DEFINITION		
	metric	
		Analysis
DEFINITION		
	norm	
		Analysis
Drigwignov		
DEFINITION		
	convex	
		Analysis

Let X be a set. A functions $d: X \times X \to \mathbb{R}$ is a <u>metric</u> on X if it satisfies the following conditions:

- $d(x,y) \ge 0$ for all $x,y \in X$ and d(x,y) = 0 if and only if x = y.
- d(x,y) = d(y,x)
- $d(x,z) \le d(x,y) + d(y,z)$

Let X be a vector space over \mathbb{R} or \mathbb{C} . Then a <u>norm</u> on X, denoted by $\|\cdot\|$ if a function $\|\cdot\|: X \to [0, \infty)$ with the following properties

- $||v|| \ge 0$ and ||v|| = 0 if and only if v = 0
- $\|\lambda v\| = |\lambda| \|v\|$ with $\lambda \in \mathbb{R}$ or \mathbb{C} .
- $||v + w|| \le ||v|| + ||w||$

A subset A of a vector space X is <u>convex</u> if for all $x, y \in A$

$$tx + (1 - t)y \in A$$

for all $t \in [0, 1]$

Definition		
	sequence	
		Analysis
Definition		
	converges	
		Analysis
DEFINITION		
	Cauchy	
		Analysis

A <u>sequence</u> in X is a function $X: \mathbb{N} \to X$, commonly denoted by $\{x_n\}_{n=1}^{\infty}$. Let $\{x_n\}_{n=1}^{\infty}$ be sequence in X and $x \in X$. Then $\{x_n\}_{n=1}^{\infty}$ converges to x provided for all $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n \geq N$ $d(x, x_n) < \epsilon$ A sequence in a metric space X is a Cauchy if for every $\epsilon > 0$, there exists N such that $d(x_n, x_m) < \epsilon$ for all n, m > N.

Definition	
complete	
	Analysis
DEFINITION	
open ball	
	Analysis
	ANALISIS
DEFINITION	
closed ball	
	Analysis

A metric space (X,d) is called $\underline{\text{complete}}$ if every Cauchy sequence in (X,d) converges in (X,d).

For all $x \in X, r > 0$, we define the open ball

$$B_r(x) := \{ y \in X : d(y, x) < r \}$$

For all $x \in X, r > 0$, we define the <u>closed ball</u>

$$\bar{B}_r(x) := \{ y \in X : d(y, x) \le r \}$$

DEFINITION		
	open	
		Analysis
DEFINITION		
	closed	
		Analysis
DEFINITION		
	closure	
		Analysis

A set $U \subset X$ is <u>open</u> provided for every $x \in U$, we can identify a r > 0 such that

$$B_r(x) \subset U$$

A set $F \subset X$ is <u>closed</u> provided $F^c = X \setminus F$ is open.

For any $A \subset X$, the <u>closure</u> of A is the smallest closed subset of X that contains A, denoted by \bar{A} . Moreover,

$$\bar{A} := \bigcap_{F \text{ closed } \subset X; A \subset F} F$$

Equivalently,

 $\bar{A} := \{x \in X : \text{ there exists a sequence in } X \text{ that converges to } x\}$

DEFINITION		
	dense	
		Analysis
DEFINITION		
	seperable	
		Analysis
Definition		
	continuous	
		Analysis

A set $A \subset X$ is dense if $\bar{A} = X$. In other words, for every point $x \in X$, there exists $\{a_n \in A\}_{n=1}^{\infty}$ such that

$$a_n \to x$$

A metric space (X, d) is separable if it has a countable dense subset.

Let $x_0 \in X$ and $f: X \to Y$ be a function. Then f is <u>continuous</u> at x_0 if for all $\epsilon > 0$, $\exists \delta > 0$ such that

$$d_X(x,x_0) < \delta \implies d_Y(f(x),f(x_0)) < \epsilon$$

That is,

$$f(B_{\delta}(x_0)) \subset B_{\epsilon}(f(x_0))$$

Further, f is continuous on a subset $E \subset X$ provided f is continuous at every point $x \in E$.

Definition	
$uniformly\ continuous$	
amiorniy continuous	
	Analysis
DEFINITION	
upper semi-continuous / lower semi-continuous	·
apper semi conomiacas / lower semi conomiacas	
	Analysis
DEFINITION	
isometric embedding	
Bometric embedding	
	Analysis

A function $f:X\to Y$ is uniformly continuous if for all $\epsilon>0$, there exists $\delta(\epsilon)>0$ such that

$$d_Y(f(x), f(y)) < \epsilon$$

whenever $d_X(x,y) < \delta$.

