COGS118A HW3

1) Convex

a) convex

b) Not convex

c) Convex

d) Not convex

e) Not convex

f) Not convex

2) Least Square Estimation

(a) Compute the gradient of g(W) with respect to W.

$$g(W) = (XW - Y)^T(XW - Y)$$
$$g(W) = W^T X^T XW - W^T X^T Y - Y^T XW + Y^T Y$$
$$\frac{d}{dW}(W^T X^T XW - W^T X^T Y - Y^T XW + Y^T Y)$$
$$\frac{dg(W)}{dW} = 2X^T XW - 2X^T Y$$

b) By setting the answer of part (a) to 0, prove the following:

$$\frac{dg(W)}{dW} = 2X^T XW - 2X^T Y = 0$$
$$2X^T XW - 2X^T Y = 0$$
$$W = \frac{2X^T Y}{2X^T X}$$
$$W = \frac{X^T Y}{X^T X}$$
$$W^* = \frac{X^T Y}{X^T X} = (X^T X)^{-1}(X^T Y)$$

c) Python: Download the file HW3.ipynb from the course website. Then, complete the TODO blocks in the Jupyter notebook.

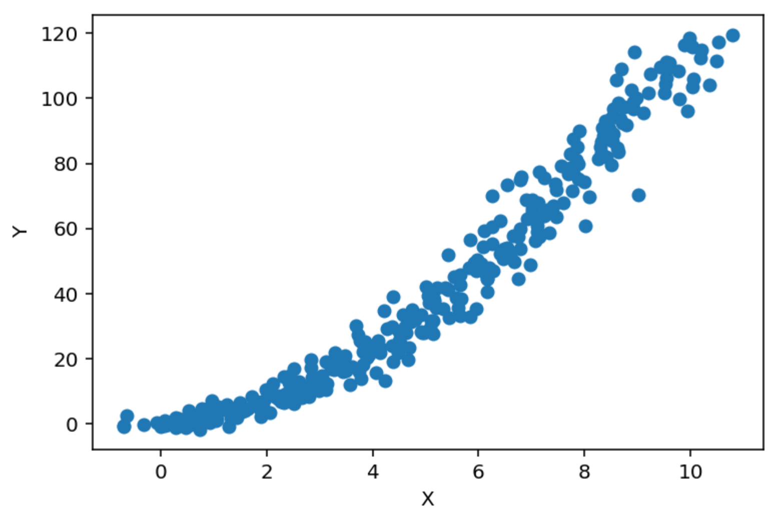
In [1]:
```
import numpy as np
from numpy.linalg import inv
import matplotlib.pyplot as plt
%config InlineBackend.figure_format = 'retina'
```

In [2]:
```
# Import packages and load data
X_and_Y = np.load('./q2-least-square.npy')
X = X_and_Y[:, 0]  # Shape: (300,)
Y = X_and_Y[:, 1]  # Shape: (300,)
```

2.1: 2D Scatterplot

In [3]:
```
# TODO: Plot the a scatter graph of data.
plt.scatter(X,Y)
plt.xlabel('X')
plt.ylabel('Y')
```

Out[3]: `Text(0,0.5,'Y')`



2.2: Compute the Least Sequare Line Using the Closed Form (Example Code)

