# Problem Solving Techniques 문제해결

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  - Program design example

# Contents

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# 1. Problem Description

## Description of Plate Packing Problem

- A manufacturer seeks to enter the competitive campus dining hall market.
- Dining halls only buy plates in a single standard size.
- The company seeks an edge in the market through its unique packing method.
- The company tries to pack the plates as many as possible, not to be broken
  The packing box size: horizontal $w$ and vertical $h$, The plate radius: $r$

# 1. Problem Description

## ❖ Description of Plate Packing Problem

- A manufacturer seeks to enter the competitive campus dining hall market.
- Dining halls only buy plates in a single standard size.
- The company seeks an edge in the market through its unique packing method.
- The company tries to pack the plates as many as possible, not to be broken
  The packing box size: horizontal $w$ and vertical $h$, The plate radius: $r$
- Question 1) Which packing method should be chosen?
- Question 2) How many plates can be packed by the method?
- Question 3) How many plates can be placing on the top of any given plate?

# 2. Plate Packing (1)

- **Question 1) & 2) Which packing method? How many plates packed?**
  - **Consider two methods!: 1) Rectangular lattices, 2) Hexagonal lattices**

# 2. Plate Packing (1)

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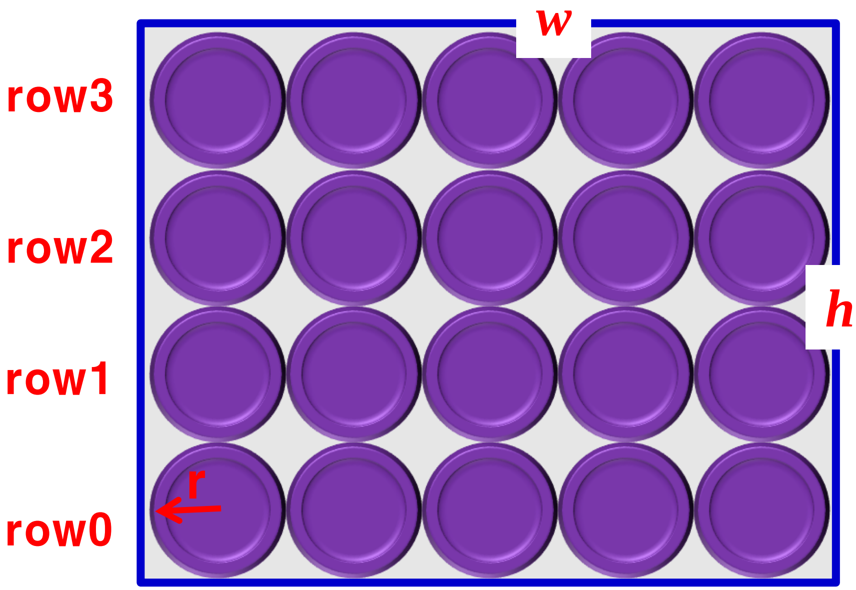



Method 1: Rectangular

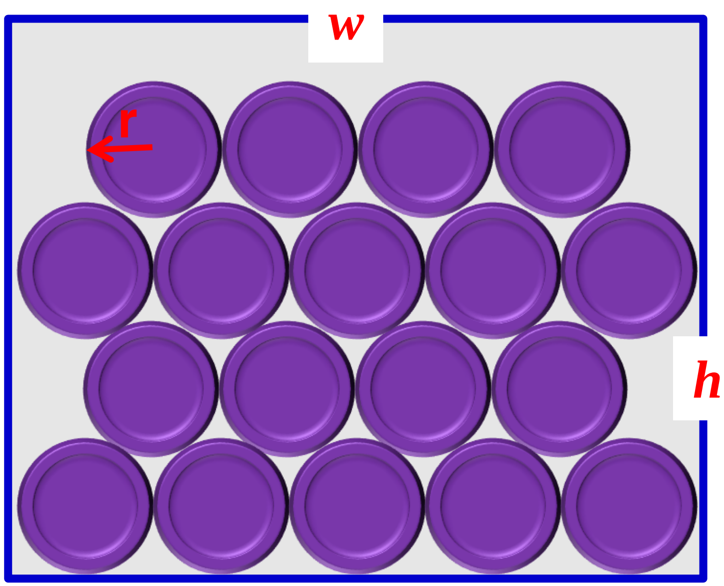
L = num. of layers $$L = \lfloor h/2r \rfloor$$

P=num. of plates per layer $$P = \lfloor w/2r \rfloor$$

T = total num. of plates $$T = \lfloor h/2r \rfloor \times \lfloor h/2r \rfloor$$

# 2. Plate Packing (1)

- **Question 1) & 2) Which packing method? How many plates packed?**
  - **Consider two methods!: 1) Rectangular lattices, 2) Hexagonal lattices**



