CSCI 2824 Discrete Structures
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Assignment 3 Solutions

1. You're looking for the case where the implication is false. In this implication, "If you didn't buy a car from us, then you paid too much" the truth table looks like:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

When P is true and Q is false, the implication is false. From the consumer perspective, this would be the case where they didn't buy the car from the company and they didn't pay too much.

2. The contrapositive of $P \rightarrow Q$ is $\neg Q \rightarrow \neg P$. Translating the slogan gives us, "If you didn't pay too much, then you bought the car from us." The truth table looks like:

P	Q	$\neg P$	$\neg Q$	$\neg Q \rightarrow \neg P$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

This implication is false when $\neg Q \rightarrow P$. From the consumer perspective, this would be the case where they didn't pay too much, and didn't buy the car from the company.

3. To determine if the statements are logically equivalent, you can set up a truth table and check that all rows are the same for the same values of P and Q. Let P be "If you don't eat your peas" and Q be "You can't have dessert". Then, "If you eat your peas" is $\neg P$, and "You can have dessert" is $\neg Q$. The truth table shows that these are not logically equivalent statements:

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg P \rightarrow \neg Q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

4. Write in propositional logic
    a. If you don't attend the concert, you will get an F for the course.
    Let P be "If you don't attend the concert". Let Q be "You will get an F for the course". $P \rightarrow Q$
    b. We will go if you will go.

Let P be “If you will go”. Let Q be “We will go”.

c. If you don’t eat breakfast, you will be hungry.
   Let P be “If you don’t eat breakfast”. Let Q be “You will go hungry”.
d. If a quadrilateral is a square, it has four equal sides and four equal angles.
   Let P be “If a quadrilateral is a square”. Let Q be “It has four equal sides”. Let R be “It has four equal angles”. $P \rightarrow Q \wedge R$

5. Rewrite as the contrapositive.
   a. If you don’t get an F for the course, then you can attend the concert.
   b. If we won’t go, then you won’t go.
   c. If you won’t be hungry, then eat breakfast.
   d. If it does not have four equal sides and four equal angles, then it is not a square.

6. Sometimes you win, sometimes you lose, sometimes it rains. You can think of this as there exists a time in all time when it is raining, or you are winning, or you are losing, but only one of these things is happening at a time. There could also be a time when nothing is happening.

   Let P(x) be time when you win.
   Let Q(x) be time when you lose.
   Let R(x) be time when it rains.
   Let Z(x) be time when nothing happens.

$$\exists x \in time, P(x) xor Q(x) xor R(x) xor Z(x)$$