Sports, Smartphones, and Simulation as an Engaging Method to Teach Projectile Motion Incorporating Air Resistance

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Much data, both published and anecdotal, have shown that students grasp scientific concepts more easily when they are directly involved in the learning via lab experiments or other hands-on activities. Hands-on or experiential learning also appears to aid in students' ability to retain scientific theory. One way to engage students in a first-year calculus-based classical physics course is with the use of smartphones to validate or illustrate a variety of concepts covered in the lectures. It is true that almost all incoming students own or have access to a smartphone. This paper presents a method for students to use a smartphone app to track the trajectory of a sports projectile fired in real time (i.e., a basketball, volleyball, shot put, etc.). This experimental result was compared to a theoretical prediction of the trajectory of the projectile using an incremental air resistance algorithm implemented in Microsoft Excel. The students were required to estimate all of the parameters within the drag force equation (i.e., the drag coefficient, density of air, etc.) that made up part of the theoretical prediction. Ultimately, the goal was for the students to compare their experimental result with their prediction and gain an appreciation for the factors affecting the agreement (or lack of) between the two results.

Literature review

Previous authors have described the use of smartphone apps to allow students to corroborate various theoretical principles taught in classical mechanics.[1-11] In many of these studies, the smartphone apps were used to experimentally verify 1D kinematics, circular motion, or projectile motion principles, without necessarily presenting a method for a theoretical prediction of the motion. The articles cited here propose different methods, i.e., some make use of smartphones or cameras to capture the video while others propose the use of accelerometers or other sensors as a data logger. Some researchers specifically studied the trajectory of sports projectiles such as shuttlecocks, rugby balls, baseballs, etc.[12-14] These investigations also presented aerodynamic models used to predict the motion of the projectile, and this result was compared to the experimental one. The predictions accounted for the drag and lift forces acting on the projectile and appropriately attempted to estimate these forces. The importance of accounting for the effect of air resistance on predicting key parameters of interest during projectile motion has also been given consideration.[15] It is expected that accounting for air resistance will result in a more accurate prediction of the actual path of the projectile.

The one drawback to some of these studies was the method used to experimentally track the motion of the projectile. In these studies, multiple digital cameras (in some cases more than four) were required and positioned at strategic locations to record the motion of the projectile. The video camera required calibration by the user, and, once the motion was recorded, further software was typically required to digitize the motion and smooth the resulting data. Although the use of multiple cameras allows for the generation of more precise 3D trajectory data, it is of interest to find simple methods that allow students to use common tools (such as a smartphone) to adequately capture and estimate projectile motion.

This paper aims to present a one-stop method for students to use, i.e., a smartphone app to directly capture and digitize the motion of a projectile. Only one cell phone is required, and calibration of the distances is performed within the app with a known length placed in the path of the trajectory. The resulting x-y trajectory data of the projectile can then be exported to Excel or another similar program for further manipulation and/or graphing. In addition, this paper also uses a simple incremental algorithm to predict the trajectory of the projectile, under the influence of air resistance, which can then be implemented in Microsoft Excel.

Methodology

The students were asked to complete the project in teams of two. There were three main components to the project:

1. Select and learn how to correctly use an appropriate app to track the x-y motion of a projectile. A few suggested apps were Vernier Video Physics and VidAnalysis. Each group was required to launch a sports projectile (i.e., basketball, volleyball, football, shot put, etc.). Ideally, no two groups would launch the same projectile. The expectation was that the drag characteristics on each of these projectiles would differ and students could gain insight into how other groups estimated the drag parameters for their specific projectile.
2. Create an Excel spreadsheet that outputs a prediction of the x-y trajectory of the projectile, based on calculations using an incremental projectile motion with air resistance algorithm.
3. Compare the experimental and predicted results.

The first part of the project involved experimentally tracking the x-y motion of the projectile via the app using the following

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method:

1. One member of the group would launch the projectile while the other member held the smartphone for capturing the video.
2. It is important to ensure that the camera was held fixed and the projectile remains approximately perpendicular to the viewing direction. It is also important that the entire trajectory remain within the camera window of the smartphone.
3. An object with known dimension, such as a meterstick, was placed in the same plane as the motion. This known dimension was used to scale the video.
4. The origin of the coordinate system was set at the point when the ball completely left the student's hands.
5. The app was used to collect the x-y position of the ball at each point of its trajectory.
6. The x-y position data file could then be opened in Excel for further analysis/graphing, etc.