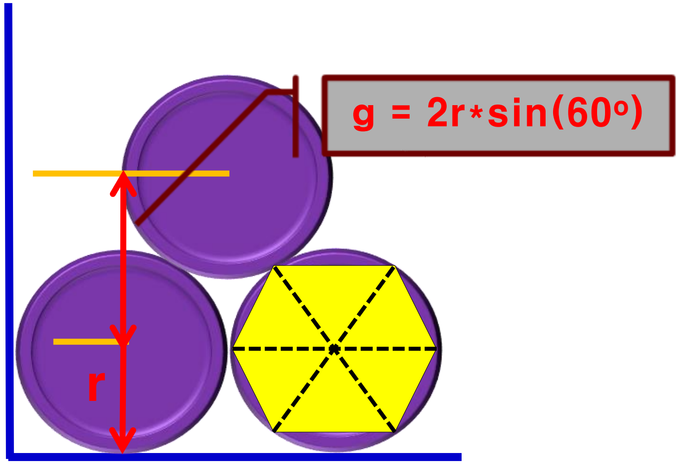

Method 2: Hexagonal

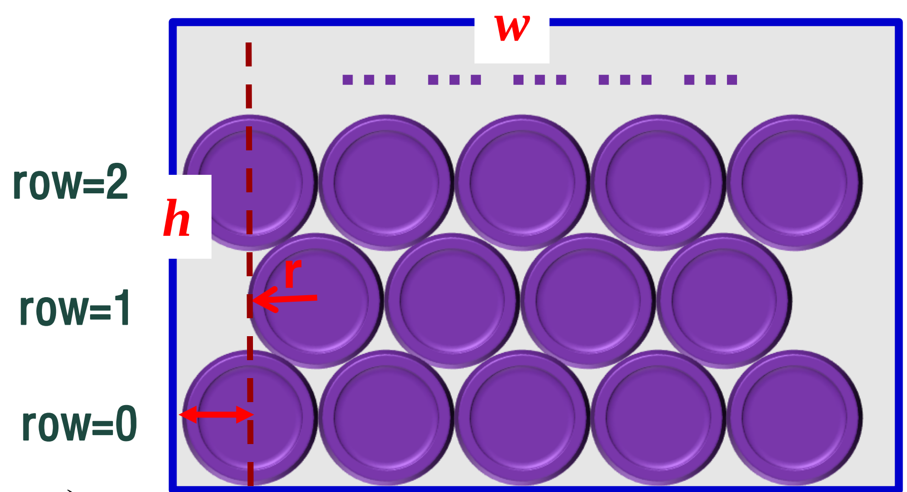
L = ?

P = ?

T = ?

# 2. Plate Packing (2)

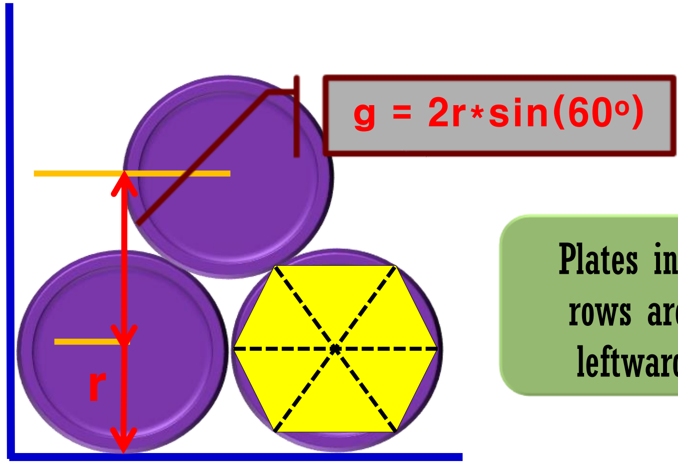



$$L = (1 + \lfloor (h-2r)/g \rfloor) = \left(1 + \left\lfloor \frac{h-2r}{2r \cdot (\sqrt{3}/2)} \right\rfloor \right)$$

```
int dense_layers(double w, double h, double r)
{
        double gap;

        if ((2*r) > h) return(0);

        gap = 2.0 * r * (sqrt(3)/2.0);
        return( 1 + floor((h-2.0*r)/gap) );
}
```

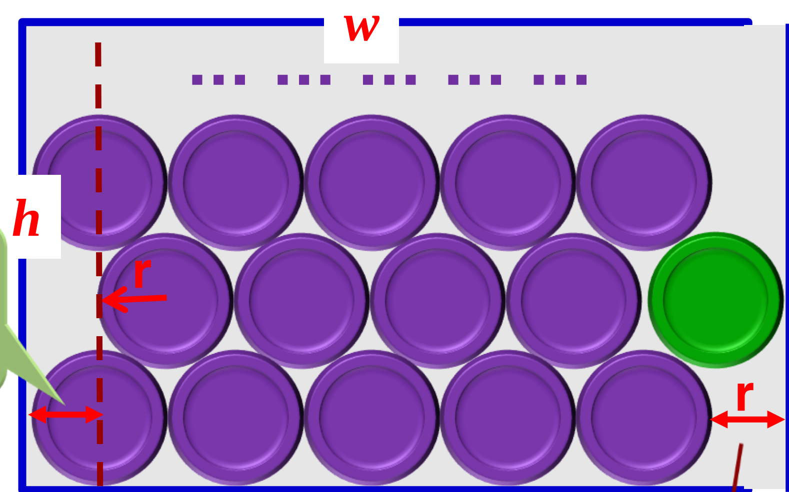
# 2. Plate Packing (2)




row=2

Plates in the even rows are skewed leftwardly by r!

row=0




Unless the remaining space in the even rows is greater than r, #plates in the odd rows is less than… by 1!

