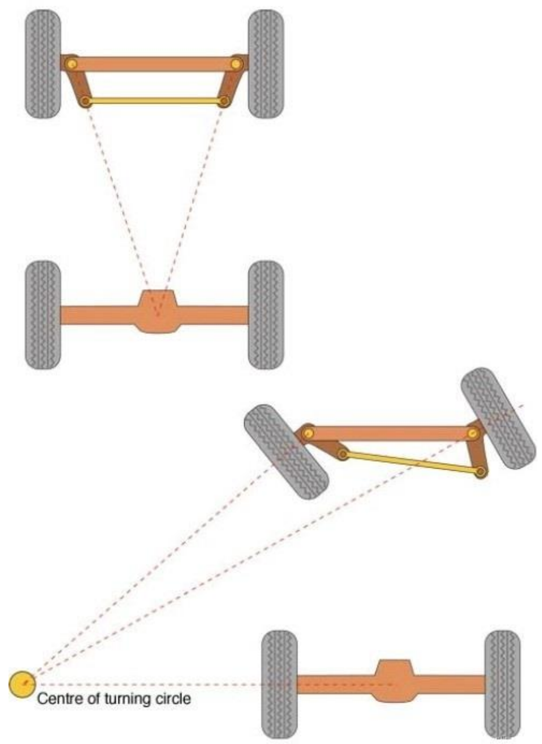
阿克曼结构小车的泊停过程

本代码主要完成阿克曼结构小车的倒车入库的代码复现，主要是对垂直停车的模拟部分。

一、阿克曼小车的结构

阿克曼小车，指的是利用阿克曼转向几何制作的小车，就是路上常见的前轮转向的结构。阿克曼转向机构（Ackermann steering）是为了解决汽车在转向时，由于左、右转向轮的转向半径不同所造成的左、右转向轮转角不同的问题，除了阿克曼结构，还有万向轮（全向轮）结构和麦克纳姆结构，以下是阿克曼舵机转向示意图：