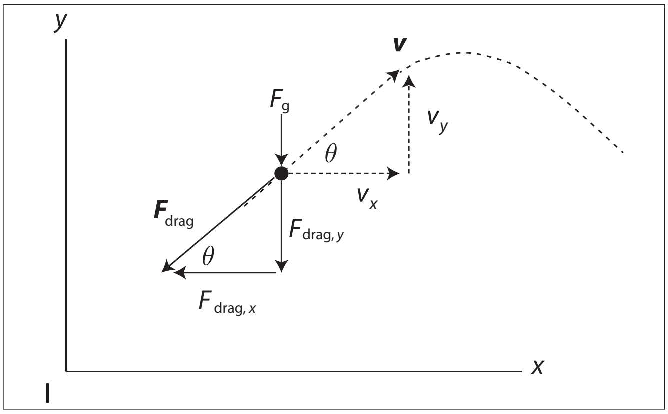

Fig. 1. Forces acting on a projectile under the influence of air resistance.

The second part of the project involved predicting the x-y trajectory of the projectile, incorporating the effect of air resistance on the motion. If air resistance is neglected, only the force of gravity is acting on the projectile, such that the acceleration along the x-direction is zero and along the y-direction is taken to be equal to $-9.8\ \mathrm{m/s^2}$.

If air resistance is considered, there are two forces acting on the projectile at any instant—the downward force due to gravity (F_g) and the drag force (F_{drag}) that is always opposed to velocity as shown in Fig. 1. Recall that the velocity vector is tangent to the projectile's path. The drag force is a variable force as it is dependent on the projectile's velocity at any instant. Consequently, the acceleration along the x- and y-directions will be variable.

Below are the equations for two-dimensional motion with constant acceleration, applied to a small increment of time Δt. If the time interval is small enough, the acceleration can be treated as a constant for this small time interval. Note that x, v, and a are position, velocity, and acceleration, respectively.

$$x_{i+1} = x_i + v_{x,i}\Delta t + \frac{1}{2}a_{x,i}\Delta t^2 \tag{1}$$

$$v_{x,i+1} = v_{x,i} + a_{x,i}\Delta t \tag{2}$$

$$y_{i+1} = y_i + v_{y,i}\Delta t + \frac{1}{2}a_{y,i}\Delta t^2 \tag{3}$$

$$v_{y,i+1} = v_{y,i} + a_{y,i}\Delta t. \tag{4}$$

If the time interval is small (1% of the total flight time), Δt^2 is smaller, so the Δt^2 terms in Eqs. (1) and (3) may be neglected and the constant acceleration equations become

$$x_{i+1} = x_i + v_{x,i}\Delta t \tag{5}$$

$$v_{x,i+1} = v_{x,i} + a_{x,i}\Delta t \tag{6}$$

$$y_{i+1} = y_i + v_{y,i}\Delta t \tag{7}$$

$$v_{y,i+1} = v_{y,i} + a_{y,i}\Delta t. \tag{8}$$

The above equations give the kinematics of the projectile's motion. It is also necessary to incorporate the dynamics, in order to relate the acceleration of the projectile to the forces acting on it.

The drag force acts on the object while it is in the air. The drag force is proportional to the speed squared, such that

$$F_{drag} = Bv^2\,. \tag{9}$$

The proportionality constant (B) depends on the drag coefficient (C), the density of air (ρ), and the cross-sectional area of the projectile (A) such that

$$B = \frac{1}{2}C\rho A. \tag{10}$$

The x- and y-components of the drag force (see Fig. 1) are given, respectively, by

$$F_{drag,x} = -Bv^2\cos\theta = -Bv^2\frac{v_x}{v} = -Bvv_x \tag{11}$$

$$F_{drag,y} = -Bv^2\sin\theta = -Bv^2\frac{v_y}{v} = -Bvv_y. \tag{12}$$

The x- and y-components of the gravitational force are given, respectively, by

$$F_{g,x} = 0 \tag{13}$$

$$F_{g,y} = -mg\,. \tag{14}$$

Note that m in Eq. (14) refers to the mass of the projectile.

The x- and y-components of the acceleration using Newton's second law are given, respectively, by:

$$a_x = \frac{F_{net,x}}{m} = \frac{F_{drag,x} + F_{g,x}}{m} = \frac{-Bvv_x}{m} \tag{15}$$

$$a_y = \frac{F_{net,y}}{m} = \frac{F_{drag,y} + F_{g,y}}{m} = \frac{-Bvv_y - mg}{m}. \tag{16}$$

Substitution of Eq. (15) into Eq. (6) and Eq. (16) into

Eq. (8) gives the equations for predicting the position and velocity of the projectile at any point in its trajectory via

$$x_{i+1} = x_i + v_{x,i}\Delta t \tag{17}$$

$$y_{i+1} = y_i + v_{y,i}\Delta t \tag{18}$$

$$v_{x,i+1} = v_{x,i} + \frac{-Bv_i v_{x,i}}{m}\Delta t \tag{19}$$

$$v_{y,i+1} = v_{y,i} + \frac{-Bv_i v_{y,i} - mg}{m}\Delta t. \tag{20}$$

The only inputs into the prediction equations given by Eqs. (17) – (20) are C, ρ, A, Δt, as well as the x- and y-components of the initial velocity. Note that the x- and y-components of the initial velocity were taken directly from the velocity vs. time graph output by the smartphone app. Students were required to set up an Excel spreadsheet, which automatically calculated the x- and y-position of the projectile based on the given inputs. This result served as their prediction, which was then compared to the experimental result obtained from the smartphone app.

