satisfy

$$v \cdot S_v = 0, \quad S_v^2 B_v = -\tfrac{3}{4} B_v; \quad \{S_v^\lambda, S_v^\sigma\} = \tfrac{1}{2}(v^\lambda v^\lambda - g^{\lambda\sigma}), \quad [S_v^\lambda, S_v^\sigma] = i\epsilon^{\lambda\sigma\alpha\beta} v_\alpha S_{v\beta}. \tag{11}$$

Then

$$\bar{B}_v \gamma_5 B_v = 0, \quad \bar{B}_v \gamma^\mu B_v = v^\mu \bar{B}_v B_v, \quad \bar{B}_v \gamma^\mu \gamma_5 B_v = 2\bar{B}_v S_v^\mu B_v,$$
$$\bar{B}_v \sigma^{\mu\nu} B_v = 2\epsilon^{\mu\nu\alpha\beta} v_\alpha \bar{B}_v S_{v\beta} B_v, \quad \bar{B}_v \sigma^{\mu\nu}\gamma_5 B_v = 2i(v^\mu \bar{B}_v S_v^\nu B_v - v^\nu \bar{B}_v S_v^\mu B_v). \tag{12}$$

Thus the effective lagrangian can be written using $v^\mu$ and $S_v^\mu$ exclusively and omitting all gamma matrices. The most general lagrangian at lowest order is

$$L_v^0 = \mathrm{i}\,\mathrm{Tr}\bar{B}_v(v\cdot\mathscr{D})B_v + 2D\,\mathrm{Tr}\,\bar{B}_v S_v^\mu\{A_\mu, B_v\} + 2F\,\mathrm{Tr}\,\bar{B}_v S_v^\mu[A_\mu, B_v] + \tfrac{1}{4}f^2\,\mathrm{Tr}\,\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger + a\,\mathrm{Tr}\,M(\Sigma+\Sigma^\dagger)\,, \qquad (13)$$

where

$$\mathscr{D}^\mu B_v = \partial^\mu B_v + [V^\mu, B_v]\,, \qquad (14)$$

$$V^\mu = \tfrac{1}{2}(\xi\,\partial^\mu\xi^\dagger + \xi^\dagger\,\partial^\mu\xi), \quad A^\mu = \tfrac{1}{2}\mathrm{i}(\xi\,\partial^\mu\xi^\dagger - \xi^\dagger\,\partial^\mu\xi)\,, \qquad (15)$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \qquad (16)$$

is the quark mass matrix. Note that the mass term $m_B\bar{B}B$ in the usual chiral lagrangian is absent because of the redefinition (2). At next order (in the chiral expansion) the baryon lagrangian is

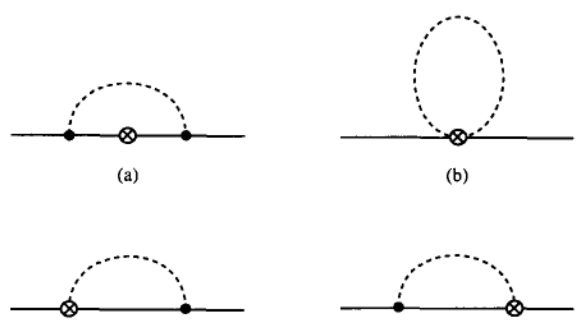
$$L_v^1 = b_D\,\mathrm{Tr}\,\bar{B}_v\{\xi^\dagger M\xi^\dagger + \xi M\xi, B_v\} + b_F\,\mathrm{Tr}\,\bar{B}_v[\xi^\dagger M\xi^\dagger + \xi M\xi, B_v] + c_1\,\mathrm{Tr}\,\bar{B}_v\mathscr{D}^2 B_v + c_2\,\mathrm{Tr}\,\bar{B}_v(v\cdot\mathscr{D})^2 B_v$$
$$+ d_1\,\mathrm{Tr}\,\bar{B}_v A^2 B_v + d_2\,\mathrm{Tr}\,\bar{B}_v(v\cdot A)^2 B_v + f_1\,\mathrm{Tr}\,\bar{B}_v(v\cdot\mathscr{D})(S_v\cdot A)B_v + f_2\,\mathrm{Tr}\,\bar{B}_v(S_v\cdot\mathscr{D})(v\cdot A)B_v$$
$$+ \mathrm{i}\epsilon^{\alpha\beta\lambda\sigma}v_\alpha\,\mathrm{Tr}\,\bar{B}_v S_{v\beta}A_\lambda A_\sigma B_v + \text{permutations}\,, \qquad (17)$$

where the permutations are over distinct orderings of the octets $B_v$, $\mathscr{D}$ and $A$.