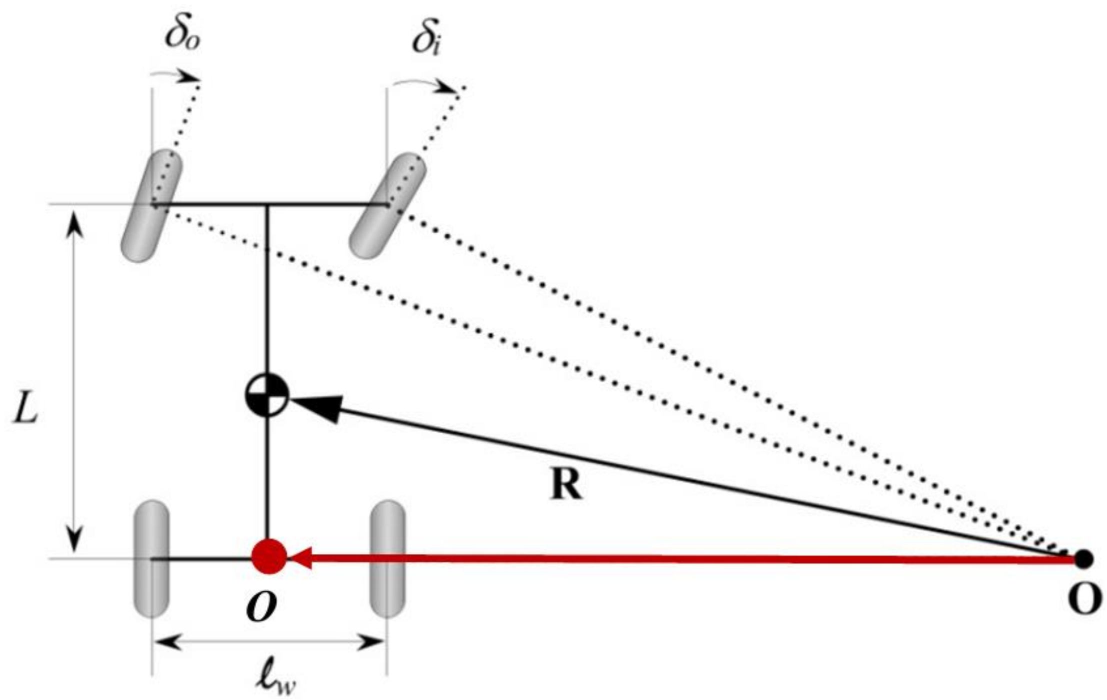


二、阿克曼转向几何

阿克曼转向几何(Ackerman Turning Geometry)是一种为了解决交通工具转弯时，内外转向轮路径指向的圆心不同的几何学。

在单车模型中，将转向时左/右前轮偏角假设为同一角度，虽然通常两个角度大致相等，但实际并不是，通常情况下，内侧轮胎转角更大。如下图所示，δ_o 和δ_i 分别为外侧前轮和内侧前轮偏角，当车辆右转时，右前轮胎为内侧轮胎，其转角δ_i 较左前轮胎转角δ_o 更大。ℓ_W为轮距，L 为轴距，后轮两轮胎转角始终为 0 °。

当以后轴中心为参考点时，转向半径 R 为下图中红线。



当滑轮角当滑移角β很小时，且后轮偏角为 0 时，我们有：

$$\frac{\dot{\psi}}{V} \approx \frac{1}{R} = \frac{\delta}{L}$$

由于内外侧轮胎的转向半径不同，因此有：

$$\delta_o = \frac{L}{R + \frac{\ell_w}{2}}$$

$$\delta_i = \frac{L}{R - \frac{\ell_w}{2}}$$

则前轮平均转角

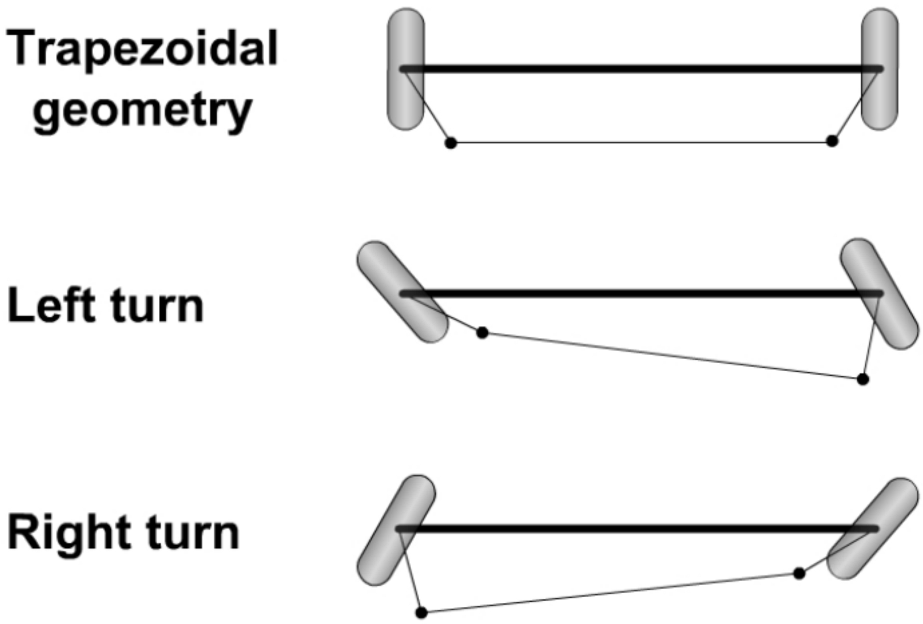
$$\delta = \frac{\delta_o + \delta_i}{2} \cong \frac{L}{R}$$

内外转角之差

$$\Delta\delta = \delta_i - \delta_o = \frac{L}{R^2}\ell_w = \delta^2\frac{\ell_w}{L}$$

因此，两个前轮的转向角的差异Δδ 与平均转向角 δ 的平方成正比。

依据阿克曼转向几何设计的车辆，沿着弯道转弯时，利用四连杆的相等曲柄使内侧轮的转向角比外侧轮大大约 2~4 度，使四个轮子路径的圆心大致上交会于后轴的延长线上瞬时转向中心，让车辆可以顺畅的转弯，如下图所示，称作梯形拉杆装置差动转向机构：



