CSCI 5454 Problem Set 5

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No Collaborators

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1. *Formulate Gru's problem as an integer programming problem and write its linear programming relaxation.*

   Integer Programming Problem

   Assume each drone can communicate with any number of the k stations. The constraint for this problem is that we have to use every drone but we don't have to use every station. For this problem we want to minimize the sum of the cost of activating a station a_j and the cost c_{ij} of each of the drones communicating with that station. In mathematical terms we are trying to

   $$minimize \sum_{j=1}^{k} a_j x_j + \sum_{j=1}^{k} x_j \sum_{i=1}^{n} c_{ij}$$

   subject to the constraints

   (a)
   $$x_j = \begin{cases} 1 & \text{if we activate station } j \\ 0 & \text{if we don't activate station } j \end{cases}$$

   (b) Let $d_{ij} = 1$ if drone i is communicating with station j then the mathematical notation for ensuring all drones are communicating with at least one station is

   $$\forall d \quad \sum_{j=1}^{k} d_{ij} x_j \geq 1$$

   Linear Programming Relaxation

   The linear programming relaxation of this problem is still trying to

   $$minimize \sum_{j=1}^{k} a_j x_j + \sum_{j=1}^{k} x_j \sum_{i=1}^{n} c_{ij}$$

but instead of $x_j \in \mathbb{Z}$ and being only 1 or 0 it can now be any number between 1 and 0. x_j is now $\in \mathbb{R}$. The question now though is how do you round x_j to make sure the constraint

$$\forall d \quad \sum_{j=1}^{k} d_{ij} x_j \geq 1$$

is still satisfied. As was discussed in lecture you should use randomized rounding.

2. *Find an $s-t$ flow of amount at least d that minimizes the total cost, or report that such a flow does not exist.*

   (a) *Show how to solve this problem using linear programming.*

   In this problem we want to send the target amount of flow d from s to t. At the same time we want to minimize the total cost of the units of flow f_e for all $e \in E$. Written mathematically this is

$$\textit{minimize} \sum_{(u,v)\in E} p_{u,v} f_{u,v}$$

   subject to the contraints

   i.
$$f_{u,v} \leq c_{u,v}$$

   The flow of an edge is less than or equal to the capacity of the edge.

   ii.
$$\sum_{v\in V} f_{s,v} = \sum_{v\in V} f_{v,t} = d$$

   The sum of the flow from the source vertex s to other vertices must equal the target flow d and the sum of the flow from other vertices to the sink vertex t must equal the target flow d.

   iii.
$$\sum_{(w,u)\in E} f_{w,u} = \sum_{(u,z)\in E} f_{u,z}$$

   The amount of flow entering u equals the amount of flow leaving u. Flow is conserved.

   If these constraints are satisfied by a graph with a particular flow then an $s-t$ flow of amount at least d that minimizes the total cost exists, otherwise no such flow exists.

   (b) *Show that the $s-t$ shortest path problem and the classic network flow problem are both special cases of this problem.*