The effective lagrangian (13) can now be used to compute chiral logarithms in the effective theory. The calculation simplifies in the heavy fermion formalism because there are no gamma matrices left. For example, the baryon propagator is $\mathrm{i}/(v\cdot k)$, and the pion–nucleon vertex is proportional to $k\cdot S_v$. We have computed the renormalization of the vector and axial vector currents using (13). The vector current is not renormalized to this order in chiral perturbation theory, which is consistent with the Ademollo–Gato theorem. The Feynman graphs which lead to renormalization of the axial vector current are shown in figs. 1 and 2. These corrections have been computed before using conventional chiral perturbation theory by Bijnens, Sonoda and Wise, so we refer the rader to ref. [6] for details. The only difference between our calculation and the calculation using the conventional baryonic chiral lagrangian is that the Feynman rules are much simpler in our case. A sample calculation is presented in the appendix. Following ref. [6], we work in the limit $m_u = m_d = 0$, and use the Gell-Mann–Okubo formula to rewrite $M_\eta^2$ as $\frac{4}{3}M_K^2$. The renormalization of the axial current

(a) (b)

Fig. 1. Wavefunction renormalization graphs. The solid dot represents vertices in chiral lagrangian (13). Graph (b) vanishes identically.



(a) (b) (c) (d)

Fig. 2. The Feynman graphs which contribute to the renormalization of the axial current. The dot represents vertices in chiral lagrangian (13) and the circled cross represents an insertion of the axial current (18).

$$
\begin{aligned}
J_\mu^A &= \tfrac{1}{2}D\,\mathrm{Tr}\,\bar{B}_v\gamma_\mu\gamma_5\{\xi T^A\xi^\dagger+\xi^\dagger T^A\xi, B_v\}+\tfrac{1}{2}F\,\mathrm{Tr}\,\bar{B}_v\gamma_\mu\gamma_5[\xi T^A\xi^\dagger+\xi^\dagger T^A\xi, B_v] \\
&\quad +\tfrac{1}{2}\,\mathrm{Tr}\,\bar{B}_v\gamma_\mu[\xi T^A\xi^\dagger-\xi^\dagger T^A\xi, B_v]+\mathrm{i}\,\tfrac{1}{2}f^2\,\mathrm{Tr}\,T^A\,(\partial^\mu\Sigma^\dagger\Sigma-\partial^\mu\Sigma\Sigma^\dagger) \\
&= D\,\mathrm{Tr}\,\bar{B}_v S_v^\mu\{\xi T^A\xi^\dagger+\xi^\dagger T^A\xi, B_v\}+F\,\mathrm{Tr}\,\bar{B}_v S_v^\mu[\xi T^A\xi^\dagger+\xi^\dagger T^A\xi, B_v] \\
&\quad +\tfrac{1}{2}v^\mu\,\mathrm{Tr}\,\bar{B}_v[\xi T^A\xi^\dagger-\xi^\dagger T^A\xi, B_v]+\mathrm{i}\,\tfrac{1}{2}f^2\,\mathrm{Tr}\,T^A(\partial^\mu\Sigma^\dagger\Sigma-\partial^\mu\Sigma\Sigma^\dagger)
\end{aligned}
\tag{18}
$$

can be written in the form

$$
\langle B_i|J_\mu^A|B_j\rangle=\left[\alpha_{ij}^A+(\beta_{ij}^A-\lambda_{ij}\alpha_{ij}^A)\,\frac{M_K^2}{16\pi^2f^2}\ln\left(\frac{M_K^2}{\mu^2}\right)\right]\bar{u}_{B_i}\gamma_\mu\gamma_5u_{B_j}\,,
\tag{19}
$$

where $\alpha_{ij}^A$ is the lowest order result, $\lambda_{ij}$ is the one-loop correction due to wavefunction renormalization (fig. 1), $\beta_{ij}^A$ is the remaining one-loop correction (fig. 2), and $u$ is a spinor. The coefficients $\alpha_{ij}^A$ are

