Formule folosite pentru calcule:

Formule:

$$\sum_{j=0}^{n} 1 = 1 + 1 + 1 + \dots + 1 = n$$

$$\sum_{j=0}^{n} i = 0 + 1 + 2 + \dots + n = \frac{n \cdot (n+1)}{2}$$

$$\sum_{j=0}^{n} i^2 = 0^2 + 1^2 + 2^2 + \dots + n^2 = \frac{n \cdot (n+1)(2n+1)}{6}$$

$$\sum_{j=0}^{n} i^3 = 0^3 + 1^3 + 2^3 + \dots + n^3 = \frac{n^2 \cdot (n+1)^2}{4}$$

$$\sum_{j=0}^{n} i^4 = 0^4 + 1^4 + 2^4 + \dots + n^4 = \frac{n \cdot (n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{j=0}^{n} p^i = 1 + p + p^2 + p^3 + \dots p^n = \frac{p^{n+1} - 1}{p - 1}$$

1. $\sum_{i=0}^{n} 1 = n \in \Theta(n)$

2. $T(n) = \begin{cases} 1 & n \leq 1 \\ T(n-1) + 1 & \text{altfel} \end{cases}$

$$\begin{aligned}
T(n) &= \cancel{T(n-1)} + 1 \\
\cancel{T(n-1)} &= \cancel{T(n-2)} + 1 \\
\cancel{T(n-2)} &= \cancel{T(n-3)} + 1 \\
&\dots \\
\cancel{T(2)} &= T(1) + 1 \quad (+) \\
\hline
T(n) &= T(1) + n - 1 = 1 + n - 1 = n \Rightarrow
\end{aligned}$$

$T(n) \in \Theta(n)$

3. $\sum_{i=0}^{n} 1 = n \in \Theta(n)$

4. $$\sum_{i=0}^{n}\sum_{j=0}^{n} 1 = \sum_{i=0}^{n} n = n^2 \in \Theta(n^2)$$

5. $$\sum_{i=0}^{n}\sum_{j=0}^{i^2} 1 = \sum_{i=0}^{n} i^2 = \frac{n\cdot(n+1)(2n+1)}{6} \in \Theta(n^3)$$

6. $$\sum_{i=0}^{n}\sum_{j=0}^{i} 1 = \sum_{i=0}^{n} i = \frac{n\cdot(n+1)}{2} \in \Theta(n^2)$$

7. $$\sum_{i=0}^{n}\sum_{j=0}^{i^2}\sum_{k=0}^{j} 1 = \sum_{i=0}^{n}\sum_{j=0}^{i^2} j = \sum_{i=0}^{n}\left(0+1+2+\ldots+i^2\right)$$

$$= \sum_{i=0}^{n}\frac{i^2\cdot(i^2+1)}{2} = \frac{1}{2}\cdot\sum_{i=0}^{n}\left(i^4+i^2\right) =$$

$$= \frac{1}{2}\cdot\left(\sum_{i=0}^{n} i^4 + \sum_{i=0}^{n} i^2\right) =$$

$$= \frac{1}{2}\cdot\left(\frac{n\cdot(n+1)(2n+1)(3n^2+3n-1)}{30} + \frac{n\cdot(n+1)(2n+1)}{6}\right)$$

$$\in \Theta(n^5)$$

8. $$\sum_{i=0}^{x}\sum_{j=0}^{y}\sum_{k=0}^{z} 1 = \sum_{i=0}^{x}\sum_{j=0}^{y} z = \sum_{i=0}^{x} z\cdot y = z\cdot y\cdot x$$
$$\in \Theta(zxy)$$

9. $$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} j =$$

$$= \sum_{i=1}^{n-1} (i+1+i+2+i+3+\dots+n)$$

$$= \sum_{i=1}^{n-1}\left(\frac{n\cdot(n+1)}{2} - \frac{i\cdot(i+1)}{2}\right) =$$

$$= \sum_{i=1}^{n-1} \frac{n^2+n}{2} - \sum_{i=1}^{n-1}\frac{i^2+i}{2} = \frac{1}{2}\cdot\sum_{i=1}^{n-1} n^2 + \frac{1}{2}\cdot\sum_{i=1}^{n-1} n$$

