Thermodynamics of the climate system

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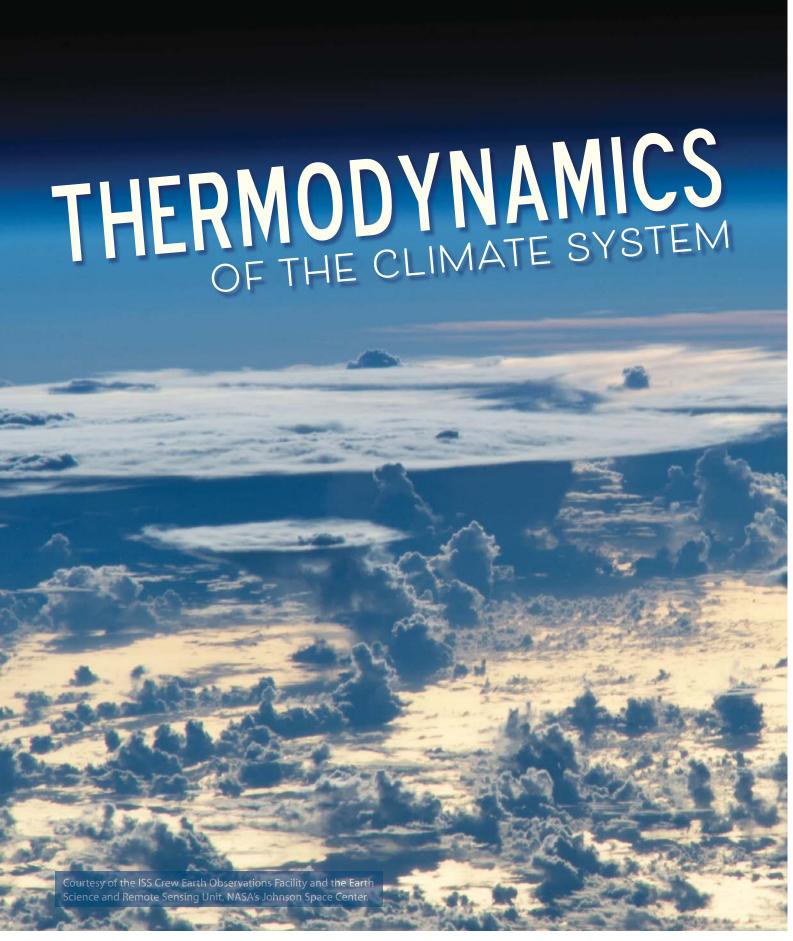
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Martin S. Singh and Morgan E O'Neill

To understand Earth's climate, think of it as a giant, planetary-scale heat engine that drives the circulation of the oceans and atmosphere.

hroughout its history, Earth has experienced vastly different climates, including "snowball Earth" episodes, during which the planet is believed to have been entirely covered in ice, and hothouse periods, during which prehistoric alligators may have roamed the Arctic. Recent anthropogenic greenhouse gas emissions are the cause of modern, rapid climatic change, which poses a growing hazard to societies and ecosystems.

The climate system comprises the fluid envelopes of Earth: the atmosphere, oceans, and cryosphere. Those constituents, along with the evolving surface properties of the solid lithosphere, are responsible for reflecting some and absorbing most radiation received from the Sun. The climate system is close to an energy balance at all times. The total energy doesn't significantly fluctuate in time because terrestrial radiation is emitted to space at approximately the same rate at which solar energy is absorbed.

Being in nearly exact energy balance with the universe allows Earth to have a relatively familiar climate tomorrow and a century from now. But over time, small deviations from a strict energy balance can induce massive changes in climate. Such small deviations are due to the diurnal and seasonal cycles, orbital variations—the Milankovitch cycles, for example (see the article by Mark Maslin, Physics Today, May 2020, page 48)—and internal forcings, such as anthropogenic emissions of carbon dioxide.

