BOSTON™
INSTITUTE OF FINANCE

Welcome to the Boston Institute of Finance Calculator Keystrokes listing. Below you will find links to the formulas and keystrokes for the calculations used throughout this program. It is important for you to be comfortable using a financial calculator to perform all of the calculations presented here when you sit for the CFP® Certification Examination. Although many of these formulas are provided on CFP Board's formula sheet, there are others you will need to memorize. To see the formulas provided and to print off this formula sheet visit CFP Board's website.

Note: There are many financial calculators available and which calculator you choose to use is largely a matter of preference. Please note, that the keystrokes provided throughout this program are written *specifically* for the HP 12C financial calculator. Because different calculators operate differently, if you are NOT using the HP 12C, these keystrokes may differ slightly from the keystrokes required for your calculator.

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Cash Flow Convention

Your financial calculator is designed to treat cash inflows into an investment as a negative number and cash outflows (or funds available) from an investment as a positive number. This is particularly important when you are making a calculation that involves both a lump sum and periodic payments. The context of a problem could require these "payments" to be either additional deposits into an account, or funds that will be withdrawn from an account. Just remember, any money that is going into an investment is negative, and money coming out (or is available to come out of from an investment) is positive.

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Solve for a Variable

If there are no periodic payments involved, you may solve for any of the following four variables, given the other three:

n i PV FV

If there are periodic payments involved, you may solve for any of the following five variables, given the other four:

n i PV PMT FV

Where:

PV	is the present value of an amount today
FV	is the future value of an amount in the future
n	is the number of compounding periods (months, quarters, and years are typical)
PMT	is the periodic payment amount (must be equal in this application - uneven cash flows will be covered later)
i	is the interest rate *per compounding period*

NOTE:

The order of input does not matter, as long as what you are solving for is entered last. What does matter, is that you respect the proper cash flow convention.

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Clearing your Calculator

Prior to entering any keystrokes, always clear your calculator. Since you will be using the financial registers frequently, it will be necessary to clear both the data registers and the financial registers. Once you turn on your calculator or begin a new calculation, immediately enter:

Keystrokes
f **REG** f **FIN**

NOTE:

f invokes the **orange** registers, referenced on top of the keys.

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Scenario Analysis

The calculator will not clear any of your entries until you input the keystrokes to do so (see: Clearing your Calculator). This feature actually works to your advantage and allows you to perform various "what if" scenarios. For example, if I am calculating what lump sum will be required today, in order to grow to $50,000 in ten years, I may start off with an assumed interest rate of 7%.

Keystrokes	
50000 **FV** 7 **i** 10 **n** **PV**	
The calculator returns:	-25,417.46

Now, without clearing the calculator, change the interest rate assumption to 8%.

Keystrokes	
8 **i** **PV**	
The calculator returns:	-23,159.67

Now, without clearing the calculator, maintain the interest rate assumption of 8%, but change the time horizon to 12 years.

Keystrokes	
12 **n** **PV**	
The calculator returns:	-19,855.69

NOTE:

To set your calculator to display two decimals, key in:

Keystrokes
f 2

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Future Value of a Lump Sum

Application
In order to achieve a goal in 5½ years from today, a client invested $12,300 in an account bearing a 4% interest rate, compounded quarterly. What will be the value of this investment in 5½ years?

Keystrokes	
12300 **CHS** **PV** 22 **n** 1 **i** **FV**	
The calculator returns:	15,310.01

Application
In order to achieve a goal in 5½ years from today, a client invested $12,300 in an account bearing a 4% interest rate, compounded monthly. What will be the value of this investment in 5½ years?

Keystrokes	
12300 **CHS** **PV** 66 **n** 4 **g** **12÷** **FV**	
The calculator returns:	15,321.14

NOTE:

g invokes the **blue** registers, referenced on bottom of the keys.

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Present Value of a Lump Sum

Application

A client requires $15,000 in 5½ years. What lump sum will need to be invested today, assuming the client can earn a rate of 4%, compounded monthly?

Keystrokes	
15000 **FV** 4 **g** **12÷** 66 **n** **PV**	
The calculator returns:	-12,042.19

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Future Value of an Annuity (equal payments - end of period)

Application

As part of a divorce settlement, Jane will receive 60 monthly payments of $1,000, beginning one month from today. If Jane plans to invest these funds in a money market earning 4%, what will be the value of this account in five years?

Keystrokes	
1000 **CHS** **PMT** 60 **n** 4 **g** **12÷** **FV**	
The calculator returns:	66,298.98

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Future Value of an Annuity (equal payments - beginning of period)

Application

As part of a divorce settlement, Jane will receive 60 monthly payments of $1,000, beginning today. If Jane plans to invest these funds in a money market earning 4%, what will be the value of this account in five years?

Keystrokes	
g **BEG** 1000 **CHS** **PMT** 60 **n** 4 **g** **12÷** **FV**	
The calculator returns:	66,519.97

NOTE:

In order to return your calculator back to the default mode (payments at the end of the period), it will be necessary to input:

Keystrokes
g **END** (this is the #8 Key)

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Present Value of a Lump Sum with Payments

Application
A client requires $30,000 in 10 years. At this point, she has managed to save $7,400 towards this goal. The money is invested in a mutual fund that is anticipated to earn an average annual rate of return over the next ten years of 8.5%. What is the additional monthly amount that will be needed in order for the client to obtain her goal? Solve for the monthly deposits needed under two scenarios.

First, deposits are to begin one month from today:

Keystrokes	
30000 **FV** 8.5 **g** **12÷** 10 **g** **12×** 7400 **CHS** **PV** **PMT**	
The calculator returns:	-67.71

and then, deposits into the fund begin immediately:

Keystrokes	
g **BEG** **PMT**	
The calculator returns:	-67.23

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Future Value of a Lump Sum with Payments

Application

Robert has just invested $20,000 in ABC mutual fund. Furthermore, he plans to invest an additional $650 per month, beginning one month from today. If the assumed after-tax annual rate of return is expected to average 8%, what will be the amount of this investment in 20 years?

Keystrokes	
20000 **CHS** **PV** 240 **n** 8 **g** **12÷** 650 **CHS** **PMT** **FV**	
The calculator returns:	481,399.33

What amount will the fund grow to if Robert can manage to deposit $750 per month?

Keystrokes	
750 **CHS** **PMT** **FV**	
The calculator returns:	540,301.37

What amount will the fund grow to in 25 years if Robert can manage to deposit $750 per month?

