

-
1. *Formulate Gru's problem as an integer programming problem and write its linear programming relaxation.*

Integer Programming Problem

Assume each drone can communicate with any number of the k stations. The constraint for this problem is that we have to use every drone but we don't have to use every station. For this problem we want to minimize the sum of the cost of activating a station a_j and the cost c_{ij} of each of the drones communicating with that station. In mathematical terms we are trying to

$$\text{minimize } \sum_{j=1}^k a_j x_j + \sum_{j=1}^k x_j \sum_{i=1}^n c_{ij}$$

subject to the constraints

(a)

$$x_j = \begin{cases} 1 & \text{if we activate station } j \\ 0 & \text{if we don't activate station } j \end{cases}$$

- (b) Let $d_{ij} = 1$ if drone i is communicating with station j then the mathematical notation for ensuring all drones are communicating with at least one station is

$$\forall d \quad \sum_{j=1}^k d_{ij} x_j \geq 1$$

Linear Programming Relaxation

The linear programming relaxation of this problem is still trying to

$$\text{minimize } \sum_{j=1}^k a_j x_j + \sum_{j=1}^k x_j \sum_{i=1}^n c_{ij}$$

but instead of $x_j \in \mathbb{Z}$ and being only 1 or 0 it can now be any number between 1 and 0. x_j is now $\in \mathbb{R}$. The problem now though how do you round x_j to make sure the constraint

$$\forall d \quad \sum_{j=1}^k d_{ij} x_j \geq 1$$

is still satisfied. As was discussed in lecture you should use randomized rounding.

2.