November 5, 2013

CSCI 2830
Solutions to Assignment 7

For full credit, you must show your work.

1. (20 points) To answer these questions, we need first to reduce A to echelon form. Using Gaussian elimination, we get

$$U = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & \frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix}.$$

If you use partial pivoting, the third pivot is $\frac{11}{3}$.

Solution:

(a) To compute a basis for $\mathrm{nul}(A)$, we have to solve the system $Ax = 0$ which is equivalent to solving $Ux = 0$. Since A and U have three columns, $x = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T$. There are three pivots in U, so there are no free variables. That means that the only vector in $\mathrm{nul}(A)$ is the zero vector. A basis is thus the zero vector.

(b) The pivot columns in U are each of the three columns, so a basis for the column space of A is the three columns of A.

(c) $\dim(\mathrm{nul}(A)) = 0$ since the only vector in the nullspace is the zero vector.

(d) Because there are three pivots, A has three linearly independent columns. Thus, $\dim(\mathrm{col}(A)) = 3$.

(e) The sum of the two dimensions should equal the number of columns, and, indeed, $0 + 3 = 3$.

(f) $\mathrm{rank}(A) = 3 = \dim(\mathrm{col}(A)) =$ number of linearly independent columns of A.

2. (15 points)

Solution: This problem is the same as Example 2 on p.241 of the textbook with different numbers.

We first give variable names to the coefficients of b_1 and b_2 in the C basis: $[b_1]_C = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $[b_2]_C = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

To find what those coefficients are, we set up and solve the two systems $\begin{pmatrix} c_1 & c_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = b_1$ $\begin{pmatrix} c_1 & c_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = b_2$

Combining those two systems into one gives

$$\begin{aligned} \begin{pmatrix} c_1 & c_2 \end{pmatrix} \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} &= \begin{pmatrix} b_1 & b_2 \end{pmatrix} \\ \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} &= \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

Solving the system gives

$$P_{C \leftarrow B} = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \end{pmatrix}.$$