A function $f:X\to Y$ is upper semi-continuous if for every sequence $\{x_n\}\to x$, we have that

$$\limsup_{n \to \infty} f(x_n) \le f(x)$$

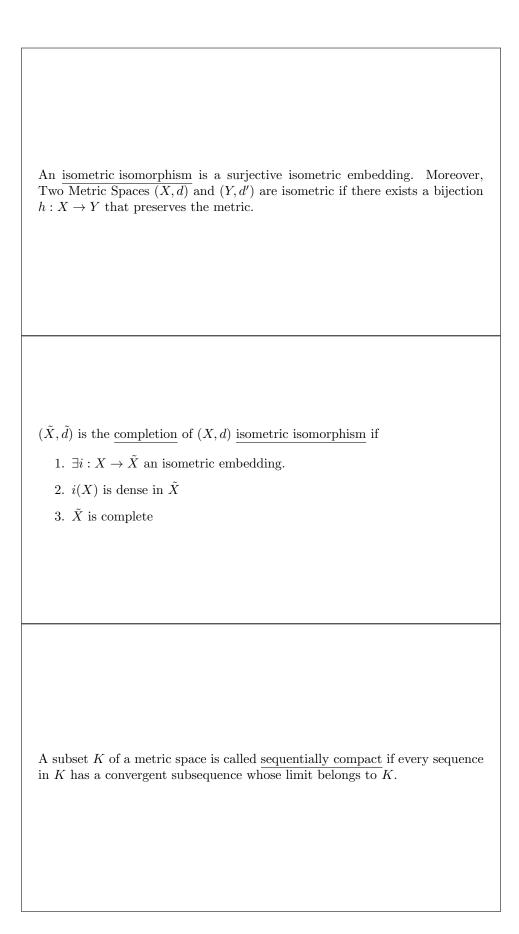
Similarly, $f: X \to Y$ is <u>lower semi-continuous</u> if for every sequence $\{x_n\} \to x$, we have that

$$\liminf_{n \to \infty} f(x_n) \ge f(x)$$

Let (X,d) and (\tilde{X},\tilde{d}) be metric spaces. Then an isometric embedding is a one-to-one map $i:X\to \tilde{X}$ such that for all $x,y\in X$

$$\tilde{d}(i(x),i(y)) = d(x,y)$$

DEFINITION		
	isometric isomorphism	
	10011101101101101110111	
		Analysis
		ANALISIS
DEFINITION		
	completion of an isometric isomorphism	
	-	
		Analysis
-		
DEFINITION		
	sequentially compact	
	_	
		Analysis
		TINALIBIS



DEFINITION		
	cover	
		Analysis
DEFINITION		
	open cover	
		Analysis
D		TIVILISIS
DEFINITION		
	subcover	
	54500701	
		Analysis

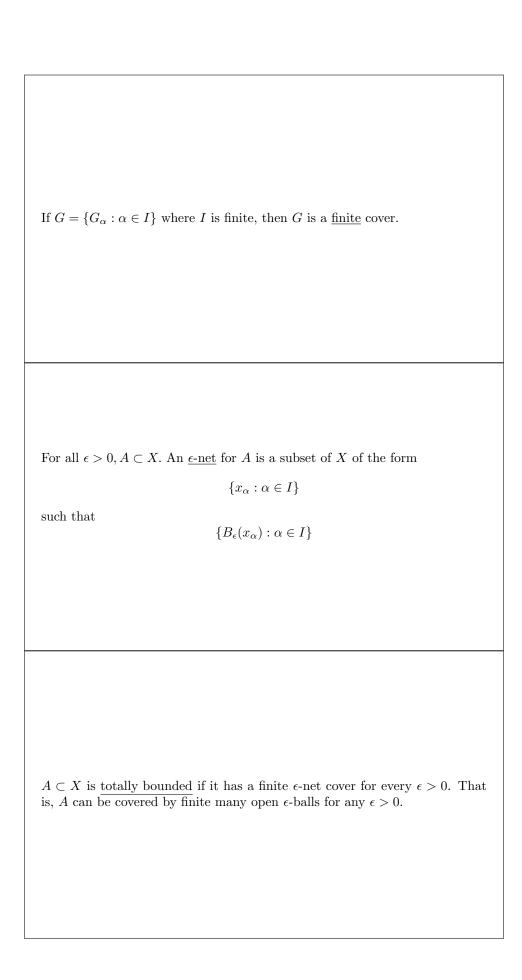
Let I be an index set. A collection $G=\{G_\alpha:\alpha\in I\}$ of subsets of X is called a <u>cover</u> of a subset $A\subset X$ if

$$A \subset \bigcup_{\alpha \in I} G_{\alpha}$$

If every G_{α} in the cover is open, we say that $\{G_{\alpha}\}$ is an open cover.

A <u>subcover</u> of $G = \{G_{\alpha} : \alpha \in I\}$ is a collection $\{G_{\alpha} : \alpha \in I_0\}$ where $I_0 \subset I$.

DEFINITION		
	finite cover	
		Analysis
DEFINITION		
	$\epsilon ext{-net}$	
		Annana
		Analysis
DEFINITION		
	totally hounded	
	totally bounded	
		Analysis



DEFINITION		
	support of a function	
		Analysis
DEFINITION		
	$C_0(X)$	
		Analysis
DEFINITION		
	Bernstein Polynomials	
	Defusion 1 olynomias	
		Analysis

The support of a function $f: X \to \mathbb{R}$ is given by

$$supp(f) := \overline{\{x \in X : f(x) \neq 0\}}$$

$$C_c(X) := \{ f \in C(X) : supp(f) \text{ is compact} \} \subset C(X)$$

Now define:

$$C_0(X) := \overline{C_c(X)}$$

and equip this set with the $\left\|\cdot\right\|_{\infty}.$ We see this set fits within the chain:

$$C_c(X) \subset C_0(X) \subset C_b(X) \subset C(X) \cap B(X)$$

The Bernstein Polynomials. We define a basis of unity:

$$\mathcal{X}_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}$$

Then by the binomial theorem:

$$1 = \sum_{k=0}^{n} \binom{n}{k} x^{k} (1-x)^{n-k}$$

So for any $f \in C([0,1])$, the Bernstein Polynomial of f is

$$B_n(x;f) := \sum_{k=0}^n f\left(\frac{k}{n}\right) \mathcal{X}_{n,k}(x)$$

DEFINITION		
	Hausdorff	
		Analysis
DEFINITION		
	equibounded	
		Analysis
		ANALISIS
DEFINITION		
	equicontinuous	
		Analysis