我们根据如上原理复现了转向的代码，如下图：

```
function [xa1, xb1, xb2, xa2, ya1, yb1, yb2, ya2]=Turn1(x,y,a,b,seta)
global wl;
global ww;
global D;
global L;
global WSeta;



xa1=x;
ya1=y;


xa2=xa1+a*cos(seta);
ya2=ya1-a*sin(seta);

xb1=x+b*sin(seta);
yb1=y+b*cos(seta);

xb2=xb1+a*cos(seta);
yb2=yb1-a*sin(seta);

patch([xa1 xa2 xb2 xb1],[ya1 ya2 yb2 yb1],[0 1 0]);

% The left back wheel
xref=x+(D-wl/2)*sin(seta);
yref=y+(D-wl/2)*cos(seta);

xa11=xref-(ww/2)*cos(seta);
ya11=yref+(ww/2)*sin(seta);

xa21=xa11+(ww)*cos(seta);
ya21=ya11-(ww)*sin(seta);

xb11=xa11+wl*sin(seta);
yb11=ya11+wl*cos(seta);

xb21=xb11+ww*cos(seta);
yb21=yb11-ww*sin(seta);

patch([xa11 xa21 xb21 xb11],[ya11 ya21 yb21 yb11],[0 0 0]);



% The Right back wheel

xref=xref+(a)*cos(seta);
yref=yref-(a)*sin(seta);

xa11=xref-(ww/2)*cos(seta);
ya11=yref+(ww/2)*sin(seta);

xa21=xa11+(ww)*cos(seta);
ya21=ya11-(ww)*sin(seta);

xb11=xa11+wl*sin(seta);
yb11=ya11+wl*cos(seta);

xb21=xb11+ww*cos(seta);
yb21=yb11-ww*sin(seta);

patch([xa11 xa21 xb21 xb11],[ya11 ya21 yb21 yb11],[0 0 0]);
```

```
% The left front wheel
xref=x+(D+L)*sin(seta);
yref=y+(D+L)*cos(seta);

KWheel=sqrt((wl/2)^2+(ww/2)^2);


xa11=xref-KWheel*sin(seta-WSeta+atan(ww/wl));
ya11=yref-KWheel*cos(seta-WSeta+atan(ww/wl));

xa21=xa11+(ww)*cos(seta-WSeta);
ya21=ya11-(ww)*sin(seta-WSeta);

xb11=xa11+wl*sin(seta-WSeta);
yb11=ya11+wl*cos(seta-WSeta);

xb21=xb11+ww*cos(seta-WSeta);
yb21=yb11-ww*sin(seta-WSeta);

patch([xa11 xa21 xb21 xb11],[ya11 ya21 yb21 yb11],[0 0 0]);


% The Right front wheel
xref=xref+(a)*cos(seta);
yref=yref-(a)*sin(seta);

KWheel=sqrt((wl/2)^2+(ww/2)^2);


xa11=xref-KWheel*sin(seta-WSeta+atan(ww/wl));
ya11=yref-KWheel*cos(seta-WSeta+atan(ww/wl));

xa21=xa11+(ww)*cos(seta-WSeta);
xa21=xa11+(ww)*cos(seta-WSeta);
ya21=ya11-(ww)*sin(seta-WSeta);

xb11=xa11+wl*sin(seta-WSeta);
yb11=ya11+wl*cos(seta-WSeta);

xb21=xb11+ww*cos(seta-WSeta);
yb21=yb11-ww*sin(seta-WSeta);

patch([xa11 xa21 xb21 xb11],[ya11 ya21 yb21 yb11],[0 0 0]);


% The Right front wheel
xref=xref+(a)*cos(seta);
yref=yref-(a)*sin(seta);

KWheel=sqrt((wl/2)^2+(ww/2)^2);


xa11=xref-KWheel*sin(seta-WSeta+atan(ww/wl));
ya11=yref-KWheel*cos(seta-WSeta+atan(ww/wl));

xa21=xa11+(ww)*cos(seta-WSeta);
ya21=ya11-(ww)*sin(seta-WSeta);

xb11=xa11+wl*sin(seta-WSeta);
yb11=ya11+wl*cos(seta-WSeta);

xb21=xb11+ww*cos(seta-WSeta);
yb21=yb11-ww*sin(seta-WSeta);

patch([xa11 xa21 xb21 xb11],[ya11 ya21 yb21 yb11],[0 0 0]);
```