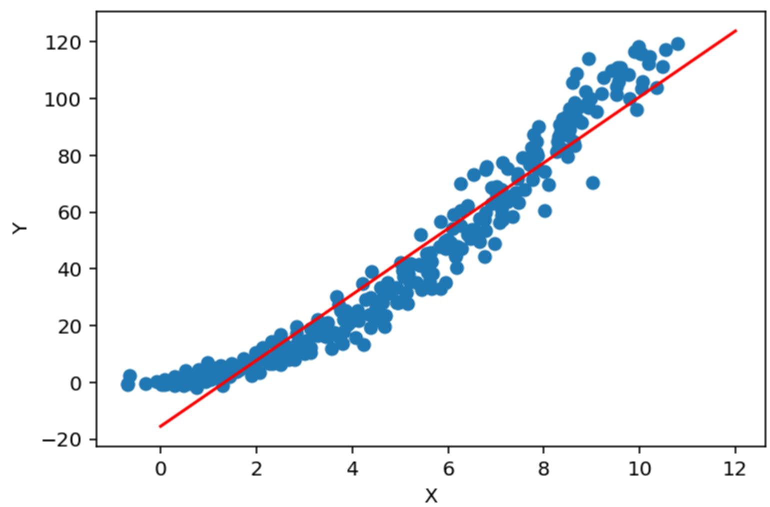
In [4]:
```
# Compute the least square line over the given data
# Assume Y = w0 + w1 * X = (w0, w1).(1, X) = W.X1
# You might find the following functions useful: np.matrix, np.hstack, n
p.ones, reshape, dot

X1 = np.matrix(np.hstack((np.ones((len(X),1)), X.reshape(-1,1))))
W = np.dot(np.dot(inv(np.dot(np.transpose(X1), X1)), np.transpose(X1)),
Y)
w0 = (W.item((0, 0)))
w1 = (W.item((0, 1)))
print('Y = {:.2f} + {:.2f}*X'.format(w0, w1))
```

```
Y = -15.47 + 11.61*X
```

2.3: 2D Scatterplot & the Estimated Least Square Line

In [5]:
```
# TODO 3.  Plot the the estimated least square line on top of the scatte
r plot in (2).
# The scatterplot and the line should be in the same figure.
plt.scatter(X, Y)
plt.plot(np.linspace(0,12,10), (w0 + w1 * np.linspace(0,12,10)), color=
'r')
plt.xlabel('X')
plt.ylabel('Y')
```

Out[5]:
```
Text(0,0.5,'Y')
```



2.4: Compute the Least Square Parabola Using the Closed Form

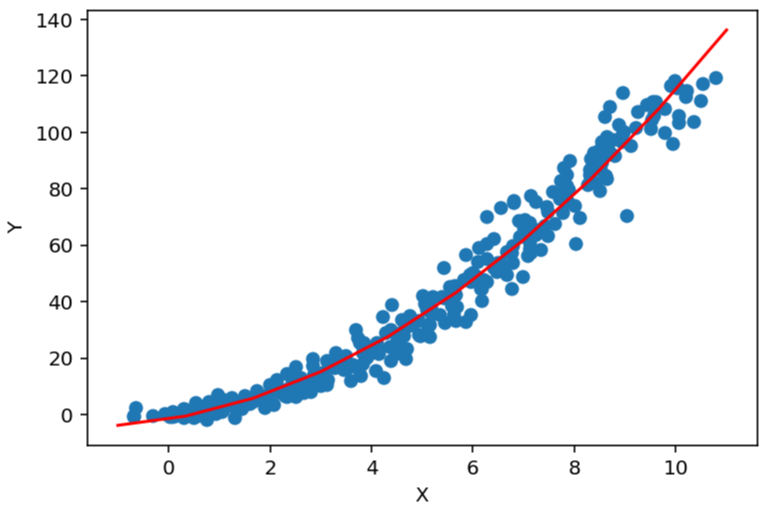
In [6]:
```
# TODO 4. Compute the least square parabola over the given data
# Assume Y = w0 + w1 * X + w2 * X^2 = (w0, w1, w2).(1, X, X^2) = W.X2
X2 = np.matrix(np.hstack((np.ones((len(X),1)), X.reshape(-1,1), np.squar
e(X.reshape(-1,1)))))
W = np.dot(np.dot(inv(np.dot(np.transpose(X2), X2)), np.transpose(X2)),
Y)
print(W.shape)
w0 = (W.item((0, 0)))
w1 = (W.item((0, 1)))
w2 = (W.item((0, 2)))
print('Y = {:.2f} + {:.2f}*X + {:.2f}*X^2'.format(w0, w1, w2))
```

```
(1, 3)
Y = -1.71 + 3.02*X + 0.87*X^2
```

