Hyperon Masses

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I. HYPERON MASS FORMULA

The mass formula are generically written as

$$M_H = M_H^{(\chi)} + \delta M_H^{\text{disc}} + \delta M_H^{q_s \neq q_s^*} \tag{1}$$

where the discretization (δM_H^{disc}) and strange quark mass mistuning ($\delta M_H^{q_s \neq q_s^*}$) are given by, respectively,

$$\delta M_H^{\text{disc}} = M_H^{(0)} \Big(\overbrace{2\epsilon_\pi^2}^{\text{LO}} + \overbrace{d_{aa}\epsilon_a^4 + d_{al}\epsilon_a^2\epsilon_\pi^2}^{\text{N}^2\text{LO}} + \cdots \Big) \,, \tag{2}$$

$$\delta M_H^{q_s \neq q_s^*} = M_H^{(0)} \Big(\overbrace{d_s \delta s_F^2}^{\text{LO}} + \overbrace{d_{as}\delta s_F^2 \epsilon_a^2 + d_{ls}\delta s_F^2 \epsilon_\pi^2 + d_{ss}\delta s_F^4}^{\text{N}^2\text{LO}} + \cdots \Big) \,. \tag{3}$$

Notice that there is no NLO correction for either of these terms. However, such a term does exist in the chiral expansion.

Below we concentrate on the chiral formula. For convenience we define the following variables

$$\Lambda_\chi = 4\pi F_\pi$$

$$\epsilon_\pi = \frac{m_\pi}{\Lambda_\chi} \qquad \epsilon_a = \frac{a}{2w_0} \qquad \epsilon_{H_1 H_2} = \frac{m_{H_2}^{(0)} - m_{H_1}^{(0)}}{\Lambda_\chi}$$

$$s_F^2 = \frac{2m_K^2 - m_\pi^2}{\Lambda_\chi} \qquad \delta s_F^2 = s_F^2 - (s_F^*)^2$$

and non-analytic functions

$$\mathcal{F}(\epsilon_\pi, \epsilon_\Delta, \mu) = -\epsilon_\Delta \left(\epsilon_\Delta^2 - \epsilon_\pi^2\right) R\left(\frac{\epsilon_\pi^2}{\epsilon_\Delta^2}\right) - \frac{3}{2}\epsilon_\pi^2 \epsilon_\Delta \log\left(\epsilon_\pi^2 \frac{\Lambda_\chi^2}{\mu^2}\right) - \epsilon_\Delta^3 \log\left(4\frac{\epsilon_\Delta^2}{\epsilon_\pi^2}\right) \,, \tag{4}$$

$$\mathcal{J}(\epsilon_\pi, \epsilon_\Delta, \mu) = \epsilon_\pi^2 \log\left(\epsilon_\pi^2 \frac{\Lambda_\chi^2}{\mu^2}\right) + 2\epsilon_\Delta^2 \log\left(4\frac{\epsilon_\Delta^2}{\epsilon_\pi^2}\right) + 2\epsilon_\Delta^2 R\left(\frac{\epsilon_\pi^2}{\epsilon_\Delta^2}\right) \,, \tag{5}$$

$$R(x) = \begin{cases} \sqrt{1-x}\log\left(\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right) \,, & 0 < x \leq 1 \\ 2\sqrt{x-1}\arctan\left(\sqrt{x-1}\right) \,, & x > 1 \end{cases} \tag{6}$$

A. $S = 1$ hyperons

The Λ:

$$
\begin{aligned}
M_\Lambda^{(\chi)} =& M_\Lambda^{(0)} && \text{(LLO)}\\
&+ \sigma_\Lambda \Lambda_\chi \epsilon_\pi^2 && \text{(LO)}\\
&- \frac{1}{2} g_{\pi\Lambda\Sigma}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_{\Lambda\Sigma}, \mu) - 2 g_{\pi\Lambda\Sigma^*}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_{\Lambda\Sigma^*}, \mu) && \text{(NLO)}\\
&+ \frac{3}{4} g_{\pi\Lambda\Sigma}^2 (\sigma_\Lambda - \sigma_\Sigma) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{\Lambda\Sigma}, \mu) && \text{(N}^2\text{LO)}\\
&\quad + 3 g_{\pi\Lambda\Sigma^*}^2 (\sigma_\Lambda - \overline{\sigma}_\Sigma) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{\Lambda\Sigma^*}, \mu)\\
&\quad + \alpha_\Lambda^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_\Lambda^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$

