

P1.**(A)**

Let $\max(2x_1 + 3x_2 - 5x_3, x_1, x_2, 2) \leq t$. We can then form the following linear program:

$$\begin{array}{llllll} \min & t & & & & \\ \text{s.t.} & +2x_1 & +3x_2 & -5x_3 & \leq & t \\ & +2x_1 & -x_2 & +x_3 & \leq & t \\ & x_1, & x_2 & & \leq & t \end{array}$$

(B)

Let $t_1, t_2, t_3, t_4 \geq 0$.

Let $|x_1 + x_2| \leq t_1, |x_2 - x_3| \leq t_2, |x_3 - x_1| \leq t_3, |x_1 + x_2 + x_3| \leq t_4$.

You now have the linear problem

$$\begin{array}{llllll} \min & +t_1 & +t_2 & +t_3 & +t_4 & \\ \text{s.t.} & +x_1 & +x_2 & & & \leq t_1 \\ & -x_1 & -x_2 & & & \leq t_1 \\ & & +x_2 & -x_3 & & \leq t_2 \\ & & -x_2 & +x_3 & & \leq t_2 \\ & -x_1 & & +x_3 & & \leq t_3 \\ & +x_1 & & -x_3 & & \leq t_3 \\ & +x_1 & +x_2 & +x_3 & & \leq t_4 \\ & -x_1 & -x_2 & -x_3 & & \leq t_4 \\ & t_1, & t_2 & t_3 & t_4 & \geq 0 \end{array}$$

(C)

Let $\max(|x_1|, |x_2|, |x_3|, |x_1 + x_2|) \leq t$.

You now have the linear problem

$$\begin{array}{llll} \min & t & & \\ \text{s.t.} & +x_1 & -x_2 & \leq 5 \\ & & +x_2 & \leq 3 \\ & +x_1 & & \leq t \\ & -x_1 & & \leq t \\ & & +x_2 & \leq t \\ & & -x_2 & \leq t \\ & & +x_3 & \leq t \\ & & -x_3 & \leq t \\ & +x_1 & +x_2 & \leq t \\ & -x_1 & -x_2 & \leq t \\ & t & & \geq 0 \end{array}$$