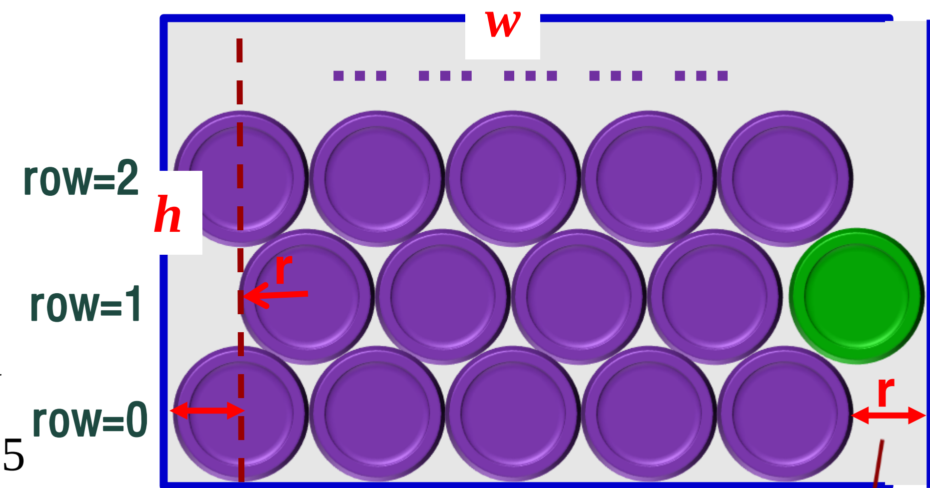
$$L = (1 + \lfloor (h-2r)/g \rfloor) = \left(1 + \left\lfloor \frac{h-2r}{2r \cdot (\sqrt{3}/2)} \right\rfloor\right)$$

```
int dense_layers(double w, double h, double r)
{
        double gap;

        if ((2*r) > h) return(0);

        gap = 2.0 * r * (sqrt(3)/2.0);
        return( 1 + floor((h-2.0*r)/gap) );
}
```

$$P = \begin{cases} \lfloor w/2r \rfloor, & \text{for even-numbered row} \\ \lfloor w/2r \rfloor, & \text{for odd-numbered row} \\ & \quad \& \, (w/2r) - \lfloor w/2r \rfloor \geq 0.5 \\ \lfloor w/2r \rfloor - 1, & \text{for odd-numbered row} \\ & \quad \& \, (w/2r) - \lfloor w/2r \rfloor < 0.5 \end{cases}$$

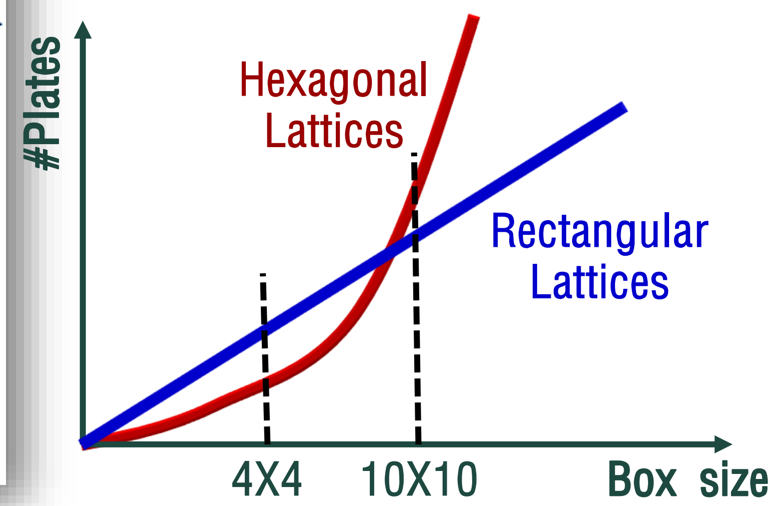
# 2. Plate Packing (3)

$$P=\begin{cases}\lfloor w/2r\rfloor, & \text{for even-numbered row}\\ \lfloor w/2r\rfloor, & \text{for odd-numbered row}\\ & \quad \&\ (w/2r)-\lfloor w/2r\rfloor \ge 0.5\\ \lfloor w/2r\rfloor-1, & \text{for odd-numbered row}\\ & \quad \&\ (w/2r)-\lfloor w/2r\rfloor < 0.5\end{cases}$$