Keystrokes	
25 **g** **12×** **FV**	
The calculator returns:	860,073.32

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Inflation Adjusted (or the Real Rate of Return)

Quite often, financial planning requires the use of what is known as the *real rate of return* or the *inflation adjusted rate*. This is particularly true in retirement planning, where the time horizon is often far off into the future. Furthermore, due to an increase in life expectancy, the actual amount of time that individuals spend in their retirement years is also expanding.

Application

John is 55 years old and has just retired. When aggregating John's IRAs, 401(k), and a cash balance plan, John's lump sum retirement assets total $1,150,000. Furthermore, John will receive Social Security benefits in the amount of $25,000 per year. These Social Security payments will be indexed with inflation and automatically increase as inflation increases. John would like to know if his retirement assets are sufficient, given the following:

- The after-tax rate of return for the retirement years will be 6%.
- The inflation rate for the retirement duration is expected to be 3%.
- John would like to plan for 35 years of retirement (live to the age of 90).
- In total, John requires $75,000 per year in today's dollars.

Keystrokes	
g **BEG** 50000 **PMT** 35 **n** 1.06 **ENTER** 1.03 ÷ 1 **-** 100 × **i** **PV**	
The calculator returns:	-1,119,893.19

NOTE:

This problem set the calculator to the beginning mode since the funds are always required at the start of the year for retirement planning purposes. It would appear that if all assumptions hold, John has enough financial stock at retirement to provide purchasing power equal to $50,000 in today's dollars. This amount, when added to the Social Security benefit amount (which will automatically grow with inflation) will generate the necessary purchasing power of $75,000 in today's dollars. Another alternative for calculating the real rate of return is to take the difference between the total return and the inflation rate as a whole number (6 - 3 = 3) and divide by 1 plus the inflation rate expressed as a decimal (1.03). That's it. In other words, you may substitute part of the formula and solve as:

Keystrokes
g **BEG** 50000 **PMT** 35 **n** 3 **ENTER** 1.03 ÷ **i** **PV**

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Weighted Betas, Returns, and Durations

The weighting function on your HP12C is an invaluable tool, which not only saves time, but also ensures the accuracy of the calculation.

Application
What is the expected return for a portfolio with the following assets and their respective returns and market values?

Assets	Cost	Current Market Value	Expected Return
S&P Index Fund	$70,000	$81,123	10%

Small Cap Growth	$15,000	$13,575	14%
Growth & Income	$10,000	$10,500	9%
Home Depot	$10,000	$14,105	20%
GE	$10,000	$11,564	10%
EMC	$10,000	$11,867	15%

Keystrokes	
10 **ENTER** 81123 **Σ+**	
The calculator displays:	1.00

14 **ENTER** 13575 **Σ+**	
The calculator displays:	2.00

9 **ENTER** 10500 **Σ+**	
The calculator displays:	3.00

20 **ENTER** 14105 **Σ+**	
The calculator displays:	4.00

10 **ENTER** 11564 **Σ+**	
The calculator displays:	5.00

15 **ENTER** 11867 **Σ+**	
The calculator displays:	6.00

g **x w** (the #6 key)	
The calculator returns:	11.71

NOTE:

The cost information has nothing to do with the weighting function. The answer represents a percentage, so the final answer is that the weighted expected return for the portfolio is 11.71%. If the calculator does not display 1.00 after your first entry, input **f** **REG** to clear the data register. This function can also be used to weight betas and in the context of a bond portfolio, weight durations as well. The key to this function is always remembering to use the **ENTER** key for the item you are weighting and the summation key **Σ+** for the weights (market values).

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Standard Deviation

CFP Board's formula sheet provides two formulas that are used to calculate the standard deviation of an asset's returns.

Formula 1:

$$\sigma_r = \sqrt{\frac{\sum_{t=1}^{n} (r_t - \bar{r})^2}{n}}$$

Formula 2:

$$S_r = \sqrt{\frac{\sum_{t=1}^{n} (r_t - \bar{r})^2}{n-1}}$$

The first formula divides certain computations by *n*, while the second formula divides by *n-1*. Using statistics, you would use the *n-1* computation when you have a sample of data from a population of a given statistic. This is the common case for the real world. Subtracting 1 from *n*, where *n* represents the number of occurrences is known as normalizing the statistic. You would only use the *n* computation if you had the entire data population for a given statistic. This is extremely rare in the real world. In fact, your HP12C only has a built in short cut for the *n-1* version. For comparative purposes, the keystrokes for the *n-1* formula will be detailed, as well as the HP12C shortcut method.

Application (using formula)
You would like to determine the standard deviation for the returns of ABC stock based on the last 4 years of data.

__**Returns of ABC**__
Year 1 17%
Year 2 4%
Year 3 -2%
Year 4 29%

Step 1 (add)	
17 **ENTER** 4 + 2 **CHS** + 29 +	
The calculator displays:	48

Step 2 (divide by 4)

4 ÷	
The calculator displays:	12 (this is the mean or average return)

Step 3 (subtract mean from each return)	
17 **ENTER** 12 -	
The calculator displays:	5 (set aside)

4 **ENTER** 12 -	
The calculator displays:	-8 (set aside)

2 **CHS** **ENTER** 12 -	
The calculator displays:	-14 (set aside)

29 **ENTER** 12 -	
The calculator displays:	17 (set aside)

Step 4 (square each result)	
5 **ENTER** 5 &#215	
The calculator displays:	25 (set aside)

8 **ENTER** 8 &#215	
The calculator displays:	64 (set aside)

14 **ENTER** 14 &#215	
The calculator displays:	196 (set aside)

17 **ENTER** 17 &#215	
The calculator displays:	289 (set aside)

Step 5 (add the results)	
25 **ENTER** 64 + 196 + 289 +	
The calculator displays:	574

Step 6 (divide by 3, which is "n-1")	
3 ÷	
The calculator displays:	191.33

Step 7 (take the square root)	
g **Square Root** (this is the y^x key)	
The calculator returns:	13.83

NOTE:

Since we used whole numbers to represent the returns, remember that the end result is also a percent. In this case, the standard deviation is 13.83%.

Application (shortcut calculator method)

Keystrokes	
17 **Σ+**	
The calculator displays:	1.00

4 **Σ+**	
The calculator displays:	2.00

2 **CHS** **Σ+**	
The calculator displays:	3.00

29 **Σ+**	
The calculator displays:	4.00

g **x (the zero key)**	
The calculator returns:	12 (this is the mean)

g **s (the decimal key)**	
The calculator returns:	13.83 (this is the standard deviation)

NOTE:

Your display will be 1 only if your calculator is properly cleared.

