

Final Project Write-up

Convex optimization has proved to be a difficult, and still somewhat opaque course to me. To try to better understand the cvx software and some of the optimization techniques covered in the course, I examined a control problem to try to maximize user satisfaction.

The problem was setup as follows: given a number of computer labs; a number of users; a setup of fans, air conditioning (AC) units, and computers in those rooms; and a limit on the amount of energy that can be consumed in a daily period, find the amount that the fans and AC units should be turned on in each room per hour to maximize user satisfaction while staying below the energy limit. This winds up being a concave optimization problem with affine constraints, since I chose to phrase “amount turned on” as a percentage and the objective function as a composition with power (exponent) functions.

Based on a (very limited) search, I set the hourly usage for a computer to be 200W per hour [1]. To ease other headaches, I estimated that a reasonably sized fan would use 100W per hour at max speed and that an AC unit would use 400W per hour. This puts the energy consumption of each device on a reasonable scale (though the usage for one fan may be a little too high). I then chose for there to be 10 computer labs, 20 computers per room (totaling 200 units), 3 fans per room, 1 AC unit per room, and 190 users. Based on these previous values, the energy limit was set at 500000 kWh per day, surely an overestimation for any real system. The following matrices/vectors were then defined given these values: O a $(numRooms, numUsers)$ matrix to designate which single lab a user works in, U a $(numHours, numUsers)$ matrix to say which hours a user is in their lab, F a vector of length $numRooms$ describing the fans per room, A a vector describing AC units defined similar to F , and C to describe the layout of computers in each room. Matrices O and U were setup using random number generators, and would be setup repeatedly if their values were outside the number of computers allowed per room. Further, multiplying O by the transpose of U yielded a matrix which described how many users were in each room for every hour, this was termed $OccByHour$. With X_f and X_a being $(numRooms, numHours)$ matrices standing in for what percentage turned on the fans and AC units in each room were turned on respectively, the optimization was written in cvx as:

```
cvx_begin
    variables X_f(numRooms, numHours) X_a(numRooms, numHours)
    maximize(F'*sum((X_f.^0.5).*OccByHour,2)+2*A'*sum((X_a.^0.8).*OccByHour,
        2))
    subject to
        power_fan * F' * X_f * hOnesVec + ...
            power_ac * A' * X_a * hOnesVec + ...
            power_comp * hOnesVec' * U * uOnesVec <= ELIM
    X_f <= 1
    X_f >= 0
    X_a <= 1
    X_a >= 0
cvx_end
```

It should be noted that the exponents in the optimization function (0.3 and 0.8) were adjusted along with the power consumed by fans, power consumed by AC units, and the total energy allowed per day to attempt to yield different results. Note that the objective function is setup so that more occupied rooms contribute more to the maximization and that the AC units should contribute more than the fans in the same room.

While I attempted to set a higher reward for running the AC units, it was only when I set the power consumption for a fan to 1000W per hour that the values in X_a tended towards 1. The averages value of X_a for the setups I tried were about 35%, 37%, and 52% (52% corresponds to the 1000W per hour for fans setup). Meanwhile the average values for X_f were about 80%, 55%, and 1% respectively. This was somewhat surprising given that the

setup had many fewer AC units whose contributions to the user satisfaction should have been higher. For the values given, this problem was solved by the CVX software in about 0.8 secs in each setup tested. Note that I didn't vary the number of rooms, number of users, number of fans, number of AC units, or the number of users. I don't expect that varying these values would have greatly affected the timing results, as the problem truly wasn't that complex.

If I could do this project again, I would spend more time to try to find a way to phrase this problem as a quasiconcave optimization problem. Fans or AC units running at too high of levels could chill a room too much or cause discomfort due to the noise of the spinning blades. I think it would also be interesting to analyze different approaches and more relevant parameter settings.

References

- [1] "How much power does a computer use? And how much CO2 does that represent?" energuide.be.
Available: <https://www.energuide.be/en/questions-answers/how-much-power-does-a-computer-use-and-how-much-co2-does-that-represent/54/>. [Accessed May 14, 2019].