A metric space X space is <u>Hausdorff</u> if for every $x \neq y$, then we can identify open subsets $U_x, V_y \subset X$ such that

$$x \in U_x, y \in V_y$$
 and $U_x \cap V_y = \emptyset$

We then say A can <u>separate</u> points if we can identify an $a \in A$ "between" two points.

Let \mathcal{F} be a family of functions

$$f:(X,d_X)\to (Y,d_Y)$$

between metric spaces. Then $\mathcal F$ is called equibounded provided for all $f\in\mathcal F$ there exists $g\in\mathcal F$ if

$$||f||_{\infty} \le M + ||g||_{\infty}$$

Let \mathcal{F} be a family of functions

$$f:(X,d_X)\to (Y,d_Y)$$

between metric spaces. Then \mathcal{F} is called equicontinuous at $x \in X$ if for every $\epsilon > 0$, there is a $\delta = \delta(\epsilon, x) > 0$ such that

$$d_X(x,y) < \delta \implies d_Y(f(x),f(y)) \le \epsilon$$

for all $f \in \mathcal{F}$.

DEFINITION		
	dimension of a linear space	
		Analysis
DEFINITION		
	Schauder Basis	
		Analysis
DEFINITION		
	linear mapping	
	FF O	
		Analysis

The $\underline{\text{dimension}}$ of a linear space is equal to the number of elements that form a (linear) basis for the space.

Let X be a separable Banach space. A <u>Schauder Basis</u> is a sequence $\{x_n\}_{n=1}^{\infty}$ such that for all $x \in X$, there exists a unique sequence $\{c_n\}_{n=1}^{\infty} \subset \mathbb{R}$ such that

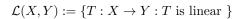
$$x = \sum_{i=1}^{\infty} c_n x_n$$

Let X, Y be linear spaces. Then $T: X \to Y$ is <u>linear</u> provided

$$T(ax + by) = aT(x) + bT(y)$$

for any $x, y \in X, a, b \in \mathbb{F}$.

DEFINITION		
	$\mathcal{L}(X,Y)$	
	, ,	
		Analysis
DEFINITION		
	bounded	
		Analysis
DEFINITION		
	$\mathcal{B}(X,Y)$	
	<i>、 </i>	
		Analysis

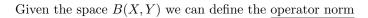


Let X,Y be normed, linear spaces. A linear map $T:X\to Y$ is bounded if there exists $M\ge 0$ such that

$$\|Tx\|_Y \leq M \, \|x\|_X$$

 $\mathcal{B}(X,Y) := \{T: X \to Y: T \text{ is a bounded, linear map from } X \text{ to } Y\}$

DEFINITION		
	operator norm	
		Analysis
DEFINITION		
	Relation "Stronger" in regards to Norms	
		Analysis
DEFINITION		
	equivalent norms	
		Analysis



$$||T|| = \inf\{M : \forall x \in X, ||Tx|| \le M ||x||\}$$

Let $\|\cdot\|_1$ and $\|\cdot\|_2$ be two norms on a linear space X. We say $\|\cdot\|_2$ is stronger than $\|\cdot\|_1$ if

$$\|x_n\|_2 \to 0 \implies \|x_n\|_1 \to 0$$

Two norms $\|\cdot\|_1$ and $\|\cdot\|_2$ are equivalent if there exists $c_1>0$ and $c_2>0$ such that

$$c_1 \|x\|_1 \le \|x\|_2 \le c_2 \|x\|_1$$

for all $x \in X$.

DEFINITION		
	Ker(T)	
		Analysis
DEFINITION		
	Range(T)	
		Analysis
DEFINITION		
	well-posed problem	
		Analysis

$$Ker(T):=\{x\in X: Tx=0\}$$

It is a subspace of X, and also called the null space of T.

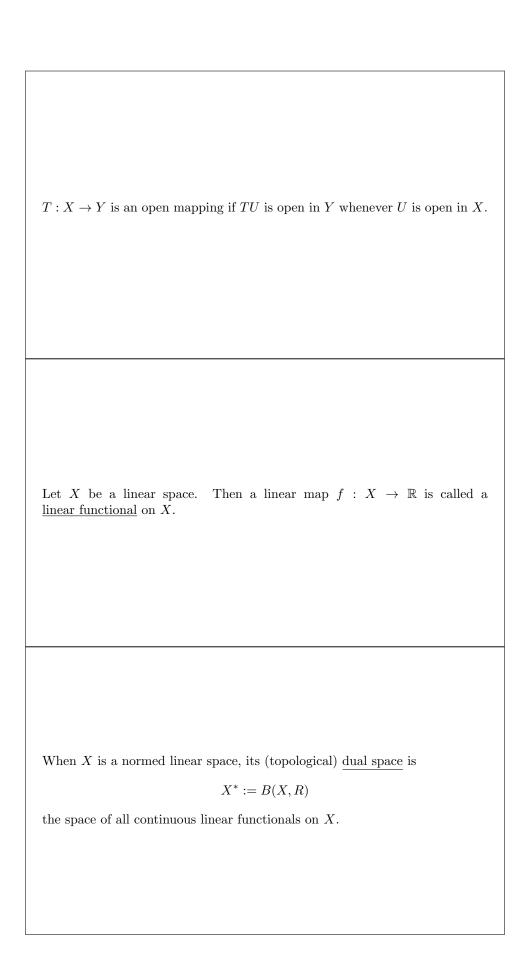
$$Range(T) := \{y \in Y : Tx = y \text{ for some } x \in X\}$$

It is a subspace of Y.