二、代码复现

转向代码已在上面给出，主函数代码如下所示：

```
clc
close all
clearvars
set(gca,'xtick',[])
set(gca,'ytick',[])
% Declare global variables

global wl;
global ww;
global D;
global L;
global WSeta;

wl=7;
ww=2;
    A=15;
    L=20;
    D=5;
    F=8;
    B=L+D+F;

% Rectangle Coordinates for Parked car 1
    Parked_Car1x1=0;
    Parked_Car1y1=40;
    Parked_Car1x2=Parked_Car1x1;
    Parked_Car1y2=75;
    Parked_Car1x3=15;
    Parked_Car1y3=Parked_Car1y2;
    Parked_Car1x4=Parked_Car1x3;
    Parked_Car1y4=Parked_Car1y1;

% Rectangle Coordinates for Parked car 2
% Rectangle Coordinates for Parked car 2

    Parked_Car2x1=40;
    Parked_Car2y1=40;
    Parked_Car2x2=Parked_Car2x1;
    Parked_Car2y2=75;
    Parked_Car2x3=55;
    Parked_Car2y3=Parked_Car2y2;
    Parked_Car2x4=Parked_Car2x3;
    Parked_Car2y4=Parked_Car2y1;


% Rectangle Coordinates for Parked car 3
    Parked_Car3x1=-40;
    Parked_Car3y1=40;
    Parked_Car3x2=Parked_Car3x1;
    Parked_Car3y2=75;
    Parked_Car3x3=-25;
    Parked_Car3y3=Parked_Car3y2;
    Parked_Car3x4=Parked_Car3x3;
    Parked_Car3y4=Parked_Car3y1;


%  Tuning Parameters

    WSeta=50;
    WSeta=(WSeta/180)*pi; % degree to radians
    xi=Parked_Car2x2-3;
    yi=Parked_Car2y2+25;
    fle=40;
```

```
% Simulation Setup

    axis([-60 100 -20 100],'equal');
    t=0:0.01:pi;
    seta=2*t;
    Xc=xi-(L/tan(WSeta));
    Yc=yi+D;


    Rbl=sqrt(D^2+(L/tan(WSeta))^2);
    phi=atan(D/(L/tan(WSeta)));

% Initialize start scenario

        i=1;
        x=Xc+Rbl*cos(seta(i)+phi);
        y=Yc-Rbl*sin(seta(i)+phi);
        patch([Parked_Car2x1 Parked_Car2x2 Parked_Car2x3 Parked_Car2x4],[Parked_Car2y1 Parked_Car2y2 Parked_Car2y3 Parked_Car2y4],[1 0 0]);
        patch([Parked_Car1x1 Parked_Car1x2 Parked_Car1x3 Parked_Car1x4],[Parked_Car1y1 Parked_Car1y2 Parked_Car1y3 Parked_Car1y4],[0 1 1]);
        patch([Parked_Car3x1 Parked_Car3x2 Parked_Car3x3 Parked_Car3x4],[Parked_Car3y1 Parked_Car3y2 Parked_Car3y3 Parked_Car3y4],[1 1 0]);
        title(" ");
        text(Parked_Car1x2,Parked_Car1y2+5,"Car 1");
        text(Parked_Car2x2,Parked_Car2y2+5,"Car 2");
        text(Parked_Car3x2,Parked_Car3y2+5,"Car 3");
        text(Parked_Car3x2,Parked_Car3y2+60,"The Parking Process of Ackerman's Vehicle",'Color','r','FontSize',14);
        Turn1(x,y,A,B,seta(i));
        pause(4);


% Car Parking Simulation Scenario

    for i=2:fle
        x=Xc+Rbl*cos(seta(i)+phi);



    for i=2:fle
        x=Xc+Rbl*cos(seta(i)+phi);
        y=Yc-Rbl*sin(seta(i)+phi);
        cla;
        patch([Parked_Car2x1 Parked_Car2x2 Parked_Car2x3 Parked_Car2x4],[Parked_Car2y1 Parked_Car2y2 Parked_Car2y3 Parked_Car2y4],[1 0 0]);
