Quota Baskets as a Quadratic Program

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January 23, 2021

1 Model

I'm using the notation from R package "quadprog" so it should be easy to implement. That model is:

$$\min \frac{1}{2} E'DE - d'E \qquad subject\ to: \quad AE \geq b \tag{1}$$

Suppose we have s species, n technologies, and m quota baskets. To use quadprog, we need the following:

- E (nx1 vector of efforts ... these are what we are trying to find)
- D (nxn symmetric matrix...used for the costs)
- d (nx1 vector ... used for revenue)
- A (mxn matrix ... used for quota basket harvest)
- b (mx1 vector ... used for quota basket caps)

Here is my proposal for how to do this:

1.1 D

D is an nxn matrix. All off-diagonal elements are 0. The i^{th} diagonal element is: $D_{ii} = 2c_i$, where c_i is the cost parameter for the i^{th} technology.

1.2 d

d is an nx1 vector, made up as follows:

- Construct an sx1 vector of prices, P. The i^{th} element is the price of species i.
- Construct an sxs matrix of stocks, B. The off-diagonal elements are 0. The i^{th} diagonal element is B_{ii} =X$_i$, which is the stock size for species i.

- Construct an sxn matrix of catchability coefficients, Z. The ij^{th} element is q_{ij}, which is the catchability of species i using technology j.

With these, you can construct the following:

$$d' = P'BZ \tag{2}$$

which has dimension 1xn, so d has dimension nx1.

1.3 A

A is an mxn matrix that defines the quota basket harvest. To construct A, do the following:

- Construct matrices B and Z from above
- Construct an mxs matrix D, where D_{ij} equals 1 if quota basket i contains species j and it equals 0 if quota basket i does not contain species j.

With these, you can construct the following:

$$A = -DBZ \tag{3}$$

which has dimension mxn. You can iterpret the ij^{th} element of A as follows: A_{ij} is the negative harvest per unit effort (of technology j) in quota basket i using technology j.[1]

1.4 b

b is a mx1 vector of negative harvest caps, where element b_i is the negative cap for quota basket i. Notice that both A and b are all negative numbers. The R function quadprod wants the harvest to be larger than the constraint, so if you take the negative of both sides, we get the desired inequality.

2 Implementation

Once you construct all of these matrices and vectors, it should be as simple as inputting them into quadprog and it will tell you the answer, which is a mx1 vector of efforts, E.

You should check all of my math - it's possible I screwed up a transpose or something else. Then you should try this for one of the problems you have already solved to make sure you get the same answer. This should work in a second or less for any size problem.

[1] I think quadprog may want you to input the transpose of A, not A itself. So you may have to replace A with A'.