2.5: 2D Scatterplot & the Estimated Parabola

In [7]:
```
# TODO 5.  Plot the the estimated parabola on top of the scatter plot in (2).
# The scatterplotand the parabola should be in the same figure
plt.scatter(X, Y)
plt.plot(np.linspace(-1,11,10), (w0 + w1 * np.linspace(-1,11,10) + w2 * (np.linspace(-1,11,10)**2)), color='r')
plt.xlabel('X')
plt.ylabel('Y')
```

Out[7]: `Text(0,0.5,'Y')`



3) Least Square Estimation via Gradient Descent

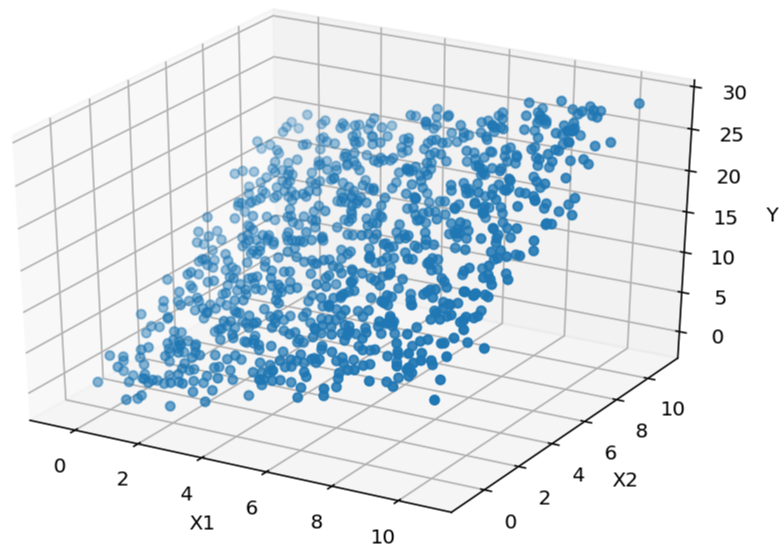
In [8]:
```
from mpl_toolkits.mplot3d import Axes3D
%config InlineBackend.figure_format = 'retina'
```

In [9]:
```
# Import packages and load data
X_and_Y = np.load('./q3-gradient-descent.npy')
X1 = X_and_Y[:, 0]    # Shape: (900,)
X2 = X_and_Y[:, 1]    # Shape: (900,)
Y  = X_and_Y[:, 2]    # Shape: (900,)
print(X1.shape, X2.shape, Y.shape)
```

```
(900,) (900,) (900,)
```

3.1: 3D Scatterplot

In [10]:
```
# TODO: Plot the a scatter graph of data.
fig = plt.figure()
ax = Axes3D(fig)
ax.scatter(X1, X2, Y)
ax.set_xlabel('X1')
ax.set_ylabel('X2')
ax.set_zlabel('Y')
plt.show()
```



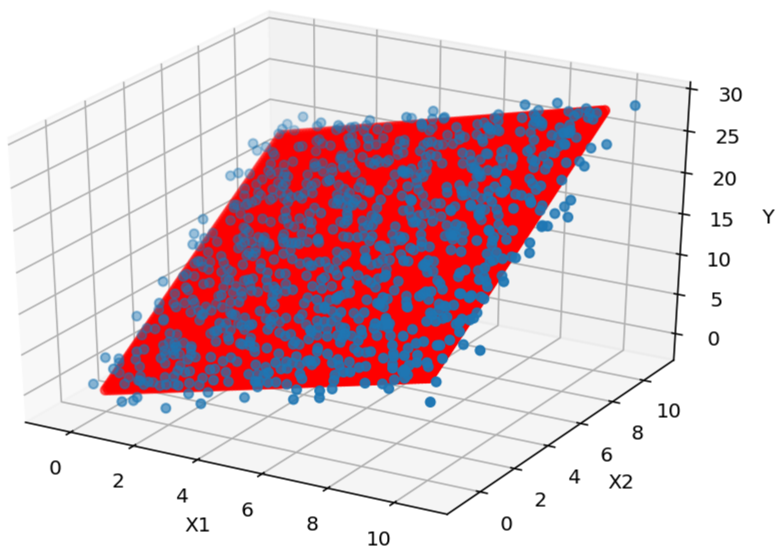
3.2 Compute the Least Square Plane Using the Closed Form