        patch([Parked_Car1x1 Parked_Car1x2 Parked_Car1x3 Parked_Car1x4],[Parked_Car1y1 Parked_Car1y2 Parked_Car1y3 Parked_Car1y4],[0 1 1]);
        patch([Parked_Car3x1 Parked_Car3x2 Parked_Car3x3 Parked_Car3x4],[Parked_Car3y1 Parked_Car3y2 Parked_Car3y3 Parked_Car3y4],[1 1 0]);
        text(Parked_Car1x2,Parked_Car1y2+5,"Car 1");
        text(Parked_Car2x2,Parked_Car2y2+5,"Car 2");
        text(Parked_Car3x2,Parked_Car3y2+5,"Car 3");
        text(Parked_Car3x2,Parked_Car3y2+60,"The Parking Process of Ackerman's Vehicle",'Color','r','FontSize',14);
        Turn1(x,y,A,B,seta(i));
        pause(.05);
    end



     xi=x+A*cos(seta(fle));
     yi=y-A*sin(seta(fle));
    Xc=xi+Rbl*cos(seta(fle));
    Yc=yi-Rbl*sin(seta(fle));
    pause(1);



    for i=fle:-1:1
        x=Xc-Rbl*cos(seta(i));
        y=Yc+Rbl*sin(seta(i));
        cla;
        patch([Parked_Car2x1 Parked_Car2x2 Parked_Car2x3 Parked_Car2x4],[Parked_Car2y1 Parked_Car2y2 Parked_Car2y3 Parked_Car2y4],[1 0 0]);
        patch([Parked_Car1x1 Parked_Car1x2 Parked_Car1x3 Parked_Car1x4],[Parked_Car1y1 Parked_Car1y2 Parked_Car1y3 Parked_Car1y4],[0 1 1]);

        patch([Parked_Car3x1 Parked_Car3x2 Parked_Car3x3 Parked_Car3x4],[Parked_Car3y1 Parked_Car3y2 Parked_Car3y3 Parked_Car3y4],
        text(Parked_Car1x2,Parked_Car1y2+5,"Car 1");
        text(Parked_Car2x2,Parked_Car2y2+5,"Car 2");
        text(Parked_Car3x2,Parked_Car3y2+5,"Car 3");
        text(Parked_Car3x2,Parked_Car3y2+60,"The Parking Process of Ackerman's Vehicle",'Color','r','FontSize',14);
        [xa1, xb1, xb2, xa2, ya1, yb1, yb2, ya2]=Turn2(x,y,A,B,seta(i));
        pause(.05);
    end


 pause(1);
 for i=1:50
 ya1=ya1-.5;
 yb1=yb1-.5;
 ya2=ya2-.5;
 yb2=yb2-.5;
 cla;
patch([Parked_Car2x1 Parked_Car2x2 Parked_Car2x3 Parked_Car2x4],[Parked_Car2y1 Parked_Car2y2 Parked_Car2y3 Parked_Car2y4],[1 0 0]);
patch([Parked_Car1x1 Parked_Car1x2 Parked_Car1x3 Parked_Car1x4],[Parked_Car1y1 Parked_Car1y2 Parked_Car1y3 Parked_Car1y4],[0 1 1]);
patch([Parked_Car3x1 Parked_Car3x2 Parked_Car3x3 Parked_Car3x4],[Parked_Car3y1 Parked_Car3y2 Parked_Car3y3 Parked_Car3y4],[1 1 0]);
patch([xa1 xb1 xb2 xa2],[ya1 yb1 yb2 ya2],[0 1 0]);
text(Parked_Car1x2,Parked_Car1y2+5,"Car 1");
text(Parked_Car2x2,Parked_Car2y2+5,"Car 2");
text(Parked_Car3x2,Parked_Car3y2+5,"Car 3");
text(Parked_Car3x2,Parked_Car3y2+60,"The Parking Process of Ackerman's Vehicle",'Color','r','FontSize',14);
 ya11=ya1+(D-wl/2);
 yb11=ya11+wl;
 ya21=ya2+(D-wl/2);
 yb21=ya21+wl;
 xa11=xa2+(ww/2);
 xb11=xa2+(ww/2);
 xa21=xa2-(ww/2);
 xb21=xa2-(ww/2);
```

