October 22, 2012

CSCI 2830
Solutions to Assignment 6

For full credit, you must show your work.

1. (15 points) Show that the determinant of the following matrix is 30 by cofactor expansion then show the same result by Gaussian elimination:

$$A = \begin{pmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 2 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix}.$$

Solution:

$$\begin{aligned}
\det A &= 1*(1)*\begin{vmatrix} 1 & 1 & 0 \\ -1 & 2 & 2 \\ 3 & 0 & 2 \end{vmatrix} + 0*(-1)*\begin{vmatrix} 0 & 1 & 0 \\ 1 & 2 & 2 \\ 1 & 0 & 2 \end{vmatrix} + 2*(1)*\begin{vmatrix} 0 & 1 & 0 \\ 1 & -1 & 2 \\ 1 & 3 & 2 \end{vmatrix} + \\
&\quad (-3)*(-1)*\begin{vmatrix} 0 & 1 & 0 \\ 1 & -1 & 2 \\ 1 & 3 & 2 \end{vmatrix} \\
&= 1*[(1*2*2+1*2*3+0)-(0+0+2*-1*1)]+0 \\
&\quad +2*[(0+1*2*1+0)-(0+0+2*1*1)] \\
&\quad +3*[(0+1*2*1+1*1*3)-(-1*1*1+0+0)] \\
&= 12+0+18 = 30.
\end{aligned}$$

I used the shorthand method for 3×3 determinants to evaluate the determinants of the minors.

By Gaussian elimination, find

$$A = LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & 3 & -5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 30 \end{pmatrix}.$$

Then $\det A = \det L \det U = 1 * 30 = 30$.

2. (10 points) A box has edges from (0,0,0) to (3,1,1) and (1,3,1) and (1,1,3). Find its volume.

Solution: volume $= \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 20.$