September 11, 2013

CSCI 2830
Solutions to Assignment 2

For full credit, you must show your work on each problem. You may work with others to figure out the problems, but you need to do your own writeup.

1. (10 points) Let $v_1 = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$.

   (a) List five vectors in Spanv_1, v_2. For each vector, show the weights on the vectors used to generate the result and list the three entries of the vector. Do not make sketches.

   (b) Give a geometric description of the span in the previous part.

   Solution: Any five linear combinations of v_1 and v_2 work here. For example, $\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$ with weights 1 (for v_1) and 1 (v_2); $\begin{pmatrix} 4 \\ 0 \\ 7 \end{pmatrix}$ with weights 2 and 1; $\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix}$ with weights -1 and 0; $\begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$ with weights 0 and -2; $\begin{pmatrix} 33 \\ 0 \\ 22 \end{pmatrix}$ with weights 11 and 0.

   The vectors in problem 16 are of dimension three, but there are only two of them. Because the two vectors are not multiples of each other span(v_1,v_2) is a plane in 3D. Because the second (y) component is zero in all the vectors, the plane is the xz-plane. (It is good enough here to say that it is a plane.)

2. (10 points) Let $u = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$ and $A = \begin{pmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{pmatrix}$. Is u in the subset of $\mathcal{R}^3$ spanned by the columns of A? Why or why not?

   Solution: The vector u is not in the span of the columns of A. Reducing A to echelon form gives a matrix with a row of zeros, e.g., with only two nonzero pivots. Augmenting A with u then reducing the result gives a matrix with three nonzero pivots. If u had been in the span, there would have been only two nonzero pivots. Alternatively, you can consider the system $Ax = u$. It has no solution, meaning that u is not a linear combination of the columns of A so u is not in the span of A.

You might be able to figure out why u cannot be formed via a linear combination of the columns of A by inspection, but you'll need some words of explanation to your approach in that case.

3. (10 points) Construct a 3×3 matrix A whose columns do *not* span $\mathcal{R}^3$. Show that the matrix you constructed has the desired property.

   Solution: You need a matrix with linearly dependent columns because three linearly independent vectors of dimension three span $\mathcal{R}^3$. One example is $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ which has one linearly independent column. (I realize that we covered linear independence after you turned in these answers. You should have been able to do them all without knowing about linear independence. I just use it in my answers here as it is more efficient to use the term.)

   Or do it by construction–start with any nonzero 3×3 matrix B in echelon form that has fewer than three nonzero pivots. Performa a row operation that creates a matrix A that is not in echelon form. Since A does not have a pivot position in every row, its columns do not span $\mathcal{R}^3$.

4. (10 points) Construct a 3×3 nonzero matrix A such that the vector $x = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is a solution of $Ax = 0$.

   Solution:

   Here, you need a matrix such that the first column plus the second column multiplied by -2 plus the third column gives a zero vector. That means that the linear combination of columns with the elements of the vectors as weights gives the zero vector. One example is the 3×3 matrix of all ones.