$$
\begin{aligned}
&\alpha_{pn}^{1+i2}=(D+F),\quad \alpha_{\Lambda\Sigma^-}^{1+i2}=\frac{2}{\sqrt{6}}D,\quad \alpha_{\Xi^0\Xi^-}^{1+i2}=(D-F)\,,\quad \alpha_{p\Lambda}^{4+i5}=-\frac{1}{\sqrt{6}}(D+3F)\,,\\
&\alpha_{\Lambda\Xi^-}^{4+i5}=-\frac{1}{\sqrt{6}}(D-3F),\quad \alpha_{n\Sigma^-}^{4+i5}=(D-F)\,,\quad \alpha_{\Sigma^0\Xi^-}^{4+i5}=\frac{1}{\sqrt{2}}(D+F)=\sqrt{2}\,\alpha_{\Sigma^+\Xi^0}^{4+i5}\,.
\end{aligned}
\tag{20}
$$

The wavefunction renormalization factors

$$
\sqrt{Z_iZ_j}=1+\lambda_{ij}\frac{M_K^2}{16\pi^2f^2}\ln\left(\frac{M_K^2}{\mu^2}\right)
$$

are obtained from fig. 1:

$$
\begin{aligned}
&\lambda_{pn}=\tfrac{17}{6}D^2-5DF+\tfrac{15}{2}F^2,\quad \lambda_{\Lambda\Sigma^-}=\tfrac{10}{3}D^2+6F^2,\quad \lambda_{\Xi^0\Xi^-}=\tfrac{17}{6}D^2+5DF+\tfrac{15}{2}F^2,\quad \lambda_{p\Lambda}=\tfrac{31}{12}D^2-\tfrac{5}{2}DF+\tfrac{33}{4}F^2\,,\\
&\lambda_{\Lambda\Xi^-}=\tfrac{31}{12}D^2+\tfrac{5}{2}DF+\tfrac{33}{4}F^2,\quad \lambda_{n\Sigma^-}=\tfrac{43}{12}D^2-\tfrac{5}{2}DF+\tfrac{21}{4}F^2,\quad \lambda_{\Sigma^0\Xi^-}=\tfrac{43}{12}D^2+\tfrac{5}{2}D+\tfrac{21}{4}F^2=\lambda_{\Sigma^+\Xi^0}\,.
\end{aligned}
\tag{21}
$$

Fig. 2 yields the coefficients $\beta_{ij}^A$:

$$
\begin{aligned}
&\beta_{pn}^{1+i2}=\tfrac{2}{9}D^3+\tfrac{2}{9}D^2F+\tfrac{2}{3}DF^2-2F^3-\tfrac{1}{2}D-\tfrac{1}{2}F,\quad \beta_{\Lambda\Sigma^-}^{1+i2}=\frac{1}{\sqrt{6}}(\tfrac{17}{9}D^3-DF^2-D)\,,\\
&\beta_{\Xi^0\Xi^-}^{1+i2}=\tfrac{2}{9}D^3-\tfrac{2}{9}D^2F+\tfrac{2}{3}DF^2+2F^3-\tfrac{1}{2}D+\tfrac{1}{2}F,\quad \beta_{p\Lambda}^{4+i5}=\frac{1}{\sqrt{6}}(\tfrac{19}{18}D^3-\tfrac{5}{2}D^2F-\tfrac{7}{2}DF^2+\tfrac{9}{2}F^3+\tfrac{5}{4}D+\tfrac{15}{4}F)\,,\\
&\beta_{\Lambda\Xi^-}^{4+i5}=\frac{1}{\sqrt{6}}(\tfrac{19}{18}D^3+\tfrac{5}{2}D^2F-\tfrac{7}{2}DF^2-\tfrac{9}{2}F^3+\tfrac{5}{4}D-\tfrac{15}{4}F),\quad \beta_{n\Sigma^-}^{4+i5}=\tfrac{7}{18}D^3-\tfrac{13}{18}D^2F+\tfrac{7}{6}DF^2+\tfrac{1}{2}F^3-\tfrac{5}{4}D+\tfrac{5}{4}F\,,\\
&\beta_{\Sigma^0\Xi^-}^{4+i5}=\frac{1}{\sqrt{2}}(\tfrac{7}{18}D^3+\tfrac{13}{18}D^2F+\tfrac{7}{6}DF^2-\tfrac{1}{2}F^3-\tfrac{5}{4}D-\tfrac{5}{4}F)=\sqrt{2}\,\beta_{\Sigma^+\Xi^0}^{4+i5}\,.
\end{aligned}
\tag{22}
$$

Our results differ from ref. [6] in that ref. [6] inadvertently omitted wavefunction renormalization for the axial current. We find that the one-loop correction is as large as the lowest order result. Bijnens et al. [6] found that the correction was only 30% because wavefunction renormalization was not included.

