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Baryon Chiral Perturbation Theory*

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Abstract

A new formulation of baryon chiral perturbation theory is developed using an effective Lagrangian for broken $SU(3)_L \times SU(3)_R$ chiral symmetry, in which the baryons appear as heavy static fermions. This formulation of baryon chiral perturbation theory has a consistent expansion in powers of momentum and light quark masses because the baryon mass does not appear in the effective Lagrangian. The chiral expansion in powers of the physical strange quark mass requires the inclusion of both the spin-1/2 octet and the spin-3/2 decuplet in the effective theory. The chiral Lagrangian for octet and decuplet baryons is used to compute the leading non-analytic in m_s corrections to baryon axial vector currents, masses, and non-leptonic decays. The results are compared with experiment. There is a large non-analytic strange quark contribution to the baryon masses which changes the value of $\langle p| \, m_s \bar{s}s \, |p\rangle$ from 411 MeV to approximately zero. The decoupling of the non-analytic contributions due to intermediate decuplet states in the chiral limit is studied in some detail. It is possible to define an $SU(6)$ symmetry of the effective Lagrangian which relates octet and decuplet states. The best fit values for the octet and decuplet pion couplings respect approximate $SU(6)$ symmetry, but the non-leptonic weak decay parameters do not.

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1. Introduction

The QCD Lagrangian for the three light quarks has an approximate $SU(3)_L \times SU(3)_R$ chiral symmetry, which is spontaneously broken by the strong interactions to the diagonal $SU(3)_V$ subgroup. There is a pseudoscalar octet of approximate Goldstone bosons, the π's, K's and η. Low momentum processes involving Goldstone bosons can be related to each other using chiral symmetry. The implications of chiral symmetry for low-momentum processes are best derived using an effective Lagrangian which implements the spontaneous breaking of chiral symmetry.[1]–[6] The Goldstone boson effective Lagrangian is the most general possible Lagrangian with broken $SU(3)_L \times SU(3)_R$ symmetry, including terms with arbitrary numbers of derivatives. Higher dimension operators in the effective Lagrangian are suppressed by inverse powers of the chiral symmetry breaking scale $\Lambda_\chi \sim 1$ GeV. Thus the pion scattering amplitudes in the effective Lagrangian are determined as a power series in k/Λ_χ, where k is the pion momentum. The explicit $SU(3)_L \times SU(3)_R$ symmetry breaking by the light quark masses can be incorporated by including factors of the quark mass matrix M in the effective Lagrangian, where M transforms as $(\mathbf{3}, \overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}}, \mathbf{3})$ under $SU(3)_L \times SU(3)_R$. The power counting scheme of dividing factors of ∂_μ and M by Λ_χ can be shown to be a consistent scheme to all orders in the loop expansion if $\Lambda_\chi \leq 4\pi f$. Even though the effective Lagrangian is non-renormalizable, and has an infinite number of coupling constants, one only needs to know a finite number of couplings to determine scattering amplitudes to any fixed order in k/Λ_χ.

Octet baryon fields can be included in the effective Lagrangian as matter fields, following the prescription of Callan *et al.*[5] The higher dimension terms in the baryon Lagrangian are also suppressed by inverse powers of Λ_χ but there is no longer a consistent derivative expansion for processes involving baryons. Consider, for example,

$$L = \overline{B}\left(i\partial\!\!\!/ - m_B\right)B + a\overline{B}\left(i\partial\!\!\!/ - m_B\right)\frac{\partial^2}{\Lambda_\chi^2}B$$

which is the baryon kinetic term plus a higher dimension term with two additional derivatives. The higher dimension term is important even for low-momentum transfer processes, because the time derivatives in $\partial^2/\Lambda_\chi^2$ produce a factor of m_B^2/Λ_χ^2 which is not small. Thus a computation of low-momentum baryon-pion scattering amplitudes requires that one first sum all the time derivatives in the effective Lagrangian; it is not consistent to just use the lowest order term. A similar problem occurs in the loop expansion. Weinberg's power counting argument fails because m_B is a dimensionful number of order Λ_χ.

Higher order loop graphs can produce amplitudes which are only “suppressed” by factors of $m_B/\Lambda_\chi \sim 1$, and are therefore as important as lower order amplitudes. A new formalism is developed in this article which circumvents these problems. The baryon is included in the chiral Lagrangian as a heavy static fermion. This theory is shown to have a consistent derivative and loop expansion. The leading non-analytic corrections to the baryon axial currents, masses, and non-leptonic decays are computed in this formalism, and the results are compared with experiment. We show that the experimental data requires that the baryon chiral Lagrangian include both the octet and decuplet baryons.

2. The Heavy Baryon Theory and Power Counting

Low-energy theorems of current algebra only require that the pion momentum be small and that the baryons are nearly on- shell; the large baryon mass is irrelevant. The heavy baryon formalism described in this section is useful under precisely the same conditions. The heavy baryon Lagrangian describes the interactions of a heavy baryon with low-momentum pions. The velocity of the baryon is nearly unchanged when it exchanges some small momentum with the pion. Thus, it is convenient to use a variant of the formalism developed recently for the study of the heavy quark limit in QCD.[8],[9] A nearly on-shell baryon with velocity v^μ has momentum

$$p^\mu = m_B v^\mu + k^\mu, \tag{2.1}$$

where m_B is the baryon mass, and $k \cdot v \ll \Lambda_\chi$ is proportional to the amount by which the baryon is off-shell. The effective theory is written in terms of baryon fields B_v with definite velocity v^μ, which are related to the original baryon fields by[9]

$$B_v(x) = e^{im_B \not v\, v_\mu x^\mu} B(x). \tag{2.2}$$

The new baryon fields obey a modified Dirac equation,

$$i\not\partial B_v = 0 \tag{2.3}$$

which no longer contains a baryon mass term. Derivatives acting on the field B_v produce factors of k, rather than p, so that higher derivative terms in the Lagrangian are suppressed by powers of k/Λ_χ, which is small. Thus, the heavy baryon Lagrangian has a consistent derivative expansion. The effective theory can be used provided the pion momentum and the off-shellness of the baryon are small compared with Λ_χ.

The heavy baryon Lagrangian also has a $1/m_B$ expansion. The $1/m_B$ effects in the original Dirac theory can be reproduced in the effective theory by including higher dimension operators suppressed by inverse powers of m_B.[10] The relative magnitude of m_B and Λ_χ is irrelevant for the validity of the chiral expansion. In general, one might want to separate terms which are formally $1/m_B$ from terms which are formally $1/\Lambda_\chi$. In this article, we will restrict our discussion to the lightest baryon multiplets, the octet and decuplet, which have masses of order Λ_χ. Thus the $1/m_B$ and $1/\Lambda_\chi$ expansions can be combined into a single expansion in $1/\Lambda_\chi$.

The heavy baryon chiral theory has a consistent power counting expansion. The proof is rather elementary, and is essentially the same as the proof of the power counting expansion for the chiral quark model.[11] The terms in the effective theory are of the form

$$f^2\Lambda_\chi^2\left(\frac{\overline{B}_v}{f\sqrt{\Lambda_\chi}}\right)^{n_1}\left(\frac{B_v}{f\sqrt{\Lambda_\chi}}\right)^{n_2}\left(\frac{\partial}{\Lambda_\chi}\right)^{n_3}\left(\frac{M}{\Lambda_\chi}\right)^{n_4}(v)^{n_5}(\Sigma)^{n_6}\left(\Sigma^\dagger\right)^{n_7} \tag{2.4}$$

with coefficients of order one, provided $\Lambda_\chi \leq 4\pi f$. Note that the Lagrangian includes explicit factors of the baryon velocity v^μ. This power counting scheme is consistent because the effective heavy baryon theory does not have any terms which contain the baryon mass m_B. Factors of m_B therefore cannot occur in any loop graph, and the dimensional analysis arguments used in the proof of (2.4) are valid.

The heavy baryon chiral theory is similar to the non-relativistic formulation of baryon chiral perturbation theory discussed recently by Weinberg.[12] The non-relativistic baryon theory is the heavy baryon theory in the Lorentz frame in which $v^\mu = (1,0,0,0)$. There are some advantages to using the heavy baryon theory. Lorentz invariance is manifest, and quantum corrections can be computed in a straightforward manner using ordinary Feynman graphs, rather than time ordered perturbation theory.

3. The Heavy Baryon Chiral Lagrangian

The heavy baryon chiral Lagrangian is written in terms of the baryon fields

$$B_v = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & \Sigma_v^+ & p_v \\ \Sigma_v^- & -\frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & n_v \\ \Xi_v^- & \Xi_v^0 & -\frac{2}{\sqrt{6}}\Lambda_v \end{pmatrix}, \tag{3.1}$$

and the Goldstone boson fields

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{3.2}$$

The fields ξ and Σ are related to the pion fields by

$$\xi = e^{i\pi/f}, \quad \Sigma = \xi^2 = e^{2i\pi/f}. \tag{3.3}$$

With this normalization, the pion decay constant is $f \approx 93$ MeV. Under a $SU(3)_L \times SU(3)_R$ transformation, these fields transform as

$$\Sigma \to L\Sigma R^\dagger, \quad B_v \to U B_v U^\dagger, \tag{3.4}$$

$$\xi \to L\xi U^\dagger = U\xi R^\dagger, \tag{3.5}$$

where U is defined implicitly by Eq. (3.5).