$$- \frac{1}{2}\cdot\sum_{i=1}^{n-1} i^2 - \sum_{i=1}^{n-1} i =$$

$$\cancel{= \frac{1}{2}\cdot\left(n^2\cdot(n-1) + n\cdot(n-1) - \frac{n\cdot(n+1)(2n+1)}{6} - n\cdot\right.}$$

$$= \frac{1}{2}\cdot\left(n^2\cdot(n-1) + n\cdot(n-1) - \frac{(n-1)\cdot n\cdot(2n-1)}{6} - \frac{(n-1)\cdot n}{2}\right)$$

$$= \frac{1}{2}\cdot\left(\frac{6n^3-6n^2+6n^2-6n-(n-1)\cdot n\cdot(2n-1) - 3n\cdot(n-1)}{6}\right)$$

$$= \frac{1}{12}\cdot\left(6n^3 - 6n - 2n^3 + 3n^2 - n - 3n^2 + 3n\right) \to \in \Theta(n^3)$$

10. $$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{j+i} 1 = \sum_{i=1}^{n}\sum_{j=1}^{i}(j+i-j+1) =$$

$$\sum_{i=1}^{n}\sum_{j=1}^{i} i+1 = \sum_{i=1}^{n}\sum_{j=1}^{i} i + \sum_{i=1}^{n}\sum_{j=1}^{i} 1 =$$

$$= \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i = \frac{n\cdot(n+1)(2n+1)}{6} + \frac{n\cdot(n+1)}{2} \in \Theta(n^3)$$

Anumite calcule poate nu sunt corecte…dar formule de la inceput ar trebui sa fie.
Am calculat separat cei 3 termeni si la final am adunat rezultatele.

11. $$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{i+j}\sum_{l=1}^{i+j-k} 1 = \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{i+j}(i+j-k) =$$

$$= \sum_{i=1}^{n}\sum_{\underset{I}{j=1}}^{i}\sum_{k=j}^{i+j} i + \sum_{i=1}^{n}\sum_{\underset{II}{j=1}}^{i}\sum_{k=j}^{i+j} j - \sum_{i=1}^{n}\sum_{\underset{III}{j=1}}^{i}\sum_{k=j}^{i+j} k =$$

$$= *$$

I $$\sum_{i=1}^{n}\sum_{j=1}^{i}(i+1)\cdot i = \sum_{i=1}^{n}\sum_{j=1}^{i} i^2+i = \sum_{i=1}^{n}\sum_{j=1}^{i} i^2 + \sum_{i=1}^{n}\sum_{j=1}^{i} i =$$

$$= \sum_{i=1}^{n} i^3 + \sum_{i=1}^{n} i^2 = \frac{n^2\cdot(n+1)^2}{4} + \frac{n\cdot(n+1)(2n+1)}{6}$$

$$= \frac{n^2\cdot(n^2+2n+1)}{4} + \frac{2n^3-n^2-2n^2+n}{6} =$$

$$= {}^{3)}\frac{n^4+2n^3+n^2}{4} + {}^{2)}\frac{2n^3-3n^2+n}{6} = \frac{3n^4+6n^3+3n^2+4n^3-6n^2+2n}{12}$$

$$= \frac{3n^4+10n^3-3n^2+2n}{12}$$

11

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{i+j} j = \sum_{i=1}^{n}\sum_{j=1}^{i}(i+1)\cdot j =$$

$$\sum_{i=1}^{n}\sum_{j=1}^{i} ij + \sum_{i=1}^{n}\sum_{j=1}^{i} j = \sum_{i=1}^{n} i\cdot\frac{i\cdot(i+1)}{2} + \sum_{i=1}^{n}\frac{i\cdot(i+1)}{2}$$

$$= \frac{1}{2}\cdot\left(\sum_{i=1}^{n}(i^3+i^2) + \sum_{i=1}^{n}(i^2+i)\right) \cdot \frac{1}{2}\left(\frac{n^2\cdot(n+1)^2}{4} + \frac{n\cdot(n+1)(2n+1)}{6} + \right.$$

$$\left. + \frac{n\cdot(n+1)(2n+1)}{6} + \frac{n\cdot(n+1)}{2}\right) =$$

$$= \frac{1}{2}\cdot\left(\frac{n^2\cdot(n^2+2n+1)}{4} + \frac{(n^2+n)(2n+1)}{3} + \frac{n^2+n}{2}\right) =$$

$$= \frac{1}{2}\cdot\left(\frac{3n^4+6n^3+3n^2+8n^3+4n^2+8n^2+4n+6n^2+6n}{12}\right)$$

$$= \frac{1}{24}\cdot\left(3n^4+14n^3+21n^2+10n\right)$$

12

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{i+j} k = \sum_{i=1}^{n}\sum_{j=1}^{i}\frac{(i+j)(i+j+1)}{2} - \frac{(j-1)\cdot j}{2} =$$