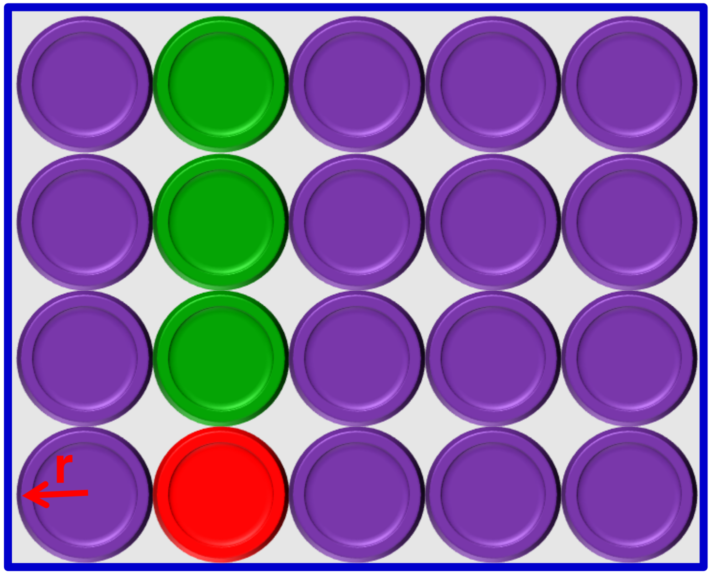

Unless the remaining space in the even vs is greater than r, #plates in the odd rows is less than… by 1!

```
int plates_per_row(int row, double w, double r)
{
        int plates_per_full_row;

        plates_per_full_row = floor(w/(2*r));

        if ((row % 2) == 0) return(plates_per_full_row);

        if (((w/(2*r))-plates_per_full_row) >= 0.5)
                return(plates_per_full_row);
        else
                return(plates_per_full_row - 1);
}
```



# 3. Plate Weight (1)

- Question 3) How many plates can be placing on the top of any given plate?
  - Consider the rectangular method

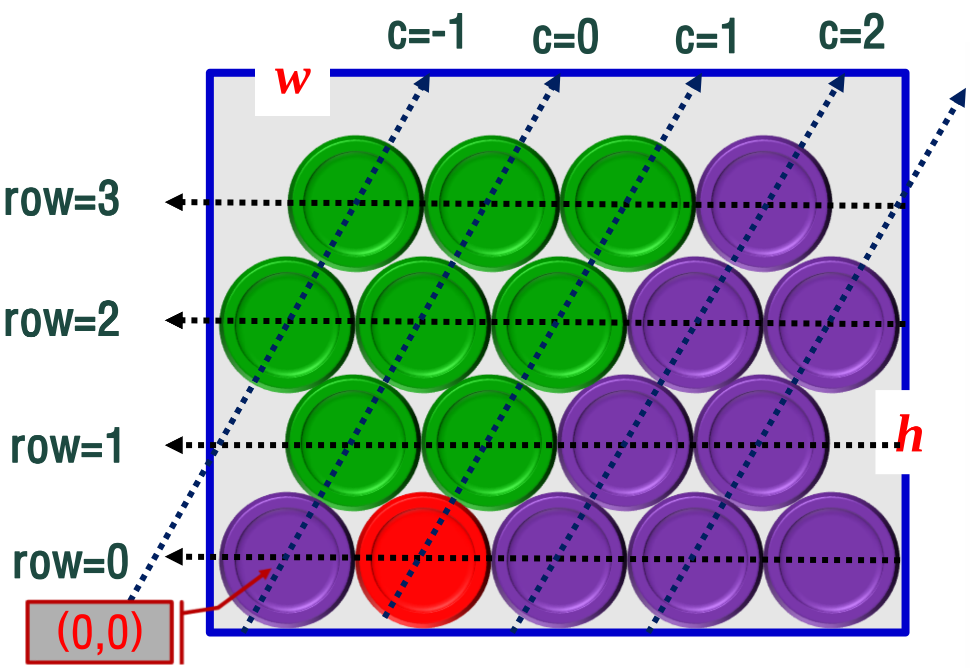


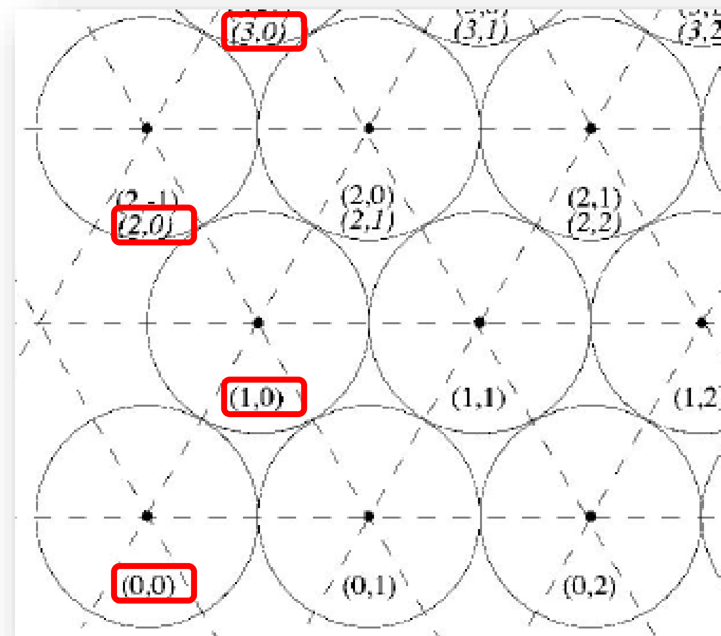

❖The red plate only receives monotonic loads from the green plates.