The Dirac structure of the heavy baryon field simplifies considerably, because the B_v field is a two-component spinor.[9] The heavy baryon field B_v is projected onto the particle portion of the spinor using the projection operator

$$P_v = \left(\frac{1+\not{v}}{2}\right), \quad B_v = P_v B_v. \tag{3.6}$$

The effects of antibaryon spinor components are represented in the effective theory by including higher dimension operators suppressed by $1/m_B$. One can also define spin operators S_v^μ that act on the baryon fields,[7] with the properties

$$v \cdot S_v = 0, \quad S_v^2\ B_v = -\tfrac{3}{4}\ B_v, \tag{3.7}$$

$$\left\{S_v^\lambda, S_v^\sigma\right\} = \tfrac{1}{2}\left(v^\lambda v^\sigma - g^{\lambda\sigma}\right), \quad \left[S_v^\lambda, S_v^\sigma\right] = i\epsilon^{\lambda\sigma\alpha\beta} v_\alpha S_{v\beta}, \tag{3.8}$$

where $\epsilon_{0123} = +1$. The spin operators reduce to the usual spin operators $\vec{\sigma}/2$ for a non-relativistic spin-1/2 particle in the rest frame where $v^\mu = (1,0,0,0)$. Spin operators are defined in an arbitrary Lorentz frame by boosting from the rest frame. The commutation relations (3.8) can be derived by taking the usual relations which are valid in the rest frame, such as $\left[S^i, S^j\right] = i\epsilon^{ijk} S^k$, and writing them in four-vector notation using v^μ. These relations are then valid in an arbitrary Lorentz frame. The Dirac structure of the theory can now be eliminated; all Lorentz tensors made from spinors can be written in terms of v^μ and S_v^μ, using the identities

$$
\begin{aligned}
&\overline{B}_v \gamma_5 B_v = 0, \quad \overline{B}_v \gamma^\mu B_v = v^\mu \overline{B}_v B_v, \quad \overline{B}_v \gamma^\mu \gamma_5 B_v = 2\overline{B}_v S_v^\mu B_v, \\
&\overline{B}_v \sigma^{\mu\nu} B_v = 2\epsilon^{\mu\nu\alpha\beta} v_\alpha \overline{B}_v S_{v\beta} B_v, \quad \overline{B}_v \sigma^{\mu\nu}\gamma_5 B_v = 2i\left(v^\mu \overline{B}_v S_v^\nu B_v - v^\nu \overline{B}_v S_v^\mu B_v\right).
\end{aligned}
\tag{3.9}
$$

$\overline{B}_v \gamma_5 B_v$ vanishes because it is order $1/m_B$. The operator $\overline{B}\gamma_5 B$ matches onto an operator involving B_v fields which is suppressed by $1/m_B$ in the heavy baryon theory,

$$
\overline{B}\gamma_5 B \to \frac{1}{m_B} \overline{B}_v \; S_v \cdot \overleftrightarrow{\partial} B_v. \tag{3.10}
$$

The most general Lagrangian at lowest order (one derivative or one M) is

$$
L = \int \frac{d^3 v}{2v^0} L_v \tag{3.11}
$$

with

$$
\begin{aligned}
L_v = {} & i \,\mathrm{Tr}\, \overline{B}_v \left(v \cdot \mathcal{D}\right) B_v + 2D \;\mathrm{Tr}\; \overline{B}_v \, S_v^\mu \left\{A_\mu, B_v\right\} + 2F \;\mathrm{Tr}\; \overline{B}_v \, S_v^\mu \left[A_\mu, B_v\right] \\
& + b_D \;\mathrm{Tr}\; \overline{B}_v \left\{\xi^\dagger M \xi^\dagger + \xi M \xi, B_v\right\} + b_F \;\mathrm{Tr}\; \overline{B}_v \left[\xi^\dagger M \xi^\dagger + \xi M \xi, B_v\right] \\
& + \sigma \;\mathrm{Tr}\; M(\Sigma + \Sigma^\dagger) \;\mathrm{Tr}\, \overline{B}_v B_v + \frac{f^2}{4} \;\mathrm{Tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + a \;\mathrm{Tr}\; M(\Sigma + \Sigma^\dagger),
\end{aligned}
\tag{3.12}
$$

where

$$
\mathcal{D}^\mu B_v = \partial^\mu B_v + [V^\mu, B_v], \tag{3.13}
$$

$$
V^\mu = \frac{1}{2}\left(\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi\right), \quad A^\mu = \frac{i}{2}\left(\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi\right), \tag{3.14}
$$

and

$$
M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \tag{3.15}
$$

is the quark mass matrix. The integration of L_v over all v in Eq. (3.11) ensures that the theory is Lorentz invariant. It also allows one to consider several baryons with different velocities simultaneously. From now on, we will write down velocity dependent Lagrangians L_v without indicating the integral over v explicitly. All the terms in Eq. (3.12) have either one factor of the symmetry breaking mass matrix M, or at least one derivative. Note that the mass term $m_B \overline{B} B$ in the usual chiral Lagrangian, which has no powers of ∂ or M, is absent because of the redefinition Eq. (2.2).

4. The Spin-3/2 Decuplet

The spin-3/2 decuplet can also be included in the effective chiral theory. The decuplet can be described by a Rarita- Schwinger field $(T^\mu)_{abc}$, which contains both spin-1/2 and spin-3/2 pieces. The spin-1/2 pieces are projected out using the constraint $\gamma^\mu T_\mu = 0$. Under $SU(3)_L \times SU(3)_R$, T^μ transforms as

$$T^\mu_{abc} \to U_a^d\, U_b^e\, U_c^f\, T^\mu_{def}, \tag{4.1}$$

where U is defined in Eq. (3.5). The field T^μ_v is defined in terms of T^μ by

$$T^\mu_v(x) = e^{im_T \not{v} v_\mu x^\mu} T^\mu(x). \tag{4.2}$$

Spin operators S_v^ν can be defined which act on the spinor indices of T^μ_v, and satisfy the same spin algebra as the baryon spin operators Eq. (3.8). The spin operator S_v^ν is not the total angular momentum operator J_v^ν of the spin-3/2 particle. The total angular momentum operator J_v^ν, which is defined by

$$(J_v^\nu\ T_v)^\mu = S_v^\nu\ T_v^\mu + i\epsilon^{\mu\nu\alpha\beta} v_\alpha\, T_{v\beta}, \tag{4.3}$$

acts on both the spinor and vector indices, and satisfies

$$J_v^2\ T_v^\mu = -\tfrac{15}{4}\ T_v^\mu. \tag{4.4}$$

The constraint $\gamma_\mu T^\mu = 0$ implies that

$$v^\mu T_\mu = 0, \qquad S_v^\mu T_\mu = 0. \tag{4.5}$$

The propagator for the Rarita-Schwinger field contains a polarization projector which projects out the four physical positive energy spinor solutions to the equation of motion $\mathcal{U}_i^\mu$, $i = 1-4$. The polarization sum is

$$P_v^{\mu\nu} = \sum_i \mathcal{U}_i^\mu \overline{\mathcal{U}}_i^\nu = (v^\mu v^\nu - g^{\mu\nu}) - \tfrac{4}{3} S_v^\mu S_v^\nu. \tag{4.6}$$

The identities

$$\begin{aligned} &P_v^{\mu\nu} P_{v\nu}{}^{\lambda} = -P_v^{\mu\lambda}, \quad P_v^{\mu\nu} v_\nu = P_v^{\mu\nu} v_\mu = 0, \quad P_v^{\mu\nu} g_{\mu\nu} = -2, \\ &P_v^{\mu\nu} S_{v\,\nu} = S_{v\,\mu} P_v^{\mu\nu} = 0, \quad P_v^{\mu\nu} S_{v\,\mu} = -\tfrac{4}{3} S_v^\nu, \quad S_{v\,\nu} P^{\mu\nu} = -\tfrac{4}{3} S_v^\mu, \end{aligned} \tag{4.7}$$

are useful for the computation of Feynman diagrams.

The lowest order decuplet Lagrangian is

$$
\begin{aligned}
L_v^{10} = & -i\,\overline{T}_v^\mu (v\cdot\mathcal{D})\, T_{v\,\mu} + \Delta m\,\overline{T}_v^\mu\, T_{v\,\mu} + \mathcal{C}\,\left(\overline{T}_v^\mu A_\mu B_v + \overline{B}_v A_\mu T_v^\mu\right) \\
& + 2\mathcal{H}\,\overline{T}_v^\mu S_{v\,\nu} A^\nu T_{v\,\mu} + c\,\overline{T}_v^\mu\,\left(\xi^\dagger M \xi^\dagger + \xi M \xi\right) T_{v\,\mu} - \tilde\sigma\ \mathrm{Tr}\ M(\Sigma+\Sigma^\dagger)\,\overline{T}_v^\mu T_{v\,\mu}.
\end{aligned}
\tag{4.8}
$$

The kinetic term for the decuplet has the opposite sign from that for the octet because the spinor solutions $\mathcal{U}^\mu$ are spacelike, so that $\mathcal{U}^2 < 0$. Eq. (4.8) uses a modified definition of T_v^μ, in which a factor of $\exp(im_B \not v\, v_\mu x^\mu)$ multiplies T^μ in Eq. (4.2). This definition avoids the introduction of factors of $\exp(i(m_T - m_B)\not v\, v_\mu x^\mu)$ into the Lagrangian in terms which contain both decuplet and octet fields. The decuplet Lagrangian thus contains an explicit decuplet mass term proportional to $\Delta m = m_T - m_B$ since only part of the decuplet mass was removed by the transformation to a velocity dependent field.

5. A Sample Calculation: $T \to B\pi$

The heavy baryon Lagrangian can now be used to compute Green's functions in the effective theory. The Feynman rules can be read off from Eqs. (3.12) and (4.8). The octet propagator is $i/\,(k\cdot v)$, the decuplet propagator is $iP_v^{\mu\nu}/\,(k\cdot v - \Delta m)$, the pion nucleon vertex is $k\cdot S_v$ times a Clebsch-Gordan coefficient, *etc.* Graphs are much easier to compute in the heavy baryon theory than in the traditional baryon chiral perturbation theory using four-component spinors, because the Dirac matrix structure has disappeared.

We compute the decay width $T \to B\pi$ using the heavy baryon Lagrangian. Instead of computing the decay width directly, we will instead compute the imaginary part of the Feynman diagram shown in fig. 1. The optical theorem implies that the desired decay rate is twice the imaginary part of fig. 1. The calculation illustrates several techniques which are useful in computing Feynman graphs in the heavy baryon theory. From Eq. (4.8) and Eq. (3.14), the $TB\pi$ vertex equals $\mathcal{C}k^\mu/\sqrt{2}f$ times a Clebsch-Gordan coefficient. The Feynman integral is

$$
I = \frac{\mathcal{C}^2}{2f^2}\,(\mathrm{Clebsch})^2 \int \frac{d^Dk}{(2\pi)^D}\ \left(\frac{i}{(k+p)\cdot v + i\overline{\epsilon}}\right)\,\left(\frac{i}{k^2 - M^2 + i\overline{\epsilon}}\right)\,\overline{T}_\mu\,(-k^\mu)\,(k^\nu)\,T_\nu,
\tag{5.1}
$$

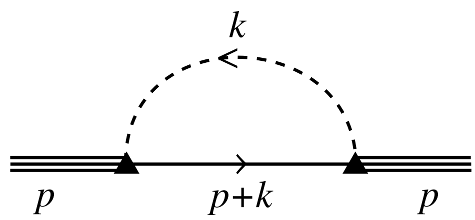

FIGURE 1.

