OpenMP Implementation of Gaussian Elimination

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1 Introduction

Let the system be

$$Ax = b \tag{1}$$

Here A is a dense n x n matrix of coefficients such that $A[i][j] = a_{i,j}$, b is an n x 1 vector and x is the desired solution vector.

A system of equations is solved in two stages. First, through a series of algebraic manipulations, the original system of equations is reduced to an upper triangular system of the form $Ux = y$, where U is a unit upper-triangular matrix. In the second stage of solving a system of linear equations, the upper-triangular system is solved for the variables in reverse order from x_{n-1} to x_0 by a procedure know as back-substitution.

Algorithm 1 *gaussian_ elimination*

```
procedure gaussian_elimination(A,b,y)
begin
    for k:=0 to n-1 do
        for j:=k+1 to n-1 do
            A[k][j]:=A[k][j]/A[k][k]; //division step
        y[k]:=b[k]/A[k][k];
        A[k][k]:=1;
        for i:=k+1 to n-1 do
            for j:=k+1 to n-1 do
                A[i][j]:=A[i][j]-A[i]k]xA[k][j]; //elimination step
            b[i]:=b[i]-A[i][k]xy[k];
            A[i][k]:=0;
        endfor;
    endfor;
end gaussian_elimination;
```

2 Pipeline Communication and Computation

The matrix is scattered to all processor so two consecutive row are in two consecutive processors.

During the k^{th} iteration processor P_k broadcast part of the k^{th} row of the matrix to processors $P_{k+1}, ..., P_{p-1}$. Assume that the processors $P_0 ... P_{p-1}$ are connected in a linear array, and P_{k+1} is the first processor to receive the k^{th} row from processor P_k. Then the processor P_{k+1} must forward this data to P_{k+2}. However, after forwarding the k^{th} row to P_{k+2}, processor P_{k+1} need not wait to perform the elimination step until all the processors up to P_{p-1} have received the k^{th} row. Similarly, P_{k+2} can start its computation as soon as is has forwarded the k^{th} row to P_{k+3}, and so on. Meanwhile, after completing the computation for the k^{th} iteration, P_{k+1} can perform the division step, and start the broadcast of the $(k+1)^{th}$ row by sending it to P_{k+2}. In this case of shared memory the receive, send and broadcast from MPI are replaced by OpenMP lock procedures but without pipeline communication.

This algorithm is made for comparing with MPI implementation.

3 Results

I have compiled the Gaussian Elimination without load balancing and pivoting program with two OpenMP compilers: Omni 1.6 and Intel C Compiler 8.0 for LINUX and the serial with gcc and Intel C Compiler 8.0 for LINUX both with maximum optimization "-O3" and for Intel C Compiler I've put also "-mcpu=pentiumpro -tpp6" for maximum optimization.

The executable were run on a dual pentium II at 500MHz with 256MB RAM and with LINUX Fedora Core 1.

The following results were made for a average or 10 runs for serial and parallel programs and with blue is plotted the results from Omni and with red the results from ICC.