We fit $D$ and $F$ to the measured hyperon semileptonic decays. The errors on the experimental values of $g_A$ were each increased by 0.2 to avoid biasing the fit to the $D+F$ value favored by neutron $\beta$ decay. The value 0.2 is a nominal value for the theoretical uncertainty in the calculation because of higher order SU(3) breaking effects. The best fit values are $D=0.56\pm0.1$, $F=0.33\pm0.06$, and $F/D=0.59\pm0.15$, with a $\chi^2=1.8$ for 12 degrees of freedom. The best fit values omitting chiral loop corrections are $D=0.80\pm0.14$, $F=0.50\pm0.12$, and $F/D=0.63\pm0.19$, with $\chi^2=1.1$. The large shift in $D$ and $F$ is a reflection of the large one-loop correction. One should probably not use a SU(3) parametrization of the axial current in view of the large one-loop symmetry breaking corrections. Note that the breakdown in the perturbation series occurs because of the large symmetry breaking $K$ mass, not due to a breakdown in the derivative expansion of chiral perturbation theory. There is no problem if one only works with an SU(2) effective theory and computes pion loop corrections.

The proton matrix element of the $T^8$ axial current is relevant for analyzing the results of a recent spin dependent $\mu$–$p$ scattering experiment [7]. Using the notation (19), we find

$$\alpha^8_{pp} = \frac{1}{\sqrt{12}}(3F-D), \quad \beta^8_{pp} = \frac{1}{\sqrt{12}}\left(-\tfrac{11}{9}D^3+3D^2F+3DF^2-3F^3-\tfrac{9}{2}F+\tfrac{3}{2}D\right), \tag{23}$$

and $\lambda_{pp}=\lambda_{pn}$ is given by eq. (21) since $Z_{pp}=Z_{pn}$ by isospin symmetry. Using the one-loop values for $D$ and $F$ above, we find $\langle p|T^8|p\rangle = 0.18\pm0.18$, which implies [8] that $\Delta s=-0.17\pm0.27$, and $\Delta u+\Delta d+\Delta s=0.12\pm0.24$. These numbers are not very reliable because of the large SU(3) breaking corrections to the axial currents.

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*Appendix.* Omitting all SU(3) Clebsch–Gordan factors, the Feynman integral for fig. 2a is

$$I=\int\frac{d^4k}{(2\pi)^4}\bar{B}_v\left((2k\cdot S_v)\frac{i}{k\cdot v}2S^\mu_v\frac{i}{k\cdot v}(-2k\cdot S_v)\right)B_v\frac{i}{k^2-M^2}=8i\bar{B}_vS^\alpha_vS^\mu_vS^\beta_vB_v\int\frac{d^4k}{(2\pi)^4}\frac{k_\alpha k_\beta}{(k\cdot v)^2(k^2-M^2)}\,.$$

Using the identity [4]

$$\frac{1}{ab^2}=\int_0^\infty\frac{8\lambda\,d\lambda}{(a+2b\lambda)^3}$$

to combine denominators, shifting the momentum integral to $l^\mu=k^\mu-\lambda v^\mu$, and using $v\cdot S_v=0$ yields

$$I=16i\bar{B}_vS^\alpha_vS^\mu_vS_{v\alpha}B_v\int_0^\infty\lambda\,d\lambda\int\frac{d^4l}{(2\pi)^4}\frac{l^2}{(l^2-M^2-\lambda^2)^3}=-16\,\bar{B}_vS^\alpha_vS^\mu_vS_{v\alpha}B_v\frac{1}{16\pi^2\epsilon}\int_0^\infty\lambda\,d\lambda\,(M^2+\lambda^2)^{-\epsilon}$$

$$=8\bar{B}_vS^\alpha_vS^\mu_vS_{v\alpha}B_v\frac{1}{16\pi^2\epsilon}(M^2)^{1-\epsilon}\,,$$

where we have dropped terms which are finite as $\epsilon\to0$. Using the identity $S^\alpha_vS^\mu_vS_{v\alpha}=\frac{1}{4}S^\mu_v$, which follows from eq. (10) and (11) gives

$$I=\frac{M^2}{16\pi^2\epsilon}(2\bar{B}_vS^\mu_vB_v)(M^2)^{-\epsilon}\,.$$

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