A problem is called well-posed if

- A solution exists
- The solution is unique
- The solution is "stable", i.e. the solution depends continuously on the data.

DEFINITION		
	open mapping	
		Analysis
DEFINITION		
	linear functional	
		Analysis
DEFINITION		
	dual space	
		Analysis



DEFINITION		
	bidual	
		Analysis
DEFINITION		
	reflexive	
		A 22.1.22.22.22
		Analysis
DEFINITION		
	wools top closes	
	weak topology	
		Analysis

Since X^* is a Banach space, we can consider its dual space

$$(X^*)^* = X^{**}$$

called the <u>bidual</u> of X.

If $X = X^{**}$, then we say that X is <u>reflexive</u>

Let X be a normed, linear space. The <u>weak topology</u> on X is defined as the weakest topology which makes all of the functionals $\phi \in X^*$ continuous. Let $\Phi := \{\phi^{-1}(U) : U \subset \mathbb{R} \text{ open }, \phi \in X^*\}$. Since

$$\bigcup \phi^{-1}(U) = X$$

there exists a unique topology $\mathcal{T}_{\text{weak}}$ on X having Φ as its sub-base. Moreover,

$$\mathcal{T}_{\text{weak}} = \bigcup_{\text{arbitrary finite}} \bigcap_{\phi^{-1}(U)$$

DEFINITION		
	strongly / weakly converging	
		Analysis
DEFINITION		
	Weak-* Convergence	
		Analysis
DEFINITION		
	inner product space	
	mile produce space	
		Analysis

Let $\{\phi_n\}_{n=1}^{\infty}$ be a sequence in X^* and $\phi \in X^*$.

- 1. Strong Convergence: $\phi_n \to \phi$ strongly in X^* provided $\|\phi_n \phi\|_{X^*} \to 0$
- 2. Weak Convergence: $\phi_n \rightharpoonup \phi$ weakly in X^* means for all $F \in X^{**}, F(\phi_n) \to F(\phi)$.

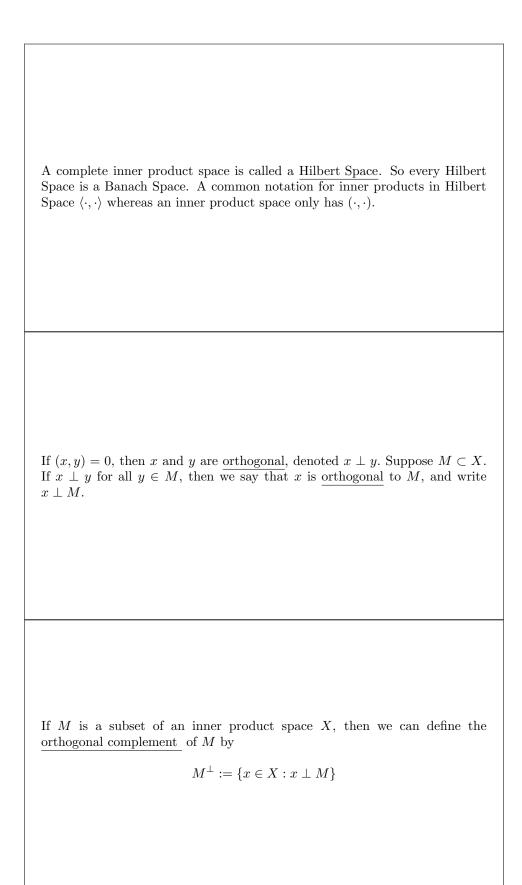
Weak-* Convergence $\phi_n \stackrel{*}{\rightharpoonup} \phi$ in X if $\phi_n(x) \to \phi(x)$ for all $x \in X$.

An inner product space on a complex linear space X is a map $\langle \cdot, \cdot \rangle : X \times X \to \mathbb{C}$ such that for all $x, y, z \in X$, and $\lambda, \mu \in \mathbb{C}$:

- 1. $\langle x, \lambda y + \mu y \rangle = \lambda \langle x, y \rangle + \mu \langle x, y \rangle$
- 2. $\langle x, y \rangle = \overline{\langle y, x \rangle}$ (Hermitian symmetric)
- 3. $\langle x, x \rangle \ge 0$ and $\langle x, x \rangle = 0 \iff x = 0$

A linear space with an inner product is called an $\underline{\text{inner product space}}$ or a pre-Hilbert space.