In [11]:
```
# TODO: Compute the least square Plane over the given data
# Assume Y = w0 + w1 * X1 + W2 * X2 = (w0, w1, w2).(1, X1, X2) = W.X
X = np.matrix(np.hstack((np.ones((len(X1),1)), X1.reshape(-1,1), X2.resh
ape(-1,1))))
W = np.dot(np.dot(inv(np.dot(np.transpose(X), X)), np.transpose(X)), Y)
w0 = (W.item((0, 0)))
w1 = (W.item((0, 1)))
w2 = (W.item((0, 2)))
print('Y = {:.2f} + {:.2f}*X1 + {:.2f}*X2'.format(w0, w1, w2))
```

```
Y = -0.70 + 0.98*X1 + 1.94*X2
```

3.3: 3D Scatterplot & the Estimated Least Square Plane

In [29]:
```
# TODO: Plot the scatter graph of data and estimated plane using the clo
sed form solution.
fig = plt.figure()
ax = Axes3D(fig)
ax.scatter(X1, X2, Y)
ax.set_xlabel('X1')
ax.set_ylabel('X2')
ax.set_zlabel('Y')
X1_plane, X2_plane = np.meshgrid(np.linspace(0,10,100), np.linspace(0,10
,100))
Y_plane = w0 + w1 * X1_plane + w2 * X2_plane
ax.scatter(X1_plane, X2_plane, Y_plane, c='r')
plt.show()
```



3.4: Compute the gradient of g(W) with respect to W.

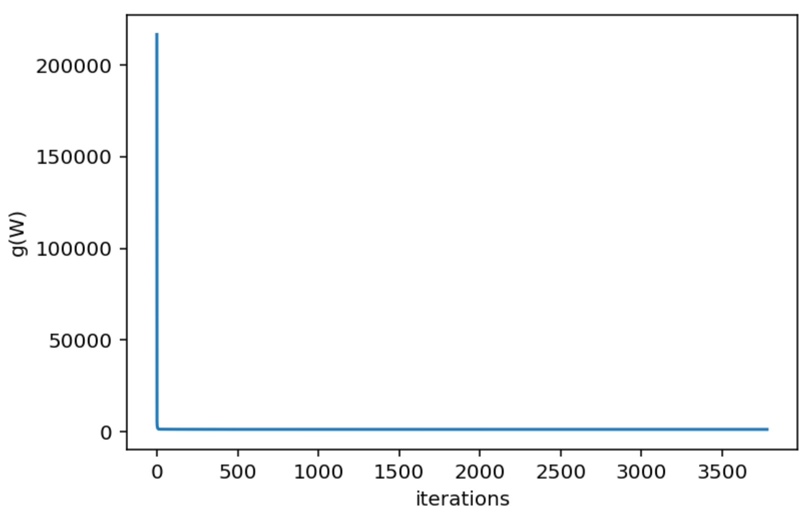
In [13]:
```
# TODO: g'(W)
def g_prime_W(X, Y, W):
    return (2*np.dot(np.dot(np.transpose(X),X), W) - 2*(np.dot(np.transp
ose(X),Y)))
```