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\frac{i^2+ij+i+ij+\cancel{j^2}+j-\cancel{j^2}+j}{2} = \sum_{i=1}^{n}\sum_{j=1}^{i}\frac{i^2+2ij+i+2j}{2} =$$

$$= \sum_{i=1}^{n}\frac{1}{2}\left(\sum_{j=1}^{i} i^2 + \sum_{j=1}^{i} 2ij + \sum_{j=1}^{i} i + \sum_{j=1}^{i} 2j\right) =$$

$$= \sum_{i=1}^{n} \frac{1}{2}\cdot\left(i^3 + \cancel{2}i\cdot\left(\frac{i\cdot(i+1)}{\cancel{2}}\right) + i^2 + 2\cdot\frac{i\cdot(i+1)}{2}\right)$$

$$= \sum_{i=1}^{n} \frac{1}{2}\left(i^3 + i^3 + i^2 + i^2 + i^2 + i\right) = \frac{1}{2}\cdot\sum_{i=1}^{n} 2i^3 + 3i^2 + i$$

$$= \frac{1}{2}\cdot\left(2\cdot\sum_{i=1}^{n} i^3 + 3\cdot\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right) = \frac{1}{2}\cdot\left(\cancel{2}\cdot\frac{n^2\cdot(n+1)^2}{\cancel{4}} + 3\cdot\frac{n\cdot(n+1)\cdot(2n+1)}{6} + \frac{n\cdot(n+1)}{2}\right) =$$

$$= \frac{1}{2}\cdot\left(\frac{n^2\cdot(n^2+2n+1)}{2} + \frac{(n^2+n)(2n+1)}{2} + \frac{n^2+n}{2}\right) =$$

$$= \frac{1}{4}\cdot\left(n^4 + 2n^3 + n^2 + 2n^3 + n^2 + 2n^2 + n + n^2 + n\right) =$$

$$= \frac{1}{4}\cdot\left(n^4 + 4n^3 + 5n^2 + 2n\right)$$

$$\frac{3n^4 + 10n^3 - 3n^2 + 2n}{12} + \frac{3n^4 + 14n^3 + 21n^2 + 10n}{24} -$$

$$\frac{1}{4}\cdot\left(n^4 + 4n^3 + 5n^2 + 2n\right) = \in \Theta(n^4)$$

12. $$\sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=i+j-1}^{n} 1 = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (n-(i+j-1)+1)$$

$$= \sum_{i=1}^{n} \sum_{j=i+1}^{n} (n-i-j+2) = \underbrace{\sum_{i=1}^{n} \sum_{j=i+1}^{n} n}_{I} - \underbrace{\sum_{i=1}^{n} \sum_{j=i+1}^{n} i}_{II} -$$

$$- \underbrace{\sum_{i=1}^{n} \sum_{j=i+1}^{n} j}_{III} + \underbrace{\sum_{i=1}^{n} \sum_{j=i+1}^{n} 2}_{IV} =$$

$$= \frac{n^3-n^2}{2} - \frac{n^3-n}{6} - \frac{n^3-n}{3} + n^2-n = \frac{3n^3-3n^2-n^3+n-2n^3+2n+6n^2-6n}{6} = \frac{3n^2-3n}{6} \in \Theta(n^2)$$

I $$\sum_{i=1}^{n} \sum_{j=i+1}^{n} n = \sum_{i=1}^{n} n\cdot(n-i-1+1) = \sum_{i=1}^{n} (n^2 - ni) =$$

$$n^3 - n\cdot\sum_{i=1}^{n} i = n^3 - n\cdot\frac{n\cdot(n+1)}{2} = \frac{2n^3-n^3-n^2}{2} = \frac{n^3-n^2}{2}$$

