Two approaches to reduce the rank reversal occurrences in the PROMETHEE II method

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1. Introduction

In a world as wide and complex as ours, one is often confronted to intricate and complex decision problems. These problems often consist of different conflicting aspects which should simultaneously be optimised. Unfortunately, there is generally no optimal solution which optimises all these aspects and choosing a best solution therefore consist in making the best trade-off. Finding these trade-offs can be a fastidious task and making a non optimal choice could lead to dramatic consequences. Therefore, the entity in charge of the decision (called the decision maker) could want to use some multi-criteria decision aid methods. These methods are aimed at giving the decision maker some insights about the problem by elaborating a mathematical model and quantifying the different solutions.

2. PROMETHEE II and rank reversals

One of these multi-criteria decision aid methods is PROMETHEE II (Preference Ranking Organisation METHod for Enrichment Evaluations) which has originally been proposed by J.P. Brans in 1982 [3].

PROMETHEE II is a ranking method based on pairwise comparisons [8]. This means that it will first build an $n \times n$ pairwise preference matrix Π which elements π_{ij} represent the pairwise preference of the ith alternative over the jth. Then, this matrix will be used to build a complete ranking of all the alternatives.

Such methods have often been criticised for being susceptible to the rank reversal phenomenon. This phenomenon is non uniquely defined but its main idea is that the relative ordering between two alternatives is influenced by the presence or absence of a third one [4]. W. De Keyser and P. Peeters [7] were the first to point out that the PROMETHEE methods suffer from rank reversal occurrences.

Indeed, if a decision aid method based on pairwise comparisons build nontransitive pairwise preferences matrices, then there will always be some rank reversals possible.

One example of such a situation is given in Figure 1, where 3 alternatives are considered.

In this illustration we can see that alternative x is pairwise preferred over y, which is preferred over z, which is itself preferred over x. One can easily figure out why it will not been possible to build a complete ranking which is not susceptible to rank reversals. Suppose the method, for instance, makes the following ranking:

$$\Pi = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \end{array}$$

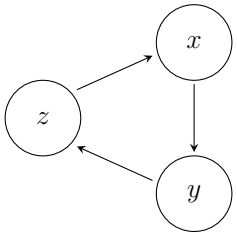

Figure 1. Example of non transitive preferences matrix and its graphical representation

$$x \succ y \succ z \tag{1}$$

If alternative y is removed, then the method will logically change the ranking to:

$$z \succ x \tag{2}$$

and a rank reversal will have happened.

The possibility of rank reversals to occur with a method can be problematic. Suppose for example that the three alternatives x, y and z were three countries which are the final candidates for the hosting of the 2024 edition of the Olympic Games. The candidate which may host the competition would be the one ranked first by the considered method (suffering from rank reversals). An initial ranking could be the one indicated in equation 1. Imagine now, that a large-scale doping scandal is discovered in the country y, and that y gets eliminated of the competition. A new candidate, t could then be reselected from the already eliminated ones.

The new pairwise preferences could lead to the following matrix:

$$\Pi = \begin{array}{c} \\ x \\ z \\ t \end{array} \begin{array}{c} \begin{array}{ccc} x & z & t \end{array} \\ \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

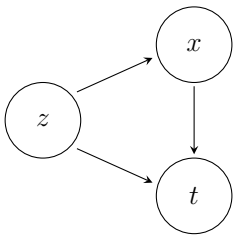

Figure 2. Example of rank reversal between x and z

The ranking method would probably now produce the following ranking:

$$z \succ x \succ t \tag{3}$$

We can conclude from this example that a rank reversal between alternatives x and z occurred due to the removal of alternative y. This has some significant consequences since the country x will not be selected as hosting country for the Olympic Games. It is not easy to find reasons why such rank reversals could occur (why should alternative y influence the relative ordering between x and z). Such situations are undesirable for most of the decision makers. They will therefore try to avoid multi-criteria methods where rank reversals occur.

For these reasons, two variations of the PROMETHEE II methods have been proposed which were aimed at reducing, or suppressing these rank reversals. The main results of the investigation of these two methods is summarised here under.

3. ROBUST PROMETHEE

The ROBUST PROMETHEE method has originally been proposed by De Smet in [9].