P2.

```

1
2 options = optimoptions('linprog', 'Algorithm', 'dual-simplex');
3
4 % Read in the csv file skipping the first row
5 M = csvread('insulinGlucose1.csv', 1);
6
7 % Insulin input values
8 u = M(:,1);
9 % Glucose levels
10 G = M(:,2);
11
12 A = [];
13 b = [];
14
15 [m,~] = size(G(12:end));
16 for i = 1:m
17     b = [b, G(i)];
18     b = [b, -G(i)];
19 end
20
21 b = b';
22
23 % Initialize the objective function which is the minimum of the sum
24 % of the absolute values of the residuals.
25 % Residuals have the form y - Ax.
26 f = ones(1,17);
27
28 % Create the constraint matrix A which is the residuals
29 % of glucose levels 12 -> 707.
30 % If u = |x| then u = x and u = -x
31 [m,~] = size(G);
32 for t = 11:(m-1)
33     a = [G(t) G(t-1) G(t-2) G(t-3) G(t-4) G(t-5) G(t-6) G(t-7) G(t-8) G(t-9) G(t-10)];
34     a = horzcat(a, [u(t) u(t-1) u(t-2) u(t-3) u(t-4) u(t-5)]);
35     % Append the constraint ax.
36     A = [A;a];
37
38     % Append the constraint -ax.
39     a = times(a, -1);
40     A = [A;a];
41 end
42
43 % Solve for the coefficients.
44 % x will have an array of the coefficient values when it is solved.
45 x = linprog(f,A,b);
46
47 % Recreate the matrix A without the additional absolute value constraints.
48 A = [];
49 for t = 11:(m-1)
50     a = [G(t) G(t-1) G(t-2) G(t-3) G(t-4) G(t-5) G(t-6) G(t-7) G(t-8) G(t-9) G(t-10)];
51     a = horzcat(a, [u(t) u(t-1) u(t-2) u(t-3) u(t-4) u(t-5)]);
52     A = [A;a];
53 end
54
55 % Create an array of the history steps.
56 t = [];
57 for i = 12:m
58     t = [t, i];
59 end
60
61 % Plot the true glucose levels at each history step t.i.
62 figure(1);
63 scatter(t,G(12:end));
64 hold on;
65
66 % Plot the best fit line of the predicted glucose levels at each
67 % history step t.i.
68 % x stores the estimated coefficients of the best fit line.
69 % |x| = sqrt(x^2)
70 A = (A*x).^2;
71 A = sqrt(A);
72 plot(t, A);
73 grid on;
74 legend('True glucose levels', 'Best fit line of predicted glucose levels');
75 title('True glucose levels and best fit line of predicted glucose levels vs history step');
76 xlabel('History step');
77 ylabel('Glucose level');
78 saveas(1, 'predictions.png');
79
80 % Plot the residuals.
81 figure(2);
82 residuals = [];
83 trueGlucoseValues = G(12:end);
84 predictedGlucoseValues = A;
85 [m,~] = size(predictedGlucoseValues);
86 for i = 1:m
87     residuals = [residuals, trueGlucoseValues(i) - predictedGlucoseValues(i)];
88 end
89
90 subplot(2,1,1);
91 scatter(t, residuals);
92 hold on;
93 plot(t, zeros(m));
94 grid on;
95 title('Residuals vs history step');
96 xlabel('History step');
97 ylabel('Residual value');
98
99 subplot(2,1,2);

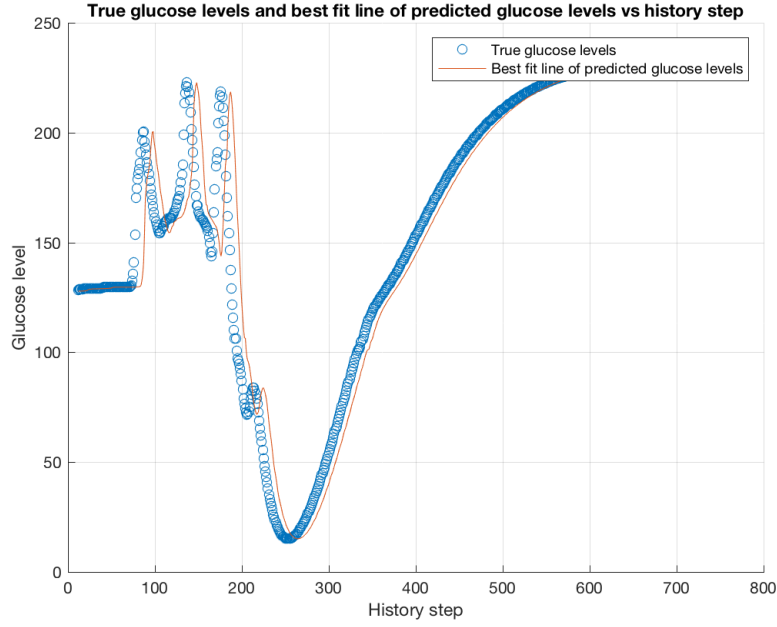
```

```

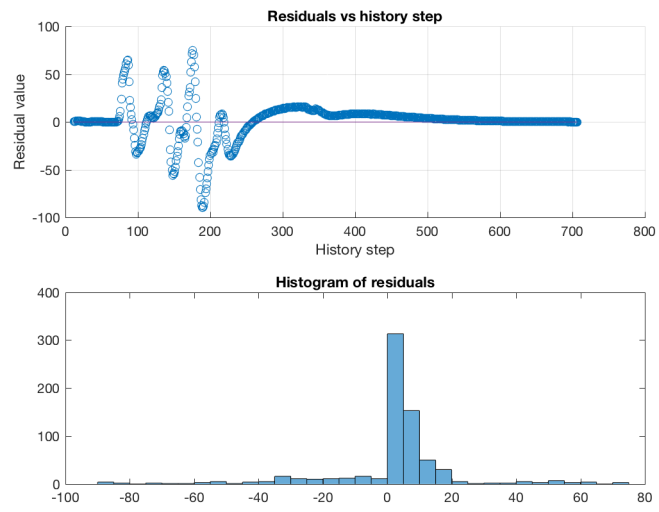
100 histogram(residuals);
101 title('Histogram of residuals');
102 saveas(2, 'residuals.png');

```

Coefficient	Value
a_0	8.704148513061227e-14
a_1	-3.375077994860476e-14
a_2	-1.474376176702208e-13
a_3	2.131628207280301e-13
a_4	-1.243449787580175e-13
a_5	-1.056932319443149e-13
a_6	1.376676550535194e-13
a_7	-3.979039320256561e-13
a_8	3.179678742526448e-13
a_9	6.430411758628907e-13
a_{10}	0.999999999999309
b_0	1.989519660128281e-13
b_1	-2.060573933704291e-13
b_2	1.847411112976261e-13
b_3	-8.526512829121202e-14
b_4	-1.207922650792170e-13
b_5	7.815970093361102e-14



(a) Linear Regression Predictions



(b) Residuals

P3.