Another characteristic of Earth's climate—indeed, any planetary climate—is that it evolves irreversibly. Imagine watching a 10-second video of a field with a leafy

tree on a sunny day. Would you notice if that video had been shown in reverse? Maybe not. Now imagine watching a 10-second clip of the same field and tree during a windy rainstorm. You could probably immediately assess whether the clip was run forward or backward in time. Some obvious tells stand out: Rain should fall toward the ground, and leaves should separate from, not attach to, the tree.

The climate system contains myriad irreversible processes, and on both a calm day and a stormy day they produce entropy. Like energy, entropy is a property of any thermodynamic system, and it can be calculated if one knows the state of the system. But unlike energy, entropy is not conserved. Rather, it is continuously produced by irreversible processes. Although physicists often consider ideal, reversible processes, all real physical processes are irreversible and therefore produce entropy.

In accordance with the second law of thermodynamics, irreversibility in the climate system permanently increases the total entropy of the universe. As in the case for total energy, though, the total entropy in the climate system is relatively steady. That's because the climate is an open system that

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receives much less entropy from the Sun than it exports to the universe (see box 1). The difference between what is imported and what is exported is produced locally, through friction, mixing, or irreversible phase changes.

Although the climate is approximately steady, it is far from thermodynamic equilibrium, which would be a very cold and boring state with no motion. Instead, the climate system may be thought of as an engine, fueled by the unequal distribution of solar radiation incident upon it. It is those gradients in energy, and the resulting gradients in temperature and pressure they produce, that allow the wind to blow.¹

Climate system as heat engine

The concept of a heat engine is familiar to engineers and students of thermodynamics. Through the transport of heat from a hot reservoir to a cold one, a heat engine produces mechanical energy that may then be used to perform useful work. Examples include steam engines, internal combustion engines, and power plants. When run in reverse, a heat engine becomes a refrigerator or a heat pump.

The efficiency of the engine provides information about how much work it can produce for a given heat input. A remarkable consequence of the second law of thermodynamics is that a theoretical upper limit to that efficiency exists, and it may be expressed as a simple function of the temperatures $T_{\rm H}$ and $T_{\rm C}$ of the hot and cold reservoirs:

$$\eta_{\rm C} = \frac{T_{\rm H} - T_{\rm C}}{T_{\rm H}} \ .$$

Named the Carnot efficiency after the scientist who first derived it, 2 $\eta_{\rm C}$ determines the maximum possible work any heat engine can perform on an external body. It is achieved by a closed, reversible (ideal) engine, known as a Carnot engine (see figure 1a). Real heat engines can never truly reach the Carnot efficiency because their work output is limited by irreversible processes (see figure 1b). The output of an internal combustion engine is limited, for instance, by frictional losses between the pistons and cylinders and by conductive losses to the surroundings.

The climate system is essentially a giant planetary-scale heat engine. It is heated by the absorption of solar radiation and cooled by the emission of radiation to space (see figure 1c). The heating is largest at the warm tropical surface, while the cooling occurs primarily in the colder troposphere and is weighted toward higher latitudes. The planetary heat engine transports heat from the warm surface source to the colder tropospheric sink by the flows of the atmosphere and oceans.

But how do climate scientists characterize the work performed by the planetary heat engine? Earth cannot push on any external body, and in the framework of a classic heat engine, its work output is identically zero! The oceans and atmosphere do, however, perform work on themselves and each other, and that work generates the familiar winds and ocean currents that scientists observe. For climate scientists, useful work is that used to drive atmospheric and oceanic circulations

Because the work performed by the planetary heat engine is internal to the engine itself, its efficiency is not limited by the Carnot efficiency. Rather, the climate system can, in principle,

