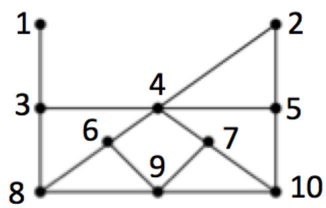
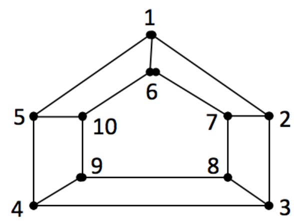
CSCI 2824 Discrete Structures
Instructor: Hoenigman
Assignment 9
Due Tuesday, December 10 at 2pm

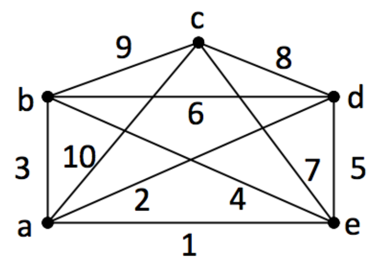
Problems



Graph G



Graph F



Graph Q

1. Refer to Graph G for this problem.
    a. How many nodes are there?
        i. 10
    b. List the degree of each node.
        i. Deg(1) = 1
        ii. Deg(2) = 2
        iii. Deg(3) = 3
        iv. Deg(4) = 5
        v. Deg(5) = 3
        vi. Deg(6) = 3
        vii. Deg(7) = 3
        viii. Deg(8) = 3
        ix. Deg(9) = 4
        x. Deg(10) = 3
    c. What is the sum of the degrees of the nodes?
        i. 30
    d. How many edges are there?
        i. 15
2. Refer to Graph G for this problem.
    a. Find two different circuits in the graph
        i. One circuit: 8->9->6->8
        ii. One circuit: 2->4->5->2
    b. Find a walk from 1 to 8 that uses 5 edges
        i. One example: 1->3->4->7->9->8
    c. Does the graph have a Hamilton path or Hamilton circuit? Explain.
        i. The graph has a Hamilton path, such as 1,3,8,6,9,7,10,5,2,4. However, the graph does not have a Hamilton circuit. Since node 1 is a leaf in the graph, there is no way to go through that node without re-visiting node 3.

d. Does the graph have an Euler path or Euler circuit? Explain.
    i. To be an Euler path, it needs to visit each edge exactly once. To do so, there cannot be nodes with an odd degree that are interior nodes. All nodes except 2 and 9 have an odd degree. Therefore, any path visiting all nodes would have to go through multiple nodes with an odd degree, so the graph cannot have an Euler path. Since it doesn't have an Euler path, it also doesn't have an Euler circuit.

3. Refer to Graph F for this problem. The graph is not currently Eulerian. Add the fewest number of edges possible to create a graph that is Eulerian.
    a. To be Eulerian, all vertices need to be even degree. Currently, all vertices have degree = 3. Five edges need to be added to the graph to give each vertex a degree of 4.
4. What is the minimum number of edges that a simple connected graph with n vertices can have? (Refer to the Lecture21 – IntroToGraphs lecture notes on Moodle for definitions of simple and connected)
    a. A simple graph has no multiple edges or loops. A connected graph has a path between any pair of vertices. The minimum number of edges is n-1.
5. Find a minimal spanning tree of Graph Q shown above.
    a. [a,d],[a,e],[a,b],[e,c]
6. Let A be the set of letters in the English alphabet. For each of the following relations on A, decide if it is reflexive, transitive, or antisymmetric. (Each can satisfy more than one of these properties.)
    a. $R_1 = \{(\alpha, \beta) \in A \times A: a\ immediately\ precedes\ \beta\ in\ alphabetical\ order\}$
        i. Not reflexive since an element cannot precede itself
        ii. It is not transitive. If a immediately precedes b and b immediately precedes c, then a cannot immediately precede c.
        iii. It is antisymetric, since a precedes b means that b cannot immediately precede a.
    b. $R_2 = \{(\alpha, \beta) \in A \times A: a\ comes\ before\ \beta\ in\ alphabetical\ order\}$
        i. Not reflexive
        ii. It is transitive. If a comes before b and b comes before c, then a comes before c.
        iii. It is antisymetric. If a comes before b then b cannot come before a.

7. Let A = {1,2,3}. Give an example of a relation R on A that is
    a. Transitive and reflexive, but not antisymmetric
        i. R= {(1,2),(2,3),(1,3),(1,1),(2,2),(3,3),(2,1),(3,2),(3,1)}
    b. Antisymmetric and reflexive but not transitive
        i. R={(1,1),(2,2),(3,3)}
    c. Antisymmetric and transitive but not reflexive
        i. R={(1,2),(2,3),(1,3),(3,2),(2,1)}