II $$\sum_{i=1}^{n} \sum_{j=i+1}^{n} i = \sum_{i=1}^{n} (n-i-1+1)\cdot i = \sum_{i=1}^{n} (in - i^2) =$$

$$n\cdot\sum_{i=1}^{n} i - \sum_{i=1}^{n} i^2 = n\cdot\frac{n\cdot(n+1)}{2} - \frac{n\cdot(n+1)\cdot(2n+1)}{6} =$$

$$= \frac{3n^3+3n^2-(2n^3+3n^2+n)}{6} = \frac{n^3-n}{6}$$

III $\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} j = \sum_{i=1}^{n}\left(\frac{n\cdot(n+1)}{2} - \frac{i\cdot(i+1)}{2}\right) =$

$$\sum_{i=1}^{n} \frac{n^2+n-i^2-i}{2} = \frac{1}{2}\left(\sum_{i=1}^{n} n^2 + \sum_{i=1}^{n} n - \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i\right)$$

$$= \frac{1}{2}\cdot\left(n^3+n^2 - \frac{2n^3+3n^2+n}{6} - \frac{n\cdot(n+1)}{2}\right) =$$

$$= \frac{1}{2}\left(\frac{6n^3+6n^2-2n^3-3n^2-n-3n^2-3n}{6}\right) = \frac{1}{\cancel{12}}\cdot(\cancel{4}n^3-\cancel{4}n)$$

$$= \frac{(n^3-n)}{3}$$

IV $\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} 2 = \sum_{i=1}^{n} 2\cdot(n-i-1+1) = 2\cdot\sum_{i=1}^{n}(n-i) =$

$$2\cdot\sum_{i=1}^{n} n - 2\cdot\sum_{i=1}^{n} i = 2\cdot n^2 - \cancel{2}\frac{(n\cdot(n+1))}{\cancel{2}} = 2n^2-n^2-n = n^2-n$$

E o problema mai speciala, pentru ca termeul de gradul maxim (n^3) dispare. Dar am verificat intr-un program care calculeaza automat suma, si asa se intampla.

13. Nu se poate cu sumă din cauza condiției $i*i <= n$

$$i*i \le n \Rightarrow i \le \sqrt{n} \in \Theta(\sqrt{n})$$

14. While se execută tot timpul de n ori ca și cum ar fi ciclu for. $\Rightarrow \Theta(n)$

15. pt: $i=1$ j: $1,2,3,\ldots,n$ $\Rightarrow$ total n

$i=2$ j: $1,3,5,7,\ldots n$ $\frac{n}{2}$

$i=3$ j: $1,4,7,10,13,\ldots,n$ $\frac{n}{3}$

$\vdots$

$i=n$ j: 1 $\frac{n}{n}$

În total: $n+\frac{n}{2}+\frac{n}{3}+\ldots+\frac{n}{n} = n\cdot\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right)$

harmonic series, poate fi estimat ca $\ln n = \log_e n$

$\Rightarrow \Theta(n\cdot\log_2 n)$

16. Cele 2 cicluri sunt independente (ciclul cu j nu depinde de i => nr. de repetiții e egal pt fiecare i)

for cu i => $\frac{n}{3}$ ori se repetă

for cu j => $\frac{n}{4}$ ori se repetă

$\Rightarrow \in \Theta(n^2)$

17. Cele 2 cicluri sunt independente.

for cu i => n ori se repetă

while $j = n$ => $j = j/2$

$j = n, \frac{n}{2}, \frac{n}{4}, \frac{n}{8}, \ldots$ etc. $\log_2 n$ repetiții

$\Rightarrow \in \Theta(n \cdot \log_2 n)$

18. pt. $i = 1$ => $\log_2 1$

$i = 2$ $\log_2 2$

$i = 3$ $\log_2 3$

$i = 4$ $\log_2 4$

$\vdots$

$i = n$ $\log_2 n$

suma logaritmilor este logaritmul produsului

$\log_2 a + \log_2 b = \log_2 a \cdot b$

$\log_2 1 + \log_2 2 + \ldots + \log_2 n \Rightarrow \log_2 (1 \cdot 2 \cdot 3 \cdot \ldots \cdot n)$

