# Computing and Plotting the Gaussian Mark

This project was done for Differential Geometry Course taught by Professor Gabriel Pripoaie at students from second year of University Spiru-Haret, Faculty of Mathematics and Computer Science.

## 1 Introduction

Let $u = \overset{o}{u} \subseteq R^n$ and $f : u \to E^{n+1}$ be a differentiable function with $x \in u$ ,with $f(x) = (f^1(x), f^2(x), ..., f^{n+1}(x))$.

**Definition 1** *The function $f : u \subset R^n \to R^{n+1}$ is called immersion if $rangJ_f(x)$ is $n$ for each $x \in u$.*

**Definition 2** *The function f is called parametrized surface if f is immersion in any domain point.*

**Notation 3** *Let note*

$$df_x(e_i) = f_{x^i}(x) = \frac{\partial f^\alpha}{\partial x^i}(x)\overline{e_\alpha}$$

**Notation 4** *Let $T_{f(x)}R^{n+1}$ be $R^{n+1}$ and $df_x(T_xR^n) = T_{f(x)}f = sp\{\frac{\partial f}{\partial x^1}(x), ..., \frac{\partial f}{\partial x^n}(x)\}$*

**Definition 5** *Let $f : u \to E^{n+1}$ be a parametrized hypersurface and $X : u \to E^{n+1}$ be a differential application we call X field of vectors along hypersurface f.*

**Definition 6** *Let $X : u \to E^{n+1}$ be a field of vectors along hypersurface f and if $\forall x \in u, X(x) \in T_{f(x)}f$ then X is tangent.*

**Definition 7** *Let $X : u \to E^{n+1}$ be a field of vectors along hypersurface f and if $\forall x \in u, X(x) \perp T_{f(x)}f$ then X is normal to f.*

**Definition 8** $N(x) = \frac{f_{x^1}(x) \times f_{x^2}(x) \times ... f_{x^n}(x)}{\left\| f_{x^1}(x) \times f_{x^2}(x) \times ... f_{x^n}(x) \right\|}$ *is called unity normal to f.*

**Definition 9** *Let $f : u \to E^{n+1}$ be a parametrized hypersurface then $\{f_{x^1}, ..., f_{x^n}, N\}$ we call Gaussian mark associated to f.*

**Definition 10** *Let $I_x(v, w) =< df_x(v), df_x(w) >= g_x(df_x(v), df_x(w)) = v^i w^j g_x(f_{x^i}, f_{x^j})$ be the first fundamental form of hypersurface f in x, and $I_x = g_x$ be the first fundamental form associated to hypersurface f.*

**Definition 11** *Let $f(X, Y) =< df_p(X), df_p(Y) >$ with $X, Y \in T_{f(p)}f$ , $\forall p \in u$ be the second fundamental associated to hypersurface f,$h_{ij}(p) =< f_{x^i x^j}(p), N(p) >$ .*

# 2 Implementation

This is implemented in MAPLE because I have there differential, solving and plotting of data.

**Algorithm 12**

- *compute the Jacobian and if the rank of the Jacobian is zero then we have immersion so we can continue else we return*
- *find the points where* $< f_{x^1}(x) \times f_{x^2}(x) \times ... f_{x^n}(x), f_x(x) > \neq 0$ *because in these points we don't have normal to surface and print these points.*
- *compute* $N$
- *compute and print the Gaussian Mark* $\{f_{x^1}, ..., f_{x^n}, N\}$
- *compute and print first and second fundamental forms.*
- *make the translation of the Gaussian Mark to the surface by symbolic adding the hypersurface.*

**Exercise 13** *Find the first and second fundamental forms and plot the curve with the Gaussian Mark. The curve is given by the following parametrized relation for* $\forall x \in [-\pi/2, \pi/2], \forall y \in [-2, 0]$

$$f(x, y) = (2 * \cos(x) * \cos(y), 2 * \cos(x) * \sin(y), 2 * \sin(x))$$

In the following point we don't have normal to surface (1.570796327,y).
The first and second fundamental form are:

$$g(x, y) = \begin{bmatrix} 4 & 0 \\ 0 & 4\cos(x)^2 \end{bmatrix}, h(x, y) = \begin{bmatrix} \frac{2\cos(x)}{|\cos(x)|} & 0 \\ 0 & 2|\cos(x)|\cos(x) \end{bmatrix}$$

The Gaussian Mark is:

$$f_{x^1}(x, y) = (-2sin(x)\cos(y), -2\sin(x)\sin(y), 2\cos(x))$$
$$f_{x^2}(x, y) = (-2\cos(x)\sin(y), 2\cos(x)\cos(y), 0)$$
$$N(x, y) = \left(-\frac{\cos(x)^2\cos(y)}{|\cos(x)|}, -\frac{\cos(x)^2\sin(y)}{|\cos(x)}, -\frac{\sin(x)\cos(x)}{|\cos(x)|}\right)$$