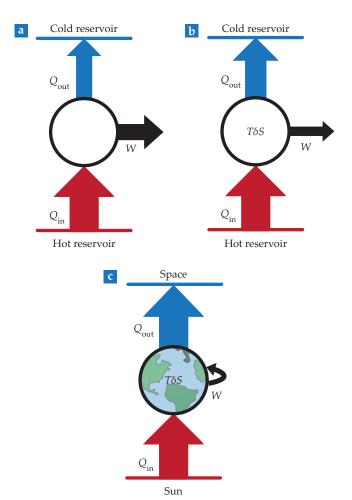


FIGURE 1. CLIMATE AS HEAT ENGINE. A heat engine produces mechanical energy in the form of work W by absorbing an amount of heat $Q_{\rm in}$ from a hot reservoir (the source) and depositing a smaller amount $Q_{\rm out}$ into a cold reservoir (the sink). (a) An ideal Carnot heat engine does the job with the maximum possible efficiency. (b) Real heat engines are irreversible, and some work is lost via irreversible entropy production $T\delta S$. (c) For the climate system, the ultimate source is the Sun, with outer space acting as the sink. The work is performed internally and produces winds and ocean currents. As a result, $Q_{\rm in} = Q_{\rm out}$.

recycle some of the heat produced by the frictional dissipation of winds and ocean currents and increase its maximum efficiency to a value

$$\eta_{\rm P}^{\rm max} \approx \frac{T_{\rm H} - T_{\rm C}}{T_{\rm C}} \,,$$

which is similar to the Carnot efficiency, except that the temperature in the denominator is replaced by that of the cold sink.³ The maximum planetary efficiency occurs when all available energy is used to drive atmospheric and oceanic currents and when the dissipation of those currents is concentrated at the warm source—for instance, through friction with Earth's surface. As we shall make clear, Earth's heat engine operates far from that limit.

Along with doing work, atmospheric and oceanic circulations are important in setting the spatial cloud and temperature distribution on Earth. As a result, the winds and currents driven by the planetary heat engine affect both its efficiency and the amount of heat that it transports. Those effects lead to an important feedback that regulates the climate: The work performed by the planetary heat engine acts to reduce the temperature gradient that drives it.

Such behavior complicates the analysis of Earth's heat engine, but it also raises tantalizing questions of planetary climate dynamics. What sets the efficiency of the planetary heat engine? Has it changed in the past, and will it change in the future? How does the operation of the planetary heat engine affect the everyday weather?

Irreversible processes

The work performed by the planetary heat engine produces in the atmosphere and oceans eddies of vastly varying scale and intensity, including tiny ripples on the ocean surface and violent winds in a tropical cyclone. Turbulence deforms such eddies into new shapes and patterns until viscosity ultimately dissipates their kinetic energy into heat. The resultant cycle of energy production and dissipation, beautifully described in 1955 by Edward Lorenz,4 implies a balance between work and frictional dissipation in the climate system.

The presence of friction does not necessarily limit the planetary heat engine's efficiency. In fact, the heat engine approaches its maximum efficiency when the frictional dissipation of winds and ocean currents is the dominant irreversible process. But other irreversible processes in the climate system compete for the available energy, as shown in figure 2. For example, heat conduction-that between the surface and atmosphere and that caused by molecular diffusion in the oceans and atmosphere-reduces the planetary efficiency just like conductive losses do in an internal combustion engine. Absorption, reflection, and emission of radiation are also irreversible processes, although they are generally not considered in discussions of the planetary heat engine (see box 1).

On Earth, an additional class of irreversible processes represents by far the most important control on the planetary heat engine. Those processes exist because of an aspect of Earth's climate that makes it habitable for life: the presence of an active hydrologic cycle.

Consider the path of a parcel of water from the ocean surface through Earth's hydrologic cycle. Heated by the Sun, the parcel initially enters the atmosphere by evaporating into the air. Like the drying of a wet shirt on a clothesline, that process of evaporation is irreversible. In its gaseous form, the parcel is at the mercy of the winds, swirling through the atmosphere and mixing with the air around it. Eventually the parcel is drawn into an updraft, cooling as it rises, until it condenses into tiny droplets in the saturated core of a cloud.

