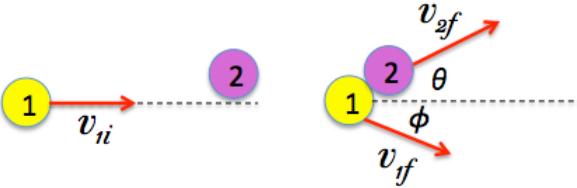


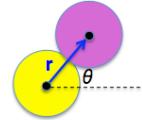
Modeling an elastic collision:

The following must be known: $m_1, v_{1i}, m_2, v_{2i}, \theta$



We can estimate θ by doing $\theta = \arctan(r_y/r_x)$,

where r is the vector connecting the center of the spheres.



Unknowns (3): v_{1f}, v_{2f}, ϕ

3 equations needed to solve the problem

$$\text{conservation of momentum x-direction: } m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos \phi + m_2 v_{2f} \cos \theta \quad (\text{A})$$

$$\text{conservation of momentum y-direction: } 0 = m_1 v_{1f} \sin \phi + m_2 v_{2f} \sin \theta \quad (\text{B})$$

$$\text{conservation of kinetic energy: } m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \quad (\text{C})$$

$$\frac{B}{A}: \quad \frac{m_1 v_{1f} \sin \phi}{m_1 v_{1f} \cos \phi} = \frac{-m_2 v_{2f} \sin \theta}{m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f} \cos \theta}$$

$$\tan \phi = \frac{-m_2 v_{2f} \sin \theta}{m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f} \cos \theta}$$

$$A^2: \quad m_1^2 v_{1f}^2 \cos^2 \phi = [(m_1 v_{1i} + m_2 v_{2i}) - m_2 v_{2f} \cos \theta]^2$$

$$B^2: \quad m_1^2 v_{1f}^2 \sin^2 \phi = m_2^2 v_{2f}^2 \sin^2 \theta$$

$$A^2 + B^2: \quad m_1^2 v_{1f}^2 (\sin^2 \phi + \cos^2 \phi) = m_2^2 v_{2f}^2 \sin^2 \theta + [(m_1 v_{1i} + m_2 v_{2i}) - m_2 v_{2f} \cos \theta]^2$$

$$m_1^2 v_{1f}^2 = m_2^2 v_{2f}^2 \sin^2 \theta + m_2^2 v_{2f}^2 \cos^2 \theta - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + (m_1 v_{1i} + m_2 v_{2i})^2$$

$$m_1^2 v_{1f}^2 = m_2^2 v_{2f}^2 (\sin^2 \theta + \cos^2 \theta) - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + (m_1 v_{1i} + m_2 v_{2i})^2$$

$$m_1^2 v_{1f}^2 = m_2^2 v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + (m_1 v_{1i} + m_2 v_{2i})^2 \quad (\text{D})$$

$$C: \quad m_1 v_{1f}^2 = m_1 v_{1i}^2 + m_2 v_{2i}^2 - m_2 v_{2f}^2$$

$$\begin{aligned} D: \quad \frac{m_1^2 v_{1f}^2}{m_1 v_{1f}^2} &= \frac{m_2^2 v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + (m_1 v_{1i} + m_2 v_{2i})^2}{m_1 v_{1i}^2 + m_2 v_{2i}^2 - m_2 v_{2f}^2} \\ m_1 &= \frac{m_2^2 v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + (m_1 v_{1i} + m_2 v_{2i})^2}{m_1 v_{1i}^2 + m_2 v_{2i}^2 - m_2 v_{2f}^2} \end{aligned}$$

$$m_1 (m_1 v_{1i}^2 + m_2 v_{2i}^2 - m_2 v_{2f}^2) = m_2^2 v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + (m_1 v_{1i} + m_2 v_{2i})^2$$

$$m_1^2 v_{1i}^2 + m_1 m_2 v_{2i}^2 - m_1 m_2 v_{2f}^2 = m_2^2 v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + m_1^2 v_{1i}^2 + m_2^2 v_{2i}^2 + 2m_1 m_2 v_{1i} v_{2i}$$

$$m_1 m_2 v_{2i}^2 - m_1 m_2 v_{2f}^2 = m_2^2 v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 v_{2f} \cos \theta + m_2^2 v_{2i}^2 + 2m_1 m_2 v_{1i} v_{2i}$$

$$(m_2^2 + m_1 m_2) v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 \cos \theta v_{2f} + m_2^2 v_{2i}^2 + 2m_1 m_2 v_{1i} v_{2i} + m_1 m_2 v_{2i}^2 = 0$$

$$(m_2^2 + m_1 m_2) v_{2f}^2 - 2(m_1 v_{1i} + m_2 v_{2i}) m_2 \cos \theta v_{2f} + m_2^2 v_{2i}^2 + 2m_1 m_2 v_{1i} v_{2i} + m_1 m_2 v_{2i}^2 = 0$$

a
b
c

$$v_{2f} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

From the \pm , start by choosing the + answer, and see if you get reasonable values.

Finally, to get v_{1f} , go back to equation (B):

$$v_{1f} = \frac{-m_2 v_{2f} \sin \theta}{m_1 \sin \phi}$$

The equations in yellow are essentially what you need to insert in the code. Insert them in steps, in whatever organized way that you prefer. You probably want to calculate θ first, then v_{2f} , and ϕ and v_{1f} .