CSCI 5654-Fall16: Assignment #5
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P1.
(A)
Solve the initial linear equation and then branch on any variables that have a fractional value. Add the constrains of the fractional variables, solve the linear equation, and repeat until the solution is integral and not fractional.

Branches	Solution	Optimal value
	$x = [-5 \quad -5 \quad 1.5 \quad -.5]$	2.5
$x_4 \leq -1$	$x = [-5 \quad -5 \quad 2 \quad -1]$	2
$x_4 \geq 0$	$x = [-2 \quad -4 \quad 0 \quad 0]$	2

The optimal value of the objective function is 2. One solution leads to this value is:

$$x_1 = 1 \quad x_2 = 1 \quad x_3 = 1 \quad x_4 = 1$$

(B)

Branches	Solution	Optimal value
	$x = [1.3333 \quad 1 \quad 1 \quad .6667]$	6.3333
$x_4 \geq 1$	$x = [1 \quad 1 \quad 1 \quad 1]$	6
$x_4 \leq 0$	$x = [0 \quad 1 \quad 1 \quad 0]$	3
$x_1 \leq 1$	$x = [1 \quad 1 \quad 1 \quad 1]$	6
$x_1 \geq 2$	Infeasible	

The optimal value of the objective function is 6. One solution leads to this value is:

$$x_1 = 1 \quad x_2 = 1 \quad x_3 = 1 \quad x_4 = 1$$

P2.
Dictionary # 1
First, we choose x_1 because it is a variable with a fractional solution. We rewrite the equation for x_1 as:

$$0.666667x_5 - 0.333333x_4 + x_1 = 0.666666666667$$

Next we rewrite the above equation in terms of an integer part and a fractional part.

$$(0x_5 - x_4 + x_1) + (0.666667x_5 + 0.777777x_4) = 0 + 0.666666666667$$

The fractional part $(0.666667x_5 + 0.777777x_4) \geq 0.666666666667$. The cutting plane is then given by the equation:

$$(0.666667x_5 + 0.777777x_4) + w_6 = 0.666666666667$$

Dictionary # 2
Equations for variables with fractional solutions:

$$\begin{aligned}
-0.333333x_8 - 0.666667x_9 + 0.333333x_3 + x_4 &= 4.33333333333 \\
0.333333x_8 - 0.333333x_9 + 2.666667x_3 + x_5 &= 8.66666666667 \\
0.333333x_8 + 0.666667x_9 - 0.333333x_3 + x_1 &= 5.66666666667 \\
-0.333333x_8 + 0.333333x_9 - 2.666667x_3 + x_2 &= 1.33333333333
\end{aligned}$$

Equations written with integral and fractional parts:

$$\begin{aligned}
(-x_8 - x_9 + 0x_3 + x_4) + (0.777777x_8 + 0.444443x_9 + 0.333333x_3) &= 4 + .33333333333 \\
(0x_8 - x_9 + 2x_3 + x_5) + (0.333333x_8 + 0.777777x_9 + 666667x_3) &= 8 + .66666666667 \\
(0x_8 + 0x_9 - x_3 + x_1) + (0.333333x_8 + 0.666667x_9 + 0.777777x_3) &= 5 + .66666666667 \\
(-x_8 + 0x_9 - 3x_3 + x_2) + (0.777777x_8 + 0.333333x_9 + 0.444443x_3) &= 1 + .33333333333
\end{aligned}$$

Cutting planes for the above equations:

$$\begin{aligned}
(0.777777x_8 + 0.444443x_9 + 0.333333x_3) + w_{10} &= .33333333333 \\
(0.333333x_8 + 0.777777x_9 + 666667x_3) + w_{11} &= .66666666667 \\
(0.333333x_8 + 0.666667x_9 + 0.777777x_3) + w_{12} &= .66666666667 \\
(0.777777x_8 + 0.333333x_9 + 0.444443x_3) + w_{13} &= .33333333333
\end{aligned}$$

P3 (20 points). Consider the graph below which shows various locations and the driving times between them in hours:

Our goal is to decide whether or not to place a hospital at each node. The following are the cost of building a hospital at various nodes in millions of dollars:

Node	Cost
1	3
2	3
3	1.5
4	1
5	1.2
6	1.3
7	0.9
8	0.8

The following constraints should apply to our placement of hospitals: each node should either have a hospital or be within 1 hour driving distance of a hospital.

For your convenience, the table of pairwise shortest path distances is given as an excel spreadsheet.

(A) Let G be a graph with n nodes and $W(i, j)$ denote the shortest path weight between nodes i and j. Finally let $\mathbf{c}$ be a $n \times 1$ vector of node costs. Write down an integer linear programming formulation of the above problem.

(B) Formulate and solve your ILP for the given example.

P4 (15 points) Consider a polyhedron P given by the constraints

$$A\mathbf{x} \leq \mathbf{b},\ \ell \ \leq \ \mathbf{x} \ \leq \ \mathbf{u}\,.$$

(a) Write down mixed integer programs that will find the point $\mathbf{x} \in P$ with the largest number of 0 entries in $\mathbf{x}$,

(b) Write down mixed integer programs that will find the point $\mathbf{x} \in P$ with the smallest number of 0 entries in $\mathbf{x}$.

(c) Write down a mixed integer program that will search for a solution $\mathbf{x} \in P$ maximizing an objective function $\mathbf{c}^t\mathbf{x}$ such that $\mathbf{x}$ *does not satisfy* $\mathbf{a} \leq \mathbf{x} \leq \mathbf{b}$, for given $\mathbf{a}, \mathbf{b}$.

P5 (10 points) We are given sets of numbers $\langle S_1, \ldots, S_k \rangle$ such that each $S_i \subseteq \{1, \ldots, n\}$. For example, $n = 10$ and the sets are

$$S_1 : \{1, 3, 6\},\ S_2 :\ \{2, 7, 8\},\ S_3 :\ \{1, 8, 9\},\ S_4 :\ \{1, 6, 5, 3\}\,.$$

Our goal is to select a subset $S \subseteq \{1, 2, \ldots, n\}$ such that $S \cap S_i \neq \emptyset$ for $i = 1, \ldots, k$ and the sum of elements in the chosen set S is minimized.

Formulate a $0-1$ ILP for the problem for given $n, k, \langle S_1, \ldots, S_k \rangle$. Also, solve it for the example above.