The imaginary part of this graph is $\Gamma(T \to B\pi)/2$.

where $\bar{\epsilon}$ has been used in the propagators, to avoid confusion with $\epsilon = 2 - D/2$, which is used in dimensional regularization. The two denominators can be combined using the identity

$$\frac{1}{ab} = 2 \int_0^\infty \frac{d\lambda}{\left(a + 2b\lambda\right)^2}, \tag{5.2}$$

so that*

$$I = \frac{\mathcal{C}^2}{f^2} \left(\text{Clebsch}\right)^2 \int_0^\infty d\lambda \int \frac{d^D k}{\left(2\pi\right)^D} \frac{\overline{T}_\mu \, k^\mu k^\nu \, T_\nu}{\left(k^2 - M^2 + 2\lambda \left(k + p\right) \cdot v + i\bar{\epsilon}\right)^2}. \tag{5.3}$$

Shifting the momentum integral, using $v_\mu T^\mu = 0$ and $D = 4 - 2\epsilon$, gives

$$\begin{aligned} I &= \frac{\mathcal{C}^2}{f^2} \left(\text{Clebsch}\right)^2 \overline{T}_\mu T^\mu \int_0^\infty d\lambda \int \frac{d^D k}{\left(2\pi\right)^D} \frac{k^2/D}{\left(k^2 - M^2 - \lambda^2 + 2\lambda p \cdot v + i\bar{\epsilon}\right)^2} \\ &= -\frac{i}{32\pi^2} \frac{\mathcal{C}^2}{f^2} \left(\text{Clebsch}\right)^2 \ \overline{T}_\mu T^\mu \ \mu^{2\epsilon} \ \Gamma(-1+\epsilon) \int_0^\infty d\lambda \left(\lambda^2 - 2\lambda \, p \cdot v + M^2 - i\bar{\epsilon}\right)^{1-\epsilon}. \end{aligned} \tag{5.4}$$

Let $\mathcal{I}(\alpha)$ be defined by

$$\mathcal{I}\left(\alpha; b, c\right) \equiv \int_0^\infty d\lambda \left(\lambda^2 + 2\lambda b + c\right)^\alpha. \tag{5.5}$$

The recurrence relation

$$\left(1 + 2\alpha\right) \mathcal{I}\left(\alpha; b, c\right) = \left.\left(\lambda^2 + 2\lambda b + c\right)^\alpha \left(\lambda + b\right)\right|_0^\infty + 2\alpha \left(c - b^2\right) \mathcal{I}\left(\alpha - 1; b, c\right), \tag{5.6}$$

* Since $\lambda \geq 0$, $\bar{\epsilon} + 2\lambda\bar{\epsilon}$ can be replaced by $\bar{\epsilon}$.

can be derived by integration by parts. In dimensional regularization,

$$\lim_{\lambda\to\infty} \lambda^{1-\epsilon}, \lambda^{-\epsilon} = 0, \tag{5.7}$$

so Eq. (5.6) relates $\mathcal{I}(1-\epsilon; -p\cdot v, M^2)$ in Eq. (5.4) to $\mathcal{I}(-1-\epsilon; -p\cdot v, M^2)$ which is finite as $\epsilon \to 0$,

$$\begin{aligned}\mathcal{I}(1-\epsilon; -p\cdot v, M^2) &= \frac{1}{3-2\epsilon}\Big[p\cdot v\left(M^2\right)^{1-\epsilon} + 2\,(1-\epsilon)\big(M^2-(p\cdot v)^2\big)\mathcal{I}(-\epsilon; -p\cdot v, M^2)\Big]\\ \mathcal{I}(-\epsilon; -p\cdot v, M^2) &= \frac{1}{1-2\epsilon}\left[p\cdot v\left(M^2\right)^{-\epsilon} + 2\,(-\epsilon)\left(M^2-(p\cdot v)^2\right)\mathcal{I}(-1-\epsilon; -p\cdot v, M^2)\right].\end{aligned} \tag{5.8}$$

We need the imaginary part of the integral Eq. (5.4) to compute the decay width $T \to B\pi$. The only term with an imaginary part is $\mathcal{I}(-1-\epsilon; -p\cdot v, M^2)$. Since this integral is finite, we can set ϵ to zero, and evaluate it for an on-shell decuplet baryon T in the rest frame, so $p = (\Delta m, 0, 0, 0)$, $p\cdot v = \Delta m$, and

$$\begin{aligned}\mathrm{Im}\,\mathcal{I}(-1; -p\cdot v, M^2) &= \mathrm{Im}\int_0^\infty \frac{d\lambda}{(\lambda^2 - 2\lambda\Delta m + M^2 - i\bar{\epsilon})}\\ &= \mathrm{Im}\,\frac{1}{2\sqrt{(\Delta m)^2 - M^2}}\ln\left(\frac{\lambda - \Delta m - \sqrt{(\Delta m)^2 - M^2 - i\bar{\epsilon}}}{\lambda - \Delta m + \sqrt{(\Delta m)^2 - M^2 + i\bar{\epsilon}}}\right)\Bigg|_0^\infty\\ &= \frac{\pi}{\sqrt{(\Delta m)^2 - M^2}}.\end{aligned} \tag{5.9}$$

Combining Eq. (5.9), Eq. (5.8), and Eq. (5.4) gives

$$\mathrm{Im}\ I = \tfrac{1}{2}\Gamma(T\to B\pi) = (\mathrm{Clebsch})^2\,\frac{\mathcal{C}^2\left((\Delta m)^2 - M^2\right)^{3/2}}{24\pi f^2}. \tag{5.10}$$

The experimentally measured $T \to B\pi$ decays are $\Delta \to N\pi$, $\Sigma^* \to \Lambda\pi$, $\Sigma^* \to \Sigma\pi$, and $\Xi^* \to \Xi\pi$, with Clebsch-Gordan coefficients 1, $1/\sqrt{2}$, $1/\sqrt{3}$, and $1/\sqrt{2}$ respectively. The coefficients $\mathcal{C}$ determined from these decay modes[7] using Eq. (5.10) are 1.8, 1.5, 1.5, and 1.3 respectively, with an average value of

$$|\mathcal{C}| = 1.53. \tag{5.11}$$

The differences in the four numbers reflect $SU(3)$ breaking effects.

6. Why the Decuplet is Important

The effective chiral theory with octet and decuplet baryons has three scales, the pseudo-Goldstone mass M, the chiral symmetry breaking scale Λ_χ, and the decuplet-octet mass difference Δm. In the chiral limit, $M \ll \Delta m$, so one can construct an effective theory with only octet baryons and pions by integrating out the decuplet fields. Decuplet effects in the original theory are reproduced in the new theory by higher dimension operators involving only octet baryons and pions. Higher dimension operators obtained by integrating out the decuplet are suppressed by powers of $\mathcal{C}^2/\Delta m$ (where $\mathcal{C}$ is the $TB\pi$ coupling), whereas higher dimension operators due to other QCD effects are suppressed by $1/\Lambda_\chi$. In the real world, the $\Delta - N$ mass difference is approximately 300 MeV, and the $TB\pi$ coupling is approximately 1.5, as determined in Eq. (5.11). Thus decuplet contributions are expected to be 6–7 times more important than other higher dimension operators in the chiral theory. This enhancement of decuplet effects occurs because the $T - B$ mass difference is much smaller than the intrinsic hadronic scale of 1 GeV. Since the decuplet effects are so important, it is best to retain explicit decuplet fields in the effective theory, rather than integrate them out.

There are higher baryon resonances such as the $N(1440)$, which is only 500 MeV above the $N(939)$. These higher resonances are not as important as the decuplet. The $N(1440) \to N\pi$ width determines the $N(1440)N\pi$ coupling constant. One can estimate that the effect of the $N(1440)$ on octet baryon amplitudes is only about 10% that of the $\Delta(1232)$. The large suppression factor is due to the larger mass of the $N(1440)$, and the much smaller $N(1440)N\pi$ coupling. The size of the $N(1440)$ contribution is comparable to the estimate of higher dimension effects. Thus all higher resonances other than the decuplet do not have to be included explicitly; their effects can be mimicked by higher dimension operator which satisfy naive dimensional analysis.[11]

The above conclusions are not surprising if one believes that the quark model provides a reasonable qualitative picture of baryons. The octet and decuplet baryons have essentially identical wave functions, differing only in the arrangement of their spins. The higher resonances, on the other hand, differ in their spatial wavefunctions. A spin-flip does not cost very much energy, because the hyperfine spin-spin interaction is rather weak. Thus it is relatively easy for an octet baryon to be converted into a decuplet baryon, but it is more difficult to convert it into other excited states.

In the calculations described here, the decuplet-octet mass difference is treated as small compared with the kaon mass M_K. All computation are performed treating Δm as a perturbation. The chiral limit in which M_K is small compared with Δm is discussed in more detail in Section 11.

7. SU(6) Symmetry and Heavy Baryons

The heavy baryon chiral theory has octet and decuplet fields labeled by a velocity vector v. One can define an $SU(6)_v$ symmetry transformation which rotates an octet baryon B_v into a decuplet baryon T_v with the same velocity. There is an independent $SU(6)$ transformation for each v, and B_v and T_v together form a **56** of $SU(6)_v$. The **56** can be described by a tensor $Q_v^{abc;\alpha\beta\gamma}$ which has three flavor indices and three spinor indices, and is symmetric under the simultaneous permutation of spinor and flavor indices. T_v is the part of Q_v which is completely symmetric in flavor and spin, and B_v has mixed symmetry under flavor and spin. The tensor Q_v only has two non-zero components for each spinor index, because it satisfies the projection relation Eq. (3.6) on each spinor index. What we have constructed is the decomposition of the **56** of $SU(6)$ into $(\mathbf{8},\mathbf{2})+(\mathbf{10},\mathbf{4})$ of $SU(3)\times SU(2)$ under the embedding $\mathbf{6}\to(\mathbf{3},\mathbf{2})$. The symmetry can be written in a relativistic theory because there are velocity dependent fields. The non-relativistic $SU(6)$ symmetry for velocity dependent fields has also been discussed in a recent paper by Carone and Georgi.[13]

There does not seem to be any way to implement the $SU(6)_v$ transformation on the pion fields. The field A_μ defined in Eq. (3.14) is an $SU(3)$ octet, and has spin one. If the field is treated as part of the **35** of $SU(6)$, then one obtains the $SU(6)_v$ symmetry relations[14]

$$F=\tfrac{2}{3}D,\quad \mathcal{C}=-2D,\quad \mathcal{H}=-3D, \tag{7.1}$$

for the various pion-baryon coupling constants. One could also obtain these relations by using non-relativistic quark model wavefunctions for the baryons. As we will see, the best fit values for these constants is very close to this $SU(6)$ prediction. Thus there appears to be some evidence that, in some sense, the real world is close to the $SU(6)$ point of Eq. (7.1).

The field A_μ couples to baryon fields with different velocities, so that the $SU(6)_v$ transformation must be the same for all values of v. It is not clear how to extend the $SU(6)$ symmetry further, or how to study the symmetry breaking in a systematic manner.