DEFINITION		
	Hilbert Space	
		Analysis
DEFINITION		
	orthogonal	
		Analysis
Definition		
	orthogonal complement	
		Analysis



DEFINITION		
	orthogonal projection	
		Analysis
DEFINITION		
	orthonormal set	
		Analysis
DEFINITION		
isomorj	phic between inner product spaces	
		Analysis

A projection $P:\mathcal{H}\to\mathcal{H}$ is called orthogonal if for all $x,y\in\mathcal{H},$

$$\langle x, Py \rangle = \langle Px, y \rangle$$

Let X be an inner product space. A subset $S=\{e_\alpha:\alpha\in A\}\subset X$ is called <u>orthonormal</u> if it satisfies

- 1. (Orthogonality) $e_{\alpha} \perp e_{\beta}$ for all $\alpha, \beta \in A, \alpha \neq \beta$.
- 2. (Normalized) $||e_{\alpha}|| = 1$ for all $\alpha \in A$

In other words, $\langle e_{\alpha}, e_{\beta} \rangle = \delta_{\alpha,\beta} = \begin{cases} 1 & \alpha = \beta \\ 0 & \alpha \neq \beta \end{cases}$

Let $(X_1,(\cdot,\cdot)_1)$ and $(X_2,(\cdot,\cdot)_2)$ be two inner product spaces. If there exists an isomorphism $T:X_1\to X_2$ such that for all $x,y\in X$:

$$(Tx, Ty)_2 = (x, y)_1$$

Then we say that the inner product space X_1, X_2 are isomorphic.

DEFINITION		
DEFINITION		
	σ -algebra	
		Analysis
DEFINITION		
DEFINITION		
	algebra	
		Analysis
DEFINITION		
DEFINITION		
	measureable space	
		Analysis

A $\sigma\text{-algebra}$ on a set X is a collection $\mathscr A$ of subets of X such that

- 1. $\emptyset \in \Sigma$
- 2. If $A \in \Sigma$, then $A^c \in \Sigma$
- 3. If A_1, A_2, \ldots , is a countable family of sets in Σ , then

$$\bigcup_{i=1}^{\infty} A_i \in \Sigma$$

An <u>algebra</u> on a set X is a collection $\mathscr A$ of subets of X such that

- 1. $\emptyset \in \Sigma$
- 2. If $A \in \Sigma$, then $A^c \in \Sigma$
- 3.

$$\bigcup_{i=1}^{N} A_i \in \Sigma$$

A measureable space (X, \mathscr{A}) is a set X and a σ -algebra Σ on X. Elements of Σ are called measurable sets.

DEFINITION		
	measure	
		Analysis
DEFINITION		
	measure space	
		Analysis
DEFINITION		
	measure	
		Analysis

A measure μ on X is a function

$$\mu: \Sigma \to [0, \infty]$$

such that

- 1. $\mu(\emptyset) = 0$
- 2. μ is countably additive:

$$\mu(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mu(A_i)$$

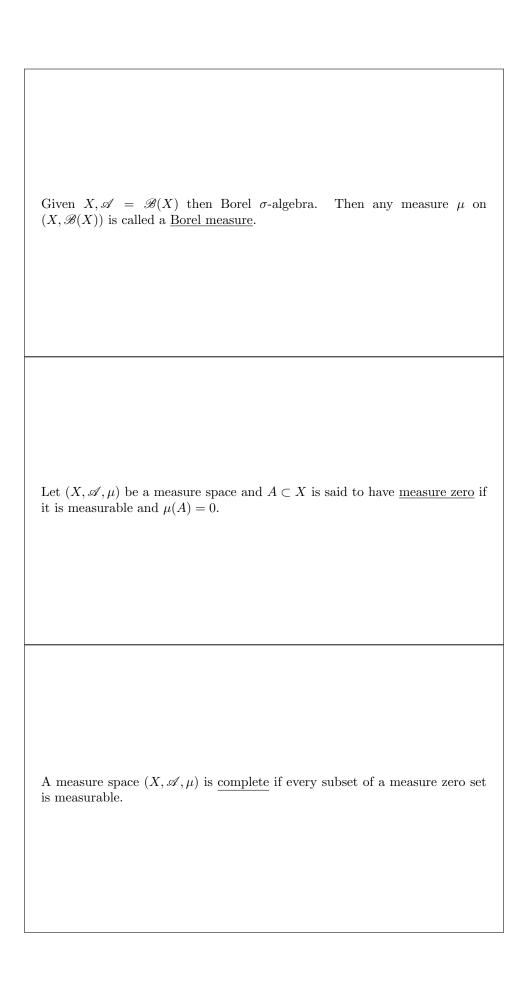
for each countable family of mutually disjoint sets $\{E_n\}_{n=1}^{\infty} \subset \Sigma$.

A measure space (X, \mathscr{A}, μ) is a set X, a σ -algebra \mathscr{A} on X, and a measure $\mu: \overline{\mathscr{A} \to [0, \infty]}$.

A measure μ is said to be

- finite if $\mu(X) < \infty$
- $\underline{\sigma\text{-finite}}$ if there exists a sequence $\{A_i\} \subset \mathscr{A}$ such that $X = \bigcup_{i=1}^{\infty} A_i$ and $\mu(A_i) < \infty$ for all n.
- probability measure if $\mu(X) = 1$.