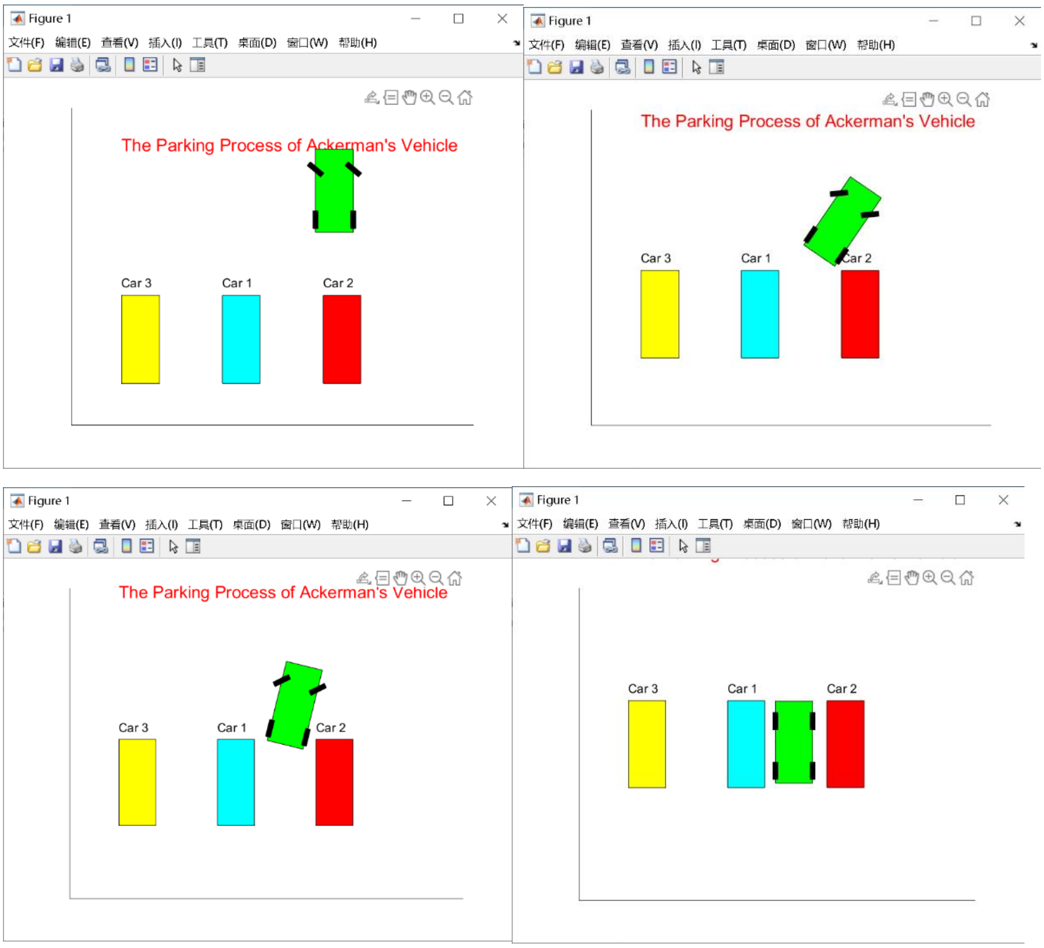
```
patch([xa11 xb11 xb21 xa21],[ya11 yb11 yb21 ya21],[0 0 0]);
xa22=xa1-(ww/2);
xb22=xa1-(ww/2);
xa12=xa1+(ww/2);
xb12=xa1+(ww/2);
patch([xa12 xb12 xb22 xa22],[ya11 yb11 yb21 ya21],[0 0 0]);
ya13=ya11+L;
yb13=yb11+L;
patch([xa11 xb11 xb21 xa21],[ya13 yb13 yb13 ya13],[0 0 0]);
patch([xa12 xb12 xb22 xa22],[ya13 yb13 yb13 ya13],[0 0 0]);

pause(.02);
end
```

四、代码运行效果

我们根据以上阿克曼小车的运动学原理复现代码，初步实现了阿克曼小车的指定位置停泊。

阿克曼小车依据以下图片所示运作，模拟将小车停放在 Car1 和 Car2 之间的位置，以下四张图片为实现停车的主要过程截图：



三、参考网址

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