# 3. Plate Weight (1)

❖ **Question 3) How many plates can be placing on the top of any given plate?**

- **Consider the hexagonal method**




❖**The red plate only receives monotonic loads from the green plates.**

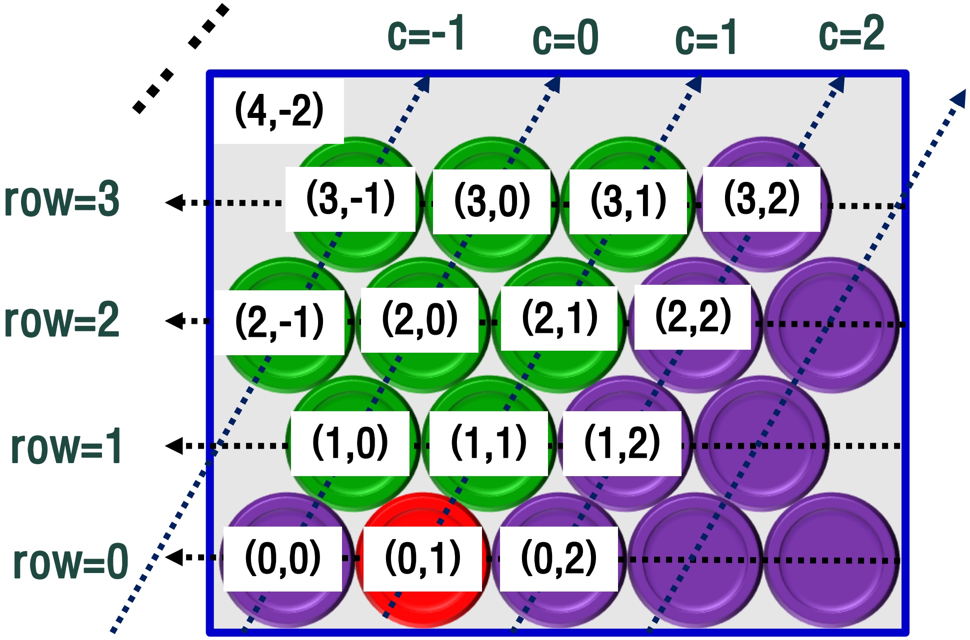
❖**In the hexagonal axis, two plates piling up the plate at (k,c) are at (k+1,c-1) & (k+1,c).**

❖**In the (k+i)the row, #plates that give the loads to the plate at (k,c) is i+1.**

❖**But, such #plates is dependent on the boundary of box.**

❖**To this end, the hexagonal coordinate is transformed into the array coordinate.**

# 3. Plate Weight (2)




- In the hexagonal one, a pattern on the increment of **negative numbers** exists.
- The following rule can be obtained.

$$x_a = x_h$$

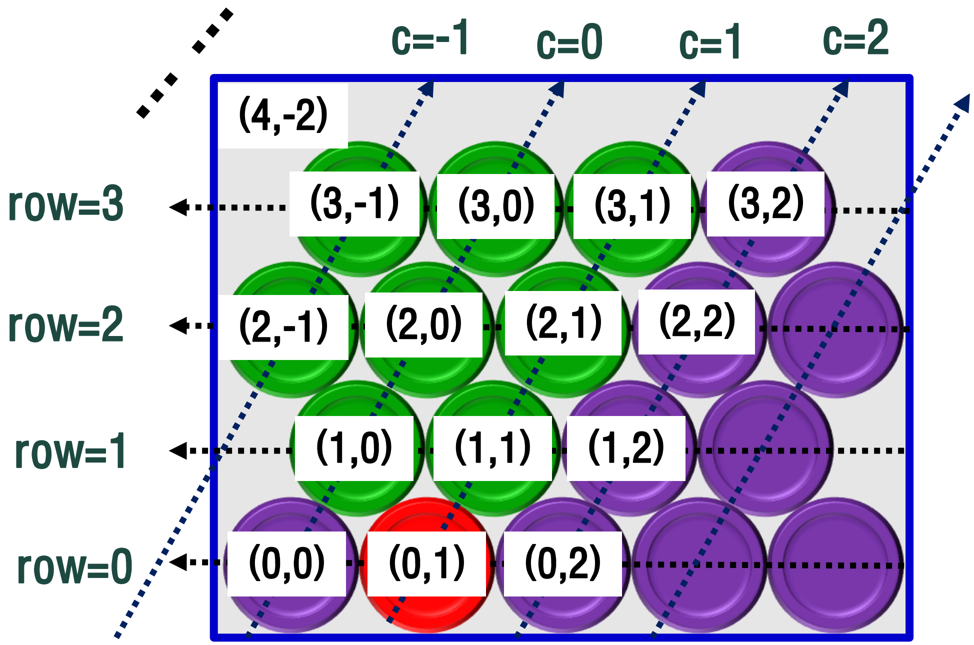
$$y_a = y_h + \left(x_h - \lceil x_h / 2 \rceil\right)$$