DEFINITION		
	Borel measure	
		Analysis
DEFINITION		
	measure zero	
		Analysis
Definition		
	complete measure space	
		Analysis



DEFINITION		
	$\overline{\mathscr{A}}$	
		Analysis
DEFINITION		
almost	everywhere property	
		Analysis
DEFINITION		
$m\epsilon$	easurable function	
		Analysis

Given a measure space (X, \mathscr{A}, μ) , we define $\overline{\mathscr{A}}$ to be the σ -algebra generated by

 $\mathcal{A} \bigcup \{ \text{subsets of measure zero sets} \}$

Then

$$\overline{\mathscr{A}} = \{A: \exists E, F \in \mathscr{A} \text{ such that } E \subset A \subset F, \mu(F \setminus F) = 0\}$$

So for all $A \in \bar{\mathcal{A}}$, define

$$\overline{\mu(A)}:=\mu(E)=\mu(F)$$

Then the complete measure space $(X, \overline{\mathscr{A}}, \overline{\mu})$ is called the <u>completion</u> of (X, \mathscr{A}, μ) .

A property that hold except on a set of measure zero is said to hold almost everywhere or <u>a.e.</u> for short. If there measure is obvious, then explicitly write $\mu-a.e.$.

Let (X, \mathscr{A}) be a measurable space. A real-valued function $f: X \to \mathbb{R}$ is a measurable function (with respect to \mathscr{A}) if for all $t \in \mathbb{R}$,

$$f^{-1}((t,\infty)) = \{x \in X : f(x) > t\}$$

is measurable.

DEFINITION		
	$(\mathscr{A}_X, \mathscr{A}_Y)$ -measurable	
		Analysis
DEFINITION		
	product measure	
		Analysis
Definition		
	L^p space	
		Analysis

A function $f:(X, \mathscr{A}_X) \to (Y, \mathscr{A}_Y)$ between two measurable spaces is called $(\mathscr{A}_X, \mathscr{A}_Y)$ -measurable if $f^{-1}(\mathscr{A}_Y) \subset \mathscr{A}_X$.

Let (X, \mathcal{A}, μ) and (Y, \mathcal{B}, ν) be two σ -finite measure spaces. For any $E \in \mathcal{A} \times \mathcal{B}$, define

$$(\mu \times \nu)(E) := \int_{X} \nu(E_x) d\mu(x) = \int_{Y} \mu(E_y) d\nu(y)$$

 $\mu \times \nu$ is a measure on $\mathscr{A} \times \mathscr{B}$, called the product measure.

Let $1 \leq p < \infty$ and let (X, \mathscr{A}, μ) be a measure space. Then

$$||f||_{L^p(X)} := \left[\int_X |f|^p \ d\mu \right]^{\frac{1}{p}}$$

Then the L^p space is defined by:

$$L^p(X) := \{f: X \to [-\infty, \infty]: f\mathscr{A} - \text{measurable with } \|f\|_{L^p(X)} < \infty\}$$

Further, $L^p(X)$ consists of equivalence classes. That is, given $f, g \in L^p(X)$, then

$$f \equiv g \iff f = ga.e.$$

DEFINITION		
	$I \infty (V)$	
	$L^{\infty}(X)$	
		Analysis
DEFINITION		
	$L^{p,\mathrm{weak}}(X)$	
	L^{r} (A)	
		Analysis
DEFINITION		
	distribution function	
	distribution function	
		Analysis

$$L^{\infty}(X):=\{f:X\rightarrow [-\infty,\infty]:f\mathscr{A}-\text{measurable with } \left\|f\right\|_{L^{\infty}(X)}<\infty\}$$

where

$$\|f\|_{L^{\infty}(X)}:=\inf\{M:\mu(\{x\in X:|f|>M\})=0\}$$

Given (X, \mathscr{A}, μ) a measure space with $1 \leq p < \infty$. We define the space

$$L^{p,\text{weak}}(X)$$

as the set of functions $f \in L^{p,\text{weak}}(X)$ provided $f: X \to \mathbb{R}$ is a \mathscr{A} -measurable function and

$$\mu(\{x \in X : |f(x)| > \lambda\}) \le \frac{C^p}{\lambda^p}$$

for all $\lambda > 0$ and some choice constant C_{λ} .

The <u>distribution function</u> $m_{\mu}(\lambda) : \mathbb{R}^+ \to [0, \infty]$ by

$$m_{\mu}(\lambda) := \mu(\{x \in X : |f(x)| > \lambda\})$$

DEFINITION		
	$\ f\ _{L^{p,\operatorname{weak}}(X)}$	
		Analysis
DEFINITION		
	simple function	
		Analysis
DEFINITION		
	step function	
		Analysis

$$||f||_{L^{p,\text{weak}}(X)} = \sup_{\lambda > 0} \left\{ \lambda m_{\mu}(\lambda)^{\frac{1}{p}} \right\}$$

A function of the form

$$\phi(x) = \sum_{i=1}^{N} a_i \mathcal{X}_{A_i}(x)$$

with $A_i \subset X$ measurable with $\mu(A_i) < \infty, |a_i| < \infty$ is a simple function.

On the Lebesgue measure space $(X, \mathcal{B}(X), \lambda)$, we define the <u>step function</u> of the form

$$\phi(x) = \sum_{i=1}^{N} a_i \mathcal{X}_{Q_i}(x)$$

where Q_i are pairwise disjoint cubes of the form:

$$Q_i := \prod_{j=1}^n [\alpha_j, \alpha_j + \ell)$$

with $\alpha_i, \ell \in \mathbb{R}^+$ and $|a_i| < \infty$.

