

Magnesium diboride (MgB_2)

- Conventional superconductor with special features
 - double gap model
 - highest T_c among BCS superconductors (39K)

MgB₂ thin films

- Two samples (#2, #3) out of three have undergone neutron scattering:
 - destroys anisotropy
 - makes film properties similar to a single-gap BCS superconductor

BCS model predictions

- Single gap, plus a phenomenological parameter (Dynes' Γ)

BCS model predictions

$$\nu_{(\Gamma, \Delta)}(E) \equiv \frac{N(E)}{N(0)} = \left| \Re \left(\frac{E - i\Gamma}{\sqrt{(E - i\Gamma)^2 - \Delta^2}} \right) \right|$$

$$f(u) = \frac{1}{\exp(u) + 1}, \text{ con } u = \frac{E}{k_B T}$$

$$\left(\frac{dI}{dV} \right)_{(\Gamma, \Delta)}(V) = \frac{1}{k_B T} \int \nu_{(\Gamma, \Delta)}(E) \left(-\frac{df}{du} \left(\frac{E - eV}{k_B T} \right) \right) dE$$

Double gap

- DoS (hence, dI/dV) as a linear combination of two single-gap Densities of States
 - “normalized” coefficients $\alpha_\pi + \alpha_\sigma = 1$
 - w/ constraint $0.66 < \alpha_\pi < 1$

$$\left(\frac{dI}{dV}\right)_{(\Gamma_\pi, \Delta_\pi, \Gamma_\sigma, \Delta_\sigma, \alpha_{\pi|\sigma})}^{MgB_2}(V) = \alpha_\pi \left(\frac{dI}{dV}\right)_{(\Gamma_\pi, \Delta_\pi)}(V) + \alpha_\sigma \left(\frac{dI}{dV}\right)_{(\Gamma_\sigma, \Delta_\sigma)}(V)$$

Computational difficulties

- The conductivity curve depends non linearly on Γ and Δ through a *numerical* integral (no analytical solution)
- Double-gap model leads to five independent parameters: Γ_{π} , Δ_{π} , Γ_{σ} , Δ_{σ} , and one coefficient e.g. α_{π}

Finding the best fit (method 1)

- Simplex
 - most powerful method, with limitations
 - doesn't need derivatives
 - χ^2 is “yet another function” to find a minimum for
 - *can't* estimate statistical errors

Finding the best fit (method 2)

- `gsl_multifit_*`
 - not a generic minimization procedure
 - less robust than *Simplex* on “irregular” functions but...
 - ...can estimate statistical covariances and sigmas!
 - $\min \chi^2(\Gamma_{\pi}, \Delta_{\pi}, \Gamma_{\sigma}, \Delta_{\sigma}, \alpha_{\pi})$ ***Levenberg-Marquardt***

Best of both worlds?

- Simplex method to find a good minimum
- Starting from the last result, `gsl_multifit_fdfsolver_iterate()` to (slightly) improve the fit, but most importantly compute statistical errors

