September 4, 2013

CSCI 2830
Solutions to Assignment 1

1. (10 points) For which values of k does

$$\begin{aligned} kx + y &= 1 \\ x + ky &= 1 \end{aligned}$$

have no solution, one solution, or infinitely many solutions? In each case, explain why.

Solution:

The two equations define the two lines (in slope-intercept form):

$$\begin{aligned} y &= -kx + 1 \\ y &= -\frac{1}{k}x + \frac{1}{k}. \end{aligned}$$

If $k = 1$, the two lines are the same ($y = -x + 1$), and the system has an infinity of solutions.

If $k = -1$, the two lines are the parallel ($y = x + 1$ and $y = x - 1$), and the system has no solutions.

For all other values of k, the two lines intersection in exactly one point, and that point is the unique solution of the system.

If this explanation seems confusing, try drawing pictures of the two lines for some selected values of k.

This problem can also be done by analyzing the coefficient matrix of the 2×2 linear system, but that way is harder than the geometric approach.

2. (5 points) Rewrite the following linear system as a coefficient matrix times a vector of unknowns equals a righthand side vector:

$$\begin{aligned} 2u + v + w &= 3 \\ 2u + 2v &= 4 \\ -v + 4w &= 4 \end{aligned}$$

Solution:

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 2 & 0 \\ 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix}$$

3. (10 points) Reduce the following matrix to row echelon form, and identify the pivot positions. (The row echelon version we are using is sometimes called upper echelon because the nonzeros are all in the upper part of the matrix.)row

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{pmatrix}.$$

Solution:

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & -16 & -34 \end{pmatrix} \to$$

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -10 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The pivot positions are (1,1), (2,2), and (4,3). Pivot columns are 1,2, and 4. (I did not use partial pivoting in this solution, but it is very good to do so.) It is fine not to scale the center row to make the (2,2) pivot be one, but it may make the arithmetic a little easier if you do.