CSCI 2824 Discrete Structures
Instructor: Hoenigman
Assignment 6 Solutions
Due Tuesday, October 22, before class

Problems

1. Let $A = \{x \in \mathbb{N} : x = 4k, k \in \mathbb{N}\}$, $B = \{y \in \mathbb{Z}^{+} : 2y \text{ is a perfect square}\}$,
$C = \{z \in \mathbb{Z} : z^2 < 1000\}$

$A = \{0, 4, 8, 12 \ldots\}$
$B = \{2, 8, 18, 32 \ldots\}$
$C = \{0, 1, 2, 3, 4 \ldots\}$

List 5 elements in each of the following sets:

1. $A \cup (B \cap C)$

$\{0, 4, 8, 12, 16\}$

2. $(A \cup B) \cap C$

$\{2, 4, 8, 16, 20\}$

3. $A \cap (B \cup C)$

$\{4, 8, 12, 16, 20\}$

2. For the following sets, write the set description in set-builder notation

a. $\{11, 33, 55, 77, \ldots\}$

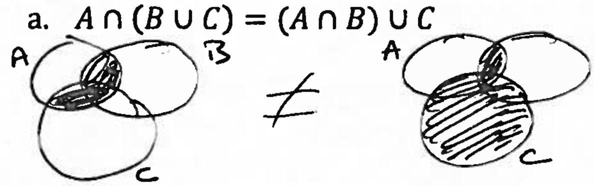
$\{x \in \mathbb{Z} : x = 11k, k = 2n+1 : n \in \mathbb{N}\}$

b. $\{1, 2, 4, 8, 16, \ldots\}$

$\{x \in \mathbb{N} : x = 2^n : n \in \mathbb{N}\}$

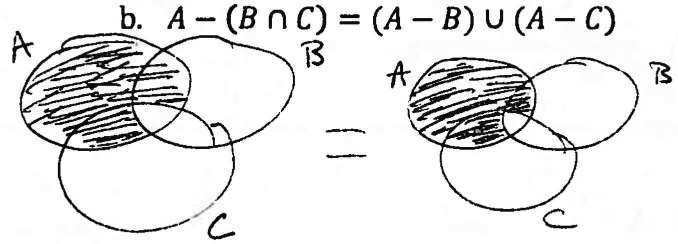
3. Use Venn diagrams to show if the following properties are true. If they are not true, provide a counterexample.

a. $A \cap (B \cup C) = (A \cap B) \cup C$



C − B not in first set.

b. $A - (B \cap C) = (A - B) \cup (A - C)$



4. Let $A = \{2k: k \in \mathbb{Z}\}$, $B = \{3k + 1: k \in \mathbb{Z}\}$, $C = \{6k + 5: k \in \mathbb{Z}\}$. Show that $\{A, B, C\}$ is not a partition of $\mathbb{Z}$.

$A = \{0, 2, 4, 6 ..\}$, $B = \{1, 4, 7, 10 ..\}$, $C = \{5, 11, 17 ...\}$

3 is not accounted for, so the partition is not valid

5. Let $A = \{2k: k \in \mathbb{Z}\}$, $B = \{4k + 1: k \in \mathbb{Z}\}$, $C = \{4k + 3: k \in \mathbb{Z}\}$. Explain why $\{A, B, C\}$ is a partition of $\mathbb{Z}$.

1. None of the sets are empty. Let $k=1$, then $A = \{2\}$, $B = \{5\}$, $C = \{7\}$
2. Show that the sets have no common elements.
   Let $2k = 4k+1$, then $k = -1/2$, which is a contradiction since $k \in \mathbb{Z}$.
   Let $2k = 4k+3$, then $k = -3/2$, which is a contradict
   Let $4k+1 = 4k+3$, then $3=1$, which is contradiction
3. $2k$ accounts for even numbers
   $4k+1 = 2 \cdot 2k+1$ which accounts for non-overlapping
   $4k+3 = 2 \cdot 2k+3$ odd numbers. All numbers are either even or odd.

6. Give a partition of A = {1, 2, 3, 4, 5, 6, 7, 8} satisfying each of the following criteria:
   a. Every part has the same size.

   $A1 = \{1, 2\}$, $A2 = \{3, 4\}$, $A3 = \{5, 6\}$
   $A4 = \{7, 8\}$

   b. There are exactly three parts, all having different sizes.

   $A1 = \{1, 2, 3, 4\}$ $A2 = \{5, 6, 7\}$
   $A3 = \{8\}$

   c. Even numbers are all in the same part.

   $A1 = \{2, 4, 6, 8\}$, $A2 = \{1, 3, 5, 7\}$

7. Let $A = \{1, 2, 3, 4, 5\}$. Use mathematical induction to prove that for all integers $n \geq 1$, if $B = \{1, 2, ... n\}$, then the number of elements in $A \times B$ is $5 \cdot n$. (Note: there is more in my lecture notes about $A \times B$).

$n(A \times B) = n(A) \cdot n(B)$
Let $n = 1$, then $A = \{1, 2, 3, 4, 5\}$ and $B = \{1\}$
$n(A \times B) = 5 \cdot 1 = 5$, so base case is true
Assume true for $m-1$: $n(A) \cdot n(B_{m-1}) = n(A \times B_{m-1})$
$B_{m-1} = \{1, 2, 3 ... m-1\}$, $5 \cdot m-1$. Add 1 more to B to get

$B_{m-1} = \{1, 2, 3, \ldots m-1\}$ and $n(A) \cdot n(B_{m-1}) =$

$5 \cdot m-1.$

$B_{m-1} + 1 = \{1, 2, 3 \ldots m-1\} + 1$

And $n(A) \cdot (n(B_{m-1}) + 1) =$

$5 \cdot (m-1+1) = 5m$

which shows that number of elements in

$A \times B = 5n.$