8. The Axial Currents

The baryon axial vector currents in the $SU(3)$ symmetry limit to lowest order in the derivative expansion are

$$\begin{aligned}J^A_\mu &= D\ \mathrm{Tr}\ \overline{B}_v\, S^\mu_v\, \{\xi T^A \xi^\dagger + \xi^\dagger T^A \xi, B_v\} + F\ \mathrm{Tr}\ \overline{B}_v\, S^\mu_v\, [\xi T^A \xi^\dagger + \xi^\dagger T^A \xi, B_v] \\ &+ \tfrac{1}{2} v^\mu\ \mathrm{Tr}\ \overline{B}_v\, [\xi T^A \xi^\dagger - \xi^\dagger T^A \xi, B_v] + i\, \tfrac{f^2}{2}\ \mathrm{Tr}\ T^A \left(\partial^\mu \Sigma^\dagger \Sigma - \partial^\mu \Sigma \Sigma^\dagger\right) \\ &+ \tfrac{1}{2} v^\mu\ \overline{T}{}^\nu_v\, (\xi T^A \xi^\dagger - \xi^\dagger T^A \xi)\, T_{v\,\nu} + \mathcal{H}\ \overline{T}{}^\nu_v\, S^\mu_v\, (\xi T^A \xi^\dagger + \xi^\dagger T^A \xi)\, T_{v\,\nu} \\ &+ \tfrac{1}{2} \mathcal{C}\ \overline{T}_{v\,\mu}\, (\xi T^A \xi^\dagger + \xi^\dagger T^A \xi)\, B_v + \tfrac{1}{2} \mathcal{C}\ \overline{B}_v\, (\xi T^A \xi^\dagger + \xi^\dagger T^A \xi)\, T_{v\,\mu}.\end{aligned} \tag{8.1}$$

The leading non-analytic correction to the axial currents is a chiral logarithmic correction of the form $m_s \ln m_s$ produced by one-loop Feynman graphs which can be found in Refs. [7] and [14]. The correction to the axial currents can be written in the form

$$\langle B_i|\, J^A_\mu\, |B_j\rangle = \left(\alpha^A_{ij} + \left(\overline{\beta}{}^A_{ij} - \overline{\lambda}_{ij} \alpha^A_{ij}\right) \frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right)\right) \overline{u}_{B_i}\, \gamma_\mu \gamma_5\, u_{B_j}, \tag{8.2}$$

where α^A_{ij} is the lowest order result, $\overline{\lambda}_{ij} = \lambda_{ij} + \lambda'_{ij}$ is the one-loop correction due to wavefunction renormalization,

$$\sqrt{Z_i Z_j} = 1 + \overline{\lambda}_{ij} \frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right), \quad \overline{\lambda}_{ij} = \tfrac{1}{2}\left(\overline{\lambda}_i + \overline{\lambda}_j\right),$$

$\overline{\beta}{}^A_{ij} = \beta^A_{ij} + \beta'{}^A_{ij}$ is the correction due to all other graphs, and u is a spinor. All coefficients are written in the form $\overline{c} = c + c'$. The total correction $\overline{c}$ equals the sum of the correction from graphs with intermediate octet lines (c) and the correction from graphs with intermediate decuplet lines (c'). Thus one can determine the correction in a theory without any decuplet fields by dropping all c' type corrections. Corrections are computed for the currents that occur in hyperon semileptonic decay, as well as for the proton matrix element of the T^8 current, which is needed for an analysis of the proton spin problem.[15] The matrix element of the current (8.2) depends on μ because Eq. (8.2) contains only the leading non-analytic term in m_s. There are also M_K^2 terms with arbitrary coefficients which come from higher dimension operators in the chiral Lagrangian. The μ dependence of Eq. (8.2) is equal to the anomalous dimensions of these coefficients, so that the total μ dependence cancels.

The lowest order coefficients α^A_{ij} are:

$$\begin{aligned}\alpha^{1+i2}_{pn} &= (D+F), & \alpha^{4+i5}_{\Lambda\Xi^-} &= -\tfrac{1}{\sqrt{6}}(D-3F), \\ \alpha^{1+i2}_{\Lambda\Sigma^-} &= \tfrac{2}{\sqrt{6}} D, & \alpha^{4+i5}_{n\Sigma^-} &= (D-F), \\ \alpha^{1+i2}_{\Xi^0\Xi^-} &= (D-F), & \alpha^{4+i5}_{\Sigma^0\Xi^-} &= \tfrac{1}{\sqrt{2}}(D+F) = \tfrac{1}{\sqrt{2}} \alpha^{4+i5}_{\Sigma^+\Xi^0}, \\ \alpha^{4+i5}_{p\Lambda} &= -\tfrac{1}{\sqrt{6}}(D+3F), & \alpha^8_{pp} &= \tfrac{1}{\sqrt{12}}(3F-D).\end{aligned} \tag{8.3}$$

The coefficients β_{ij}^{A} are:

$$
\begin{aligned}
\beta_{pn}^{1+i2} &= \tfrac{2}{9}D^3 + \tfrac{2}{9}D^2F + \tfrac{2}{3}DF^2 - 2F^3 - \tfrac{1}{2}D - \tfrac{1}{2}F,\\
\beta_{\Lambda\Sigma^-}^{1+i2} &= \tfrac{1}{\sqrt{6}}(\tfrac{17}{9}D^3 - DF^2 - D),\\
\beta_{\Xi^0\Xi^-}^{1+i2} &= \tfrac{2}{9}D^3 - \tfrac{2}{9}D^2F + \tfrac{2}{3}DF^2 + 2F^3 - \tfrac{1}{2}D + \tfrac{1}{2}F,\\
\beta_{p\Lambda}^{4+i5} &= \tfrac{1}{\sqrt{6}}(\tfrac{19}{18}D^3 - \tfrac{5}{2}D^2F - \tfrac{7}{2}DF^2 + \tfrac{9}{2}F^3 + \tfrac{5}{4}D + \tfrac{15}{4}F),\\
\beta_{\Lambda\Xi^-}^{4+i5} &= \tfrac{1}{\sqrt{6}}(\tfrac{19}{18}D^3 + \tfrac{5}{2}D^2F - \tfrac{7}{2}DF^2 - \tfrac{9}{2}F^3 + \tfrac{5}{4}D - \tfrac{15}{4}F),\\
\beta_{n\Sigma^-}^{4+i5} &= \tfrac{7}{18}D^3 - \tfrac{13}{18}D^2F + \tfrac{7}{6}DF^2 + \tfrac{1}{2}F^3 - \tfrac{5}{4}D + \tfrac{5}{4}F,\\
\beta_{\Sigma^0\Xi^-}^{4+i5} &= \tfrac{1}{\sqrt{2}}(\tfrac{7}{18}D^3 + \tfrac{13}{18}D^2F + \tfrac{7}{6}DF^2 - \tfrac{1}{2}F^3 - \tfrac{5}{4}D - \tfrac{5}{4}F) = \frac{1}{\sqrt{2}}\beta_{\Sigma^+\Xi^0}^{4+i5},\\
\beta_{pp}^{8} &= \tfrac{1}{\sqrt{12}}(-\tfrac{11}{9}D^3 + 3D^2F + 3DF^2 - 3F^3 - \tfrac{9}{2}F + \tfrac{3}{2}D),
\end{aligned}
\tag{8.4}
$$

and the coefficients ${\beta'}_{ij}^{A}$ are:

$$
\begin{aligned}
{\beta'}_{pn}^{1+i2} &= -\tfrac{10}{81}\mathcal{H}\mathcal{C}^2 + (\tfrac{2}{3}D + \tfrac{2}{9}F)\mathcal{C}^2,\\
{\beta'}_{\Lambda\Sigma^-}^{1+i2} &= -\tfrac{5}{27\sqrt{6}}\mathcal{H}\mathcal{C}^2 + \tfrac{8}{3\sqrt{6}}(D+F)\mathcal{C}^2,\\
{\beta'}_{\Xi^0\Xi^-}^{1+i2} &= \tfrac{20}{81}\mathcal{H}\mathcal{C}^2 + (\tfrac{14}{27}D + 2F)\mathcal{C}^2,\\
{\beta'}_{p\Lambda}^{4+i5} &= \tfrac{5}{9\sqrt{6}}\mathcal{H}\mathcal{C}^2 - \tfrac{1}{\sqrt{6}}(D+F)\mathcal{C}^2,\\
{\beta'}_{\Lambda\Xi^-}^{4+i5} &= -\tfrac{5}{9\sqrt{6}}\mathcal{H}\mathcal{C}^2 + \tfrac{1}{\sqrt{6}}(\tfrac{34}{18}D - F)\mathcal{C}^2,\\
{\beta'}_{n\Sigma^-}^{4+i5} &= \tfrac{5}{81}\mathcal{H}\mathcal{C}^2 + (-\tfrac{1}{27}D + F)\mathcal{C}^2,\\
{\beta'}_{\Sigma^0\Xi^-}^{4+i5} &= -\tfrac{55}{81\sqrt{2}}\mathcal{H}\mathcal{C}^2 + \tfrac{1}{\sqrt{2}}(\tfrac{19}{9}D + \tfrac{17}{9}F)\mathcal{C}^2 = \tfrac{1}{\sqrt{2}}{\beta'}_{\Sigma^+\Xi^0}^{4+i5}.\\
{\beta'}_{pp}^{8} &= \tfrac{1}{\sqrt{3}}(D-F)\mathcal{C}^2.
\end{aligned}
\tag{8.5}
$$

The wavefunction renormalization coefficients $\overline{\lambda}_i$ are given in Section 9.

The chiral corrections to the axial octet currents were computed by Bijnens, Sonoda and Wise (BSW).[16] The above results differ from BSW because decuplet contributions have been included, and also because BSW inadvertently omitted wavefunction renormalization in their calculations. Wavefunction renormalization is the biggest part of the chiral correction, and hence makes a significant difference in a fit to the experimental results. A fit to the experimental data neglecting all chiral corrections gives[15] $D = 0.80$ and $F = 0.50$. The chiral corrections are large ($\gtrsim 100\%$) if one only includes contributions from loop graphs involving internal octet field propagators. The experimental results have very small $SU(3)$ breaking effects. Thus chiral corrections including only octet chiral logarithms are in conflict with experiment. The situation is rather different if one includes both octet and decuplet graphs. There is a significant cancellation between octet and decuplet chiral logarithms, so that the $SU(3)$ breaking effects are much smaller. There are two new parameters when decuplets are included, $\mathcal{C}$ and $\mathcal{H}$. $\mathcal{C}$ was determined from $T \to B\pi$ decay, Eq. (5.11). There is therefore only one additional parameter, $\mathcal{H}$, for the decuplet fields. The best fit to the axial currents gives[14]

$$F = 0.40 \pm 0.03, \quad D = 0.61 \pm 0.04, \quad \mathcal{H} = -1.91 \pm 0.7. \tag{8.6}$$

H is not determined very well since it only occurs in the loop correction. The chiral corrections including the decuplet are all less than 30%, with the exception of $\Sigma \to n$ (67%) and $\Lambda \to p$ (46%). The theoretical and experimental results agree to within errors. Note that the values Eq. (8.6) and Eq. (5.11) approximately satisfy the $SU(6)$ relations Eq. (7.1).