The Σ:

$$
\begin{aligned}
M_\Sigma^{(\chi)} =& M_\Sigma^{(0)} && \text{(LLO)}\\
&+ \sigma_\Sigma \Lambda_\chi \epsilon_\pi^2 && \text{(LO)}\\
&- \pi g_{\pi\Sigma\Sigma}^2 \Lambda_\chi \epsilon_\pi^3 - \frac{1}{6} g_{\pi\Lambda\Sigma}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, -\epsilon_{\Lambda\Sigma}, \mu) && \text{(NLO)}\\
&\quad - \frac{2}{3} g_{\pi\Sigma^*\Sigma}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_{\Sigma\Sigma^*}, \mu)\\
&+ g_{\pi\Sigma^*\Sigma}^2 (\sigma_\Sigma - \overline{\sigma}_\Sigma) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{\Sigma\Sigma^*}, \mu) && \text{(N}^2\text{LO)}\\
&\quad + \frac{1}{4} g_{\pi\Lambda\Sigma}^2 (\sigma_\Sigma - \sigma_\Lambda) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, -\epsilon_{\Lambda\Sigma}, \mu)\\
&\quad + \alpha_\Sigma^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_\Sigma^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$

The Σ^*:

$$
\begin{aligned}
M_{\Sigma^*}^{(\chi)} =& M_{\Sigma^*}^{(0)} && \text{(LLO)}\\
&+ \overline{\sigma}_\Sigma \Lambda_\chi \epsilon_\pi^2 && \text{(LO)}\\
&- \frac{5\pi}{9} g_{\pi\Sigma^*\Sigma^*}^2 \Lambda_\chi \epsilon_\pi^3 - \frac{1}{3} g_{\pi\Sigma^*\Sigma}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, -\epsilon_{\Sigma\Sigma^*}, \mu) && \text{(NLO)}\\
&\quad - \frac{1}{3} g_{\pi\Lambda\Sigma^*}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, -\epsilon_{\Lambda\Sigma^*}, \mu)\\
&+ \frac{1}{2} g_{\pi\Sigma^*\Sigma}^2 (\overline{\sigma}_\Sigma - \sigma_\Sigma) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, -\epsilon_{\Sigma\Sigma^*}, \mu) && \text{(N}^2\text{LO)}\\
&\quad + \frac{1}{2} g_{\pi\Lambda\Sigma^*}^2 (\overline{\sigma}_\Sigma - \sigma_\Sigma) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, -\epsilon_{\Lambda\Sigma^*}, \mu)\\
&\quad + \alpha_{\Sigma^*}^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_{\Sigma^*}^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$

B. $S = 2$ hyperons

The Ξ:

$$
\begin{aligned}
M_{\Xi}^{(\chi)} = & M_{\Xi}^{(0)} && \text{(LLO)} \\
& + \sigma_{\Xi} \Lambda_\chi \epsilon_\pi^2 && \text{(LO)} \\
& - \frac{3\pi}{2} g_{\pi\Xi\Xi}^2 \Lambda_\chi \epsilon_\pi^3 - g_{\pi\Xi^*\Xi}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_{\Xi\Xi^*}, \mu) && \text{(NLO)} \\
& + \frac{3}{2} g_{\pi\Xi^*\Xi}^2 (\sigma_\Xi - \overline{\sigma}_\Xi) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_{\Xi\Xi^*}, \mu) && \text{(N}^2\text{LO)} \\
& \quad + \alpha_{\Xi}^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_{\Xi}^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$

The Ξ^*:

$$
\begin{aligned}
M_{\Xi^*}^{(\chi)} = & M_{\Xi^*}^{(0)} && \text{(LLO)} \\
& + \overline{\sigma}_{\Xi} \Lambda_\chi \epsilon_\pi^2 && \text{(LO)} \\
& - \frac{5\pi}{6} g_{\pi\Xi^*\Xi^*}^2 \Lambda_\chi \epsilon_\pi^3 - \frac{1}{2} g_{\pi\Xi^*\Xi}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, -\epsilon_{\Xi\Xi^*}, \mu) && \text{(NLO)} \\
& + \frac{3}{4} g_{\pi\Xi^*\Xi}^2 (\overline{\sigma}_\Xi - \sigma_\Xi) \Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, -\epsilon_{\Xi\Xi^*}, \mu) && \text{(N}^2\text{LO)} \\
& \quad + \alpha_{\Xi^*}^{(4)} \Lambda_\chi \epsilon_\pi^4 \log \epsilon_\pi^2 + \beta_{\Xi^*}^{(4)} \Lambda_\chi \epsilon_\pi^4
\end{aligned}
$$