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Standard Deviation of a Two-Asset Portfolio

This calculation involves two formulas, both of which are included on CFP Board's formula sheet. The covariance formula is simple and will serve as an input into the formula for the standard deviation of a two-asset portfolio. To calculate the covariance between assets *A* and *B*, you multiply the standard deviation of asset *A* times the standard deviation of asset *B*, and then multiple that result times the correlation coefficient of combined asset *AB*. If you choose to use whole numbers (as opposed to decimals) for the standard deviations in the covariance formula, you must use whole numbers for the standard deviations in the formula for the standard deviation of a two-asset portfolio.

Application
Calculate the standard deviation for the two-asset portfolio that follows:

	Asset A	Asset B
Return	15%	11%
Relative Weight	60%	40%
Beta	1.2	1.05
Standard Deviation	20%	14%

Correlation Coefficient .22

The first step is to calculate the covariance:

$COV_{AB} = SD_A \times SD_B \times r_{AB}$, where r_{AB} is the correlation coefficient between securities A and B.

Keystrokes	
20 **ENTER** 14 × .22 ×	
The calculator returns:	61.60 (this will be an input for the next calculation)

Now, calculate the standard deviation for the portfolio:

$$\sigma_p = \sqrt{W_i^2\sigma_i^2 + W_j^2\sigma_j^2 + 2W_iW_jCOV_{ij}}$$

Keystrokes	
20 **ENTER** 20 × .60 × .60 ×	
The calculator displays:	144

	(set aside)

14 **ENTER** 14 × .40 × .40 ×	
The calculator displays:	31.36 (set aside)

2 **ENTER** .60 × .40 × 61.60 ×	
The calculator displays:	29.57 (set aside)

Add the 3 prior results and take the square root:

Keystrokes	
144 **ENTER** 31.36 + 29.57 + **g** **Square Root** (this is the **y^x** key)	
The calculator returns:	14.32

Since we used whole numbers to represent the standard deviations throughout this calculation, it is now necessary to state the standard deviation as a percentage. That is, the combined standard deviation of portfolio AB is 14.32%. Also note that the original information provided a statistic for Beta and the annual returns of each asset. This information is known as a distracter and is not needed for the calculation.

Application
To further demonstrate the power of the correlation coefficient statistic, let's assume the same facts presented in the previous example, except now the correlation coefficient is negative .22 (- .22). It will not be necessary to calculate the problem in full because the rest of the data is the same. Changing signs on the correlation coefficient would lead to a covariance of -61.60. This would result in the third calculation above to be -29.57. Now let's input the last step to calculate the standard deviation.

Add the three prior results and take the square root:

Keystrokes	
144 **ENTER** 31.36 + 29.57 - **g** **Square Root** (this is the **y^x** key)	
The calculator returns:	12.07

The difference of a correlation coefficient of -.22 versus +.22, reduces the portfolio standard deviation from 14.32% to 12.07%.

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Intrinsic Value of a Bond

Application

What is the intrinsic value of a bond that matures in 7 years with a maturity value of $1,000, pays a 6% coupon (paid semi-annually), and market rates for comparable bonds are 7%?

Keystrokes	
1000 **FV** 14 **n** 30 **PMT** 3.5 **i** **PV**	
The calculator returns:	-945.40

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Intrinsic Value of a Zero Coupon Bond

Application

What is the intrinsic value of a bond that matures in 12 years with a maturity value of $1,000, does not pay a coupon, and market rates for comparable bonds are 6%?

Keystrokes	
1000 **FV** 24 **n** 0 **PMT** 3 **i** **PV**	
The calculator returns:	-491.93

NOTE:

Always assume semi-annual compounding unless told otherwise.

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Yield to Maturity (YTM) of a Bond

Application

What is the YTM of a bond that matures in 9 years with a maturity value of $1,000, pays a 7% coupon (paid semi-annually), and the current price is $1,075.42?

Keystrokes	
1000 **FV** 18 **n** 35 **PMT** 1075.42 **CHS** **PV** **i**	
The calculator returns:	2.95

2 ×	
The calculator returns:	5.91

NOTE:

The initial answer of 2.95 represents a semi-annual rate; therefore it must be multiplied times 2 in order to annualize.

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Yield to Maturity (YTM) of a Zero Coupon Bond

Application

What is the YTM of a zero coupon bond that matures in 8 years with a maturity value of $1,000, does not pay a coupon, and the current price is $504.20?

Keystrokes	
1000 **FV** 16 **n** 504.20 **CHS** **PV** **i**	
The calculator returns:	4.37

2 ×	
The calculator returns:	8.75

NOTE:

The initial answer of 4.37 represents a semi-annual rate; therefore it must be multiplied times 2 in order to annualize. Also, as long as the calculator is properly cleared, it is not necessary to input a zero value into the payment **PMT** key.

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Yield to Call (YTC) of a Bond

Application

What is the YTC of a bond that matures in 12 years with a maturity value of $1,000, pays a 7% coupon (paid semi-annually), and the current price is $1,105.22? The bond is callable in 5 years at a special call price of $1,035.

Keystrokes	
1035 **FV** 10 **n** 35 **PMT** 1105.22 **CHS** **PV** **i**	

The calculator returns:	2.60

2 ×	
The calculator returns:	5.20

NOTE:

The initial answer of 2.60 represents a semi-annual rate; therefore it must be multiplied times 2 in order to annualize.

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Duration

Application

The formula to calculate duration (Macaulay) is detailed on CFP Board's formula sheet as:

$$D = \frac{\sum_{t=1}^{n} \frac{c_t(t)}{(1+i)^t}}{\sum_{t=1}^{n} \frac{c_t}{(1+i)^t}}$$

This formula accommodates either annual or semi-annual compounding, where:

y = the market rate of interest per compounding period

T = the number of compounding periods

c = the coupon rate expressed as a percentage per compounding period

Application (semi-annual coupon payments)

Calculate the duration for a bond that matures in 22 years at a price of $1,000, pays a 6% coupon that is paid semi-annually, and the yield or market rate for comparable instruments is currently 7%.

First, solve the first part of equation and set result aside. (Set to four decimals, **f** 4)

Keystrokes	
1.035 **ENTER** .035 ÷	
The calculator returns:	29.5714

Then, solve for numerator of the expression after the minus sign and set aside.

Keystrokes

.03 **ENTER** .035 - 44 × 1.035 +	
The calculator returns:	.8150

Now, solve for the denominator.

Keystrokes	
1.035 **ENTER** 44 **y^x** 1 - .03 × .035 +	
The calculator returns:	.1413

Finally, divide numerator by denominator and subtract result from result set aside in step one.

