CSCI 2824 Discrete Structures
Instructor: Hoenigman
Assignment 7 Solutions
Due Tuesday, Nov 5 at 2pm.

Problems

1. How many different license plates are available if each plate contains a sequence of three letters followed by three digits? (No sequences of letters or numbers are prohibited.)

We're assuming the plate has 6 elements exactly and first 3 are letters and last 3 are numbers.
$26^3 \cdot 10^3$

2. How many strings of three digits
   a. Do not contain the same digit three times?

   In each position, there are 10 possible digits, so in a 3-digit string, there are 10^3 permutations. However, we need to subtract strings with all same digits giving us $10^3 - 10$.

   b. Begin with an odd digit?

   There are 5 possible odd digits, and 10 possible digits for the other positions, giving us $5 \cdot 10^2$

   c. Have exactly two digits that are 4s?

   If 2 of the positions are 4s, then there is only one position left and 9 ways to choose the digit for that position. There are 3 ways to place two 4's, so there are $3 \cdot 9 = 27$ possible strings

3. Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible ways can the saleswoman visit these cities?

$7!$ She starts in one city and has 7 possible cities to choose. From the next city, she can choose from 6 to go to next. Then, 5 to choose from, and so on. This gives us $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

4. A club has 25 members.
   a. How many ways are there to choose four members of the club to serve on an executive committee?

   $C(25,4) = \frac{25!}{4!(21)!}$ Order doesn't matter here, we just ~~the~~ need 4 of the 25.

b. How many ways are there to choose a president, vice president, secretary, and treasurer?

Here, order does matter because a solution that puts person X at treasurer is different than putting person X as president. Therefore, we have $P(25,4) = \frac{25!}{21!}$

5. How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries.

$C(45,3) \cdot C(57,4) \cdot C(69,5)$. We don't have any indication that order matters, making this a combination. $\frac{45!}{3!42!} \cdot \frac{57!}{4!53!} \cdot \frac{69!}{5!64!}$

6. A student has three mangos, two papayas, and two kiwi fruits. If the student eats one piece of fruit each day, and only the type of fruit matters, in how many different ways can these fruits be consumed.

There are 7 days to fill and we want to put the different fruits into a slot representing each day. For the mangos, we chose 3 of the 7 days and the order of the mangos doesn't matter. In the remaining 4 days we need to place 2 papayas and 2 kiwi.

$C(7,3) \cdot C(4,2) \cdot C(2,2)$

7. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. A dozen croissants

We need to choose 12 from 4 types where there are indistinguishable types. All croissants of the same type are equivalent.

$C(n+k-1, k) = C(6+12-1, 12) = \frac{17!}{12!5!}$

b. Two dozen croissants with at least five chocolate croissants and at least three almond croissants.

Since we need to have 5 chocolate and 3 almond, we remove those from our combinations of the two dozen, which leaves 16 left to choose. We're assuming all other ~~types~~ 16 can be chosen from the 6 types, which gives us $C(6+16-1, 16)$

$= \frac{21!}{16!5!}$