FLAG 1902.08191

(10,7) 1510.06039

(17) 0810.2464

$$M_\pi^2 = 2B\hat{m}\left(1 + \delta^{NLO}_{M_\pi^2}\right)$$

$$\partial_0 = \frac{\partial}{\partial 0}$$

$$\hat{m}\,\partial_{\hat{m}} = \hat{m}\left(\frac{\partial M_\pi^2}{\partial \hat{m}}\right)\partial_{M_\pi^2}$$

$$= \hat{m}\cdot 2B(1+NLO)\,\partial_{M_\pi^2}$$

$$= M_\pi^2\left(1 - \delta^{NLO}_{M_\pi^2}\right)\left(1 + \hat{m}\,\delta^{NLO}_{\frac{\partial M_\pi^2}{\partial \hat{m}}}\right)\frac{\partial}{\partial M_\pi^2}$$

$$= \left(1 - \delta^{NLO}_{M_\pi^2}\right)\left(1 + \hat{m}\,\delta^{NLO}_{\frac{\partial M_\pi^2}{\partial m}}\right) M_\pi^2 \frac{\partial}{\partial M_\pi^2}$$

$$= (\quad)(\quad)\,\frac{M_\pi}{2}\partial_{M_\pi}$$

$y = \sqrt{M^2}$

$\partial_{M^2} = \partial_y \cdot \frac{\partial y}{\partial M^2}$

$= \partial_y \cdot \frac{1}{2}\frac{1}{M}$

$M^2\partial_{M^2} = \frac{M}{2}\partial_M$

$\varepsilon_\pi\,\partial_{\varepsilon_\pi}$

@NLO

$$M_\pi^2 = M^2\left(1 - \frac{1}{2}\frac{M^2}{(4\pi F)^2}\ln\left(\frac{\Lambda_3^2}{M^2}\right)\right), \quad M^2 = 2B\hat{m}$$

$$\bar{l}_3 = \ln\frac{\Lambda_3^2}{M_{\pi,phys}^2}$$

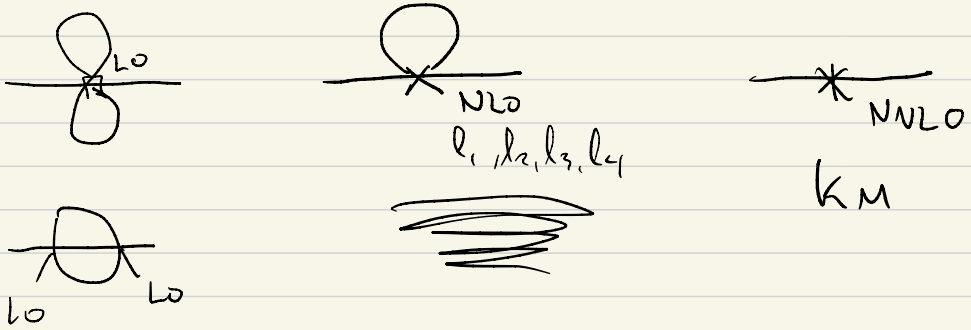
$$\ln\Lambda_3^2 = \bar{l}_3 + \ln M^2_{\pi,phys}$$

$$= M^2\left[1 + \frac{M^2}{(4\pi F)^2}\left(\frac{1}{2}\ln\frac{M^2}{\mu^2} + 2\cdot(4\pi)^2 l_3^r(\mu)\right)\right]$$

$$\delta^{NLO}_{M_\pi^2} = \frac{M^2}{(4\pi F)^2}\frac{1}{2}\left[\ln\frac{M^2}{\mu^2} + 4\bar{l}_3^r\right]$$

$$\hat{m}\,\delta^{NLO}_{\frac{\partial M_\pi^2}{\partial m}} = \frac{M^2}{(4\pi F)^2}\frac{1}{2}\left[\ln\frac{M^2}{\mu^2} + 4\bar{l}_3^r\right] + \frac{M^2}{(4\pi F)^2}\frac{1}{2}\cdot\frac{1}{M^2}\cdot\hat{m}\cdot 2B$$

N^2LO



$M^4(\ln^2 M^2 + \text{finite})$ $\quad M^2(\ln M^2 + \text{finite})$ $\quad M^4$

$$M_\pi^2 = 2B\hat{m}\left(1 + \delta^{NLO} + \delta^{N^2LO} + \cdots\right)$$

$$\hat{m}\,\partial_{\hat{m}} M_\pi^2 = 2B\hat{m}\left(1 + \delta'^{NLO} + \delta'^{N^2LO} + \cdots\right)$$

$$\left(1 - \delta^{NLO} - \delta^{N^2LO} + \cdots\right)\left(1 + \delta'^{NLO} + \delta'^{N^2LO} + \cdots\right)$$

$$= 1 - \delta^{NLO} + \delta'^{NLO} - \delta^{NLO}\delta'^{NLO} + \delta'^{N^2LO} - \delta^{N^2LO} + \cdots$$

$\delta^{NLO} + \delta'^{NLO}$: NLO

$\delta^{NLO}\delta'^{NLO} + \delta'^{N^2LO} - \delta^{N^2LO}$: N^2LO