In [21]:
```
W = np.matrix(np.zeros((3,1)))
Y = Y.reshape(-1, 1)
learning_rate = .00001
itr = [0]
g_W = [np.dot(np.transpose(np.dot(X,W) - Y), (np.dot(X,W) - Y))]
for i in range(10000):
        dW = g_prime_W(X, Y, W)
        Wnew = W - learning_rate * dW

        g_W.append(np.dot(np.transpose(np.dot(X,Wnew) - Y), (np.dot(X,Wn
ew) - Y)))

        itr.append(i+1)

        if np.linalg.norm(Wnew - W, ord = 1) < 0.0000001:
            break
        W = Wnew

w0 = (W.item((0, 0)))
w1 = (W.item((1, 0)))
w2 = (W.item((2, 0)))
print('Y = {:.2f} + {:.2f}*X1 + {:.2f}*X2'.format(w0, w1, w2))
```

```
Y = -0.70 + 0.98*X1 + 1.94*X2
```

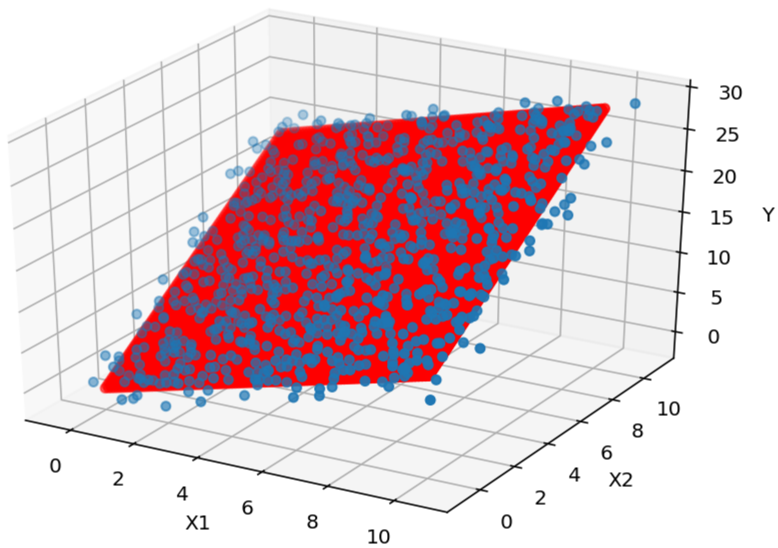
3.6 Plot the training curve

In [22]:
```
plt.xlabel('iterations')
plt.ylabel('g(W)')
plt.plot(itr, np.array(g_W).reshape(-1, 1))
plt.show()
```



3.7 Plot the scatter graph of data and estimated plane using the gradient descent solution

In [30]:
```
# TODO: Plot the scatter graph of data and estimated plane
X1_plane, X2_plane = np.meshgrid(np.linspace(0,10,100), np.linspace(0,10
,100))
Y_plane = w0 + w1 * X1_plane + w2 * X2_plane
fig = plt.figure()
ax = Axes3D(fig)
ax.scatter(X1, X2, Y)
ax.scatter(X1_plane, X2_plane, Y_plane, c='r')
ax.set_xlabel('X1')
ax.set_ylabel('X2')
ax.set_zlabel('Y')
plt.show()
```



4) Concepts

Select the correct option(s). Note that there might be multiple correct options.

1. What are the most significant difference between regression and classification?

A. unsupervised learning vs. supervised learning

B. prediction of continuous values vs. prediction of class labels $\rightarrow TRUE$

C. least square estimation vs. gradient descent

D. convex vs. non-convex problem E. higher vs. lower error $\rightarrow TRUE$

2. What are true about solving regression problem with gradient descent compared to closed-form solution?

A. matrix inverse could be expensive when the dataset is large $\rightarrow TRUE$

B. gradient descent is slower

C. gradient descent will give you the exactly the same result as closed-form solution

D. it's hard to set a good learning rate for gradient descent

3. Is gradient descent guaranteed to find the global optimal in a convex problem? What about non-convex problem?

A. yes for a convex problem

B. no for a convex problem $\rightarrow TRUE$

C. yes for a non-convex problem $\rightarrow TRUE$

D. no for a non-convex problem

4. What are true about local optimal and global optimal?

A. local optimal is better

B. There can exist multiple local optimal $\rightarrow TRUE$

C. gradient descent is able to find the global optimal $\rightarrow TRUE$

D. least square solution finds the global optimal

In [ ]: