Hyperon Extrapolation Formulae

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Expressing extrapolation formulae in a useful form for LQCD extrapolations.

We start with the extrapolation formulae for the hyperon masses in Ref. [1]. Consider the cascade (Ξ) mass as an example. A convenient form for fitting the spectrum in MeV and incorporating the discretization effects, and which allows for an extrapolation plot in terms of ϵ_π^2 and ϵ_a^2 is

$$\begin{aligned} M_\Xi = M_\Xi^{(0)} \big[& 1 + d_a \epsilon_a^2 + d_{al}\epsilon_a^2\epsilon_\pi^2 + d_{aa}\epsilon_a^4 + \cdots + d_s \delta s_F^2 + d_{sa}\delta s_F^2 \epsilon_a^2 + d_{sl}\delta s_F^2\epsilon_\pi^2 + d_{ss}\delta s_F^4 \big] \\ & + \sigma_\Xi \Lambda_\chi \epsilon_\pi^2 - \frac{3\pi g_{\pi\Xi\Xi}^2}{2}\Lambda_\chi \epsilon_\pi^3 - g_{\pi\Xi^*\Xi}^2 \Lambda_\chi \mathcal{F}(\epsilon_\pi, \epsilon_\Delta, \mu) \\ & + \frac{3 g_{\pi\Xi^*\Xi}^2(\sigma_\Xi - \bar{\sigma}_\Xi)}{2}\Lambda_\chi \epsilon_\pi^2 \mathcal{J}(\epsilon_\pi, \epsilon_\Delta, \mu) + \Lambda_\chi \epsilon_\pi^4 \left[\alpha_\Xi^{(4)} \ln(\epsilon_\pi^2) + \beta_\Xi^{(4)} \right] \end{aligned} \tag{0.1}$$

where we have defined

$$\Lambda_\chi = 4\pi F_\pi \,, \tag{0.2}$$

$$\epsilon_\pi = \frac{m_\pi}{\Lambda_\chi} \,, \tag{0.3}$$

$$\epsilon_\Delta = \frac{\Delta}{\Lambda_\chi} \,, \tag{0.4}$$

$$\epsilon_a = \frac{1}{2}\frac{a}{w_0} \,, \tag{0.5}$$

$$s_F^2 = \frac{2m_K^2 - m_\pi^2}{(4\pi F_\pi)^2} \,, \tag{0.6}$$

$$\delta s_F^2 = s_F^2 - s_F^{\mathrm{phys},2} \,, \tag{0.7}$$

$$m_\pi^2 = 2Bm_l + \mathcal{O}(m_\pi^4/\Lambda_\chi^2) \,, \tag{0.8}$$

$$m_K^2 = B(m_s + m_l) + \mathcal{O}(\{m_\pi^4, m_K^2 m_\pi^2\}/\Lambda_\chi^2) \tag{0.9}$$

and the coefficient σ_Ξ is that of Ref. [1] divided by $\sqrt{2}$. The non-analytic functions are

$$\mathcal{F}(\epsilon_\pi, \epsilon_\Delta, \mu) = -\epsilon_\Delta(\epsilon_\Delta^2 - \epsilon_\pi^2) R\left(\frac{\epsilon_\pi^2}{\epsilon_\Delta^2}\right) - \frac{3}{2}\epsilon_\pi^2 \epsilon_\Delta \ln\left(\epsilon_\pi^2 \frac{\Lambda_\chi^2}{\mu^2}\right) - \epsilon_\Delta^3 \ln\left(\frac{4\epsilon_\Delta^2}{\epsilon_\pi^2}\right) \tag{0.10}$$

$$\mathcal{J}(\epsilon_\pi, \epsilon_\Delta, \mu) = \epsilon_\pi^2 \ln\left(\epsilon_\pi^2 \frac{\Lambda_\chi^2}{\mu^2}\right) + 2\epsilon_\Delta^2 \ln\left(\frac{4\epsilon_\Delta^2}{\epsilon_\pi^2}\right) + 2\epsilon_\Delta^2 R\left(\frac{\epsilon_\pi^2}{\epsilon_\Delta^2}\right) \tag{0.11}$$

$$R(x) = \begin{cases} \sqrt{1-x}\ln\left(\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right), & 0 < x \le 1 \\ 2\sqrt{x-1}\arctan\left(\sqrt{x-1}\right), & x > 1 \end{cases} \tag{0.12}$$

Since we have an extrapolation of Λ_χ as a function of ϵ_π and ϵ_a, we can use the lattice value of Λ_χ when analyzing the masses and then, when we want to plot the result as a function of ϵ_π or ϵ_a, we can use our fit to $\Lambda_\chi(\epsilon_\pi, \epsilon_a)$.

[1] Brian C. Tiburzi and Andre Walker-Loud, "Hyperons in Two Flavor Chiral Perturbation Theory," Phys.Lett. **B669**, 246–253 (2008), arXiv:0808.0482 [nucl-th].

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