DEFINITION		
	$C_c(\mathbb{R}^n)$	
		Analysis
DEFINITION		
	bounded linear functional	
		Analysis
DEFINITION		
	absolutely continuous	
		Analysis

 $C_c(\mathbb{R}^n)$ = the space of continuous functions of compact support.

A bounded linear functional on $L^p(X)$ is a mapping $\ell:L^p(X)\to\mathbb{R}$ with

$$\ell(af + bg) = a\ell(f) + b\ell(g)$$

and

$$|\ell(f)| \le C \|f\|_{L^p}$$

for some constant C. Moreover, we define

$$\|\ell\| := \inf\{C : |\ell(f)| \le C \|f\|_{L^p} \, \forall f \in L^p\}$$

Suppose we have a measurable space (X,\mathscr{A}) equipped with two finite measures μ,ν . ν is absolutely continuous with respect to μ , denoted $\nu\ll\mu$, provided

$$\mu(B) = 0 \implies \nu(B) = 0$$

DEFINITION		
	signed measure	
		Analysis
DEFINITION		
	Hardy-Littlewoord Maximal Function	
		Analysis
DEFINITION		
	f * g	
		Analysis

A function ν on a measurable space (X, \mathscr{A}) is a signed measure provided:

- 1. For any $E \in \mathscr{A}, \nu(E) \in [-\infty, \infty]$
- 2. $\nu(\emptyset) = 0$
- 3. ν is countably additive.

Let $f \in L^1_{loc}(\mathbb{R}^n)$ (i.e.

$$\int_K |f| d\mu < \infty$$

for any compact set $K\subset \mathbb{R}^n),$ then we can define the Hardy-Littlewoord Maximal Function by

$$\mu f(x) := \sup_{r>0} \frac{1}{\mu(B_r(x))} \int_{B_r(x)} |f(y)| \ dy$$

If $f, g: \mathbb{R}^n \to \mathbb{R}$, then

$$(f * g)(x) := \int_{\mathbb{R}^n} f(x - y)g(y) \ dy$$

DEFINITION		
	approximation identity	
		Analysis
DEFINITION		
	$W^{k,p}(\Omega)$	
		Analysis
DEFINITION		
	C^1 -boundary	
	C -Doundary	
		Analysis

Suppose $K \in L^1$ satisfies the following properties:

1.
$$\lim_{r\to 0} \int_{|x|>\delta} |K_r(x)| dx = 0$$

2.
$$\int K(x) dx = 1$$

Then K is called an approximation identity.

Given $\Omega \subset \mathbb{R}^n$ open, with $1 \leq p \leq \infty$ and $K \in \mathbb{Z}_{\geq 0}$, then

$$W^{k,p}(\Omega):=\{u:\Omega\to\mathbb{R}^n:D^\alpha u \text{ exists } \forall |\alpha|\leq k \text{ and } D^\alpha u\in L^p(\mathbb{R}^n) \ \forall |\alpha|\leq k\}$$

Further, we equip this space with the norm:

$$||f||_{W^{k,p}(\Omega)} = \sum_{|\alpha| \le k} ||D^{\alpha}f||_{L^p(\Omega)}$$

Given a region Ω , a $\underline{C^1$ -boundary $\partial\Omega$ is one such that for every $x_0\in\partial\Omega$, there exists r>0 and an $\underline{C^1$ -embedding-function

$$\gamma: \mathbb{R}^{n-1} \to \mathbb{R}^n$$

such that

$$B_r(x_0) \cap \Omega = \{(x_1, \dots, x_n) \in B_r(x_0) : x_n > \gamma(x_1, \dots, x_{n-1})\}$$

DEFINITION		
	boundary value of $u \in W^{1,p}(\Omega)$	
		Analysis
Definition		
	Hölder α -continuous	
		Analysis
Definition		
	$C^{0,lpha}(\Omega)$	
		Analysis

The boundary value of $u \in W^{1,p}(\Omega)$ is defined as

$$u|_{\partial\Omega} := \lim_{\epsilon\downarrow 0} Eu * \phi_{\epsilon}|_{\partial\Omega}$$

where ϕ_{ϵ} is a standard mollifier.

A function $f:\Omega\to\mathbb{R}$ is <u>Hölder α -continuous</u> provided there exists a constant C such that

$$|f(x) = f(y)| \le C|x - y|^{\alpha}$$

for all $x, y \in \Omega$.

The space $C^{0,\alpha}(\Omega)=C^\alpha(\Omega)=$ is the space of Hölder α -continuous functions equipped with the finite norm

$$||f||_{C^{\alpha}(\Omega)} = \sup_{x \in \Omega} |f(x)| + \sup_{x,y \in \Omega, x \neq y} \frac{|f(x) - f(y)|}{|x - y|}$$

Definition		
	Fourier Transform	
		Analysis
DEFINITION		
	$H^s(\mathbb{R}^n)$	
		Analysis
		ANALISIS
DEFINITION		
	function	
	типстоп	
		Analysis

Suppose $f \in L^1(\mathbb{R}^n)$, we define the <u>Fourier Transform</u> by

$$\hat{f}(\xi) = \int_{\mathbb{R}^n} f(x)e^{-2\pi i \xi x} \ dx$$

Given $s \in \mathbb{R}$, define the space

$$H^{s}(\mathbb{R}^{n}) := \{ f \in \mathcal{S}(\mathbb{R}^{n}) : (\sqrt{1 + |\xi|^{2}})^{s} \hat{f}(\xi) \in L^{2}(\mathbb{R}) \}$$

- $s > 0 \implies$ fractional differentiation.
- $s < 0 \implies$ fractional integration.