Results

The results from one group's project will be presented in this section. One member of the group shot a basketball (see Fig. 2), and the other member used the Vernier Video Physics app to capture the motion. In terms of the prediction, the students were required to make reasonable estimates of the drag coefficient for the projectile as well as the density of air based on a search of the literature. Students made direct measurements of quantities where possible, i.e., the mass of the projectile and its cross-sectional area. The following were the constants used by the group:

Mass = m = 0.624 kg

Drag coefficient = C = 0.47

Density of air = ρ = 1.225 kg/m^3

Area of projectile = A = 0.04622 m^2

Figure 3 shows a comparison of the experimental result with the predicted result with and without incorporating the effects of air resistance. For the "Predicted without air resistance" result, the drag force term went to zero. The results from Fig. 3 show that the prediction incorporating the effects of air resistance appears to have better agreement with the experimental result compared with the prediction not incorporating the effects of air resistance. This is particularly true on the downward trajectory of the projectile, where the velocity is increasing and hence the drag forces are increasing. It should still be noted that reasons for the slight disagreement between the prediction incorporating air resistance and the experimental result could be due to, but not limited to, the value used for the drag coefficient. Also it is assumed that the drag force is proportional to the square of the velocity, as opposed to assuming it is proportional to a more general v^n. In addition, the accumulation of error in the numerical algorithm could also have contributed to the disagreement between the two results on the downward trajectory. Students

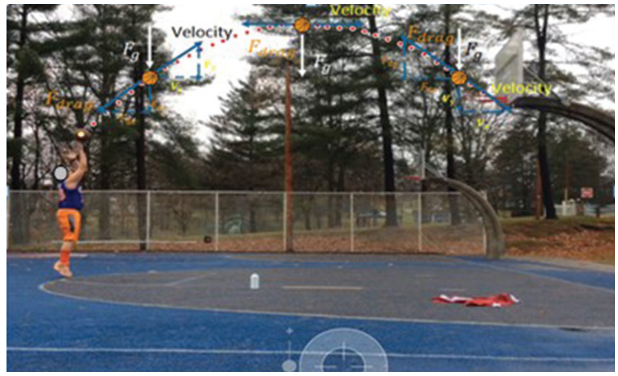

Fig. 2. Basketball shot for projectile motion analysis.

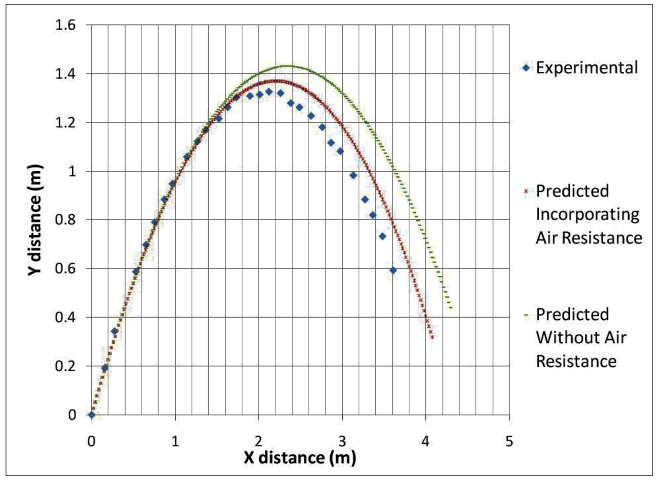

Fig. 3. Predicted x- and y-position of basketball with and without incorporating the effects of air resistance.

were required to note these factors when discussing the lack of agreement between the experimental result and the prediction incorporating air resistance.

Conclusions

This paper presents a method for incorporating smartphones and numerical simulations into a first-year classical physics course. The method allows students to take an active role in every stage of the learning process, from experimental projectile motion characterization to prediction of the motion incorporating the effects of air resistance. The intent was that the exercise would increase students' awareness of the power of predictive models. These models, when correctly validated with experimental data, can be used to predict behavior under various alternative conditions, without the need for time-consuming experimental trials. A comparison of the experimental result with the predicted result indicates that incorporating the effects of air resistance is important and necessary, specifically during the downward trajectory of the projectile. The agreement between the experimental result and the predicted result (both with and without the incorporation of air resistance) appears to be good during the upward trajectory of the projectile. The students gained an appreciation for the importance of correctly quantifying the constants in the drag force equation in order to obtain a good prediction of the trajectory and overall horizontal distance traveled. They also gained a better understanding of the factors that affected the disagreement between the two results.

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