(A)

1. $\{x_1, x_2, x_3, w_1, w_2, w_3\}$.

x_1	+2	$-x_4$	$+x_5$	$-x_6$			$+w_6$
x_2	-8		$+2x_5$	$+x_6$	$+w_4$	$+w_5$	$+w_6$
x_3	+4	$+x_4$	$-2x_5$			$-w_5$	$-w_6$
w_1	-7	$+x_4$	$+x_5$	$+3x_6$	$+w_4$	$+w_5$	
w_2	-3	$+2x_4$		$+x_6$			$-w_6$
w_3	2	$+x_4$	$-2x_5$	$+x_6$		$-w_5$	$-w_6$
ζ	16		$-6x_5$		$-3w_4$	$-2w_5$	$-5w_6$

2. $\{x_1, x_2, x_5, w_3, w_5, w_6\}$.

x_1	-1	$-x_3$	$+x_4$	$+2x_6$	$-w_1$		$+w_4$
x_2	-4	$-x_3$	$+x_4$	$+x_6$			$+w_4$
x_5	0	$-x_3$		$+2x_6$	$-w_1$	$+w_2$	$+w_4$
w_3	-2	$+x_3$		$+x_6$			
w_5	7	$+x_3$	$-x_4$	$-5x_6$	$+2w_1$	$-w_2$	$-2w_4$
w_6	-3		$2x_4$	$+x_6$		$-w_2$	
ζ	14	$+4x_3$	$-6x_4$	$-6x_6$	$+2w_1$		$-5w_4$

3. $\{x_1, x_2, x_6, w_4, w_5, w_6\}$.

x_1	-1		$+x_4$	$+x_5$		$-w_2$	
x_2	-6	$+x_3$	$+x_4$	$+x_5$	$+w_1$	$-w_2$	$-w_3$
x_6	2	$-x_3$					$+w_3$
w_4	-4	$+3x_3$		$+x_5$	$+w_1$	$-w_2$	$-2w_3$
w_5	5		$-x_4$	$-2x_5$		$+w_2$	$-w_3$
w_6	-1	$-x_3$	$+2x_4$			$-w_2$	$+w_3$
ζ	22	$-5x_3$	$-6x_4$	$-5x_5$	$-3w_1$	$+5w_2$	$+4w_3$

(B) Perform one step of the revised simplex method for the dictionary with the basis:

$$\{x_3, x_4, x_5, w_1, w_2, w_6\}$$

1. Compute the constant column and objective rows of this dictionary.

x_3	2			$-x_6$	$+w_3$		
x_4	6		$+x_2$	$-2x_6$	$+w_3$	$-w_4$	
x_5	4	$-x_1$			$-w_3$		$-w_5$
w_1	3	$-x_1$	$+x_2$	$+x_6$			
w_2	9	$-2x_1$	$+x_2$	$-2x_6$		$-w_4$	$-w_5$
w_6	0	$+2x_1$	$+x_2$	$-x_6$	$+2w_3$	$-w_4$	$+w_5$
ζ	-8	$-2x_1$	$-4x_2$	$+4x_6$	$-2w_3$	$+w_4$	

2. Choose the entering variable with the largest coefficient.

x_6 will be the entering variable because it has the largest coefficient.

3. Set up the equations to construct the column for the entering variable.

The equation for the entering variable column a_i is $a_i = -B^{-1}Ne_i$.

4. Choose the leaving variable.

The leaving variable will be w_6 because it constrains the entering variable the most.

5. Write down the basic variables in the next dictionary.

The basic variables will be $x_3, x_4, x_5, w_1, w_2, x_6$.

(C)

x_5	1	$+x_1$			$-x_4$	$+w_2$	
x_6	4		$+x_2$	$+x_3$	$-x_4$		$-w_4$
w_1	7	$-x_1$	$+2x_2$	$+x_3$	$-x_4$		$-w_4$
w_3	2		$+x_2$	$+2x_3$	$-x_4$		$-w_4$
w_5	1	$-2x_1$	$-x_2$	$-2x_3$	$+2x_4$	$-w_2$	$+w_4$
w_6	1		$+x_2$	$+x_3$	$+x_4$	$-w_2$	$-w_4$
ζ	4	$-2x_1$	$-2x_2$		$-2x_4$		$-w_4$

(D)

No solution.