If it reaches high enough altitude, the parcel encounters subfreezing temperatures of the upper atmosphere and the droplets spontaneously and irreversibly freeze. As the frozen droplets grow, they begin to fall, first as snowflakes and later as raindrops. As they fall, the droplets irreversibly lose gravitational potential energy and partially evaporate as they pass through subsaturated air.

The various irreversible processes in the hydrologic cycle limit the work performed by the planetary heat engine. The effect may be quantified by considering the contributions of those processes to the irreversible entropy production of the climate system. Although such contributions are difficult to constrain observationally-an exception is the dissipation caused by falling precipitation, which may be estimated using satellites, as shown in figure 3-one may use models of the climate system to estimate their magnitude.

In 2002, Olivier Pauluis and Isaac Held used such an approach to demonstrate that irreversible processes associated with the hydrologic cycle,5 including phase changes, mixing, and precipitation, account for most of the irreversibility in the atmosphere and in Earth's climate system more broadly (see figure 2). Those so-called moist processes limit the entropy production associated with frictional dissipation, and they reduce the planetary heat engine's efficiency. Indeed, moist

BOX 1. ENTROPY OF RADIATION

Like matter, radiation obeys the second law of thermodynamics. The concepts of entropy and irreversibility are therefore just as relevant to photons as they are to atoms and molecules. But although the second law was developed for matter using the techniques of classical thermodynamics by Sadi Carnot,² Rudolf Clausius,15 and others in the middle of the 19th century, a full account of the entropy of radiation had to wait for Max Planck's theory of heat radiation.16 According to Planck, the entropy carried by a beam of radiation is dependent on its frequency spectrum, angular distribution, and polarization. A given amount of radiant energy carries the greatest amount of entropy when it is low frequency, isotropic, and unpolarized.

Earth scrambles a focused beam of solar radiation into a diffuse beam made of reflected solar radiation and terrestrial radiation at much lower frequency. As such, the radiative interactions, including absorption, emission, and reflection, are irreversible on Earth and contribute to the planet's entropy production. A simple analysis of that production allows one to quickly reject the notion-sometimes seen in contemporary discussions of global warmingthat the greenhouse effect is in violation of the second law of thermodynamics (see the article by Raymond Pierrehumbert, Physics Today, January 2011, page 33).

In fact, irreversible entropy production by radiative processes is the dominant source of irreversibility on the planet. Most studies of the second law applied to Earth, however, consider only matter (atoms and molecules) to be a part of the climate system, whereas radiation (photons) is considered a part of the surroundings. In that view, radiation is treated as an external and reversible heat source or sink, and the irreversibility of radiative processes does not enter discussions of the planetary heat engine.

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processes exert a profound influence on various atmospheric circulations, including individual clouds and the global circulation.

Drivers of global circulation

Imagine it's late morning in a tropical paradise. The Sun is starting to heat the ground and produce warm, rising bubbles of clear air known as thermals. Those thermals are replaced by slowly sinking air that has lost energy because of radiative cooling. Such vertical exchanges, or circulations, of air are a local version of the planetary heat engine, and climate scientists expect the work done by such dry thermals to scale with the surface heating rate.

Later in the day, the surface has warmed sufficiently to make stronger thermals. They can reach and exceed the lifting condensation level, where the water vapor in the air cools enough to condense as liquid water. That process introduces a phase change. The presence of a hydrologic cycle means that rising air can be seen as it forms clouds, and the clouds themselves indicate a local dominance of irreversible entropy production from moist processes.

If the system is defined to include both clouds and the surrounding, slowly sinking air, the total work available to drive motions can potentially be much smaller, and it no longer scales with the surface heating rate. Instead, the updrafts in clouds decouple from the heating rate, and their properties depend on microscopic details of cloud processes, such as the speed at which raindrops fall through the air and the rate at which moist, cloudy air is mixed into the dry surroundings at the cloud edge.