Keystrokes	
.8150 **ENTER** .1413 ÷ **CHS** 29.5714 +	
The calculator returns:	23.8035

NOTE:

This result represents compounding periods, which in this case is not years. The duration is 23.8035 *semi-annual compounding periods*. Since duration is always expressed in years, the result is 11.9018 (23.8035 / 2) years.

Application (Annual Coupons)

Calculate the duration for a bond that matures in 22 years at a price of $1,000, pays a 6% coupon that is paid annually, and the yield or market rate for comparable instruments is currently 7%.

First, solve the first part of equation and set result aside.

Keystrokes	
1.07 **ENTER** .07 ÷	
The calculator returns:	15.2857

Then solve for numerator of the expression after the minus sign and set aside.

Keystrokes	
.06 **ENTER** .07 - 22 × 1.07 +	
The calculator returns:	.8500

Now solve for the denominator.

Keystrokes	
1.07 **ENTER** 22 **y^x** 1 - .06 × .07 +	
The calculator returns:	.2758

Finally, divide numerator by denominator and subtract result from result set aside in step one.

Keystrokes						
.8500	**ENTER**	.2758	÷	**CHS**	15.29	+
The calculator returns:				12.2081		

NOTE:

This result represents compounding periods, which in this case is years. The duration is approximately 12.21 years.

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Modified Duration

Now solve for modified duration using the above calculations for duration (semi-annual coupon payments of 11.90 years and annual payments of 12.21 years.) This is important, because it is modified duration that is indexed to a 1% (or 100 basis points which is equal to the decimal equivalent of .0100) change in interest rates, wherein a 1% change in interest rates will lead to the bond changing in price (approximately) by the duration expressed as a percentage. As an example, look at the first application below where it is determined that the modified duration for the semi-annual coupon paying bond is 11.50 years. If the interest rate were to change by 1%, the bond's price would change by approximately 11.50%. If interest rates were to change by ½% (50 basis points), the bond would change in price by approximately 5.75% (11.50/2). If interest rates changed by 1.8% (180 basis points), the expected change in the bonds price would be 20.70% (11.50 x 1.8). Of course, the inverse relationship between a bonds price and interest rates still apply. That is, an increase in interest rates would reduce a bonds price and vice versa.

The formula to convert Macaulay duration (as calculated above to modified duration) is to simply divide the Macaulay duration by 1 plus the market rate, where the market rate is expressed per compounding period.

$$Modified\ Duration = \left[\frac{Macauley\ Duration}{1 + \frac{YTM}{n}}\right]$$

Application (semi-annual compounding)

What is the modified duration of a semi-annual coupon-paying bond that has Macaulay duration of 11.90 years?

Keystrokes	
11.90 **ENTER** 1.035 ÷	
The calculator returns:	11.50

Application (annual compounding)

What is the modified duration of an annual coupon-paying bond that has Macaulay duration of 12.21 years?

Keystrokes	
12.21 **ENTER** 1.07 ÷	
The calculator returns:	11.41

NOTE:

The adjustment on the annual pay coupon is more significant than the semi-annual paying bond. This is due to the shape and the price/yield function and the fact that since the coupons are more frequent with semi-annual paying bonds, the Macaulay duration was more precise before the modification adjustment. In all cases, modified duration will always be lower than the Macaulay duration.

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Geometric Returns

The formula for geometric returns is:

$$Geometric\ Return = \sqrt[n]{(1+r_1)\times(1+r_2)\times\ldots(1+r_n)} - 1$$

r = rate of return
n = number of periods

This formula is not included on CFP Board's formula sheet.

Application
What is the geometric return of a fund over the least 7 years that has experienced the following annual returns:

Year 1	12%
Year 2	10%

Year 3	7%
Year 4	13%
Year 5	- 6%
Year 6	- 2%
Year 7	4%

Keystrokes	
1.12 **ENTER** 1.10 × 1.07 × 1.13 × .94 × .98 × 1.04 × 7 **1/x** y^x 1 - **f** 4	
The calculator returns:	.0521 (which is also expressed as 5.21%)

Application

What is the geometric return of a fund that reports the following month-end net asset values?

Month	NAV ($)
March	15.50
April	15.80
May	14.85
June	14.55
July	14.98
August	15.42
September	16.12
October	16.40
November	16.44
December	16.48

Keystrokes	
15.50 **CHS** **PV** 16.48 **FV** 9 **n** **i**	
The calculator returns:	.6835

12 ×	
The calculator returns:	8.20

NOTE:

The .6835 figure is *monthly*. Multiplying times 12 simply annualizes the return. Also, the geometric return is also known as a *time-weighted return.*

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Uneven Cash Flow Applications

The uneven cash flow keys are your most important keys on your calculator. They allow for the calculation of complex scenarios such as Internal Rates of Return (IRR), Net Present Value (NPV) calculations, solving for the present value of future cash flows by using the NPV key, solving for the future value of an uneven cash flow, life insurance needs analysis, retirement funding analysis, and college funding requirements. The keys that will be used are:

g CF_0

g CF_j

g N_j

f NPV

f IRR

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Cash Flow Time Lines

The most important part of using the uneven cash flow keys is to establish a cash flow time line. Time zero (0) in finance is now; time one (1) is a year from now; time two (2) is two years from now, etc. This is why it is not necessary to deal with the beginning and end modes, as the end of one year is the beginning of another. What is critical is that you input a cash flow for every point on the cash flow line, even if the cash flow is zero, and that you have only one discreet cash flow number for each point on the cash flow line. That is, multiple cash flows at any point in time must be netted to a single number.

Application

Create a cash flow line for the following situation:

An investor has an opportunity to purchase a two family house as an investment property. The investor has performed an analysis and has the following expected after-tax cash flows:

Initial outlay (including transaction and closing costs) = $516,450.

Year one after-tax cash outflows are expected to exceed cash inflows by $2,200.

Year two cash flows are expected to net to zero.

Years three through years six after tax cash flows are expected to be $18,000 per year.

In year seven, after tax cash flows are anticipated to be $21,000. Also, at the end of the seventh year, the investor expects to be able to sell the property for a total $880,000, net of transaction costs and taxes.

Cash Flow Line

Time:	0	1	2	3	4	5	6	7
Cash Flows:	**(516,450)**	**(2,200)**	**0**	**18,000**	**18,000**	**18,000**	**18,000**	21,000
								880,000
								901,000

NOTE:

For purposes of conservatism, standard practice in finance is to assume all annual cash flows occur at the end of the year. The bold figures represent the actual calculator input and will be demonstrated in the examples that follow.

