CSCI 2824 – Discrete Structures
Midterm 1

Name____________________________

1. Let P and Q be the propositions:
   P: You drive over 65 miles per hour
   Q: You get a speeding ticket

   Write these propositions using P and Q and the logical connectives.

   a. You drive over 65 miles an hour, but you do not get a speeding ticket.

      $P \wedge \neg Q$

   b. If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

      $\neg P \rightarrow \neg Q$

   c. Whenever you get a speeding ticket, you are driving over 65 miles per hour.

      $Q \rightarrow P$

2. Determine whether $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ are logically equivalent. Show the truth table and explain your results.

P	Q	R	$(P \wedge Q) \wedge R$	$P \wedge (Q \wedge R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Yes they are. All rows in truth table identical

3. Let P(x), Q(x), and R(x) be the statements "x is a lion", "x is fierce", "x drinks coffee". Write the following English statements in predicate logic.

   a. Some lions are fierce

      $\exists x\ P(x) \wedge Q(x)$

   b. Some fierce lions do not drink coffee.

      $\exists x\ P(x) \wedge Q(x) \wedge \neg R(x)$

4. Let $Q(x)$ be the statement $x + 1 > 2x, x \in \mathbb{Z}$. What is the truth value of $\exists x Q(x)$? Provide an example.

True. $x = 0$, then $1 > 0$

5. Prove the statement, "If n is an odd integer, then 5n + 6 is an odd integer".

Let n be an odd integer, then

$n = 2k+1$, k is an integer.

$5(2k+1)+6 = 10k+5+6 = 10k+11 = 2(5k+5)+1$

Since k is an integer, $5k+5$ is also an integer, and $5n+6$ is written in the form of an odd integer.

6. Prove the statement, "The product of any two consecutive integers is even"

Let any two consecutive integers be given as n and $n+1$. Their product is $n \times (n+1)$, Let $n = 2k$ and $n+1 = 2k+1$, where k is an integer.

$2k \cdot (2k+1) = 4k^2 + 2k = 2(2k^2+k)$

Since k is an int, $2k^2+k$ is an int and $n \cdot n+1$ is even.

7. Prove the statement, "If n is not divisible by 3, then n^2+2 is divisible by 3".

Let n be an integer not divisible by 3. therefore

$n = 3k+1$ or $n = 3k+2$, $k \in$ Integers

Case 1: $n = 3k+1$, then $n^2+2 = (3k+1)^2+2 = 9k^2+6k+1+2$
$= 3(3k^2+2k+1)$

Case 2: $n = 3k+2$, then $n^2+2 = (3k+2)^2+2 =$
$9k^2+12k+4+2 = 3(3k^2+4k+2)$

Since both cases can be written as $3(Q)$, where Q is an integer, n^2+2 is divisible by 3.

8. Show that the sequence defined by $b_k = 4b_{k-1} + 3, for\ k \geq 2,$ where $b_1 = 3$ is equivalently described by the closed formula $b_n = 2^{2n} - 1$.

First, show that the base case is true.

$b_1 = 2^{2} - 1 = 4 - 1 = 3$

Assume true for $m-1$, for some integer m.
Show that its true for m

$b_m = 4(2^{2(m-1)} - 1) + 3$
$= 4 \cdot 2^{2m-2} - 4 + 3 = 2^2 \cdot 2^{2m-2} - 1 = 2^{2m} - 1$

showing that its true for m.