(2,-1)→(2,0), (2,0)→(2,1), …

(3,-1)→(3,0), (3,0)→(3,1), …

(4,-2)→(4,0), (4,-1)→(4,1), …

# 3. Plate Weight (2)

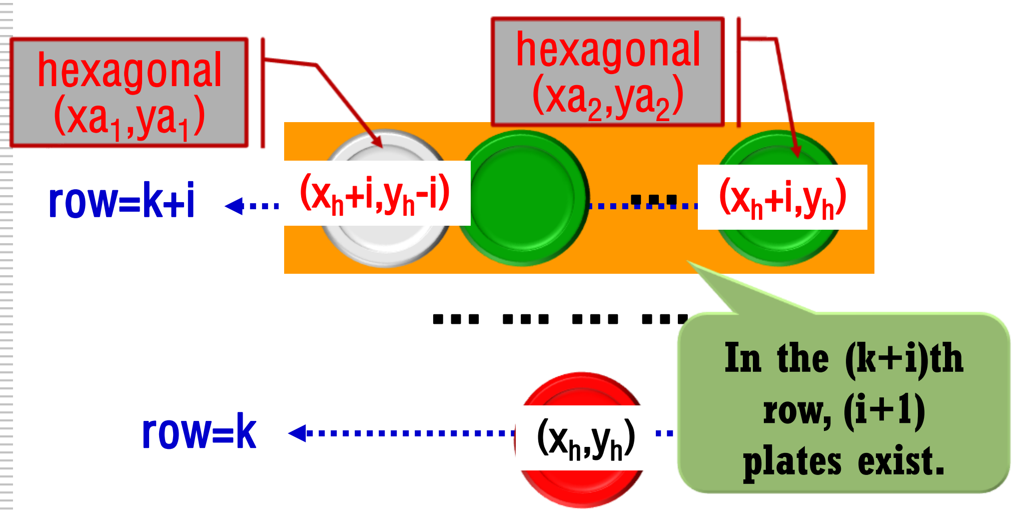



◆In the hexagonal one, a pattern on the increment of **negative numbers** exists.

◆The following rule can be obtained.

$$x_a = x_h$$
$$y_a = y_h + (x_h - \lceil x_h / 2 \rceil)$$

(2,-1)→(2,0), (2,0)→(2,1), …
(3,-1)→(3,0), (3,0)→(3,1), …
(4,-2)→(4,0), (4,-1)→(4,1), …




if $(ya_1<0)$ $ya_1 = 0$;
If $(ya_2>row\_length)$ $ya_2 = row\_length$;

◆In the **(k+i)th row**, #plates giving the loads the plate at $(x_h,y_h)$ becomes $ya_2 - ya_1 + 1$.

◆Thus, the **total load** is computed by all the plates w.r.t. all the rows.

# 3. Plate Weight (3)

```
int plates_on_top(int xh, int yh, double w, double l, double r)
{
        int number_on_top = 0;          /* total plates on top */
        int layers;                     /* number of rows in grid */
        int rowlength;                  /* number of plates in row */
        int row;                        /* counter */
        int xla,yla,xra,yra;            /* array coordinates */

        layers = dense_layers(w,l,r);

        for (row=xh+1; row<layers; row++) {
            rowlength = plates_per_row(row,w,r) - 1;

            hex_to_array(row,yh-row,&xla,&yla);
            if (yla < 0) yla = 0;               /* left boundary */

            hex_to_array(row,yh,&xra,&yra);
            if (yra > rowlength) yra = rowlength;  /* right boundary */

            number_on_top += yra-yla+1;
        }

        return(number_on_top);
}
```

```
int dense_layers(double w, double h, double r)
{
        double gap;

        if ((2*r) > h) return(0);

        gap = 2.0 * r * (sqrt(3)/2.0);
        return( 1 + floor((h-2.0*r)/gap) );
}
```

```
int plates_per_row(int row, double w, double r)
{
        int plates_per_full_row;

        plates_per_full_row = floor(w/(2*r));

        if ((row % 2) == 0) return(plates_per_full_row);

        if (((w/(2*r))-plates_per_full_row) >= 0.5)
                return(plates_per_full_row);
        else
                return(plates_per_full_row - 1);
}
```

```
hex_to_array(int xh, int yh, int *xa, int *ya)
{
        *xa = xh;
        *ya = yh + xh - ceil(xh/2.0);
}
```

# Contents

- Chapter 12 – Grid
  - Exercise

# How many squares/rectangles and cubes/boxes?

- You can see a (4x4) grid below. Can you tell me how many squares and rectangles are hidden there? Perhaps one can count it by hand but can you count it for a (100x100) grid or a (10000x10000) grid. Can you do it for higher dimensions? That is can you count how many cubes or boxes of different size are there in a (10x10x10) sized cube. Remember that your program needs to be very efficient. You can assume that squares are not counted as rectangles, and cubes are not counted as boxes.