ApplicationOn December 31, 2003, an investor opened an IRA and deposited $3,000 into a growth mutual fund. On December 31, 2004, the investor deposited an additional $3,000 into the account, but the investor did not invest any funds during 2005. On June 30, 2006, the account had grown to $9,040. What is the correct cash flow line?

Cash Flow Line

Date:	12/31/03	06/30/04	12/31/04	06/30/05	12/31/05	06/30/06
Time:	0	1	2	3	4	5
Cash Flows:	**(3,000)**	**0**	**(3,000)**	**0**	**0**	**9,040**

NOTE:

This example has incorporated the actual date into the cash flow line, in order to demonstrate an important concept. Since the question is focused on the ending date at the end of June, and all of the investments into the fund took place at the end of December, the intervals of the time line must be every six months.

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Internal Rate of Return

Also known as a *dollar weighted return*, the IRR is the rate that equates the outflow at time zero to the present value of the future inflows discounted at the IRR rate. In other words, the IRR is the discount rate that would create a Net Present Value (NPV) of zero. Please refer to the online content for additional information.

Application
Calculate the IRR for the following situation:

An investor has an opportunity to purchase a two family house as an investment property. The investor has performed an analysis and has the following expected after-tax cash flows:
Initial outlay (including transaction and closing costs) = $516,450.
Year one after-tax cash outflows are expected to exceed cash inflows by $2,200.
Year two cash flows are expected to net to zero. Years three through year six after tax cash flows are expected to be $18,000 per year.
In year seven, after tax cash flows are anticipated to be $21,000. Also, at the end of the seventh year, the investor expects to be able to sell the property for a total $880,000, net of transaction costs and taxes.

The first step is to establish a cash flow line:

Time:	0	1	2	3	4	5	6	7
Cash Flows:	**(516,450)**	**(2,200)**	**0**	**18,000**	**18,000**	**18,000**	**18,000**	21,000
								880,000
								901,000

Keystrokes	
516450 **CHS** **g** **CF_0** 2200 **CHS** **g** **CF_j** 0 **g** **CF_j** 18000 **g** **CF_j** 4 **g** **N_j** 901000 **g** **CF_j** **f** **IRR**	
The calculator returns:	9.71

The IRR for this investment is 9.71%

NOTE:

One of the big flaws of the IRR is known as the reinvestment assumption. In calculating the IRR, the assumption is that all of the future cash flows can be reinvested at the IRR rate. In this example, in order to generate a return of 9.71%, all of the future cash flows must also be reinvested and generate a 9.71% return as well.

Application
On December 31, 2003, an investor opened an IRA and deposited $3,000 into a growth mutual fund. On December 31, 2004, the investor deposited an additional $3,000 into the account, but the investor did not invest any funds during 2005. On June 30, 2006, the account had grown to $9,040. What is the IRR of this investment?

Cash Flow Line

Date:	12/31/03	06/30/04	12/31/04	06/30/05	12/31/05	06/30/06
Time:	0	1	2	3	4	5
Cash Flows:	**(3000)**	**0**	**(3000)**	**0**	**0**	**9040**

Keystrokes	
3000 **CHS** **g** **CF_0** 0 **g** **CF_j** 3000 **CHS** **g** **CF_j** 0 **g** **CF_j** 2 **g** **N_j** 9040 **g** **CF_j** **f** **IRR**	
The calculator returns:	10.65

Keystrokes	
2 ×	
The calculator returns:	21.30

The IRR for this investment is 21.30%

NOTE:

The initial answer of 10.65 represents a semi-annual rate; therefore it must be multiplied times 2 in order to annualize.

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Net Present Value

NPV is a better measure of the desirability of an investment. When using a NPV analysis, the underlying reinvestment assumption is that an investor would be able to reinvest future cash flows at the appropriate discount rate. In the context of a firm or a borrower, the appropriate discount rate would be at the firm's cost of capital or the borrower's cost of funds. In the context of an investor, the discount rate would be the investor's opportunity cost. NPV compares the outflow at time zero, with the present value of the future cash flows using the appropriate discount rate. If the NPV is positive, the discounted value of the future cash flows exceed the initial outlay. The relationship between NPV and IRR is such that if the discount rate is equal to the IRR, the NPV will be zero. If the discount rate exceeds the IRR, the NPV will be negative, and if the discount rate is less than the IRR, the NPV will be positive. Projects with positive NPVs are desirable, while projects with negative NPVs are considered undesirable. Please refer to the online content for a further discussion.

Application
Using the same example for the IRR calculation, let's assume that the investor's opportunity cost is 7%.

Calculate the NPV for the following situation:
An investor has an opportunity to purchase a two family house as an investment property. The investor

has performed an analysis and has the following expected after-tax cash flows:

Initial outlay (including transaction and closing costs) = $516,450.

Year one after-tax cash outflows are expected to exceed cash inflows by $2,200.

Year two cash flows are expected to net to zero.

Years three through year six after tax cash flows are expected to be $18,000 per year.

In year seven, after tax cash flows are anticipated to be $21,000. Also, at the end of the seventh year, the investor expects to be able to sell the property for a total $880,000, net of transaction costs and taxes.

The first step is to establish a cash flow line:

Time:	0	1	2	3	4	5	6	7
Cash Flows:	**(516,450)**	**(2,200)**	**0**	**18,000**	**18,000**	**18,000**	**18,000**	21,000
								880,000
								901,000

Keystrokes	
516450 **CHS** **g** **CF_0** 2200 **CHS** **g** **CF_j** 0 **g** **CF_j** 18000 **g** **CF_j** 4 **g** **N_j** 901000 **g** **CF_j** 7 **i** **f** **NPV**	
The calculator returns:	95,844.83

NOTE:

Had the investor's opportunity cost been 9.71%, NPV would have been zero. Had the investor's opportunity cost been higher than 9.71%, NPV would have been negative, (that is, a reduction in the investor's wealth.) It is also important to realize that the present value of the future inflows is $612,294.83 ($516,450.00 + $95,844.83) since the NPV was + $95,844.83. Also, note that since NPV and IRR are related functions, the basic initial keystrokes are the same. In fact, it is possible to calculate both the NPV and IRR with one set of input:

Keystrokes	
516450 **CHS** **g** **CF_0** 2200 **CHS** **g** **CF_j** 0 **g** **CF_j** 18000 **g** **CF_j** 4 **g** **N_j** 901000 **g** **CF_j** **f** **IRR**	
The calculator returns:	9.71

7 **i** **f** **NPV**	
The calculator returns:	95,844.83

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Present Value using the NPV Function

Quite often in finance, we need to determine the present value of a future uneven cash flow stream that has nothing to do with an initial outlay. However, by inputting zero as the initial outlay, we are essentially manipulating the NPV function to reveal the present value of a future cash flow stream.

