OpenMP Implementation of Jacobi with Dominant Row

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1 Introduction

Let

$$Ax = a \tag{1}$$

be our system of linear equations, with $A = (a_{ij})_{i,j\in\{1,...,m\}}$, $a = (a_1, ..., a_m)$ and $x = (x_1, ..., x_m)^T$.

Suppose $a_{ii} \neq 0$ for any $i \in \{1, ..., m\}$ then we can note $D = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{mm} \end{pmatrix}$ and $D^{-1} = \begin{pmatrix} 1/a_{11} & 0 \\ 0 & 1/a_{mm} \end{pmatrix}$.

The system (1) could be transform in $D^{-1}Ax = D^{-1}a$ or

$$\left(I - \left(\underbrace{I - D^{-1}A}_{B}\right)\right)x = \underbrace{D^{-1}a}_{b}$$

which is equivalent with

$$(I - B)x = b \tag{2}$$

Theorem 1 *If*

$$|a_{ii}| > \sum_{j=1,j\neq i}^{m} |a_{ij}|, \forall i \in \{1, ..., m\} \tag{3}$$

then the system (2) has the unique solution z and $\forall x^{(0)} \in R^m$ the string $(x^{(n)})_{n\in N}$ with $x^{(n+1)} = Bx^{(n)} + b$ convert to z and it takes palaces the following relations:

$$\left\|x^{(n)} - z\right\|_\infty \leq \frac{q}{1-q}\left\|x^{(n)} - x^{(n-1)}\right\|_\infty \leq \frac{q^n}{1-q}\left\|x^{(1)} - x^{(0)}\right\|_\infty \tag{4}$$

where

$$q = \max_{1\leq i\leq m} \sum_{j=1,j\neq i}^{m} \left|\frac{a_{ij}}{a_{ii}}\right|$$

2 Implementation of generator C

Where in conformity with system (1) :mat is matrix A of coefficients, y is the free vector a,x is the solution x of the system, err is the error and rez is the correct solution.

Because I want to have a random solution but also to can verified the solution and the system to be compatible with systems solved by Jacobi with dominant row I use the following generator which will assure me that I have the following solution rez=(1,2,...,m) for a m dimensional problem.

```
for(i=0;i<dim;i++) rez[i]=(double)i+1;
for(i=0;i<dim;i++)
    for(j=0;j<dim;j++) if(i!=j) {
            mat[i][j]=20000*rand()/(double)RAND_MAX;
            if((rand()/(double)RAND_MAX)<0.5) mat[i][j]=-mat[i][j]; }
for(i=0;i<dim;i++) {
    temp=0.0;
    for(j=0;j<dim;j++) if(j!=i) temp+=fabs(mat[i][j]);
    mat[i][i]=temp+(20000+temp)*rand()/(double)RAND_MAX+0.00001;
    if((rand()/(double)RAND_MAX)<0.5) mat[i][i]=-mat[i][i]; }
for(i=0;i<dim;i++) {
    y[i]=0.0; x[i]=0.0;
    for(j=0;j<dim;j++) y[i]+=mat[i][j]*rez[j]; }
```

3 Results

I have compile the parallel program with two openMP compilers: Omni 1.6 and Intel C Compiler 8.0 for LINUX and the serial with gcc and Intel C Compiler 8.0 for LINUX both with maximum optimization "-O3" and for Intel C Compiler I've put also "-mcpu=pentiumpro -tpp6" for maximum optimization.

The executable were run on a dual pentium II at 500MHz with 256MB RAM and with LINUX Fedora Core 1.

The following results were made for a average or 10 runs for serial and parallel programs and with red is plotted the results from ICC and with blue the results from Omni.