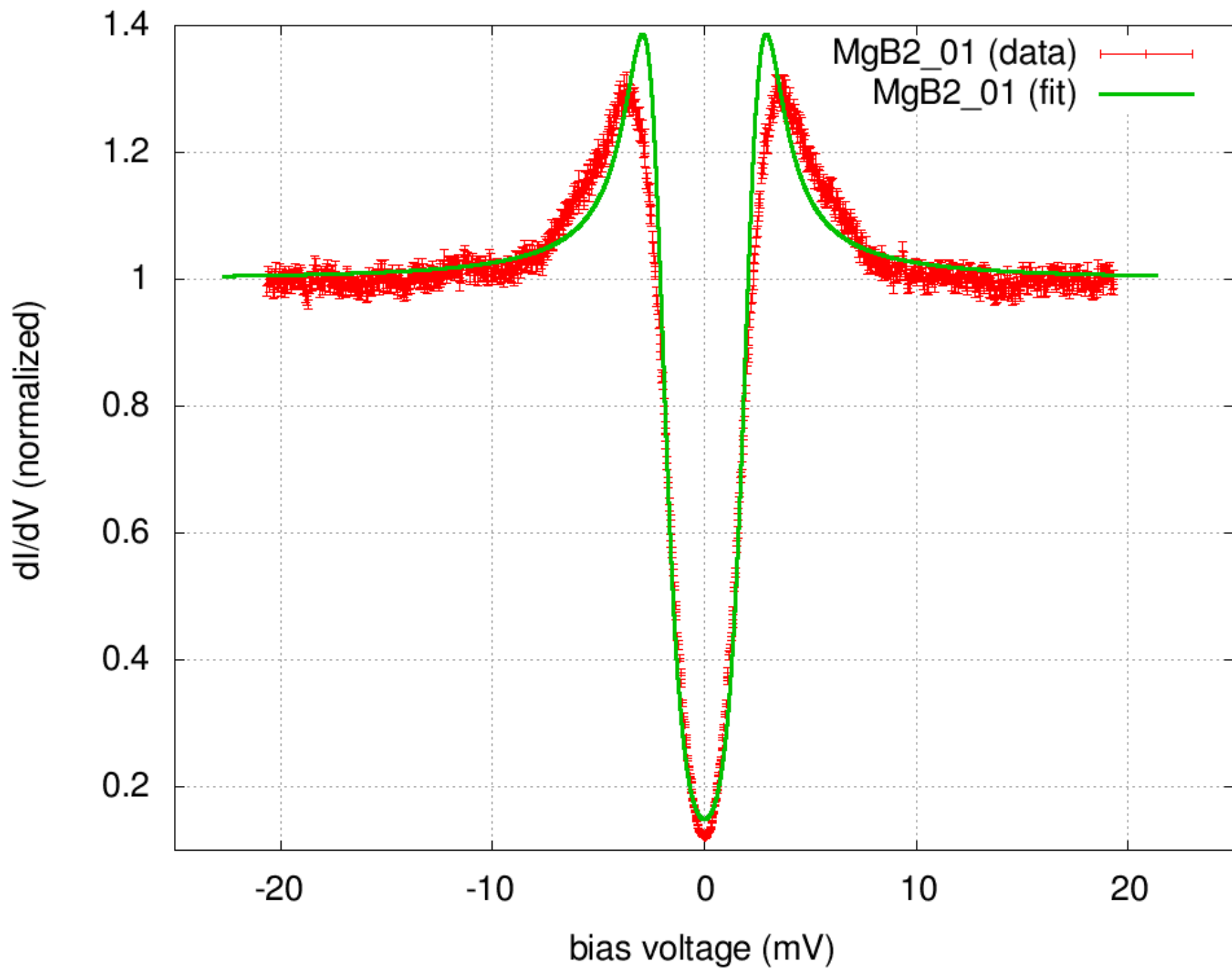


<https://www.youtube.com/watch?v=LLLq7-0DivI>

<https://bitbucket.org/gderosa/didvsuperc/src>