Application

A homeowner would like to set aside sufficient funds today to take care of the next 5 years of foreseeable home repairs. In two years, the homeowner is expected to replace the roof with an estimated cost of $9,000. The homeowner also expects to spend $1,000 per year for the next 5 years on paint and wallpaper. Finally, in five years time, the furnace and hot water heater will have to be replaced with an estimated cost of $9,600. All of these estimates are based on quotes today and therefore are expressed in today's dollars. The estimated rate of inflation over the next 5 years is projected to average 3%. If the homeowner invests these funds in a money market account that is expected to average a nominal rate of return of 4% over the five-year horizon, how much must the homeowner set aside today?

Establish a cash flow line:

Time:	0	1	2	3	4	5
Cash Flows:	**0**	**1,000**	1,000	**1,000**	**1,000**	1,000
			9,000			9,600
			10,000			**10,600**

NOTE:

The item being solved for is the outflow at time zero. Considering that today's lump sum investment will grow at 4%, what is the amount needed to fund the projected expenses, taking into account that this projected amount will increase with inflation? Since NPV compares the present value of the projected future cash flows with the outflow at time zero, you are effectively manipulating the NPV function to calculate the PV of the repair expenses. By using the real (inflation adjusted) rate of return as the discount rate, inflation of the cost of these repairs will be accounted for.

Keystrokes	
0 **g** **CF_0** 1000 **g** **CF_j** 10000 **g** **CF_j** 1000 **g** **CF_j** 2 **g** **N_j** 10600 **g** **CF_j** 1 **ENTER** 1.03 ÷ **i** **f** **NPV**	
The calculator returns:	22,832.61

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Future Value of Uneven Cash Flows

Your calculator is not designed to calculate the future value of uneven cash flows in one step. The easiest (and quickest) method is to calculate the present value and then grow that present value (as a lump sum) over the time horizon at the appropriate discount rate. This is made possible by the very nature of time value of money. The discount rate and the number of compounding periods always connect two lump sums at different points in time. Alternatively, you could calculate the future value of each cash flow and then add them together. However, this method is time consuming, involves many more keystrokes and has an enhanced likelihood of an error. In the application that follows, both methods will be detailed.

Application

A company would like to know the value of their retirement plan in 5 years. The company is planning to make contributions into the plan of $550,000 one year from today, $400,000 in two years, and then contribute an additional $375,000 at the end of years 3 through 5. The company can earn a 6% return on these funds over the next 5 years. What is the value in 5 years of these contributions to the retirement plan?

Step one (for either method) is to establish the cash flow line (000.)

Time:	0	1	2	3	4	5
Cash Flows:	**0**	**550**	**400**	**375**	**375**	**375**

Long Method

Keystrokes	
550000 **CHS** **PV** 4 **n** 6 **i** **FV**	
The calculator returns:	694,362.33 (set aside)

400000 **CHS** **PV** 3 **n** 6 **i** **FV**	
The calculator returns:	476,406.40 (set aside)

375000 **CHS** **PV** 2 **n** 6 **i** **FV**	
The calculator returns:	421,350.00 (set aside)

375000 **CHS** **PV** 1 **n** 6 **i** **FV**	
The calculator returns:	397,500.00 (set aside)

375000 **CHS** **PV** 0 **n** 6 **i** **FV**	

The calculator returns:	375,000.00 (set aside)

(Please note that the last step did not need to be calculated, in that it had no time to grow.)

Finally, add all five figures together to arrive at answer:

Keystrokes	
694362.33 **ENTER** 476406.40 + 421350 + 397500 + 375000 +	
The calculator returns:	2,364,618.73

Quick Method

Please input **f** 4 to set your display to four decimals

Keystrokes	
550 **g** **CF_j** 400 **g** **CF_j** 375 **g** **CF_j** 3 **g** **N_j** 6 **i** **f** **NPV**	
The calculator returns:	1,766.9807

CHS **PV** 5 **n** **FV**	
The calculator returns:	2,364.6187 (set aside)

1000 ×	
The calculator returns:	2,364,618.73 (set aside)

NOTE:

There were two additional shortcuts taken in the quick method. First, to minimize keystrokes, the cash flows were inputted without the zeros (000), which is why it was necessary to multiply end result by 1000. Second, by ignoring the 0 **g** **CF_0** entry, your calculator assumes the cash flow to be zero at time zero.

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Life Insurance Needs Analysis

As it relates to the replacement of income, there are two basic methods to determine an adequate amount of life insurance. The *human life value* approach is the one that is most likely to be tested on CFP certification examination. The reason is that it is simple to calculate, and it tends to generate a more conservative number. However, the *needs approach* is much more likely to be used in the real world. Under this approach, the amount of insurance required is much more exact, tends to be less than the human value approach, and quite frankly is a better "convincer" for your clients.

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Human Life Value Approach

Application

Your client currently has $500,000 of life insurance. He has come to you seeking your opinion as to the adequacy of the amount. Your client is 30 years old, anticipates working for the next 37 years, and currently earns a salary of $70,000. Furthermore, the expectation for salary increases over the next 37 years is 5% per year. Assuming a tax bracket of 25% and an investment rate of return of 7%, calculate the total amount of insurance needed using the human value approach.

Keystrokes	
37 **n** 70000 **PMT** **g** **BEG** 2 **ENTER** 1.05 ÷ **i** **PV**	
The calculator returns:	1,881,814.43

.75 ×	
The calculator returns:	1,411,360.82

Your client has $500,000 of coverage and according to this analysis; he should purchase an additional $900,000 of coverage. Doing so will replace his life income in the event the client dies.

NOTE:

The interest rate is treated like a real rate of return. In this case, the difference between the projected salary increases (5%) and the investment rate of return (7%) is 2%. This amount when divided by 1.05 generates an interest rate of 1.9048%. Since the lump sum insurance payment is not taxed (unlike salary), 1 minus the marginal tax rate of 25% is the 75% that is applied to the present value of the annuity due. The beginning mode is used because income replacement and retirement funding always require the funds at the beginning of the period.