One can think of a developing cumulus cloud as a heat engine that does work on itself and the surrounding atmosphere. But not all clouds behave like a heat engine. Imagine, for example, a thin sheet of cirrus (an ice cloud) high in the atmosphere that is simply being advected by the wind. No potential energy is being released to perform work on the surroundings.

The heat engine analogy of a single cloud, however, can be usefully applied to organized clusters of convective clouds, which can take the form of thunderstorms, midlatitude storms, and tropical cyclones. Also known as hurricanes and typhoons, tropical cyclones in particular have long been conceived of as Carnot heat engines (see the Quick Study by Kerry Emanuel, Physics Today, August 2006, page 74). In reality, those storms are irreversible and extremely inefficient.

On global scales, the atmospheric circulation is driven by the differential heating associated with the Sun's angle. It man-

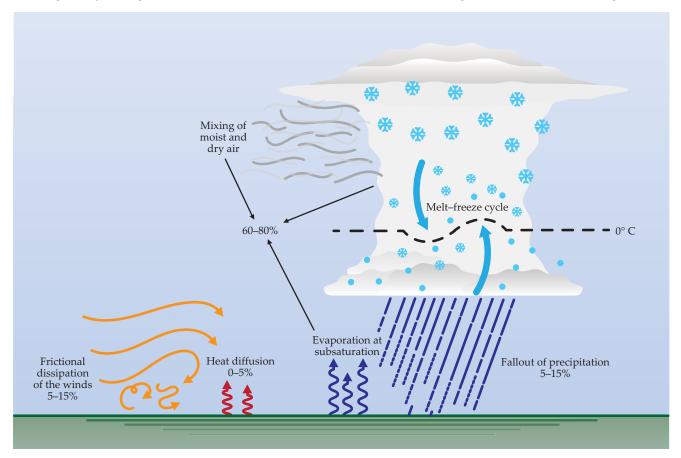


FIGURE 2. IRREVERSIBLE PROCESSES in the atmosphere. Neglecting radiative processes (not shown here), the largest sources of irreversibility in the atmosphere are those associated with the hydrologic cycle: evaporation, the mixing of moist and dry air, and the melt–freeze cycle (60–80% collectively), and the fallout of precipitation (5–15%). Those contributions limit the entropy generated by frictional dissipation of the winds (5–15%), which ultimately places a limit on the work performed by the atmospheric heat engine in generating circulations. Percentages are estimated based on global climate simulations¹² and idealized high-resolution simulations.⁸

BOX 2. HEAT ENGINES ON OTHER PLANETS

The gross characteristics of Earth's climate are unique to its rotation rate, planetary and orbital radii, mean temperature, and water content. Other planets in our solar system or in orbit around other stars have dramatically different climates. Earth's heat engine is one example of a wide range of possibilities on planets with fluid envelopes. The giant planets (Jupiter, Saturn, Uranus, and Neptune), for example, are all presumed to have water clouds, but they likely also have ammonia, ammonium hydrosulfide, and hydrogen sulfide clouds. Saturn's moon Titan has an active hydrologic cycle with methane clouds and rain (see the article by Ralph Lorenz, PHYSICS TODAY, August 2008, page 34). The presence of exotic condensation and evaporation suggests that those planetary heat engines are highly inefficient and produce considerable entropy via the whole suite of irreversible processes.

Other surprising differences exist between the climates of Earth and its neighbors. Take Mars, for example. Other than a carbon dioxide cycle that deposits snow on the winter pole and the occurrence of wispy water-ice clouds near the equator, Mars's thin atmosphere is extremely dry. One might assume that it could thus be relatively efficient, given the lack of a planetary-scale hydrologic cycle. Mars, however, has periodic, planetary-scale dust storms, which represent a major source of drag inside the atmosphere. Settling dust reduces the atmosphere's gravitational energy and converts it directly into internal energy. The process reduces the efficiency of the Martian heat engine.