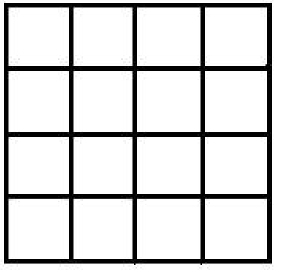


Fig: A 4x4 Grid

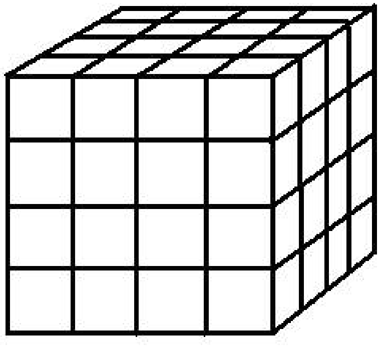


Fig: A 4x4x4 Cube

# How many squares/rectangles and cubes/boxes?

- Input
  - The input contains one integer N (1<=N<=100).

- Output
  - The output contains four integers S2, R2, S3, R3 where S2 means the number of squares of different size in (NxN) two-dimensional grid, and R2 means the number of rectangles of different size in (NxN) two-dimensional grid. S3 and R3 respectively mean the number of cubes and boxed of different size in (NxNxN) three-dimensional cube.

# How many squares/rectangles and cubes/boxes?

- Sample input/output 1

```
1
1 0 1 0
```

- Sample input/output 1

```
2
5 4 9 18
```

- Sample input/output 1

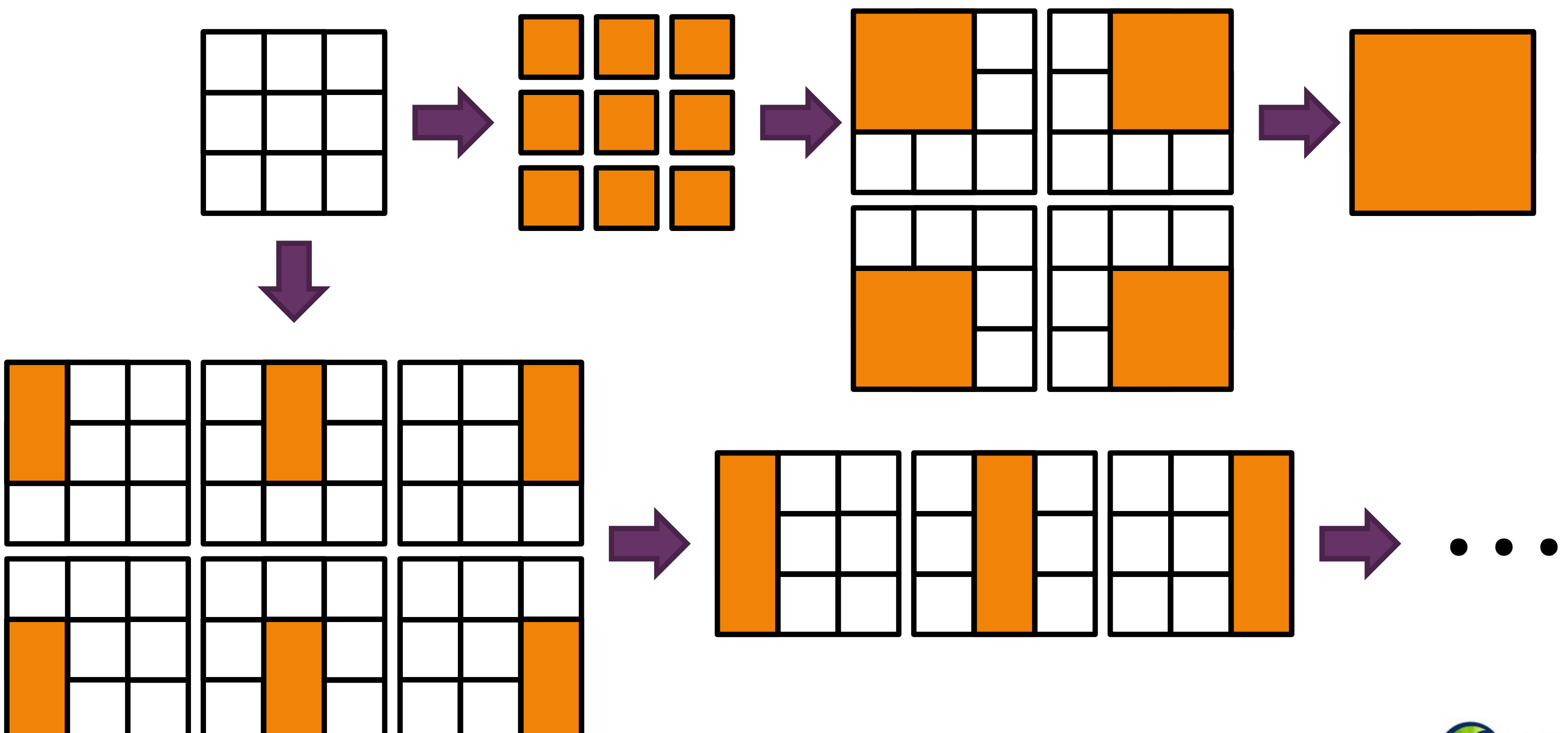
```
3
14 22 36 180
```

# How many squares/rectangles and cubes/boxes?

- Understanding the problem
  - N=2
    - How many squares in a (2x2) two-dimensional grid?
      - 1 (2x2) and 4 (1x1) squares
    - How many rectangles in a (2x2) two-dimensional grid?
      - 2 (2x1) and 2 (1x2) rectangles
    - How many cubes in a (2x2x2) sized cube?
      - 8 (1x1x1) and 1 (2x2x2) cubes
    - How many boxes in a (2x2x2) sized cube?
      - 4 (1x1x2), 4 (2x1x1), 4 (1x2x1), 2 (1x2x2), 2 (2x1x2) and 2 (2x2x1) boxes