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Needs Approach

Application

Calculate the life insurance requirements for both spouses. Detailed below are the pertinent assumptions:

- Both spouses are 30 years old, plan to retire at age 67, and earn an annual salary of $70,000.
- Emergency fund needs to be increased by $8,000 for the survivor.
- Education fund needed is $102,000.
- Funeral and administrative expenses will be $50,000.
- If either spouse dies, the surviving spouse will continue to work until age 67.
- Life insurance proceeds can be invested at a rate of 7% now, and continue at this rate in retirement.
- Husband currently has $500,000 of life insurance.
- Wife has no life insurance.
- Couple has one child; age is 2 months.
- Survivor income needs are $9,000 per month (today's dollars) for the next 22 years.
- From age 52 - 67, survivor needs will be $7,000 per month (today's dollars).
- Retirement needs for the survivor will be $5,000 per month (today's dollars) and will be needed until age 90.
- Inflation will average 3% over the entire time horizon, including retirement.
- Social Security will provide $1,600 per month to survivor at retirement, and $2,400 per month until child is 18 years old.
- Couple has no current savings or retirement plans so life insurance must fully fund the above scenario.

	Dependency Period	First 4 Years of Black Out	Remaining Black Out	Retirement
Duration	18 Years	4 Years	15 Years	23 Years
+ Cash Needs	$9,000/mo.	$9,000/mo.	$7,000/mo.	$5,000/mo.
Survivor Salary	$5,833/mo.	$5,833/mo.	$5,833/mo.	$ 0/mo.
+ Social Security	$2,400/mo.	$ 0/mo.	$ 0/mo.	$1,600/mo.
PMT (AD) mo.	$ 767/mo.	$3,167/mo.	$1,167/mo.	$3,400/mo.
PMT (AD) yr.	$9,204/yr.	$38,004/yr.	$14,004/yr.	$40,800/yr.

The second step is to develop a cash flow time line. The $160,000 of liquidly requirements (education, emergency fund, and funeral/administrative expenses) will be needed at time zero, as well as the first annual cash requirement.

Time:	0	1-17	18-21	22-36	37-59
Cash Flows:	160,000	**9,204**	**38,004**	**14,004**	**40,800**
	9,204				
	169,204				

Keystrokes
169204 g CF_0 9204 g CF_j 17 g N_j 38004 g CF_j 4 g N_j 14004 g CF_j

15 **g** **N_j** 40800 **g** **CF_j** 23 **g** **N_j** 4 **ENTER** 1.03 ÷ **i** **f** **NPV**	
The calculator returns:	580,677.69

The husband needs an additional $80,000 of life insurance and the wife needs $580,000 of coverage.

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Retirement Needs Analysis

Application

John is 47 years old and is married to Paula, age 32. They would like to know:

a. how much of a lump sum would be needed today to fund their retirement, and
b. how much of a lump sum would be needed when John reaches age 67, which is the time they plan to retire.

John and Paula want to assume the following:

1. They both reach age 90.
2. They can earn a 7% rate of return on their investments throughout both the pre-retirement years and the retirement years.
3. Inflation will average 3% over the entire time horizon.
4. Income requirements will be $80,000 (in today's dollars) while they are both alive.
5. Income requirements will be reduced for Paula to $60,000 (in today's dollars) after John is deceased.
6. They would prefer that Social Security benefits be excluded from the analysis.

Solution:

Establish the cash flow line

Time:	0	1-19	20-42	43-57
Cash Flow:	$0	$0	$80,000	**$60,000**

Solve for lump sum needed today (and do not clear calculator)

Keystrokes	
0 **g** **CF_0** 0 **g** **CF_j** 19 **g** **N_j** 80000 **g** **CF_j** 23 **g** **N_j** 60000 **g** **CF_j** 15 **g** **N_j** 4 **ENTER** 1.03 ÷ **i** **f** **NPV**	
The calculator returns:	718,746.70 (This is the lump sum required today to fund retirement.)

Solve for the lump sum required in 20 years (when John is age 67.)

Keystrokes

PV 20 **n** 7 **i** **FV**	
The calculator returns:	2,781,322.94 (This is the amount required at the beginning of retirement.)

NOTE:

Since inflation was already imbedded in the present value today ($718,746.70), it is necessary to convert to lump sum in the future based on the full (nominal) rate of return.

Application

Building on the last example, now let's take Social Security and the couple's current 401(k) balance into account.

John is 47 years old and is married to Paula, age 32. Their current 401(k) balance is $330,100. They would like to know how much they need to save at the end of each month between now and when John reaches age 67, which is the time they plan to retire.

John and Paula want to assume the following:

- They both reach age 90.
- They can earn a 7% rate of return on their investments throughout both the pre-retirement years and the retirement years.
- Inflation will average 3% over the entire time horizon.
- Income requirements will be $80,000 (in today's dollars) while they are both alive.
- Income requirements will be reduced for Paula to $60,000 (in today's dollars) after John is deceased.
- They have projected that their Social Security benefit will be $1,600 per month (when John reaches age 67) while they are both alive. Benefits will be reduced to $800 per month after John is deceased.

Solution:

Establish the cash flow line

Time:	0	1 - 19	20 - 42	43 - 57
Cash Flow:	**($330,100)**	**$0**	$80,000	$60,000
Less SS			<u>$19,200</u>	<u>$ 9,600</u>
			$60,800	**$50,400**

As the first step, solve for lump sum needed today (and do not clear calculator).

Keystrokes	
330100 **CHS** **g** **CF_0** 0 **g** **CF_j** 19 **g** **N_j** 60800 **g** **CF_j** 23 **g** **N_j** 50400 **g** **CF_j** 15 **g** **N_j** 4 **ENTER** 1.03 ÷ **i** **f** **NPV**	
The calculator returns:	227,008.57

This figure represents the shortfall. In other words, in addition to their current 401(k) balance of $330,100, the couple needs an additional $227, 009. The addition of these numbers ($330,100 + $227,009 = $557,109) is the lump sum required today to fund their retirement.

Now, solve for the monthly payments needed to compensate for the shortfall.

Keystrokes	
PV 20 **g** **12×** 7 **g** **12÷** **PMT**	
The calculator returns:	1,759.99

To properly fund their retirement, $1,759.99 must be saved monthly.

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College Funding

There are several methods for determining college-funding requirements. The method that is presented here is known as the "uneven cash flow" method and is the method that is recommended for use.

Please compare this method with the one presented in the *Financial Aid* lesson of the *Educational Funding* module of Course One. The method that is presented there is calculated per child, which is time consuming when determining funding needs for multiple children. In contrast, the uneven cash flow method is easier to use, contains less steps, and therefore requires less keystrokes. Perhaps the most compelling reason to learn and adopt this method is that it is the same method that is applied for both life insurance needs analysis and retirement funding analysis.

Quite often only one step is required. This occurs when the question asks, "how much must the client set aside today to fund their children's educational requirements?" However, if the question asks how much must the client save monthly, quarterly, annually, or for any specific period, a second step will be needed to convert a lump sum today into a payment (deposit) stream. This one or two-step process is all that is needed regardless of the amount of children you are funding!

