TAM 470 / CSE 450

Homework 2

Instructions: For problems that you are instructed to complete **by hand**, show all work and ensure that hand-written work is neat and easy to read; if you use computer code to make any calculations, submit a screenshot of your code. Problems hosted on **PrairieLearn** do not require any written work to be submitted unless the problems instructions indicate otherwise.

Problem 1 (10 points)

Consider the one-sided difference approximation for the **second** derivative f_j'' at a grid point x_j on a uniformly spaced grid with spacing h:

$$f_j'' = \frac{f_j - 2f_{j-1} + f_{j-2}}{h^2} + \tau$$

Determine the expression for the leading error term τ for this scheme, and state the order of the scheme.

Problem 2 (10 points)

Find the most accurate formula for the first derivative at x_i utilizing known values of f at x_{i-1}, x_i, x_{i+1}, and x_{i+2}. The points are uniformly spaced with spacing h. Find the expression for the leading error term and state the order of the method.

You can use a symbolic solver (e.g. `sympy` in Python, or an online tool) if you wish. If you do, include screenshots of the code/tool used to produce the solution.

Problem 3 (10 points)

Please go to the Homework 2 set on PrairieLearn to complete this question:

Write a function that implements a fourth-order Padé scheme with a third-order scheme for boundary nodes (i.e. the matrix equation (2.18) from the Moin text) to numerically calculate the derivative of a function $f(x)$ on a grid of equally-spaced points.

Problem 4 (10 points)

(**a**) Use your function from Problem 3 to compute the Padé approximated derivative of $f(x) = \cos(2x^2)$ on the interval $[0, \pi]$ for $N = 6, 11$, and 21 grid points on this interval. Submit 3 plots, each containing the exact derivative (smoothly plotted, using approx 100 grid points on the interval $[0, \pi]$) and the Padeé derivative approximations at the N uniformly distributed gird points on the interval $[0, \pi]$.

(**b**) Generate a log-log plot of truncation error vs grid spacing h for the points $x = 0$ and $x = \frac{\pi}{2}$, using the same function and interval as discussed in part (a). Be sure compute the truncation error for enough grid spacings h to produce the expected linear relationship on a log-log plot. Estimate the slopes of the log-log plots by using `numpy.polyfit` or similar (see Jupyter notebook example from class) and comment on whether the error behavior in the log-log plots matches your expectations for this Padé scheme at both interior and boundary grid points.

Problem 5 (10 points): <u>4 credit-hour students only</u>

Solve Moin textbook Exercise 4 from Chapter 2. Complete both parts (a) and (b) (5 pts each).

You can use a symbolic solver (e.g. `sympy` in Python, or an online tool) if you wish. If you do, include screenshots of the code/tool used to produce the solution.