9. Baryon Masses

Leading non-analytic corrections to baryon octet and decuplet masses[17] are calculated using Lagrangians L_v and L_v^{10}. The one-loop formulæ for baryon masses can be written in the form

$$\begin{aligned} M_i =& \overline{m} - 2\overline{\sigma} m_s + \overline{\alpha}_i m_s - \overline{\beta}_i \frac{M_K^3}{16\pi f^2} + \left(\overline{\gamma}_i - \overline{\lambda}_i \overline{\alpha}_i\right) m_s \frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right) \\ & \pm 2{\lambda'}_i (\sigma - \tilde{\sigma}) m_s \frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right) \pm {\lambda'}_i \Delta m \frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right), \end{aligned} \tag{9.1}$$

where plus and minus signs apply for octet and decuplet masses, respectively. The term $\overline{m}$ is the $SU(3)$ invariant mass of the baryon multiplet; $\overline{m} = m_B$ for octet baryons and $\overline{m} = m_T = m_B + \Delta m$ for decuplet baryons. The quantity $-2\overline{\sigma} m_s$ is the tree-level contribution to the baryon masses from sigma terms; $\overline{\sigma} = \sigma$ for octet baryons and $\overline{\sigma} = \tilde{\sigma}$ for decuplet baryons. The tree-level coefficients $\overline{\alpha}_i = \alpha_i$

$$\begin{aligned} \alpha_N &= -2(b_D - b_F), \qquad & \alpha_\Delta &= 0, \\ \alpha_\Sigma &= 0, & \alpha_{\Sigma^*} &= \tfrac{2}{3} c, \\ \alpha_\Lambda &= -\tfrac{8}{3} b_D, & \alpha_{\Xi^*} &= \tfrac{4}{3} c, \\ \alpha_\Xi &= -2(b_D + b_F), & \alpha_{\Omega^-} &= 2c, \end{aligned} \tag{9.2}$$

describe baryon mass splittings which are linear in m_s. Octet and decuplet masses depend on distinct parameters at tree level. The four octet masses are given in terms of the three parameters $(m_B - 2\sigma m_s)$, $b_D m_s$, and $b_F m_s$, whereas the four decuplet masses are written in terms of the two parameters $m_T - 2\tilde{\sigma} m_s$ and $c m_s$. The tree-level masses satisfy the baryon mass relations,

$$\tfrac{3}{4} M_\Lambda + \tfrac{1}{4} M_\Sigma - \tfrac{1}{2}\left(M_N + M_\Xi\right) = 0, \tag{9.3}$$

$$M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_{\Omega^-} - M_{\Xi^*}. \tag{9.4}$$

These two mass relations, the Gell-Mann–Okubo formula for octet masses and the equal spacing rule for decuplet masses, are a direct consequence of the fact that $SU(3)$ symmetry breaking is purely octet.

Three non-analytic terms appear at one loop — contributions to the baryon masses which vary as $m_s^{3/2}$, $m_s^2 \ln m_s$, and $(\Delta m) m_s \ln m_s$. The $(\Delta m) m_s \ln m_s$ non-analytic mass contribution is a new term which arises only if octet and decuplet baryons are treated together in the chiral Lagrangian. There is no non-analytic correction proportional to $m_B m_s \ln m_s$ since m_B does not appear in the chiral Lagrangian for heavy baryon fields.

The coefficients $\overline{\beta}_i = \beta_i + \beta_i'$ parametrize the non-analytic $m_s^{3/2}$ contribution:

$$\begin{aligned}
&\beta_N = (\tfrac{5}{3} D^2 - 2DF + 3F^2) + \tfrac{4}{9\sqrt{3}}(D^2 - 6DF + 9F^2), && \beta_N' = \tfrac{1}{3}\mathcal{C}^2,\\
&\beta_\Sigma = 2(D^2 + F^2) + \tfrac{16}{9\sqrt{3}} D^2, && \beta_\Sigma' = (\tfrac{10}{9} + \tfrac{8}{9\sqrt{3}})\mathcal{C}^2,\\
&\beta_\Lambda = \tfrac{2}{3} D^2 + 6F^2 + \tfrac{16}{9\sqrt{3}} D^2, && \beta_\Lambda' = \tfrac{2}{3}\mathcal{C}^2,\\
&\beta_\Xi = (\tfrac{5}{3} D^2 + 2DF + 3F^2) + \tfrac{4}{9\sqrt{3}}(D^2 + 6DF + 9F^2), && \beta_\Xi' = (1 + \tfrac{8}{9\sqrt{3}})\mathcal{C}^2,
\end{aligned} \tag{9.5}$$

$$\begin{aligned}
&\beta_\Delta = (\tfrac{5}{27} + \tfrac{20}{81\sqrt{3}})\mathcal{H}^2, && \beta_\Delta' = \tfrac{1}{3}\mathcal{C}^2,\\
&\beta_{\Sigma^*} = \tfrac{40}{81}\mathcal{H}^2, && \beta_{\Sigma^*}' = (\tfrac{2}{9} + \tfrac{4}{9\sqrt{3}})\mathcal{C}^2,\\
&\beta_{\Xi^*} = (\tfrac{5}{9} + \tfrac{20}{81\sqrt{3}})\mathcal{H}^2, && \beta_{\Xi^*}' = (\tfrac{1}{3} + \tfrac{4}{9\sqrt{3}})\mathcal{C}^2,\\
&\beta_{\Omega^-} = (\tfrac{10}{27} + \tfrac{80}{81\sqrt{3}})\mathcal{H}^2, && \beta_{\Omega^-}' = \tfrac{2}{3}\mathcal{C}^2.
\end{aligned} \tag{9.6}$$

Wavefunction renormalization

$$Z_i = 1 + \overline{\lambda}_i \frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right)$$

determines the coefficients $\overline{\lambda}_i = \lambda_i + \lambda_i'$,

$$\begin{aligned}
&\lambda_N = \tfrac{17}{6} D^2 - 5DF + \tfrac{15}{2} F^2, && \lambda_N' = \tfrac{1}{2}\mathcal{C}^2,\\
&\lambda_\Sigma = \tfrac{13}{3} D^2 + 3F^2, && \lambda_\Sigma' = \tfrac{7}{3}\mathcal{C}^2,\\
&\lambda_\Lambda = \tfrac{7}{3} D^2 + 9F^2, && \lambda_\Lambda' = \mathcal{C}^2,\\
&\lambda_\Xi = \tfrac{17}{6} D^2 + 5DF + \tfrac{15}{2} F^2, && \lambda_\Xi' = \tfrac{13}{6}\mathcal{C}^2,
\end{aligned} \tag{9.7}$$

$$
\begin{aligned}
\lambda_\Delta &= \tfrac{25}{54}\mathcal{H}^2, & \lambda'_\Delta &= \tfrac{1}{2}\mathcal{C}^2,\\
\lambda_{\Sigma^*} &= \tfrac{20}{27}\mathcal{H}^2, & \lambda'_{\Sigma^*} &= \tfrac{2}{3}\mathcal{C}^2,\\
\lambda_{\Xi^*} &= \tfrac{55}{54}\mathcal{H}^2, & \lambda'_{\Xi^*} &= \tfrac{5}{6}\mathcal{C}^2,\\
\lambda_{\Omega^-} &= \tfrac{35}{27}\mathcal{H}^2, & \lambda'_{\Omega^-} &= \mathcal{C}^2.
\end{aligned} \tag{9.8}
$$

The coefficients $\overline{\beta}$ are determined by the same diagrams as wavefunction renormalization coefficients $\overline{\lambda}$. These coefficients are equal to Clebsch-Gordan coefficients times a power of the Goldstone boson mass. The wavefunction renormalization coefficient is multiplied by M^2, whereas the $m_s^{3/2}$ contribution is multiplied by M^3. Thus the η and K contributions to $\overline{\beta}$ and $\overline{\lambda}$ are proportional to each other, except that the η contribution to $\overline{\beta}$ has an additional factor of $\sqrt{4/3}$, because $M_\eta^2 = 4M_K^2/3$. The constant of proportionality is determined by explicit computation to be $2/3$. Thus

$$
\overline{\beta} = \tfrac{2}{3}\left(\overline{\lambda}(K) + \tfrac{2}{\sqrt{3}}\overline{\lambda}(\eta)\right) = \tfrac{2}{3}\left(\overline{\lambda} + (\tfrac{2}{\sqrt{3}} - 1)\overline{\lambda}(\eta)\right),
$$

where $\overline{\lambda} = \overline{\lambda}(K) + \overline{\lambda}(\eta)$ equals the sum of contributions from diagrams with K and η exchange. The coefficients $\overline{\gamma}_i = \gamma_i + \gamma'_i$ are:

$$
\begin{aligned}
\gamma_N &= \tfrac{43}{9}b_D - \tfrac{25}{9}b_F - b_D(\tfrac{4}{3}D^2 + 12F^2) + b_F(\tfrac{2}{3}D^2 - 4DF + 6F^2) + \tfrac{52}{9}\sigma,\\
\gamma_\Sigma &= 2b_D - b_D(6D^2 + 6F^2) - b_F(12DF) + \tfrac{52}{9}\sigma,\\
\gamma_\Lambda &= \tfrac{154}{27}b_D - b_D(\tfrac{50}{9}D^2 + 18F^2) + b_F(12DF) + \tfrac{52}{9}\sigma,\\
\gamma_\Xi &= \tfrac{43}{9}b_D + \tfrac{25}{9}b_F - b_D(\tfrac{4}{3}D^2 + 12F^2) - b_F(\tfrac{2}{3}D^2 + 4DF + 6F^2) + \tfrac{52}{9}\sigma,
\end{aligned} \tag{9.9}
$$

$$
\gamma'_N = \tfrac{1}{3}c\,\mathcal{C}^2, \quad \gamma'_\Sigma = \tfrac{8}{9}c\,\mathcal{C}^2, \quad \gamma'_\Lambda = \tfrac{4}{3}c\,\mathcal{C}^2, \quad \gamma'_\Xi = \tfrac{29}{9}c\,\mathcal{C}^2, \tag{9.10}
$$

$$
\begin{aligned}
\gamma_\Delta &= -c + \tfrac{5}{27}c\mathcal{H}^2 + \tfrac{52}{9}\tilde{\sigma}, & \gamma'_\Delta &= 0,\\
\gamma_{\Sigma^*} &= -\tfrac{52}{27}c + \tfrac{40}{81}c\mathcal{H}^2 + \tfrac{52}{9}\tilde{\sigma}, & \gamma'_{\Sigma^*} &= -\tfrac{2}{3}b_D\mathcal{C}^2,\\
\gamma_{\Xi^*} &= -\tfrac{77}{27}c + \tfrac{95}{81}c\mathcal{H}^2 + \tfrac{52}{9}\tilde{\sigma}, & \gamma'_{\Xi^*} &= -(\tfrac{4}{3}b_D + \tfrac{2}{3}b_F)\mathcal{C}^2,\\
\gamma_{\Omega^-} &= -\tfrac{34}{9}c + \tfrac{20}{9}c\mathcal{H}^2 + \tfrac{52}{9}\tilde{\sigma}, & \gamma'_{\Omega^-} &= -2(b_D + b_F)\mathcal{C}^2.
\end{aligned} \tag{9.11}
$$

The one-loop formulæ for the baryon masses depend on five parameters: $m_B - 2\sigma m_s + \frac{52}{9}\sigma m_s x$, $m_T - 2\tilde{\sigma}m_s + \frac{52}{9}\tilde{\sigma}m_s x$, $b_D m_s$, $b_F m_s$, and $c m_s$, where $x = (M_K^2/16\pi f^2)\ln M_K^2/\mu^2$. A sixth parameter $\Delta m + 2(\sigma - \tilde{\sigma})m_s$ is equal to the difference of the first two parameters up to terms which are higher order in the strange quark mass expansion.