(E)

No solution.

```

1
2 A = [+1 -1 0 0 0 -1 +1 0 0 0 0 0;...
3       +1 0 0 -1 -1 0 0 +1 0 0 0 0;...
4       0 0 -1 0 0 -1 0 0 +1 0 0 0;...
5       0 -1 -1 +1 0 +1 0 0 0 +1 0 0;...
6       +1 0 +1 0 +1 +1 0 0 0 0 +1 0;...
7       -1 0 0 -1 +1 -1 0 0 0 0 0 +1];
8
9 b = [3 -1 -2 +4 +6 -2]';
10
11 c = [-2 -3 -1 -1 0 +1 0 0 0 0 0 0];
12
13 % P3 a1
14 xB = [1 2 3 7 8 9];
15 xN = [4 5 6 10 11 12];
16 B = A(:, xB);
17 N = A(:, xN);
18 Binv = inv(B);
19 % Dictionary constant value column.
20 Binv*b;
21 % Dictionary coefficient value rows.
22 -Binv*N;
23
24 cB = c(:, xB);
25 cN = c(:, xN);
26 % Objective function constant value.
27 cB*(Binv*b);
28 % Object function coefficient values.
29 cN - cB*(Binv*N);
30
31 % P3 a2
32 xB = [5 6 7 8 9 10];
33 xN = [1 2 3 4 11 12];
34
35
36 B = A(:, xB);
37 N = A(:, xN);
38 Binv = inv(B);
39 % Dictionary constant value column.
40 Binv*b;
41 % Dictionary coefficient value rows.
42 -Binv*N;
43
44 cB = c(:, xB);
45 cN = c(:, xN);
46 % Objective function constant value.
47 cB*(Binv*b);
48 % Object function coefficient values.
49 cN - cB*(Binv*N);
50
51 % P3 a3
52 xB = [1 2 6 10 11 12];
53 xN = [3 4 5 7 8 9];
54
55 B = A(:, xB);
56 N = A(:, xN);
57 Binv = inv(B);

```

```

58 % Dictionary constant value column.
59 Binv*b;
60 % Dictionary coefficient value rows.
61 -Binv*N;
62
63 cB = c(:, xB);
64 cN = c(:, xN);
65 % Objective function constant value.
66 cB*(Binv*b);
67 % Object function coefficient values.
68 cN - cB*(Binv*N);
69
70 % p3 b
71 xB = [3 4 5 7 8 12];
72 xN = [1 2 6 9 10 11];
73
74 B = A(:, xB);
75 N = A(:, xN);
76 Binv = inv(B);
77 % Dictionary constant value column.
78 Binv*b;
79 % Dictionary coefficient value rows.
80 -Binv*N;
81
82 cB = c(:, xB);
83 cN = c(:, xN);
84 % Objective function constant value.
85 cB*(Binv*b);
86 % Object function coefficient values.
87 cN - cB*(Binv*N);
88
89 % Choose the third variable as the entering variable.
90 % Get the column of the entering variable.
91 a_i = -Binv*N(:, 3);
92
93 % Determine the leaving variable row index by finding the value that
94 % constraints the entering variable the most.
95 -(Binv*b)./a_i;
96
97 % Get the leaving variable row.
98 a_j = -(Binv*N);
99 a_j = a_j(6,:);
100
101 % P3 c
102 f = [-2 -3 -1 -1 0 1];
103 Y = [1 -1 0 0 0 -1;1 0 0 -1 -1 0;0 0 -1 0 0 -1;0 -1 -1 1 0 1;1 0 1 0 1 1;-1 0 0 -1 1 -1];
104 x = [3 -1 -2 4 6 -2];
105 options = optimoptions('linprog','Algorithm','dual-simplex');
106 [x,fval] = linprog(-f, Y, x, [], [], zeros(size(f)), [], options)
107
108 xB = [5 6 7 9 11 12];
109 xN = [1 2 3 4 8 10];
110
111 B = A(:, xB);
112 N = A(:, xN);
113 Binv = inv(B);
114 % Dictionary constant value column.
115 Binv*b
116 % Dictionary coefficient value rows.
117 -Binv*N
118
119 cB = c(:, xB);
120 cN = c(:, xN);
121 % Objective function constant value.
122 cB*(Binv*b)
123 % Object function coefficient values.
124 cN - cB*(Binv*N)

```