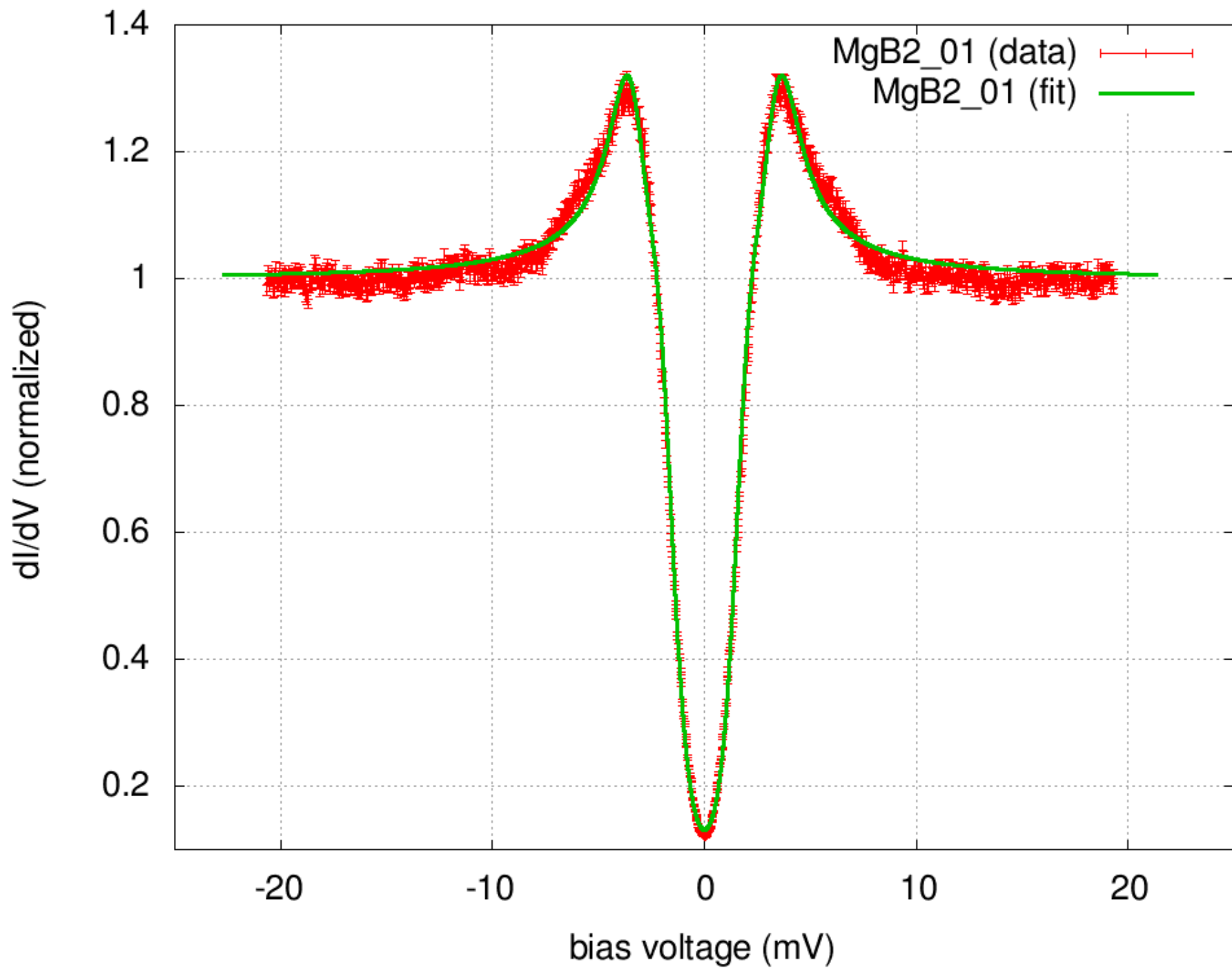
Results

- Sample #1; single gap
 - $\Gamma = 0.2639 \pm 0.0023$
 - $\Delta = 2.2327 \pm 0.0023$
 - $\chi^2/\text{DoF} = 17.669$