The corrections to the mass relations Eqs. (9.3), (9.4) are

$$
\begin{aligned}
\tfrac{3}{4}M_\Lambda + \tfrac{1}{4}M_\Sigma - \tfrac{1}{2}\left(M_N + M_\Xi\right) &= \left(\tfrac{2}{3}(D^2 - 3F^2) - \tfrac{1}{9}\mathcal{C}^2\right)\left(1 - \tfrac{2}{\sqrt{3}}\right)\frac{M_K^3}{16\pi f^2} \\
&- \left(\tfrac{16}{3}b_D D^2 + \left(\tfrac{2}{3}b_D + \tfrac{5}{3}b_F\right)\mathcal{C}^2 + \tfrac{5}{9}c\,\mathcal{C}^2\right) m_s \frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right)
\end{aligned}
\tag{9.12}
$$

$$
\begin{aligned}
&\left(M_{\Sigma^*} - M_\Delta\right) - \left(M_{\Xi^*} - M_{\Sigma^*}\right) = \left(M_{\Xi^*} - M_{\Sigma^*}\right) - \left(M_{\Omega^-} - M_{\Xi^*}\right) \\
&= \left(\tfrac{2}{9}\mathcal{C}^2 - \tfrac{20}{81}\mathcal{H}^2\right)\left(1 - \tfrac{2}{\sqrt{3}}\right)\frac{M_K^3}{16\pi f^2} + \left(\tfrac{2}{9}\mathcal{C}^2 c + \tfrac{2}{3}\mathcal{C}^2 b_F\right) m_s \frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right).
\end{aligned}
\tag{9.13}
$$

The correction to the GMO formula (with only octet diagrams) was computed previously by BSW.[16] Corrections to the GMO formula and the equal spacing rule depend only on the non-singlet and non-octet portions of the chiral corrections. Wavefunction coefficients λ_i and λ_i' which are purely singlet and octet automatically satisfy the mass relations, and hence all terms proportional to σ and $\tilde\sigma$ and the logarithmic mass contribution proportional to Δm cancel out of the mass relations. In addition, to $O(m_s^2)$, differences of decuplet mass splittings are simply related, so that there is a new mass relation

$$
\tfrac{1}{2}\left(M_{\Sigma^*} - M_\Delta\right) - \left(M_{\Xi^*} - M_{\Sigma^*}\right) + \tfrac{1}{2}\left(M_{\Omega^-} - M_{\Xi^*}\right) = 0,
\tag{9.14}
$$

which is valid upto second order in symmetry breaking, including all leading non-analytic corrections.

The quantity $M_K^3/16\pi f^2 \approx 279$ MeV is numerically quite large. This term, however, always appears with the suppression factor $(2/\sqrt{3} - 1)$ in the baryon mass relations, since the coefficients $\overline{\beta}_i$ would be proportional to the wavefunction coefficients $\overline{\lambda}_i$ if the contribution of η exchange graphs did not receive an extra factor of $2/\sqrt{3}$ from the M^3 behavior of the graph. The quantity $(2/\sqrt{3} - 1)(M_K^3/16\pi f^2) \approx 43$ MeV is the generic correction expected from the M_K^3 piece. The chiral logarithm $(M_K^2/16\pi^2 f^2)\ln\left(M_K^2/\mu^2\right)$ multiplies baryon mass differences which are typically of order (100 MeV), and hence this term should give corrections of order 25 MeV. Finally, the calculation of chiral corrections to the mass relations ignores corrections of order m_s^2, whose expected magnitude is $M_K^2/16\pi^2 f^2$ times a baryon mass difference of $O(100$ MeV$)$, or ≈ 20 MeV. Thus, our calculation can only be trusted at the 20 MeV level. Experimentally, the GMO formula works to a precision of 6.5 MeV. Using the best fit values of the parameters $D = 0.61$, $F = 0.40$, obtained from a fit to baryon octet semileptonic weak decays with $|\mathcal{C}| = 1.6$ as determined previously, the $M_K^3/16\pi f^2$ correction is numerically ≈ 15.4 MeV and the logarithmic correction is numerically $\approx$ 3.8 MeV. The total theoretical correction is less than 20 MeV and agrees with experiment to within the accuracy of the theoretical calculation. The relation (9.14) is satisfied to -3.6 MeV experimentally, while the three differences of mass differences are measured to be 2.6 MeV, 9.8 MeV, and 12.4 MeV, respectively. The calculated M_K^3 and logarithmic corrections to the equal spacing rule are 13.9 MeV and 9.3 MeV for the parameter values $D = 0.61$, $F = 0.40$, $|\mathcal{C}| = 1.6$, and $\mathcal{H} = -1.9$. Agreement with experiment to within 20 MeV is again obtained.

The M_K^3 contribution to the baryon masses is only suppressed by the factor $(2/\sqrt{3}-1)$ in the mass relations Eqs. (9.3) and (9.4). There is no such suppression factor in the contribution to the individual mass terms. The contribution of the M_K^3 term is ≈ 279 MeV times a Clebsch-Gordan coefficient. The Clebsch-Gordan coefficients can be quite large, so that the M_K^3 term contributes as much as -1 GeV to some of the baryon masses. However, we have already seen that the dominant contribution of the M_K^3 term is $SU(3)$ singlet and octet. The **27** piece, which contributes to the GMO formula and the equal spacing rule, is suppressed by $(2/\sqrt{3}-1)$, and is small. Thus there is a large non-linear m_s dependence in the baryon masses which is purely $SU(3)$ singlet or octet, and therefore does not affect the mass relations. The large $m_s^{3/2}$ term does not invalidate the chiral expansion in m_s. The proton matrix element $\langle p|\, m_s \overline{s} s \,|p\rangle$, obtained by differentiating the proton mass with respect to m_s, gets a large contribution from the $m_s^{3/2}$ term so that

$$\langle p|\, m_s \overline{s} s \,|p\rangle = m_s \frac{\partial m_p}{\partial m_s} \approx 0 \pm 150 \text{ MeV} \tag{9.15}$$

instead of the usual value quoted of around 411 MeV. A more detailed discussion can be found in Ref. [18].

10. Hyperon Non-leptonic Decays

According to the $\Delta I = \frac{1}{2}$ rule, $\Delta S = 1$ nonleptonic weak decays are dominated by an interaction which transforms as $(8,1)$ under $SU(3)_L \times SU(3)_R$. This component is represented in the chiral Lagrangian by

$$\begin{aligned} L_v^{\Delta S=1} &= h_D \operatorname{Tr} \overline{B}_v \left\{\xi^\dagger h \xi, B_v\right\} + h_F \operatorname{Tr} \overline{B}_v \left[\xi^\dagger h \xi, B_v\right] \\ &+ h_C \overline{T}_v^\mu \left(\xi^\dagger h \xi\right) T_{v\,\mu} + h_\pi \frac{f^2}{4} \operatorname{Tr} h \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \end{aligned} \tag{10.1}$$

to leading order in the derivative and quark mass expansions. The $s \to d$ transition is specified by the matrix

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} . \tag{10.2}$$

The Lagrangian $L_v^{\Delta S=1}$ contains three undetermined parameters h_D, h_F, and h_C which have dimensions of mass. The parameter h_π is determined from $\Delta S = 1$ K decays, $h_\pi = 3.2 \times 10^{-7}$. Although formally higher order in the derivative expansion, the term proportional to h_π is included because it produces corrections to the hyperon decay amplitudes which are order one compared to the effects of h_D, h_F and h_C, since h_π is over an order of magnitude larger than its expected value based on dimensional analysis.

The heavy baryon calculation of hyperon nonleptonic decay produces decay amplitudes of the form

$$\mathcal{A}(B_i \to B_j \pi) = G_F m_{\pi^+}^2 \, \overline{u}_{B_j} \left\{ \mathcal{A}_{ij}^{(S)} + 2(k \cdot S_v) \mathcal{A}_{ij}^{(P)} \right\} u_{B_i} , \tag{10.3}$$

where k is the outgoing momentum of the pion. The decay amplitude reduces to the non-relativistic amplitude

$$\mathcal{A}(B_i \to B_j \pi) = G_F m_{\pi^+}^2 \, \overline{u}_{B_j} \left\{ \mathcal{A}_{ij}^{(S)} + |\vec{k}| (\hat{k} \cdot \vec{\sigma}) \mathcal{A}_{ij}^{(P)} \right\} u_{B_i} , \tag{10.4}$$

in the rest frame of the heavy baryon where $v^\mu = (1,0,0,0)$ and $S^\mu_{v=0} = (0, \frac{1}{2}\vec{\sigma})$. The dimensionless amplitudes $s = \mathcal{A}_{ij}^{(S)}$ and $p = -|\vec{k}| \mathcal{A}_{ij}^{(P)}$ are order one in both the m_B and m_s expansions and are the same amplitudes s and p which are used to define the decay asymmetry parameters α, β, and γ. The heavy baryon P-wave amplitude p is related to the conventional amplitude by

$$p = \frac{|\vec{k}| \mathrm{A}_{ij}^{(P)}}{(E_j + M_j)} = \left\{ \frac{(M_i - M_j)^2 - m_\pi^2}{(M_i + M_j)^2 - m_\pi^2} \right\}^{1/2} \mathrm{A}_{ij}^{(P)} \tag{10.5}$$

where E_j is the energy of the final baryon and

$$|\vec{k}| = \frac{1}{2M_i} \left\{ \left[(M_i + M_j)^2 - m_\pi^2 \right] \left[(M_i - M_j)^2 - m_\pi^2 \right] \right\}^{1/2} . \tag{10.6}$$

The conventional definition of the P-wave amplitude $\mathrm{A}^{(P)}$ involves multiplication of p by an unnatural factor which is $O(m_B/m_s)$. Multiplication by this factor obscures the comparison of S- and P-wave hyperon decay amplitudes since P-wave amplitudes appear to be enhanced by an order of magnitude.