Two different applications are presented. The first application is the same example taken from the "educational funding" (above-mentioned) page. The second application will be for three children and will make use of a cash flow (time) line.

Application
Ricky Jones would like to plan for his daughter's college education. His daughter was born today and will attend the state university at age eighteen for four years. The cost of the university in today's dollars is $18,000 and is expected to increase at a rate of 6% annually. Inflation is expected to be at 3% annually. Ricky is expecting to earn an after-tax return of 9% on his investments. How much must Ricky save at the end of each year if he expects to make his last payment at the beginning of his daughter's first year of college?

Step one is to determine what amount is needed today (lump sum):

Keystrokes	
0 **g** **CF_0** 0 **g** **CF_j** 17 **g** **N_j** 18000 **g** **CF_j** 4 **g** **N_j** 3 **ENTER** 1.06 ÷ **i** **f** **NPV**	
The calculator returns:	41,801.47

Don't clear the calculator.

Step two is needed because you have to determine how much must be saved over the next 18 years.

With 41801.47 still showing on your calculator's display, input:

Keystrokes	
PV 18 **n** 9 **i** **PMT**	
The calculator returns:	-4,774.24

NOTE:

The reason why you use the full nominal return (real + inflation) when annuitizing to a deposit stream, is that inflation has already been fully accounted for in the lump sum number.

Application

David and Susan would like to provide $20,000 per year for the education of each of their three children, in today's dollars. The children are currently 13, 10, and 8 years old. The general rate of inflation is 4%, but education costs are expected to increase at a rate of 6.5% for the foreseeable future. All children are expected to start college at age 18 and graduate in four years. Assume that the after-tax return on the education funds will be 8.5%. How much of David's and Susan's assets are needed to provide this money, if they were to set aside a lump sum today?

Develop one cash flow number line for all three children, combined.

2nd Child Age: 10 11 12 13 14 15 16 17 **18** **19** **20** **21** 3rd Child Age: 8 9 10 11 12 13 14 15 16 17 **18** **19** **20** **21** **Today's $ (000)** **0 0 0 0 0 20 20 20 40 20 40 40 20 20**

Time Line:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1st Child Age:	13	14	15	16	17	**18**	**19**	**20**	**21**					

Keystrokes (taken directly from cash flow line)	
0 **g** **CF_0** 0 **g** **CF_j** 4 **g** **N_j** 20 **g** **CF_j** 3 **g** **N_j** 40 **g** **CF_j** 20 **g** **CF_j** 40 **g** **CF_j** 2 **g** **N_j** 20 **g** **CF_j** 2 **g** **N_j** 2 **ENTER** 1.065 ÷ **i** **f** **NPV**	
The calculator returns:	202.561

NOTE:

To see all the decimals enter: **f 3**. Also remember to multiply the result by 1000 since we removed the zeros to save keystrokes. The total amount that the couple must set aside now is $202,561.

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Amortization

The HP12C's amortization function is an invaluable time-saving tool. It can help you answer questions such as, "What will the principal balance be on the client's mortgage after 1, 5, and 10 years?" Another essential application pertains to installment sales, when you need to determine what will be taxed as ordinary income and what will be taxed as capital gains for a given year.

Application
A client would like to know what balance on her mortgage will be due to the bank at the end of 1, 5, and 10 years. She just borrowed $275,000 at a fixed rate of 7.25%, compounded monthly, for 30 years.

Enter loan information to calculate the payment: (set calculator to 2 decimals **f 2**)

Keystrokes	
360 **n** 275000 **CHS** **PV** 7.25 **g** **12÷** **PMT**	
The calculator returns:	1,875.98

Don't clear the calculator, instead, input the following:

Keystrokes	
12 **f** **AMORT**	
The calculator returns:	19,850.22 (this is the interest for the first year)

x><y	
The calculator returns:	2,661.54 (this is principal paid in the first year)

RCL PV	
The calculator returns:	272,338.46 (this is the balance at the end of the first year)

Keystrokes

48 f **AMORT**	
The calculator returns:	77,250.54 (this is the interest for four *more* years)

x><y	
The calculator returns:	12,796.50 (this is principal paid in those four additional years)

RCL PV	
The calculator returns:	259,541.96 (this is the balance at the end of the fifth year)

Keystrokes	
60 f **AMORT**	
The calculator returns:	90,371.09 (this is the interest for five *more* years)

x><y	
The calculator returns:	22,187.71 (this is principal paid in those five additional years)

RCL PV	
The calculator returns:	237,354.25 (this is the balance at the end of the tenth year)

Application

On April 30, 200X John sold a rare art piece that he bought several years ago at a cost of $25,000. The terms of the sale were: A sale price of $60,000, with a 20% down payment on April 30, with the balance to paid monthly over the next 5 years. The annual interest rate is 8%, compounded monthly. The first payment will be received on May 31st. What are the total amounts that John must report in 200X as 1) ordinary income, and 2) capital gains?

First, calculate the Gross Profit Percentage (GPP). The gross profit is $60,000 - $25,000 = $35,000. The GPP is the gross profit divided by the sale amount or, $35,000 / $60,000 = .5833. This percentage must be multiplied by the down payment and the principal received on the loan payments; the aggregation of these amounts is required to be reported as capital gain.

Second, calculate the down payment, multiply the down payment times the GPP, and set aside the result as the first step in determining the taxable gain. The down payment is 20% of $60,000 or $12,000. Multiplying that times the GPP, results in $12,000 × .5833 = $6,999.60 ($7,000 rounded.) This amount is the first step of the capital gain calculation.

Third, calculate the loan payment based on the terms:

Keystrokes	
48000 **CHS** **PV** 60 **n** 8 **g** **12÷** **PMT**	
The calculator returns:	973.27

Don't clear the calculator, instead, input the following:

Keystrokes	
8 **f** **AMORT** (8 months of payments (May 31 - Dec. 31) of the loan)	
The calculator returns:	2,436.41

John must **report as ordinary income: $2,436.41** (or $2,436 rounded.) This is the interest portion of the principal and interest (P&I.)

Now calculate the principal received.

Keystrokes	
x><y	
The calculator returns:	$5,349.75 (or $5,350 rounded)

This amount represents the principal received from the loan payments, and must be allocated between capital gain and return of basis. Multiplying the principal times the GPP, yields $5,350 X .5833 = $3,120.66 (or $3,121 rounded.) This figure represents step 2 in calculating the capital gain. Add $7,000 (the gain on the down payment from step one) to $3,121 (The gain on the principal of the total payments from step two) to determine the aggregated **total of $10,121 that must be reported as capital gains.**

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