

Given the following relation and functional dependencies:

$R = (A, B, C, D, E, F)$

$FDs = \{ A \rightarrow C, A \rightarrow D, B \rightarrow E, B \rightarrow D, A \rightarrow A, AB \rightarrow A, A \rightarrow F \}$

1.1 Using 3NF, decompose R into the proper tables — Show your steps

(1)

$A \rightarrow C$

$A \rightarrow D$

$B \rightarrow E$

$B \rightarrow D$

$A \rightarrow A$

$AB \rightarrow A$

$A \rightarrow F$

(2)

$A \rightarrow CDF$

$B \rightarrow ED$

$AB \rightarrow A$

(3)

$A \rightarrow CDF$

$B \rightarrow ED$

ACDF // BED

1.2

(1)

$A \rightarrow C$

$A \rightarrow D$

$B \rightarrow E$

$B \rightarrow D$

$A \rightarrow A$

$AB \rightarrow A$

$A \rightarrow F$

$C \rightarrow C$

$D \rightarrow D$

$E \rightarrow E$

$F \rightarrow F$

(2)

$A \rightarrow ACDF$

$B \rightarrow BDE$

$C \rightarrow C$

$D \rightarrow D$

$E \rightarrow E$

$F \rightarrow F$

$AB \rightarrow AB$

(3)

A → ACDF (non-trivial)

B → BDE (non-trivial)

C → C (trivial)

D → D (trivial)

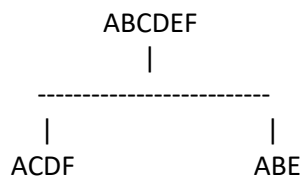
E → E (trivial)

F → F (trivial)

AB → ABCDEF (super key)

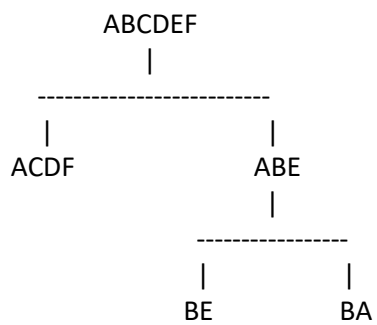
Breaking on A

$A + (ABCDEF - ACDF) = ABE$



Breaking on B

$B + (ABE - BE) = BA$



Thus, R(ABCDEF) becomes ACDF // BE // BA

```
R = (J, K, L, M, N)
FDs = { J -> JKLMN, K -> MN, N -> L, M -> N }
```

2.1 - Using 3NF, decompose R into the proper tables — Show your steps

(1)

J -> JKLMN

K -> MN

N -> L

M -> N

(2)

J -> KLMN

K -> MN

N -> L

M -> N

(3)

J -> K

K -> M

N -> L

M -> N

JK // KM // NL // MN

2.2 - Using BCNF, decompose R into the proper tables — Show your steps

(1)

$J \rightarrow JKLMN$

$K \rightarrow MN$

$L \rightarrow L$

$M \rightarrow N$

$N \rightarrow L$

(2)

$J \rightarrow JKLMN$

$K \rightarrow KMN$

$L \rightarrow L$

$M \rightarrow MN$

$N \rightarrow NL$

(3)

$J \rightarrow JKLMN$ (super key)

$K \rightarrow KMNL$ (non-trivial)

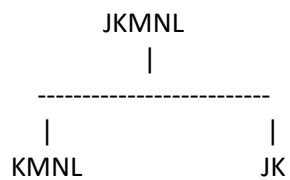
$L \rightarrow L$ (trivial)

$M \rightarrow MNL$ (non-trivial)

$N \rightarrow NL$ (non-trivial)

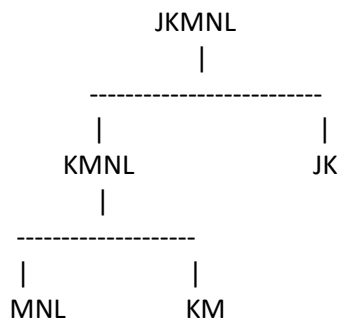
Breaking on K

$K + (JKMNL - KMNL) = KMNL$



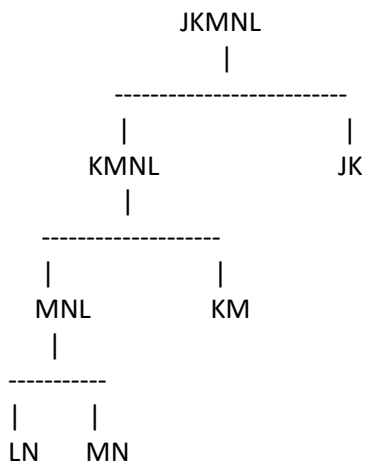
Breaking on M

$$M + (KMNL - MNL) = MNL$$



Breaking on N

$$N + (MNL - NL) = MN$$



Thus, $R(JKLMN)$ becomes $LN // MN // KM // JK$

[40 points] Consider the relation $Courses(C, T, H, L, S, G)$, whose attributes may be thought of informally as course_number, teacher_id, hour, location, student_id, and grade. Let the set of FDs for Courses be

```
R = (C, T, H, L, S, G)
FDs = { C -> T, HL -> C, HT -> L, HS -> L, CS -> G }
```