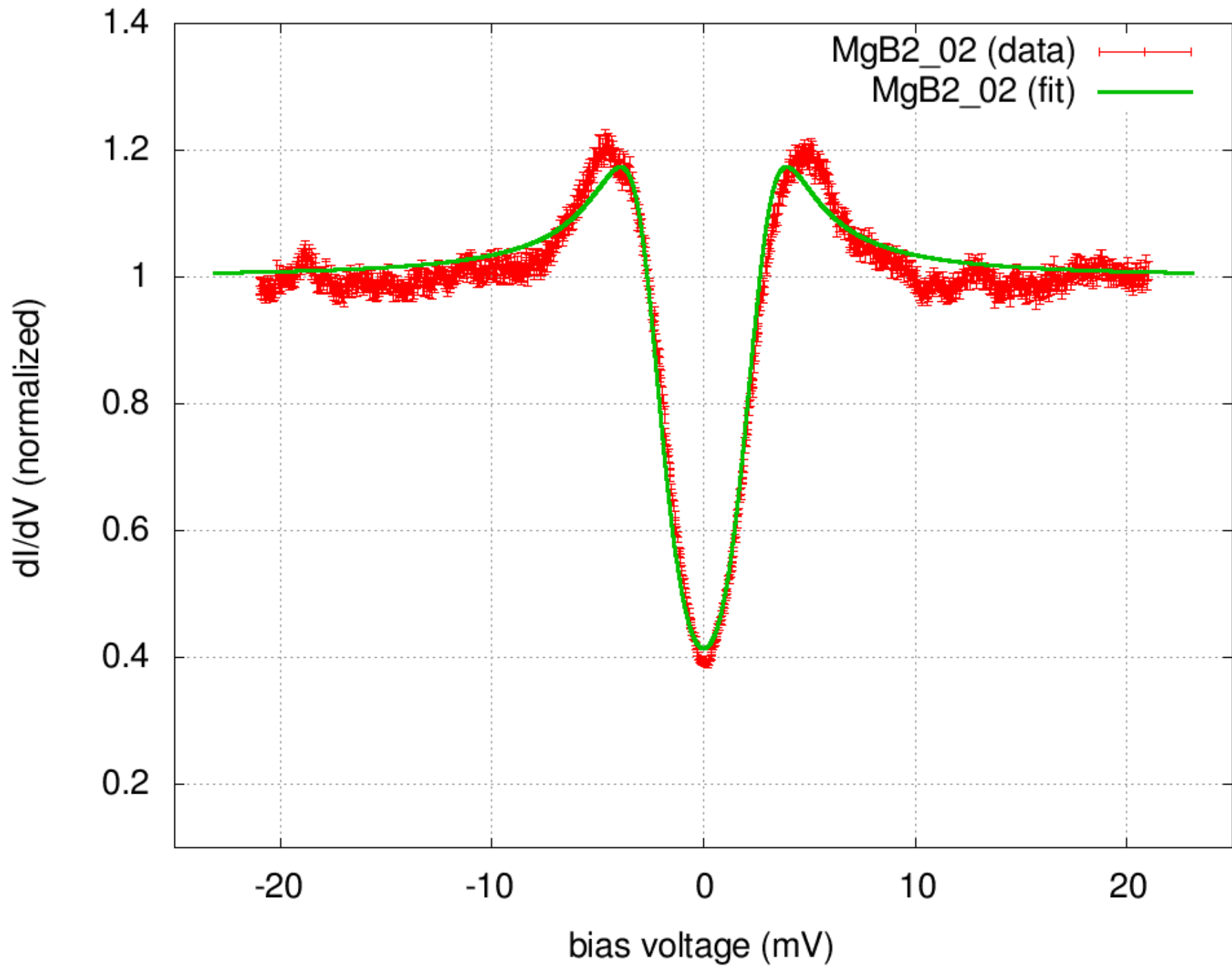
A better fit with a double gap

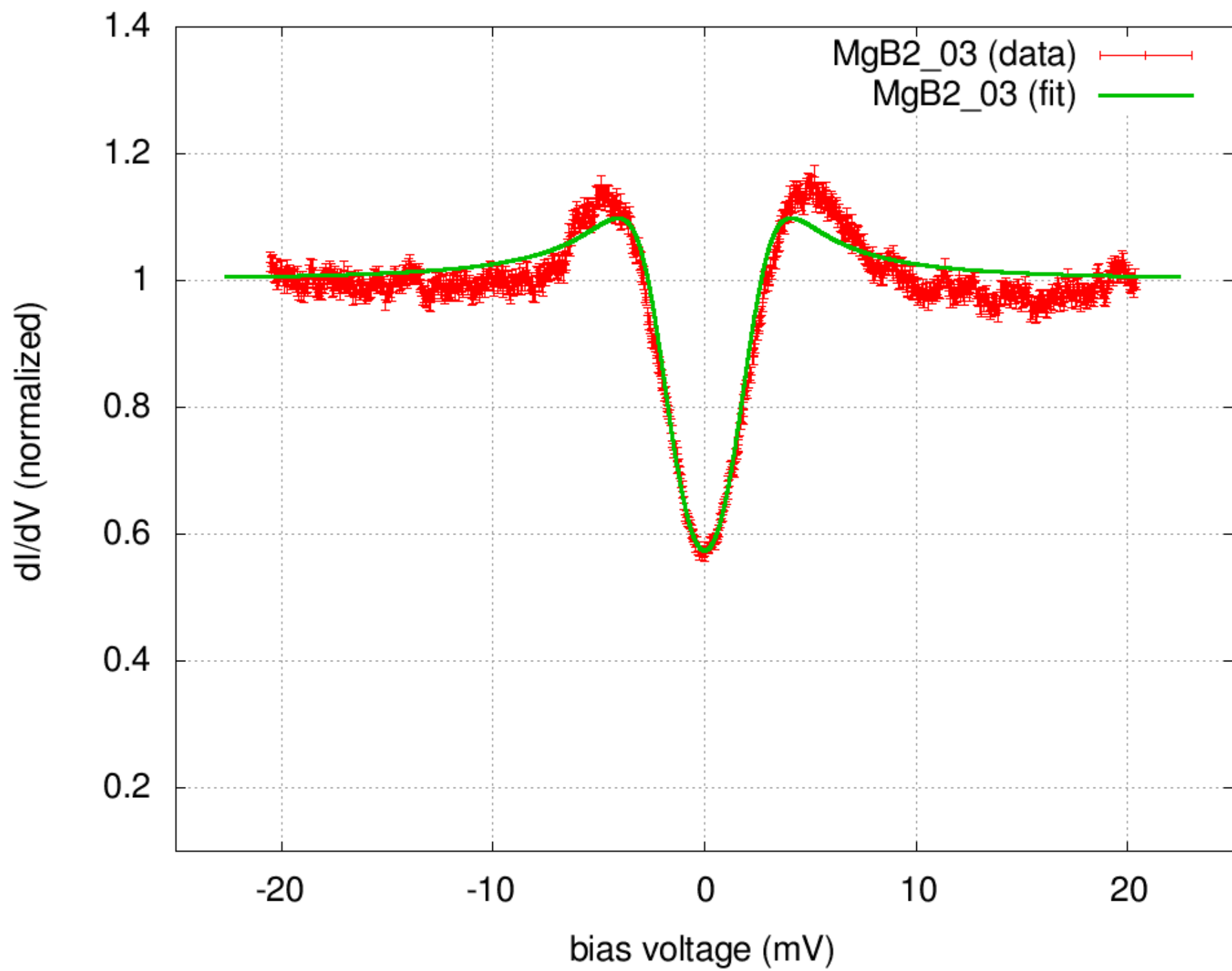
- Sample #1; double gap
 - $\Gamma = 0.1866 \pm 0.0019$
 - $\Delta_{\pi} = 1.7895 \pm 0.0050$
 - $\Delta_{\sigma} = 3.2348 \pm 0.0073$
 - $\alpha_{\pi} = 0.6660 \pm 0.0027$
 - $\chi^2/\text{DoF} = 3.4737$