The S- and P-wave hyperon nonleptonic decay amplitudes are calculated to one loop using the weak Lagrangian $L_v^{\Delta S=1}$ and the baryon-pion Lagrangians L_v and L_v^{10}. The one-loop S- and P-wave decay amplitudes can be written in the form

$$\mathcal{A}_{ij\phi} = \frac{i}{\sqrt{2}f}\left(\overline{\alpha}_{ij} + \left(\overline{\beta}_{ij} - \overline{\lambda}_{ij\phi}\,\overline{\alpha}_{ij}\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right)\right), \tag{10.7}$$

where f is related to the physical pion decay constant $f_\pi \approx 93$ MeV by

$$f_\pi = f\left(1 - \frac{1}{2}\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right)\right).$$

The coefficients $\overline{\alpha}_{ij} = \alpha_{ij}$ are tree-level amplitudes. The wavefunction coefficients $\overline{\lambda}_{ij\phi} = \overline{\lambda}_{ij} + \lambda_\phi$ include pion wavefunction renormalization,

$$\sqrt{Z_\phi} = 1 + \lambda_\phi \frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right),$$

where the pion wavefunction renormalization coefficient

$$\lambda_\pi = -\tfrac{1}{6}. \tag{10.8}$$

Four independent hyperon decay amplitudes remain after imposition of isospin. These amplitudes are taken to be $\Sigma^+ \to n\pi^+$, $\Sigma^- \to n\pi^-$, $\Lambda \to p\pi^-$, and $\Xi^- \to \Lambda\pi^-$. The other amplitudes are determined by the isospin relations

$$\begin{aligned}
&\sqrt{2}\,\mathcal{A}(\Sigma^+ \to p\pi^0) - \mathcal{A}(\Sigma^+ \to n\pi^+) + \mathcal{A}(\Sigma^- \to n\pi^-) = 0,\\
&\mathcal{A}(\Lambda \to p\pi^-) + \sqrt{2}\,\mathcal{A}(\Lambda \to n\pi^0) = 0,\\
&\mathcal{A}(\Xi^- \to \Lambda\pi^-) + \sqrt{2}\,\mathcal{A}(\Xi^0 \to \Lambda\pi^0) = 0,
\end{aligned} \tag{10.9}$$

where eqs. (10.9) apply to S- and P-wave amplitudes separately.

The tree-level $SU(3)$ symmetry predictions are

$$\begin{aligned}
\alpha^{(S)}_{\Sigma^+ n} &= 0,\\
\alpha^{(S)}_{\Sigma^- n} &= -h_D + h_F,\\
\alpha^{(S)}_{\Lambda p} &= \tfrac{1}{\sqrt{6}}(h_D + 3h_F),\\
\alpha^{(S)}_{\Xi^- \Lambda} &= \tfrac{1}{\sqrt{6}}(h_D - 3h_F),
\end{aligned} \tag{10.10}$$

for the S-wave amplitudes and

$$\begin{aligned}
\alpha^{(P)}_{\Sigma^+ n} &= -D(h_D - h_F)/(M_\Sigma - M_N) - \tfrac{1}{3} D(h_D + 3h_F)/(M_\Lambda - M_N), \\
\alpha^{(P)}_{\Sigma^- n} &= -F(h_D - h_F)/(M_\Sigma - M_N) - \tfrac{1}{3} D(h_D + 3h_F)/(M_\Lambda - M_N), \\
\alpha^{(P)}_{\Lambda p} &= \tfrac{2}{\sqrt{6}} D(h_D - h_F)/(M_\Sigma - M_N) + \tfrac{1}{\sqrt{6}}(D+F)(h_D + 3h_F)/(M_\Lambda - M_N), \\
\alpha^{(P)}_{\Xi^- \Lambda} &= -\tfrac{2}{\sqrt{6}} D(h_D + h_F)/(M_\Xi - M_\Sigma) - \tfrac{1}{\sqrt{6}}(D-F)(h_D - 3h_F)/(M_\Xi - M_\Lambda).
\end{aligned} \tag{10.11}$$

for the P-wave amplitudes. Elimination of the two parameters h_D and h_F amongst the three non-vanishing S-wave amplitudes yields one $SU(3)$ symmetry relation, the Lee-Sugawara relation,

$$\tfrac{3}{\sqrt{6}}\, \mathcal{A}^{(S)}(\Sigma^- \to n\pi^-) + \mathcal{A}^{(S)}(\Lambda \to p\pi^-) + 2\, \mathcal{A}^{(S)}(\Xi^- \to \Lambda\pi^-) = 0. \tag{10.12}$$

There is no $SU(3)$ relation for the P-wave amplitudes. The formulæ for S- and P-wave coefficients $\overline{\beta}_{ij}$ are quite lengthy and will not be given here. The coefficients are listed in Ref. [19].

Experimental S- and P-wave hyperon nonleptonic decay amplitudes are compared with theory in Table 1. The theoretical formulæ have been evaluated using parameter values determined from a best fit to one-loop S-wave formulæ. The theoretical S-wave amplitudes are in good agreement with experimental values. Chiral logarithmic corrections to tree-level S-wave amplitudes are larger than anticipated. The correction from octet graphs is small for all amplitudes. The large correction from decuplet graphs is dominated by the contribution of wavefunction renormalization coefficients times tree-level amplitudes. Since the wavefunction coefficients λ' contain a large singlet piece, most of the chiral logarithmic correction is not $SU(3)$ violating. The correction to the Lee-Sugawara relation -0.31 is in fairly good quantitative agreement with the experimental value of -0.24. The correction to the Lee-Sugawara relation can be naturally smaller than typical corrections to individual S-wave amplitudes since only the $SU(3)$-violating portion of the chiral logarithmic corrections contributes.

The tree-level $SU(3)$ symmetric P-wave amplitudes are suppressed compared to S-wave amplitudes. The two terms in the tree-level formulæ Eq. (10.11) cancel against each other for the parameter values determined by the S-wave fit. Because the tree-level contribution is suppressed, chiral logarithmic corrections are order one compared to tree-level amplitudes. Thus, it is not surprising that $SU(3)$ symmetry predictions fail for P-wave amplitudes. Because the neglected $O(m_s)$ correction is similar in magnitude to the computed $O(m_s \ln m_s)$ correction (and the chiral logarithmic correction is order one compared to the tree-level contribution), one expects corrections to the one-loop theoretical calculation of order 100%. Despite such a large uncertainty in the theoretical calculation, some of the features of the experimental P-wave amplitudes are reproduced by the one-loop theoretical calculation.

Nonleptonic $\Delta S = 1$ decays of the Ω^- can also be computed to one-loop in terms of the parameters of the octet weak Lagrangian $L_v^{\Delta S=1}$. Tree-level formulæ are given in Ref. [19].

11. Decuplet Decoupling

The calculations performed in Sections 8–10 exploited the near degeneracy of octet and decuplet baryons in the real world. These calculations were done in the parameter regime $\Delta m < M_K$. The decuplet-octet mass difference Δm was dropped from the decuplet propagator and treated as a explicit vertex in the chiral Lagrangian. This approximation keeps only terms linear in the small parameter $\Delta m/M_K$; higher order terms are neglected. In the limit of exact degeneracy, the decuplet contributes to chiral logarithmic corrections to octet processes and to the leading non-analytic M_K^3 correction to baryon octet masses. For nearly degenerate octet and decuplet baryons, an additional non-analytic baryon mass correction arises, the $\Delta m M_K^2 \ln M_K^2$ correction. Although the above approximation scheme is sufficient for comparison with experiment, the approximation breaks down in the chiral limit. As the chiral limit is approached, the inequality $\Delta m < M_K$ is replaced by the opposite inequality $M_K \ll \Delta m$. In the chiral limit, the decuplet cannot contribute to the non-analytic corrections for octet processes since these corrections come from infrared divergences. The crossover from the degeneracy regime to the chiral limit is studied in detail in this section. The precise manner in which the decuplet decouples in the chiral limit is explained.

The decuplet M_K^3 and $\Delta m M_K^2 \ln M_K^2$ corrections to octet masses are produced by the Feynman diagram shown in fig. 2(a). For momenta small compared with Δm, the decuplet can be integrated out explicitly. The higher dimension operators represented in fig. 2(b) are generated by expanding the decuplet propagator

$$\frac{1}{k \cdot v - \Delta m} = -\frac{1}{\Delta m}\left(1 + \frac{k \cdot v}{\Delta m} + \ldots\right). \tag{11.1}$$

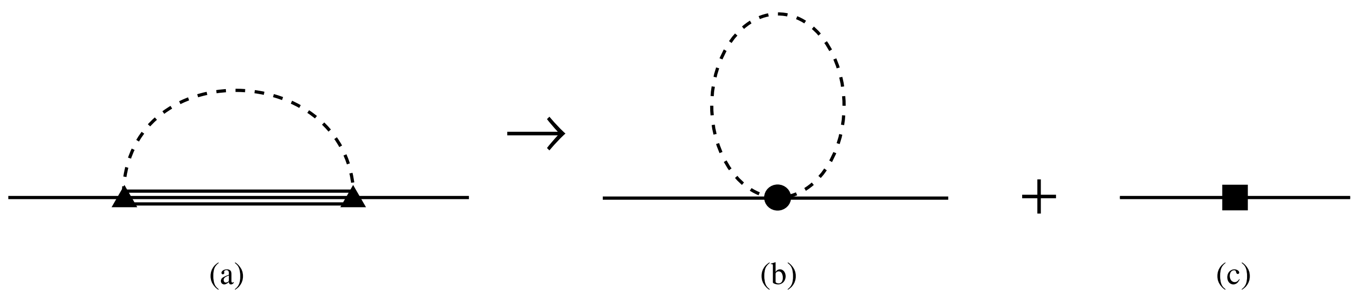

FIGURE 2.

Decoupling of the decuplet: Diagram (a) produces M_K^3 and $\Delta m M_K^2 \ln M_K^2$ corrections to octet masses when $\Delta m < M_K$. In the opposite limit, $M_K \ll \Delta m$, the intermediate decuplet baryon can be integrated out of the theory. Contracting the decuplet line produces graphs (b), which are suppressed by powers of $1/\Delta m$, where the dot in (b) represents higher dimension operators. Matching between the two theories at $\mu = \Delta m$ also produces a counterterm (c).