Another curiosity is the lack of a known, well-defined bottom boundary on the giant planets. On rocky planets like Earth, the frictional surface is the primary source of dissipation of winds. What sets the brakes on the winds of giant planets if their fluid envelopes just get denser on the way down? Hypotheses include wave breaking and magnetic field effects.

Observations demonstrate that Earth is close to entropy and energy balance at all times. That need not be true of other planets. Jupiter, Saturn, and Neptune are all losing more heat to space than they receive from the Sun, which indicates that they are still cooling and shrinking over time. Just as energy balance is not an inevitable planetary characteristic, the same holds true for the entropy budget: Those gas giants could also be losing net entropy to space. That would be consistent with the second law of thermodynamics because planets are open systems.

ifests as large overturning cells and jet streams. All planets in orbit around a star are heated most strongly at any given moment at the substellar point, where the planet's surface is directly perpendicular to the star's radiation. Because Earth's day is short relative to its orbital period around the Sun, the planet is primarily warmed in the tropics (±30° latitude), and that heat is redistributed by the oceans and atmosphere toward the poles. The polar regions therefore lose more radiation to space than they receive from the Sun. For the global circulation, the characteristic input and output temperatures of the planetary heat engine are controlled by two temperature gradients: the surface-to-upper-atmosphere gradient and the equatorto-pole gradient.

Climate scientists quantify the efficiency of the global circulation, which, as we have seen, is a strong function of the moist, irreversible processes occurring within it. One of the most robust theoretical predictions of climate change is that the total amount of water vapor in the atmosphere will increase with warming-by about 7% per kelvin.6 If the magnitude of moist processes also increases with the vapor content, scientists might expect the climate heat engine to become less efficient on a warmer planet. A study of global climate models shows that, indeed, the mechanical efficiency of simulated future climates may go down and decrease the net energy available to drive winds.⁷ More detailed modeling on local scales, however, shows the opposite.8 Which one is right? And what does it

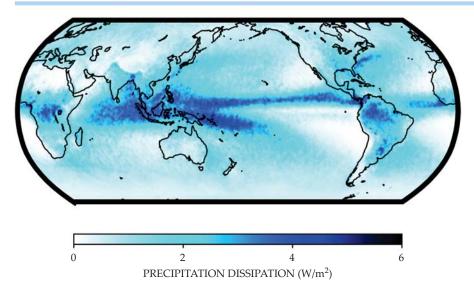


FIGURE 3. FALLING PRECIPITATION.

One of the most important sources of dissipation in the atmosphere occurs when raindrops fall, a process that reduces their gravitational potential energy. Using satellite information from NASA's Global Precipitation Measurement mission, we estimated the dissipation for the years 2015-20 with the method outlined in reference 13. The largest dissipation rates occur where precipitation rates themselves are highest—in the tropical western Pacific Ocean and in a band around the globe known as the intertropical convergence zone.



FIGURE 4. NUMERICAL MODELS are crucial for conducting climate research and estimating irreversible processes in the atmosphere and oceans. This image is a snapshot of clouds from an idealized high-resolution simulation made with the System for Atmospheric Modeling. ¹⁴ The simulation spans a 100 x 100 km² region of ocean using a horizontal grid size of 250 m. It captures many details of cloud morphology, including the tiny boundary-layer clouds that form a ring in the foreground of the image and the wispy cirrus clouds in the extreme bottom right. Processes that produce entropy irreversibly—such as mixing, evaporation, and the falling of raindrops—are not resolved and must be estimated through submodels called parameterizations.

mean for Earth's future climate? Those are outstanding questions in climate science; answering them requires fundamental advances in scientists' ability to model irreversibility in the climate system.