It is based on the repetition, R times, of PROMETHEE II on samplings of size m. All the repetitions are used to build a probability matrix P which elements p_{ij} represent the probability that an alternative a_i would be ranked before a_j in one random subset. This probability matrix is then used to build a complete ranking. ROBUST PROMETHEE is therefore also a ranking method based on pairwise comparisons, but it is hoped that the matrix P will be more transitive than Π and that there will therefore be less rank reversals.

Some empirical tests have been performed to verify this assertion on random samples of alternatives taken from the EPI [6], HDI [2] and SHANGHAI [1] data sets. The results of these tests show that ROBUST PROMETHEE indeed seems to reduce the quantity of rank reversals, but only when used with appropriate values of m. Furthermore, these appropriate values of m depend on the set of alternatives on which the method is applied and can vary from 30 to 80% quantity of alternatives of the problem. This could be a serious obstacle in the practical usage of this method.

4. REFERENCED PROMETHEE

REFERENCED PROMETHEE is the second variation of PROMETHEE II which has been analysed. It has initially been proposed by Doan and De Smet in [5].

This method is not a ranking method based on pairwise comparisons, but compares each alternative of the problem to a predefined set of reference profiles. This method therefore does not suffer from rank reversals: the alternatives are ranked according to a score computed with fixed reference profiles, each alternative's score is therefore independent of any other alternative.

Unfortunately, this method comes with the additional cost of having to find a set of reference profiles. It has been seen in the master thesis, that the rankings produced with REFERENCED PROMETHEE were strongly dependent on these sets of profiles. Furthermore, it has been seen that it is generally possible to produce rankings similar to the one produced by PROMETHEE II using only few reference profiles (the rankings produced by PROMETHEE II can be considered as satisfactory).

Two techniques aimed at finding sets of reference profiles were studied in the master thesis, but none of them is providings sets of reference producing "PROMETHEE II-similar" rankings for all decision problems.

5. Conclusion

The problem of the rank reversal phenomenon is a complex problem and the issue of it's legitimacy is still addressed by numerous authors [4]. Nevertheless, two variations of PROMETHEE II aimed at avoiding rank reversal were analysed in the master thesis. Both of them succeed in reducing the rank reversal occurrences. There are also serious obstacles for the practical utilisation of both these methods which consist in the selection of appropriate values for their respective parameters (the size of the samplings and the sets of reference profiles).

References

[1] Academic ranking of world universities. Available: http://www.shanghairanking.com/ARWU2016.html, last consulted: 24-05-2017.

[2] Human development reports. Available: http://hdr.undp.org/, last consulted: 24-05-2017.

[3] J.P. Brans. L'ingénièrie de la décision; Elaboration d'instruments d'aide à la décision. La méthode PROMETHEE. In R. Nadeau and M. Landry, editors, *L'aide à la décision: Nature, Instruments et Perspectives d'Avenir*, pages 183–213, Québec, Canada, 1982. Presses de l'Université Laval.

[4] J.P. Brans and Y. De Smet. *PROMETHEE Methods*, pages 187–219. Springer New York, New York, NY, 2016.

[5] N.A.V. Doan and Y. De Smet. On the use of reference profiles to compute alternative promethee ii rankings: A preliminary study. In *Industrial Engineering and Engineering Management (IEEM), 2016 IEEE International Conference on*, pages 326–330. IEEE, 2016.

[6] A. Hsu and al. 2016 environmental performance index. New Haven, CT: Yale University. Available: www.epi.yale.edu, last consulted: 24-05-2017.

[7] W. De Keyser and P. Peeters. A note on the use of promethee multicriteria methods. *European Journal of Operational Research*, 89(3):457–461, 1996.

[8] B. Mareschal, Y. De Smet, and Ph. Nemery. Rank reversal in the promethee ii method: some new results. In *2008 IEEE International Conference on Industrial Engineering and Engineering Management*, pages 959–963. IEEE, 2008.

[9] Y. De Smet. About the computation of robust promethee ii rankings: Empirical evidence. In *Industrial Engineering and Engineering Management (IEEM), 2016 IEEE International Conference on*, pages 1116–1120. IEEE, 2016.