$= \log_2 n! \Rightarrow n \cdot \log_2 n \Rightarrow \in \Theta(n \log_2 n)$

Stirling's approximation

19. $\sum_{i=0}^{n} 1 = n \in \Theta(n)$

20. caz favorabil : $\Theta(1)$

caz defavorabil : $\Theta(n)$

caz mediu $n+1$ cazuri $p = \frac{1}{n+1}$

$$\frac{1}{n+1} \cdot 1 + \frac{1}{n+1} \cdot 2 + \dots + \frac{1}{n+1} \cdot n + \frac{1}{n+1} \cdot n =$$

$$\frac{1}{n+1} \cdot (1+2+\dots+n) + \frac{n}{n+1} = \frac{1}{\cancel{n+1}} \cdot \frac{n \cdot \cancel{(n+1)}}{2} + \frac{n}{n+1} \in \Theta(n)$$

Complexitatea totală $O(n)$

21. $$\sum_{i=1}^{n^2} \sum_{j=0}^{i} 1 = \sum_{i=1}^{n^2} i = 1+2+\dots+n^2 = \frac{n^2 \cdot (n^2+1)}{2} \in \Theta(n^4)$$

j este inițializat cu i și se scade 1 din el, până ajunge la 0. E similar (din perspectiva numărului de repetiții) cu un ciclu de la 0 la i

22. În while j pornește de la i și la fiecare pas este împărțit la 10.

De ex: $j = 12.345 \Rightarrow 1234 \Rightarrow 123 \Rightarrow 12 \Rightarrow 1 \Rightarrow 0$

De câte ori pot împărți j la 10 $\Rightarrow \log_{10} j \approx \log_2 j$

$$\sum_{i=1}^{n^2} \log_2 i = \log_2 1 + \log_2 2 + \log_2 3 + \dots \log_2 n^2 \Rightarrow$$

Stirlings approximation

$\hookrightarrow n^2 \cdot \log_2 n^2$ (exponentul din logaritm iese în față)

$(\hookrightarrow \log_2 n^2 = 2 \cdot \log_2 n)$

$2n^2 \cdot \log_2 n \in \Theta(n^2 \cdot \log_2 n)$

<u>Obs</u>: $\log_2 n^2 = 2 \cdot \log_2 n$ | Nu sunt egale

$\log_2^2 n = \log n * \log n$

23. Recursiv:

$$T(n) = \begin{cases} 1 & n \leq 1 \\ 4 \cdot T\left(\frac{n}{2}\right) + 1 & \text{altfel} \end{cases}$$

$T(n) = 4 \cdot T\left(\frac{n}{2}\right) + 1$ presupunem $n = 2^k$

$$T(2^k) = 4 \cdot \cancel{T(2^{k-1})} + 1$$

$$\cancel{4 \cdot T(2^{k-1})} = 4^2 \cdot \cancel{T(2^{k-2})} + 4 \qquad T(2^{k-1}) = 4 \cdot T(2^{k-2}) + 1$$

$$\cancel{4^2 \cdot T(2^{k-2})} = \cancel{4^3 \cdot T(2^{k-3})} + 4^2 \qquad T(2^{k-2}) = 4 \cdot T(2^{k-3}) + 1$$