Modeling irreversibility

Models of the climate system come in various forms, such as general-circulation models that simulate the entire atmosphere or ocean and detailed large-eddy-simulation models that capture processes associated with individual clouds (see figure 4). Such models are used in numerous ways, such as forecasting the weather and probing the climate of alien worlds, as discussed in box 2. Regardless of their application, the general characteristics of climate models remain the same. The atmosphere or ocean is discretized, and a set of equations representing physical laws, such as conservation of mass, momentum, and energy, is numerically solved on the model grid.

Since weather and climate models are based on fundamental physics, one can naturally expect them to satisfy the second law of thermodynamics. Indeed, analysis of the entropy budget of climate models has allowed scientists to probe the climate system's irreversibility far beyond what observations alone would allow. Such studies have shed light on the role played by moist processes in governing how Earth's planetary heat engine may respond to climate change.

A challenge in climate modeling is representing processes that act on scales smaller than the model's grid length. For example, the large-eddy simulation represented in figure 4 has a horizontal grid spacing of 250 m. It can resolve the air movements of a given cloud, but it cannot resolve processes at smaller scales in, for example, turbulence that leads to irreversible mixing or the formation of individual raindrops. The effect of those subgrid processes must be accounted for using submodels called parameterizations.⁹

Beyond classical thermodynamics

So far, we have remained largely in the world of classical thermodynamics, having explored the conceptual model of the climate system as an irreversible heat engine. The second law of thermodynamics and the idea of irreversibility, however, may be interpreted more generally. The field of statistical mechanics, for example, has proven valuable to the study of certain long-lived flow phenomena in our solar system, such as Jupiter's famous Great Red Spot and Earth's stratospheric polar vortices.

Such problems require that researchers discard the heat engine model entirely and consider the system of interest as thermodynamically isolated and in contact with a single thermal reservoir rather than two. One can then generalize the concept of entropy to be a measure of the number of microscale arrangements of fluid particles that produce a given large-scale fluid behavior. By maximizing this Boltzmann entropy, scientists find the most likely long-term structures of the flow.

Although the Boltzmann entropy is widely known to provide the equilibrium distribution of molecular speeds in an ideal gas, it predicts counterintuitive and stunningly beautiful behavior, such as jets and vortices, when applied to planetary fluid envelopes. That's because high-Reynolds-number fluids that are dominated by stratification and rotation, which characterize most planetary fluid envelopes, exhibit quasi two-dimensional behavior that leads to an upscale energy cascade.

Instead of producing ever-smaller eddies that are lost to viscosity, 2D turbulence produces ever-larger structures that persist in time. Two-dimensional fluids pose a particular challenge theoretically because they conserve an infinite number of variables, which substantially constrains their evolution. That technical challenge was surmounted by the Robert-Sommeria-Miller (RSM) theory. (See the review in reference 11.)

The RSM theory and related statistical mechanical treatments of fluid flow provide a method to retrieve the long-time steady solutions for an inviscid fluid. But all real fluids have viscosity, and any real steady-state jet or vortex must be at least weakly forced because it is at least weakly damped by dissipation. It is remarkable then that some examples of real, large-scale vortices in the solar system can be predicted by inviscid theory for flows in thermodynamic equilibrium.

How can climate scientists reconcile a conceptual model of a planetary heat engine, which requires a temperature gradient to induce an overturning circulation, with the fact that observed large-scale vortices can be predicted by models that forbid temperature gradients? Tropical cyclones certainly have an important overturning circulation that responds to surface heating and upper-level cooling, but the much larger stratospheric polar vortex does not: It is a 2D phenomenon that is amenable to description using Boltzmann entropy. The most useful interpretation of the second law of thermodynamics is evidently feature-dependent in the climate system.

The swirling, circulating components of a planetary climate continue to inspire and confound. Understanding the drivers of a climate requires using a hierarchy of conceptual, analytical, and numerical models. Climate scientists have had to be creative and borrow from statistical mechanics, economics, and other fields to make sense of a spectacularly complex moving target. Amid a period of rapid anthropogenic climate change, it is more important than ever to make sure that climate science is accessible to the broadest possible coalition of researchers.

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