Since all terms are suppressed by $1/\Delta m$, these graphs cannot produce the M_K^3 and $\Delta m M_K^2 \ln M_K^2$ terms. The counterterm fig. 2(c) can be proportional to positive powers of Δm. This counterterm cannot produce the above terms, however, since an $SU(3)$ symmetric mass term cannot generate any chiral logarithms because it can be removed from the Lagrangian,* and any $SU(3)$ violating counterterm with factors of the mass matrix M produces corrections of order $M_K^4 \ln M_K^2$. Hence there is no term in the effective theory without decuplets which can produce the terms required. The M_K^3 and $\Delta m M_K^2 \ln M_K^2$ terms must therefore disappear as one crosses over to the parameter regime $M_K \ll \Delta m$. Similar reasoning leads to the conclusion that other chiral logarithmic corrections due to the decuplet also must disappear in the chiral limit.

The decoupling of the decuplet can be seen explicitly by computing fig. 2 for arbitrary

* This observation is equivalent to the result that no $m_B M_K^2 \ln M_K^2$ term appears in Eq. (9.1).

values of Δm and M_K. The Feynman integral for this diagram is proportional to

$$
\begin{aligned}
I &= \int \frac{d^D k}{(2\pi)^D} \frac{i}{(k\cdot v - \Delta m)} \frac{i}{(k^2 - M^2)} k_\mu(-k_\nu) \quad \overline{B}_v\, P_v^{\mu\nu} B_v \\
&= 2\int_0^\infty d\lambda \int \frac{d^D k}{(2\pi)^D} \frac{k^\mu k^\nu}{\left[k^2 - M^2 + 2\lambda(k\cdot v) - 2\lambda\Delta m\right]^2} \quad \overline{B}_v\, P_v^{\mu\nu} B_v \\
&= -\frac{4}{D}\int_0^\infty d\lambda \int \frac{d^D \ell}{(2\pi)^D} \frac{\ell^2}{\left[\ell^2 - \lambda^2 - 2\lambda\Delta m - M^2\right]^2} \quad \overline{B}_v B_v \\
&= \frac{4}{D}\frac{i}{16\pi^2}\frac{\Gamma(1+D/2)\Gamma(1-D/2)}{\Gamma(D/2)\Gamma(2)} \int_0^\infty d\lambda \left[\lambda^2 + 2\lambda\Delta m + M^2\right]^{(D/2)-1} \ \overline{B}_v B_v,
\end{aligned}
\tag{11.2}
$$

where the momentum integral is shifted to $\ell^\mu = k^\mu + \lambda v^\mu$ in the third line, and the identities $v_\mu P_v^{\mu\nu} = 0$ and $g_{\mu\nu} P_v^{\mu\nu} = -2$ are used. Setting $D = 4 - 2\epsilon$ and using the identity Eq. (5.6) gives

$$
\begin{aligned}
I &= \frac{2i}{16\pi^2}\Gamma(-1+\epsilon)\int_0^\infty d\lambda \left[\lambda^2 + 2\lambda\Delta m + M^2\right]^{1-\epsilon} \\
&= \frac{i}{24\pi^2}\left[4\left(M^2 - (\Delta m)^2\right)^2 \int_0^\infty d\lambda \frac{1}{\left[\lambda^2 + 2\lambda\Delta m + M^2\right]} \right. \\
&\qquad \left. + \left(2(\Delta m)^3 - 3M^2\Delta m\right)\ln M^2/\mu^2 + 4M^2\Delta m - \frac{10}{3}(\Delta m)^3\right]
\end{aligned}
\tag{11.3}
$$

The remaining integral can be evaluated explicitly,

$$
\begin{aligned}
&\int_0^\infty d\lambda \frac{1}{\left[\lambda^2 + 2\lambda\Delta m + M^2\right]} = \int_0^\infty d\lambda \frac{1}{\left[(\lambda + \Delta m)^2 + M^2 - (\Delta m)^2\right]} \\
&= \frac{\pi}{2\sqrt{M^2 - (\Delta m)^2}} - \frac{1}{\sqrt{M^2 - (\Delta m)^2}} \tan^{-1}\left(\frac{\Delta m}{\sqrt{M^2 - (\Delta m)^2}}\right); \quad (M > \Delta m) \\
&= -\frac{1}{2\sqrt{(\Delta m)^2 - M^2}} \ln\left(\frac{\Delta m - \sqrt{(\Delta m)^2 - M^2}}{\Delta m + \sqrt{(\Delta m)^2 - M^2}}\right); \quad (\Delta m > M).
\end{aligned}
\tag{11.4}
$$

The exact expression (11.3) and (11.4) can be studied in the two limits $\Delta m \ll M$ and $M \ll \Delta m$. Expanding the integral for $\Delta m \ll M$, and keeping only the leading terms gives

$$
I = \frac{i}{24\pi^2}\left[2\pi M^3 - 3M^2\Delta m \ln(M^2/\mu^2)\right]. \tag{11.5}
$$

The non-analytic terms are of the form included in Eq. (9.1). Note that the M_K^3 term comes from a $\tan^{-1}\infty = \pi/2$, and thus has an extra factor of π. This is a general result;

all odd powers of M_K arise from integrals similar to Eq. (11.4), and have an additional factor of π. The opposite limit, $M \ll \Delta m$ yields the expression

$$\begin{aligned} I = \frac{i}{24\pi^2} \Bigg[&-2\left((\Delta m)^2 - M^2 \right)^{3/2} \ln\left(\frac{M^2}{4(\Delta m)^2} \right) \\ &+ \left(2(\Delta m)^3 - 3M^2 \Delta m \right) \ln M^2/\mu^2 + \text{analytic in } M \Bigg]. \end{aligned} \tag{11.6}$$

Expanding $\left((\Delta m)^2 - M^2 \right)^{3/2}$,

$$\begin{aligned} I &= \frac{i}{24\pi^2} \Bigg[\left(-2\left(\Delta m\right)^3 + 3M^2 \Delta m + \mathcal{O}\left(\frac{1}{\Delta m} \right) \right) \ln \frac{M^2}{4(\Delta m)^2} \\ &\qquad + \left(2(\Delta m)^3 - 3M^2 \Delta m \right) \ln M^2/\mu^2 + \text{ analytic in } M \Bigg] \\ &= \frac{i}{24\pi^2} \Bigg[\left(-2\left(\Delta m\right)^3 + 3M^2 \Delta m \right) \ln \frac{\mu^2}{4(\Delta m)^2} \\ &\qquad + \mathcal{O}\left(\frac{1}{\Delta m} \right) \ln \frac{M^2}{4(\Delta m)^2} + \text{ analytic in } M \Bigg]. \end{aligned} \tag{11.7}$$

Note that there is no longer any decuplet contribution to the M_K^3 term, nor any $\Delta m M_K^2 \ln M_K^2$ term. The $\ln\left(\mu^2/4(\Delta m)^2\right)$ term can be reabsorbed into local counterterms, as can the analytic terms. The only infrared non-analytic pieces left are of order $1/\Delta m$, which are precisely the pieces that can be generated by the higher dimension operators produced on integrating out the decuplet.

The decoupling of the decuplet non-analytic corrections in the limit that $M_K \ll \Delta m$ agrees with theorems on the behavior of non-analytic corrections in the chiral limit.[20] The precise nature of the decoupling is rather subtle, and requires the complete expression for the Feynman integrals as a function of $\Delta m/M_K$. We have explicitly computed the decoupling in this section for the baryon masses. Similar results also hold for the axial currents and the non-leptonic decays. In the real world, $\Delta m < M_K$, so we have simplified our computations by only retaining the terms linear in Δm. The expressions given in Sections 8–10 cannot be used to study the chiral limit, as can be seen in the example above. However, correct formulæ in the extreme chiral limit can be readily obtained from these expressions by dropping all non-analytic corrections produced by graphs with decuplet intermediate states. For computations of chiral $SU(2)$ breaking effects due to the pion mass, it is important to retain the complete form of the integrals, because Δm is not small compared with M_π.

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TABLE 1

Decay	$s_{\rm expt}$	$s_{\rm theory}$	$s_{\rm tree}$	$\Delta s_{\rm loop}$	$\Delta s_{\rm octet}$	$\Delta s_{\rm decup}$
$\Sigma^+ \to n\pi^+$	0.06	−0.09	0.00	−0.09	0.13	−0.22
$\Sigma^+ \to p\pi^0$	−1.43	−1.41	−0.85	−0.55	−0.04	−0.51
$\Sigma^- \to n\pi^-$	1.88	1.90	1.21	0.69	0.18	0.51
$\Lambda \to p\pi^-$	1.42	1.44	0.91	0.53	0.16	0.37
$\Lambda \to n\pi^0$	−1.04	−1.02	−0.64	−0.37	−0.11	−0.27
$\Xi^- \to \Lambda\pi^-$	−1.98	−2.04	−1.19	−0.84	−0.14	−0.71
$\Xi^0 \to \Lambda\pi^0$	1.52	1.44	0.84	0.60	0.10	0.50
Decay	$p_{\rm expt}$	$p_{\rm theory}$	$p_{\rm tree}$	$\Delta p_{\rm loop}$	$\Delta p_{\rm octet}$	$\Delta p_{\rm decup}$
$\Sigma^+ \to n\pi^+$	1.81	0.82	−0.06	0.89	0.16	0.72
$\Sigma^+ \to p\pi^0$	1.17	0.36	−0.13	0.49	−0.06	0.55
$\Sigma^- \to n\pi^-$	−0.06	0.34	0.13	0.21	0.26	−0.05
$\Lambda \to p\pi^-$	0.52	−0.52	−0.28	−0.24	−0.31	0.08
$\Lambda \to n\pi^0$	−0.39	0.38	0.21	0.17	0.23	−0.05
$\Xi^- \to \Lambda\pi^-$	0.48	0.35	0.11	0.24	0.30	−0.07
$\Xi^0 \to \Lambda\pi^0$	−0.33	−0.24	−0.07	−0.16	−0.21	0.05

S- and P-wave amplitudes for parameter values determined from a fit to one-loop S-wave formulæ. The fit was performed setting $D = 0.61$, $F = 0.40$, $\mathcal{C} = 1.6$, and $\mathcal{H} = -1.9$. The best fit parameters are $h_D = -0.35 \pm 0.09$, $h_F = 0.86 \pm 0.05$, and $h_C = -0.36 \pm 0.65$ in units of $G_F m_{\pi^+}^2 \sqrt{2} f_\pi$. Quoted errors reflect the 20% theoretical error of the one-loop calculation. Experimental amplitudes are given in column 2. The total theoretical one-loop amplitudes (column 3) are the sum of tree-level amplitudes (column 4) and chiral logarithmic corrections (column 5). The chiral logarithmic correction is the sum of the chiral correction produced by octet and decuplet graphs, respectively. The chiral correction which results from renormalization of the pion decay constant is contained in the octet chiral correction. The theoretical S-wave amplitudes $s_{\rm theory}$ should agree with measured values to within a nominal theoretical error of 0.30 in amplitude. Agreement with experiment for P-wave amplitudes is only expected to within corrections which are comparable to $\Delta p_{\rm loop}$.