$$\vdots$$

$$\cancel{4^{k-1} \cdot T(2^1)} = 4^k \cdot T(2^0) + 4^{k-1} \quad (+)$$

$$T(2^k) = 1 + 4 + 4^2 + \dots 4^k \cdot T(1) + 4^{k-1}$$

$$= 4^0 + 4^1 + 4^2 + \dots 4^{k-1} + 4^k = \frac{4^k \cdot 4 - 1}{4 - 1}$$

$$= \frac{4 \cdot (2^k)^2 - 1}{3} = \frac{4 \cdot n^2 - 1}{3} \in \Theta(n^2)$$

24. Recursiv

$$T(n) = \begin{cases} 1 & n \le 0 \\ T(n-1)+1 & \text{altfel} \end{cases}$$

$$T(n) = \cancel{T(n-1)} + 1$$

$$\cancel{T(n-1)} = \cancel{T(n-2)} + 1$$

$$\cancel{T(n-2)} = \cancel{T(n-3)} + 1$$

$$\vdots$$

$$\cancel{T(1)} = T(0) + 1 \quad (+)$$

$$T(n) = \underbrace{1+1+1+1}_{n \text{ ori}} + T(0) = n+1 \in \Theta(n)$$

25 Recursiv

$$T(n) = \begin{cases} 1 & n \le 0 \\ T(n-5)+1 & \text{altfel} \end{cases}$$

presupunem $n = 5 \cdot k$

$$\Downarrow$$

$$k = \frac{n}{5}$$

$$T(n) = \cancel{T(n-5)} + 1$$

$$\cancel{T(5 \cdot k)} = \cancel{T(5 \cdot (k-1))} + 1$$

$$\cancel{T(5 \cdot (k-1))} = \cancel{T(5 \cdot (k-2))} + 1$$

$$\cancel{T(5 \cdot (k-2))} = T(\cancel{5 \cdot (k-3)}) + 1$$

$$\vdots$$

$$\cancel{T(5 \cdot 1)} = T(5 \cdot 0) + 1 \quad (+)$$

$$1+1+1+T(0) = k+1 = \frac{n}{5}+1 \in \Theta(n)$$

26. Recursiv:

$$T(n) = \begin{cases} 1 & \text{dacă } n \le 0 \\ T\left(\frac{n}{5}\right) + 1 & \text{altfel} \end{cases}$$

$$T(n) = T\left(\frac{n}{5}\right) + 1$$

presupunem: $n = 5^k$

$\Downarrow$

$k = \log_5 n$

$$T(5^k) = T(5^{k-1}) + 1$$

$$T(\cancel{5^{k-1}}) = T(5^{k-2}) + 1$$

$$T(\cancel{5^{k-2}}) = T(5^{k-3}) + 1$$

$$\vdots$$

$$T(5^1) = T(5^0) + 1 \quad (+)$$

$$T(5^k) = 1 + 1 + 1 + 1 + T(5^0) = k+1 = \log_5 n + 1$$

$$\in \Theta(\log_2 n)$$

$$\log_5 n = \frac{\log_2 n}{\log_2 5} \Leftarrow \text{constantă}$$

27. Deși avem n m și o recursivitatea depinde doar de n.

$$T(n) = \begin{cases} 1 & n \le 0 \\ 2 \cdot T(n-1) + 1 & \text{altfel} \end{cases}$$

$$T(n) = 2 \cdot \cancel{T(n-1)} + 1$$

$$\cancel{2^1 \cdot T(n-1)} = 2^2 \cdot \cancel{T(n-2)} + 2$$

$$\cancel{2^2 \cdot T(n-2)} = 2^3 \cdot \cancel{T(n-3)} + 2^2$$

$$\vdots$$

$$\cancel{2^{n-1} T(1)} = 2^n \cdot T(0) + 2^{n-1} \quad \oplus$$

$$T(n) = 1 + 2 + 2^2 + \dots + 2^{n-1} + 2^n \cdot T(0)$$

$$= \frac{2^{n+1} - 1}{1} \in \Theta(2^n)$$

28. Recursiv:

$$T(n) = \begin{cases} 1 & n \le 0 \text{ (dacă } n=0 \text{ for-ul nu se execută)} \\ T(n-5) + n & \leftarrow \text{ciclul for (dacă avem un "+n", "+1" nu mai trebuie)} \end{cases}$$

$T(n) = T(n-5) + n$

presupunem: $n = 5 \cdot k$

$\Downarrow$

$k = \frac{n}{5}$

$T(5k) = T(\cancel{5 \cdot (k-1)}) + 5k$

$\cancel{T(5(k-1))} = T(\cancel{5(k-2)}) + 5 \cdot (k-1)$

$\cancel{T(5 \cdot (k-2))} = T(\cancel{5 \cdot (k-3)}) + 5 \cdot (k-2)$

$\vdots$

$\cancel{T(5 \cdot 1)} = T(5 \cdot 0) + 5 \cdot 1$ (+)

$T(5k) = 5k + 5 \cdot (k-1) + 5 \cdot (k-2) + \dots + (5 \cdot 1) + 1$

$= 5 \cdot (\underbrace{k + k-1 + k}-2 + \dots 1) + 1$

$= 5 \cdot (k \cdot k - (1 + 2 + 3 + \dots + k-1)) + 1$

$= 5 \cdot \left(k^2 - \frac{(k-1) \cdot k}{2}\right) + 1 = 5 \cdot \left(\frac{2k^2 - k^2 + k}{2}\right) + 1$

$= \frac{5}{2} \cdot (k^2 + k) + 1 = \left[\left(\frac{n}{5}\right)^2 + \left(\frac{n}{5}\right)\right] \cdot \frac{5}{2} + 1 \in \Theta(n^2)$