

CSCI 5654-Fall16: Assignment #5 (Reading: Slides on ILP, Vanderbei Chapter 23).

Due Date: Friday, Nov. 4, 2016 (before class)

In-class: Assignment should be submitted on paper – no emails.

Distance Students: Assignment may be submitted on paper or by email.

Your Name: _____

P1. (10 points) Solve each of the problems below using a branch-and-bound method. Write down the solution obtained and the enumeration tree obtained. You may use any LP solver of your choice to solve the subproblems. Please do not use a ILP solver directly.

(A)

$$\begin{array}{llllll}
 \max & 2x_1 & -3x_2 & -2x_3 & -x_4 & \\
 \text{s.t.} & x_1 & -x_2 & +x_3 & & \leq 5 \\
 & x_1 & -2x_2 & -x_3 & +x_4 & \leq 3 \\
 & x_1 & -x_2 & -x_3 & -x_4 & \leq -1 \\
 & x_1, & x_2, & x_3, & x_4 & \in [-5, 5] \\
 & x_1, & x_2, & x_3, & x_4 & \in \mathbb{Z}
 \end{array}$$

Hint: Use x_4 as the variable to branch on.

Branches	Solution	Optimal value
	$x = [-5 \quad -5 \quad 1.5 \quad -.5]$	2.5
$x_4 \leq -1$	$x = [-5 \quad -5 \quad 2 \quad -1]$	2
$x_4 \geq 0$	$x = [-2 \quad -4 \quad 0 \quad 0]$	2

The optimal value of the objective function is 2. One solution leads to this value:

$$x_1 = 1 \quad x_2 = 1 \quad x_3 = 1 \quad x_4 = 1$$

(B)

$$\begin{array}{llllll}
 \max & 2x_1 & & +3x_3 & +x_4 & \\
 \text{s.t.} & x_1 & -x_2 & & +x_4 & \leq 1 \\
 & & 2x_2 & & -x_4 & \leq 2 \\
 & x_1 & & -x_3 & -2x_4 & \leq -1 \\
 & -x_1 & & & +x_4 & \leq 1 \\
 & x_1 & & & & \in \{-2, -1, 0, 1, 2\} \\
 & & x_2 & & & \in \{-1, 0, 1\} \\
 & & & x_3 & & \in \{0, 1\} \\
 & & & & x_4 & \in \{-1, 0, 1\}
 \end{array}$$

Branches	Solution	Optimal value
	$x = [1.3333 \quad 1 \quad 1 \quad .6667]$	6.3333
$x_4 \geq 1$	$x = [1 \quad 1 \quad 1 \quad 1]$	6
$x_4 \leq 0$	$x = [0 \quad 1 \quad 1 \quad 0]$	3
$x_1 \leq 1$	$x = [1 \quad 1 \quad 1 \quad 1]$	6
$x_1 \geq 2$	Infeasible	

The optimal value of the objective function is 6. One solution leads to this value:

$$x_1 = 1 \quad x_2 = 1 \quad x_3 = 1 \quad x_4 = 1$$

P2. (10 points) Consider the final dictionaries for the LP relaxation of a few ILPs. Assuming all variables are integers, write down all the cutting planes:

Dictionary # 1:

x_1	0.666666666667	$-0.666667x_5 + 0.333333x_4$
x_2	1	$-1x_5$
x_3	2	$+4x_5 \quad -1x_4$
z	1	$-1x_5$

Dictionary # 2:

x_4	4.33333333333	$+0.333333x_8 + 0.666667x_9 - 0.333333x_3$
x_5	8.66666666667	$-0.333333x_8 + 0.333333x_9 - 2.666667x_3$
x_6	10	$-1x_3$
x_7	3	$-3x_8 \quad +1x_9 \quad -18x_3$
x_1	5.66666666667	$-0.333333x_8 - 0.666667x_9 + 0.333333x_3$
x_2	1.33333333333	$+0.333333x_8 - 0.333333x_9 + 2.666667x_3$
z	7	$-1x_9 \quad -2x_3$

P3 (20 points). Consider the graph below which shows various locations and the driving times between them in hours:

Our goal is to decide whether or not to place a hospital at each node. The following are the cost of building a hospital at various nodes in millions of dollars:

Node	Cost
1	3
2	3
3	1.5
4	1
5	1.2
6	1.3
7	0.9
8	0.8

The following constraints should apply to our placement of hospitals: each node should either have a hospital or be within 1 hour driving distance of a hospital.

For your convenience, the table of pairwise shortest path distances is given as an excel spreadsheet.

(A) Let G be a graph with n nodes and $W(i, j)$ denote the shortest path weight between nodes i and j . Finally let \mathbf{c} be a $n \times 1$ vector of node costs. Write down an integer linear programming formulation of the above problem.

(B) Formulate and solve your ILP for the given example.

P4 (15 points) Consider a polyhedron P given by the constraints

$$A\mathbf{x} \leq \mathbf{b}, \ell \leq \mathbf{x} \leq \mathbf{u}.$$

- (a) Write down mixed integer programs that will find the point $\mathbf{x} \in P$ with the largest number of 0 entries in \mathbf{x} ,
- (b) Write down mixed integer programs that will find the point $\mathbf{x} \in P$ with the smallest number of 0 entries in \mathbf{x} .
- (c) Write down a mixed integer program that will search for a solution $\mathbf{x} \in P$ maximizing an objective function $\mathbf{c}^t \mathbf{x}$ such that \mathbf{x} *does not satisfy* $\mathbf{a} \leq \mathbf{x} \leq \mathbf{b}$, for given \mathbf{a}, \mathbf{b} .

P5 (10 points) We are given sets of numbers $\langle S_1, \dots, S_k \rangle$ such that each $S_i \subseteq \{1, \dots, n\}$. For example, $n = 10$ and the sets are

$$S_1 : \{1, 3, 6\}, S_2 : \{2, 7, 8\}, S_3 : \{1, 8, 9\}, S_4 : \{1, 6, 5, 3\}.$$

Our goal is to select a subset $S \subseteq \{1, 2, \dots, n\}$ such that $S \cap S_i \neq \emptyset$ for $i = 1, \dots, k$ and the sum of elements in the chosen set S is minimized.

Formulate a 0–1 ILP for the problem for given $n, k, \langle S_1, \dots, S_k \rangle$. Also, solve it for the example above.