Intuitively, we can interpret the FDs as

C -> T	a course has a unique teacher
HL -> C	only one course can meet in a given location at a given hour
HT -> L	a teacher can be in only one location at a given time (to teach a course)
HS -> L	a student can be in only one location at a given time (to attend a course)
CS -> G	a student gets only one grade in a course

3.1 - List all the superkeys for the Courses relation

Reasoning FDs (below), we can conclude that HS is the only superkey.

C -> T

HL -> CHLT

HT -> HTLC

HS -> HSLCTG

CS -> CSGT

3.2 - Verify that the given set of FDs is a minimal basis. Explain or discuss to show that it is (or is not) a minimal basis. (hint: Canonical Cover, Fc)

In order to verify that the given set of FDs is a minimal basis, we first check to see if any of the FDs can be removed. It appears we can remove HL -> C and still have a basis for the original set of FDs. Thus, it is not a minimal basis.

3.3

$C \rightarrow T, HL \rightarrow C, HT \rightarrow L, HS \rightarrow L, CS \rightarrow G$

First, we write all of the LHS: $C \rightarrow, HL \rightarrow, HT \rightarrow, HS \rightarrow, CS \rightarrow$.

Second, we copy the FDs as is: $C \rightarrow T, HL \rightarrow C, HT \rightarrow L, HS \rightarrow L, CS \rightarrow G$

Third, we check to see if there are any extraneous attributes on the left-hand side. For this step, we will look for any functional dependencies (FD) that have two or more attributes on the left-hand side and see if we can eliminate any. This involves the following examinations:

- The first FD we're going to look at is $HL \rightarrow C$. We will look at H closure (H^+) and L closure (L^+). H^+ includes only H, and L^+ includes only L. Since it appears that we cannot eliminate either the H or the L, there are no changes we can make to $HL \rightarrow C$.
- The second FD we're going to look at is $HT \rightarrow L$. We will look at H closure (H^+) and T closure (T^+). H^+ includes only H, and T^+ includes only T. Since it appears that we cannot eliminate either the H or the T, there are no changes we can make to $HT \rightarrow L$.
- The third FD we're going to look at is $HS \rightarrow L$. We will look at H closure (H^+) and S closure (S^+). H^+ includes only H, and S^+ includes only S. Since it appears that we cannot eliminate either the H or the S, there are no changes we can make to $HS \rightarrow L$.
- The fourth FD we're going to look at is $CS \rightarrow G$. We will look at C closure (C^+) and S closure (S^+). C^+ includes only C and T, and S^+ includes only S. Since it appears that we cannot eliminate either the C or the S, there are no changes we can make to $CS \rightarrow G$.

Since there are not extraneous attributes that can be eliminated, we are left with $C \rightarrow T, HL \rightarrow C, HT \rightarrow L, HS \rightarrow L, CS \rightarrow G$. This translates to $CT // HLC // HTL // HSL // CSG$.

3.4

To verify, we need to show that the decomposed relations satisfy:

Lossless Join

- It satisfies lossless join property, as you can take the intersection of $C \rightarrow T$ and $HL \rightarrow T$, and find C which is a super key for $C \rightarrow T$.

Dependency Preservation

- It can verify the dependency preserving properties, as $C \rightarrow T$ can be verified with CT, $HL \rightarrow C$ can be verified with HLC, $HT \rightarrow L$ can be verified with HTL, $HS \rightarrow L$ can be verified with HSL, and $CS \rightarrow G$ can be verified with CSG.

Since both properties are satisfied, it is in 3NF.

3.5

In order to verify, we need to show that the decomposed relations satisfy:

Lossless Join

In 3.4, we already verified that it satisfies the lossless join property.

For every non-trivial FD, $X \rightarrow \text{Attribute(s)}$, X is a super key

- Consider CT, we know that $C \rightarrow T$. No non-key dependency in this decomposed relation.
- Consider HLC, we know that $HL \rightarrow C$. No non-key dependency in this decomposed relation.
- Consider HTL, we know that $HT \rightarrow L$. No non-key dependency in this decomposed relation.
- Consider HSL, we know that $HS \rightarrow L$. No non-key dependency in this decomposed relation.
- Consider CSG, we know that $CS \rightarrow G$. No non-key dependency in this decomposed relation.

Since both properties are satisfied, there is no BCNF violation.