# How many squares/rectangles and cubes/boxes?

- Understanding the problem
  - N=3
    - How many squares in a (3x3) two-dimensional grid?
      - 1 (3x3), 4 (2x2) and 9 (1x1) squares
    - How many rectangles in a (3x3) two-dimensional grid?
      - 3 (3x1), 3 (1x3), 6 (2x1), 6 (1x2), 2 (2x3) and 2 (3x2) rectangles



# How many squares/rectangles and cubes/boxes?

- Understanding the problem
  - N=3
    - How many cubes in a (3x3x3) sized cube?
      - 27 (1x1x1), 8 (2x2x2), 1 (3x3x3) cubes
    - How many boxes in a (3x3x3) sized cube?
      - …

# How many squares/rectangles and cubes/boxes?

- Approaches for counting the number of squares and cubes
  - N=3
    - How many squares in a (3x3) two-dimensional grid?
      - 1 (3x3), 4 (2x2) and 9 (1x1) squares
    - How many cubes in a (3x3x3) sized cube?
      - 1 (3x3x3), 8 (2x2x2) and 27 (1x1x1) cubes
    - Can you derive the recurrence relation?
      - $S_n = \sum_{i=1}^{N} (N+1-i)^n$
        - n: dimension
      - $S_2$ = 3^2 + 2^2 + 1^2 = 9+4+1 = 14
        - $S_2$ = N*(N+1)*(2N+1)/6
      - $S_3$ = 3^3 + 2^3 + 1^3 = 27+8+1 = 36
        - $S_3$ = (N*(N+1)/2)^2

# How many squares/rectangles and cubes/boxes?

- Approaches for counting the number of rectangles and boxes
  - N=3
    - How many rectangles in a (3x3) two-dimensional grid?
      - Size (1x2): 3*2=6
      - Size (2x1): 2*3=6
      - Size (3x2): 1*2=2
      - Size (2x3): 1*2=2
      - Size (AxB): (3+1-A)*(3+1-B)

# How many squares/rectangles and cubes/boxes?

- Approaches for counting the number of rectangles and boxes
  - N=3
    - How many boxes in a (3x3x3) sized cube?
      - Size (2x1x1):2*3*3 = 18
      - Size (1x2x1):3*2*3 = 18
      - Size (1x1x2): 3*3*2 = 18
      - Size (2x2x1): 2*2*3 = 12
      - Size (2x1x2): 2*3*2 = 12
      - Size (1x2x2): 3*2*2 = 12
      - Size (3x1x1): 1*3*3 = 9
      - Size (1x3x1): 3*1*3 = 9
      - Size (1x1x3): 3*3*1 = 9
      - Size (3x2x1): 1*2*3 = 6
      - Size (2x3x1): 2*1*3 = 6
      - Size (1x2x3): 3*2*1 = 6
      - Size (1x3x2): 3*1*2 = 6
      - Size (3x1x2): 1*3*2 = 6
      - Size (2x1x3): 2*3*1 = 6
      - Size (3x3x2): 1*1*2 = 2
      - Size (3x2x3): 1*2*1 = 2
      - Size (2x3x3): 2*1*1 = 2
      - Size (3x2x2): 1*2*2 = 4
      - Size (2x2x3): 2*2*1 = 4
      - Size (2x3x2): 1*2*1 = 4
      - Size (3x3x1): 1*1*3 = 3
      - Size (3x1x3): 1*3*1 = 3
      - Size (1x3x3): 3*1*1 = 3
      - Size (AxBxC): (3+1-A)*(3+1-B)*(3+1-C)

$$\sum_{C=1}^{N}\sum_{B=1}^{N}\sum_{A=1}^{N}(N+1-A)(N+1-B)(N+1-C) - \sum_{i=1}^{N}(N+1-i)^3$$

# How many squares/rectangles and cubes/boxes?

- Approaches for counting the number of rectangles and boxes
  - N=3
    - How many boxes in a (3x3x3) sized cube?

$$\sum_{C=1}^{N}\sum_{B=1}^{N}\sum_{A=1}^{N}(N+1-A)(N+1-B)(N+1-C) - \sum_{i=1}^{N}(N+1-i)^3$$

=1*1*1 + 2*1*1 + 3*1*1 + 1*2*1 + 2*2*1 + 3*2*1 + … + 3*3*3 – (1*1*1 – 2*2*2 – 3*3*3)

= 216 – 36 = 180

      - (1+2+…+N)^3 – (N*(N+1)/2)^2

- Approaches for counting the number of squares and cubes
  - N=3
    - How many rectangles in a (3x3) sized square?
      - (1+2+…+N)^2 – (N*(N+1)*(2N+1)/6)
      - For N=3, the number is 6*6 – 3*4*7/6 = 22