Academic Assistant Agent - Solved Assignment

Question 1:

Calculate the de Broglie wavelength of (a) 46g mg gold ball with velocity of 30m/s. and (b) an electron with a velocity (v) of 10⁷ m/s. Analyze the value obtained to understand wave behaviour of moving electron. Consider the value v<<c.

Answer:

(a) Gold ball:

The de Broglie wavelength is given by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant ($6.626 \times 10^{-34} Js$), m is the mass, and v is the velocity.

Given: $m = 46 \times 10^{-3} kg$, $v = 30 m/s$

$$\lambda = \frac{6.626 \times 10^{-34} Js}{(46 \times 10^{-3} kg)(30 m/s)} \approx 4.8 \times 10^{-34} m$$

This wavelength is incredibly small and far beyond any measurable scale, demonstrating that the wave nature of macroscopic objects is insignificant.

(b) Electron:

Given: $m_e = 9.11 \times 10^{-31} kg$, $v = 10^7 m/s$

$$\lambda = \frac{6.626 \times 10^{-34} Js}{(9.11 \times 10^{-31} kg)(10^7 m/s)} \approx 7.27 \times 10^{-11} m$$

This wavelength is much larger than that of the gold ball, and is in the X-ray region of the electromagnetic spectrum. This signifies that the wave nature of electrons is significant at this velocity and is observable through phenomena like electron diffraction.

Analysis: The vast difference in wavelengths highlights the significance of the de Broglie wavelength in understanding wave-particle duality. Macroscopic objects have wavelengths too small to observe wave-like behavior, while microscopic particles like electrons exhibit significant wave properties at even moderate velocities.

Question 2:

A 50g marble is in a box of 10cm across. Find the permitted energies and show that in the domain of everyday experience, quantum effects are imperceptible.

Answer:

The permitted energies of a particle in a 1-D box are given by:

$$E_n = \frac{n^2 h^2}{8mL^2}$$

where n is the quantum number ($n=1, 2, 3,...$), h is Planck's constant, m is the mass, and L is the length of the box.

Given: $m = 50 \times 10^{-3} kg$, $L = 0.1 m$

For $n=1$:

$$E_1 = \frac{(1)^2 (6.626 \times 10^{-34} Js)^2}{8(50 \times 10^{-3} kg)(0.1 m)^2} \approx 1.1 \times 10^{-66} J$$

For $n=2$:

$$E_2 = 4E_1 \approx 4.4 \times 10^{-66} J$$

The energy difference between levels is extremely small. To put this into perspective, consider the thermal energy of the marble at room temperature ($\approx 300 K$). The average thermal energy is given by $\frac{3}{2}k_BT$, where k_B is the Boltzmann constant ($1.38 \times 10^{-23} J/K$).

$$\frac{3}{2}k_BT \approx \frac{3}{2}(1.38 \times 10^{-23} J/K)(300 K) \approx 6.21 \times 10^{-21} J$$

This thermal energy is vastly greater than the energy level spacing. Thus, the quantum effects (discrete energy levels) are completely overwhelmed by thermal fluctuations, making them imperceptible in everyday experience.

Question 3:

A hydrogen atom is 5.3 x 10⁻¹¹ m in radius. Estimate the minimum energy an electron can have in this atom. Analyze the value obtained.

Answer:

We can use the uncertainty principle to estimate the minimum energy. The uncertainty in position is approximately the radius of the atom, $\Delta x \approx 5.3 \times 10^{-11} m$. The uncertainty principle states:

$$\Delta x \Delta p \ge \frac{h}{4\pi}$$

Assuming the minimum uncertainty, we have:

$$\Delta p \approx \frac{h}{4\pi \Delta x} = \frac{6.626 \times 10^{-34} Js}{4\pi (5.3 \times 10^{-11} m)} \approx 9.9 \times 10^{-25} kg m/s$$

The minimum momentum is approximately Δp, and the minimum kinetic energy is:

$$E_{min} \approx \frac{p^2}{2m_e} = \frac{(9.9 \times 10^{-25} kg m/s)^2}{2(9.11 \times 10^{-31} kg)} \approx 5.4 \times 10^{-19} J$$

Converting to electron volts ($1 eV = 1.6 \times 10^{-19} J$):

$$E_{min} \approx \frac{5.4 \times 10^{-19} J}{1.6 \times 10^{-19} J/eV} \approx 3.4 eV$$

This is a rough estimate, but it's on the same order of magnitude as the ground state energy of a hydrogen atom (-13.6 eV). The positive sign indicates that this is the minimum kinetic energy; the total energy will be negative due to the potential energy.

Question 4:

An excited atom gives up its excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and time it radiates is 10ns. Find the inherent uncertainty in the frequency of the photon. Analyze the value obtained.

Answer:

The uncertainty principle relating energy and time is:

$$\Delta E \Delta t \ge \frac{h}{4\pi}$$

Since $E = hf$, where f is the frequency, we have $\Delta E = h \Delta f$. Therefore:

$$h \Delta f \Delta t \ge \frac{h}{4\pi}$$

$$\Delta f \ge \frac{1}{4\pi \Delta t}$$

Given $\Delta t = 10 ns = 10 \times 10^{-9} s$:

$$\Delta f \ge \frac{1}{4\pi (10 \times 10^{-9} s)} \approx 8 \times 10^6 Hz$$

This uncertainty in frequency is relatively small compared to typical atomic transition frequencies (which are on the order of $10^{14} Hz$). However, it demonstrates that the frequency of the emitted photon is not perfectly defined due to the finite lifetime of the excited state.

Question 5:

Find the de Broglie wavelength of the 40keV electrons used in the electron microscopy. Analyze the value obtained.

Answer:

First, convert the energy to Joules:

$$E = 40 keV = 40 \times 10^3 eV \times 1.6 \times 10^{-19} J/eV = 6.4 \times 10^{-15} J$$

The kinetic energy of the electron is given by:

$$E = \frac{1}{2}mv^2$$

We can solve for the velocity:

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2(6.4 \times 10^{-15} J)}{9.11 \times 10^{-31} kg}} \approx 1.18 \times 10^8 m/s$$

Now we can calculate the de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{m_ev} = \frac{6.626 \times 10^{-34} Js}{(9.11 \times 10^{-31} kg)(1.18 \times 10^8 m/s)} \approx 6.16 \times 10^{-12} m$$

This wavelength is on the order of picometers, which is a suitable scale for resolving atomic structures in electron microscopy. The relatively short wavelength allows for high resolution imaging.