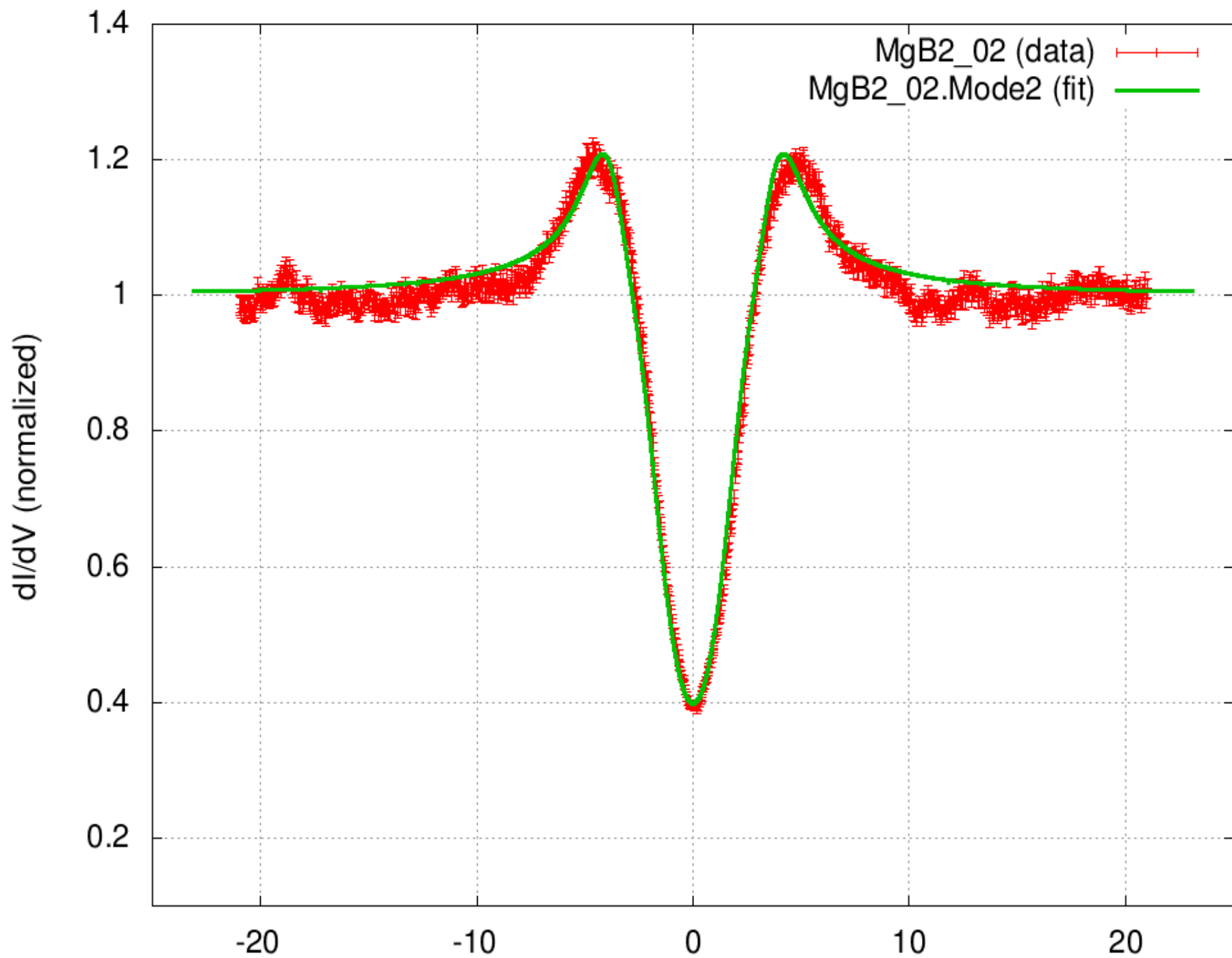


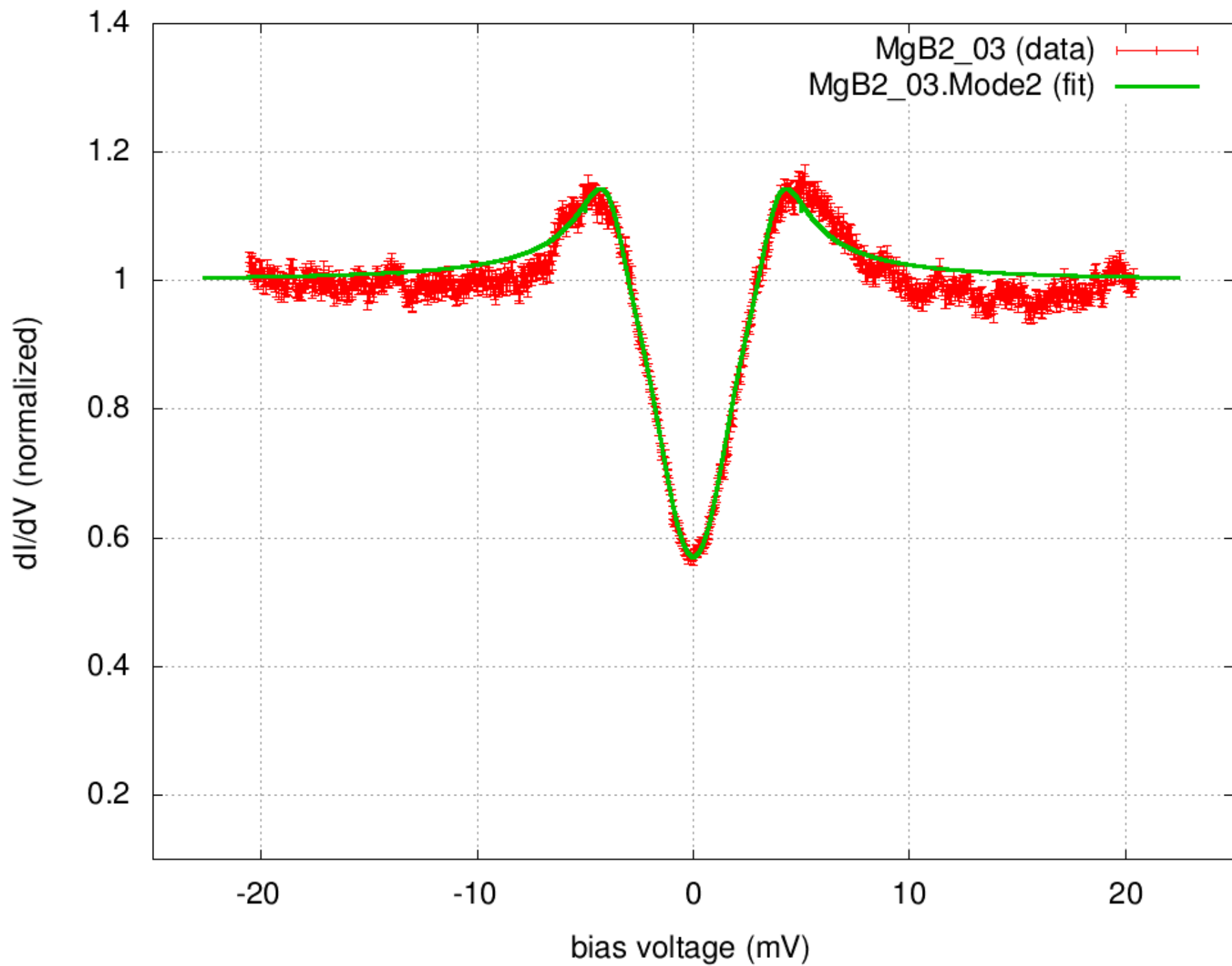
The single gap model fits better
with “disordered” (neutron-
scattered) films

Samples #2 and #3, single-gap first (Γ, Δ),
then double-gap ($\Gamma_{\pi}, \Delta_{\pi}, \Gamma_{\sigma}, \Delta_{\sigma}, \alpha_{\pi}$)









Results

	MgB2_01	MgB2_02	MgB2_03
Γ	0.2639 ± 0.0023	1.0753 ± 0.0037	1.4726 ± 0.0054
Δ	2.2327 ± 0.0023	2.5983 ± 0.0035	2.2969 ± 0.0053
$\tilde{\chi}^2$	17.669	5.0787	4.8334
Γ	0.1866 ± 0.0019	0.901 ± 0.013	0.590 ± 0.017
Δ_π	1.7895 ± 0.0050	2.018 ± 0.022	0.8620 ± 0.0094
Δ_σ	3.2348 ± 0.0073	3.433 ± 0.032	3.406 ± 0.019
α_π	0.6660 ± 0.0027	0.6661 ± 0.0064	0.6687 ± 0.0054
$\tilde{\chi}^2$	3.4737	4.4335	4.3942
Γ_π	0.2013 ± 0.0055	0.946 ± 0.012	1.297 ± 0.021
Γ_σ	0.086 ± 0.028	0.350 ± 0.075	0.452 ± 0.081
Δ_π	1.7591 ± 0.0063	2.025 ± 0.024	1.643 ± 0.031
Δ_σ	3.178 ± 0.011	3.664 ± 0.039	3.650 ± 0.042
α_π	0.666 ± 0.011	0.794 ± 0.019	0.813 ± 0.017
$\tilde{\chi}^2$	3.2813	4.1305	3.5091