CSCI 2824 Discrete Structures
Instructor: Hoenigman
Assignment 5
Due Tuesday, October 8 in class.

Problems

1. If L(n) is "$n^2 < 2$", write L(1), L(2), and L(3). Which, if any, are true?

$L(1) = 1^2 < 2$ T
$L(2) = 2^2 < 2$ F
$L(3) = 3^2 < 2$ F

2. For the recursive sequence defined by $a_k = a_{k-1} + (k + 4)$ for $k \geq 2$, where $a_1 = 5$, show that the sequence can be equivalently described by the closed formula $a_n = \frac{n(n+9)}{2}$.

$$a_n = \frac{(n-1)(n-1+9)}{2} + (n+4) = \frac{(n-1)(n+8)}{2}$$

$$= \frac{n^2+8n-n-8+2n+8}{2} = \frac{n^2+9n}{2} = \frac{n(n+9)}{2}.$$

3. For the recursive sequence defined by $b_k = 2b_{k-1} + k$ for $k \geq 2$, where $b_1 = 3$, show that the sequence can be equivalently described by the closed formula $b_n = 3 * 2^n - n - 2$.

$$b_n = 2(3 \cdot 2^{n-1} - (n-1) - 2) + n$$
$$= 3 \cdot 2^n - 2n + 2 - 4 + n$$
$$= 3 \cdot 2^n - n - 2$$

4. Use induction to prove each of the following. As part of your proof, write and verify each statement for at least *n=1* and *n=2*.

a. $\sum_{i=1}^{n}(2i+4) = n^2 + 5n \ for\ each\ n \geq 1$

Start w/ base case $n=1$: $1+5=6$ $n=2$: $4+10=14$

Let m positive $\mathbb{Z}$ be given, then

$S_m = S_{m+1} + 2m + 4$

$= (m-1)^2 + 5(m-1) + 2m + 4.$

$= (m^2 - 2m + 1) + 5m - 5 + 2m + 4 = m^2 + 5m$

b. $1 + 3 + 6 + 10 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$

(Note: it may be helpful to write this this one in sigma notation first, and then solve.)

$S_n = \sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$

$S_m = S_{m-1} + \frac{m(m+1)}{2}$

$= \frac{(m-1)(m-1+1)(m-1+2)}{6} + \frac{m(m+1)}{2}$

$= \frac{m(m-1)(m+1) + 3m^2 + 3m}{6}$

$= \frac{m^3 + 3m^2 + 2m}{6} = \frac{m(m+1)(m+2)}{6}$