We equip this space with the norm:

$$||f||_{H^s(\mathbb{R}^n)} := \left\| (\sqrt{1+|\xi|^2})^s \hat{f}(\xi) \right\|_{L^2(\mathbb{R}^n)}$$

For $0 < \alpha < n$, we define the generalized Riesz Potential

$$(I_{\alpha}f)(x) := \int_{\mathbb{R}^n} \frac{f(x)}{|x - y|^{n - \alpha}} dy$$

Definition		
Space of Schwartz Functions		
	Analysis	
	ANALISIS	
DEFINITION		
Weak Convergence / Strong Convergence in $(X, \ \cdot\)$		
	Analysis	
DEFINITION		
spectral radius		
	Analysis	

The Space of Schwartz Functions, denoted $S(\mathbb{R}^n)$, is defined as:

$$\mathcal{S}(\mathbb{R}^n) := \{ u \in C^\infty(\mathbb{R}^n) : \|u\|_{\alpha,\beta} = \sup_{x \in \mathbb{R}^n} |x^\alpha D^\beta u| < \infty \ \forall \alpha, \beta \in \mathbb{Z}^n_{\geq 0} \}$$

where we equip this space with a semi-norm:

$$[f]_{\alpha,\beta} := \sup_{x \in \mathbb{R}^n} |x^{\alpha} D^{\beta} u|$$

Let $(X, \|\cdot\|)$ be a normed linear space. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in X. We say

1. x_n converges strongly to x if

$$||x_n - x|| \to 0$$

denoted $x_n \to x$

2. x_n converges weakly to x if

$$\phi(x_n) \to \phi(x)$$

for all $\phi \in X^*$. Denoted $x_n \rightharpoonup x$.

For any $A\in B(X),$ the number $r(A)=\sup_{\lambda\in\sigma(A)}|\lambda|$ is called the spectral radius of A.

Definition		
	resolvent	
		Analysis
DEFINITION		
	spectrum	
		Analysis
DEFINITION		
	convex combination	
		Analysis

Let X be a Banach space and $A \in B(X)$. Then

$$\rho(A) = \{ \lambda \in \mathbb{C} : \lambda I - A \text{ is invertible} \}$$

For all $\lambda \in \rho(A)$, let

$$R_{\lambda} = (\lambda I - A)^{-1}$$

be the <u>resolvent</u> of A at λ .

The spectrum of A is the set

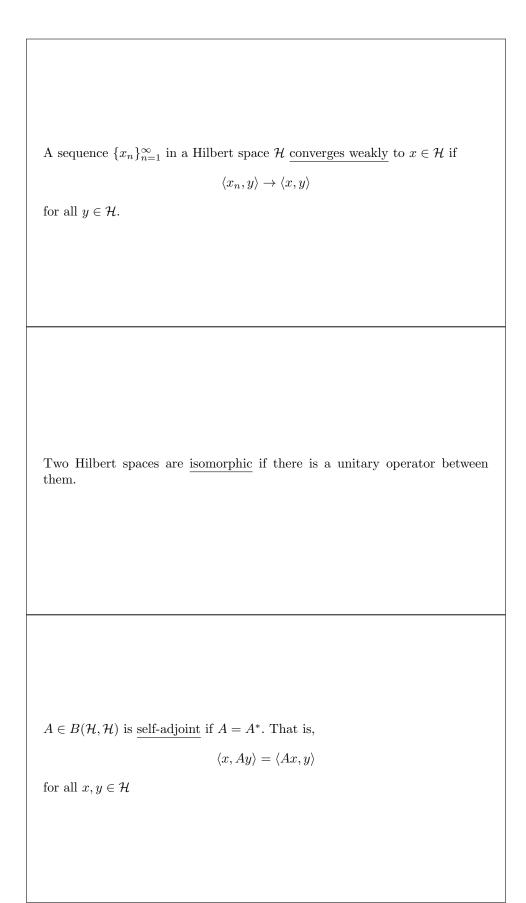
$$\sigma(A) := \mathbb{C} \setminus \rho(A) = \{\lambda \in \mathbb{C} : \lambda I - A \text{ is not invertible}\}$$

Given $\{x_1, x_2, \dots, x_n\}$ of a linear space X, we define a the set

$$C = \left\{ y = \sum_{k=1}^{n} \lambda_k x_k : \lambda_k \ge 0, \sum_{k=1}^{n} \lambda_k = 1 \right\}$$

each y is called a <u>convex combination</u> of $\{x_1, x_2, \dots, x_n\}$.

DEFINITION	weak convergence in a Hilbert Space	
		Analysis
DEFINITION		
	Isomorphic Hilbert Spaces	
		Analysis
DEFINITION		
	self-adjoint	
		Analysis



DEFINITION		
	adjoint operator	
		Analysis

Let $A:\mathcal{H}\to\mathcal{H}$ be a bounded linear operator on a Hilbert space \mathcal{H} . There exists a unique bounded linear operator $A^*:\mathcal{H}\to\mathcal{H}$ such that $\langle x, Ay \rangle = \langle A^*x, y \rangle$ for all $x, y \in \mathcal{H}$. A^* is called the adjoint of the operator.