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On Dante, Hyperspheres, and the Curvature of the Medieval Cosmos

William Egginton

In the course of his lectures on medieval literature at Oxford University in the 1950s C. S. Lewis would ask students to walk alone at night, gaze at the star-filled sky, and try to imagine how it might look to a walker in the Middle Ages. It would not likely have occurred to him that some forty years later several astrophysicists would ask that their readers perform the same activity, at least figuratively—not, however, with the intention of experiencing medieval conceptions of the cosmos but rather in order to begin to imagine the shape of our own. One of the astrophysicists in question is Stanford Professor Emeritus Robert Osserman, whose 1995 book *The Poetry of the Universe* is an attempt to acquaint lay readers with what the most recent research suggests is the shape of the universe by leading them through a brief history of the mathematical and technological developments that made such boundary-breaking thought possible. Paradoxically, when it comes to trying to imagine that all-encompassing form which is derived from a four-dimensional mathematical model called a hypersphere, he turns not to ultra-modern computer-generated images nor to dry but technically accurate exegeses of mathematical formulae but rather to the verses of a fourteenth-century Italian poet, Dante Alighieri. His claim is that in the third canticle of the Divine Comedy, which details his ascent through the heavens, Dante describes the cosmos with astonishing precision as a hypersphere, exactly as modern physics has determined it to be.

If the cosmology of the present makes use of Dante heuristically, however, it never asks the most obvious question: Why? Why does one need to go back to the fourteenth century for a poetic description of how the universe appears to modern science? Osserman assumes it to be a mere coincidence, and why should he not? What possible access would a thinker—a poet even—of the European Middle Ages have to the highest achievements of almost six centuries of subsequent thought? In answering this question, I am not proposing to naively interpret the past in our own image, claiming Dante to be a mathematical genius before his time. I am, however, suggesting that there is an excellent explanation for why one must look to the fourteenth century for models of expression adequate to the challenges posed by nineteenth and twentieth-century advances in mathematics: namely, that in the course of the Scientific Revolution, wherein were laid the epistemological foundations for the discoveries of modern science, certain possibilities of thought and imagination were discarded, forgotten, and certain abilities were, if only temporarily, lost. The Middle Ages has long been painted as a period whose culture was hostile to free thought and new knowledge; perhaps it is now time to recognize that it was also a time in which phenomena could sustain mutually exclusive and contradictory explanations, when the battle between faith and science had not yet been conclusively won by either side, and when the efforts to bridge the stories they told could produce conceptual edifices whose mere possibility would be unthinkable only 200 years later. This essay is about one such conceptual edifice and how openness to its perspective allowed a poet to put into words what could not be imagined by his descendants, and what eludes us even today.

The Shape of the Modern Universe

Imagine that you are a completely flat creature, living happily in your two-dimensional world.[1] As observers we are outside this world, and we can see you milling about with your flat friends, blissfully unaware of the uncharted regions one dimension away on either side of you. Now imagine that this plane is not flat—in the sense of rectilineal—as you might have created it in your mind, but rather is curved; in fact it is a sphere. Assuming that this sphere is large enough, there is no reason that you, an inhabitant of this surface, should be aware of its curvature. Even we, the three-dimensional observers, if close enough to the surface, would not recognize its curvature. One could conjecture, however, that for you, the inhabitant, such recognition would be impossible; a line could be seen to curve to the left or the right (assuming that those are the available dimensions), but curvature along an up/down axis would not be an option. If the sphere were made small enough, you would encounter some bizarre effects, such as continually returning to the same place you just left despite not having changed directions. Or if the sphere were really small, you could scratch your own back by simply stretching your arm out in front of your body. Now imagine that this world is not small but rather immeasurably large and that we are now all inhabitants of this space, a space that has, in fact, three dimensions. The space in which we live is still curved, it is still the surface of a gigantic sphere, but this sphere is no longer a two-dimensional surface. It is rather what mathematics calls a hypersphere, a sphere whose surface is extended in three dimensions.

[1] The scenario is reminiscent of Edwin Abbott's *Flatland* (San Francisco, 1980).

What are some of the properties of such a universe? To begin with, we inhabitants have no way of perceiving in our day-to-day experience the curvature of space for many of the same reasons that the curvature of the surface of our own Earth is not obvious to the average observer. When we arm ourselves with some of the tools provided by modern science, however, we begin to see strange things indeed. For one, when we aim our increasingly-powerful telescopes toward the edges of the universe, we find that the images we receive, in terms of the density of galaxies and the prevalence of what is called background radiation, are almost perfectly uniform *no matter in what direction we choose to gaze*. And yet our observations also inform us that our universe is expanding, that the distance between us and every other object we observe is growing at a rate proportional to that distance. These two relatively simple facts become remarkably difficult to reconcile on the basis of a three-dimensional model of space. One imagines matter spraying out through an infinite expanse of this space, with its origin in an enormous explosion which scientists call the Big Bang. Yet if this were an accurate model, the only way to explain the evidence that all other objects are receding from us at a rate proportional to their distance is to posit that we occupy the center of the universe, i.e., the point of the Big Bang, a hypothesis that is in hopeless conflict with observations of more nearby phenomena, such as our apparent position in our own galaxy, the Milky Way.

If on the other hand we imagine the universe as a three-dimensional spherical surface, the difficulty is removed, for any point on the sphere can now be conceived as its center, and from that point all lines—of vision, for example—must be drawn as meridians on a globe, converging on one point on the opposite side of the sphere. As we train our vision on the distant reaches of space, we are also looking back in time, such that the light that has traveled the furthest is also the oldest; and the uniformity we see at the edges of the visible cosmos is the uniformity of the cosmos itself, a relatively short time after its birth. Osserman calls the hypersphere that conforms to this structure a retroverse, in that its poles stretch from the here-and-now to a remote past some 20 billion light years away.[2] The seeming paradox of the retro-verse—and the aspect that makes it most difficult for us to grasp—is that from our point of reference, as from any point in the universe, we appear to be at the center of an enormous sphere, the circumference of which is in fact a single point, the point that marks the beginning of time.

According to Osserman, one of the reasons why this is so difficult for us to conceptualize is that, while "we have learned to overcome our flat-earth mentality in measurements on a global scale, we have not yet overcome our tendency to think in terms of a flat universe."[3] Clearly, advances in mathematics have enabled physicists to propose such a model, despite our apparent inability

[2] R. Osserman, *The Poetry of the Universe* (New York, 1995), 112-13.

[3] *Ibid.*, 85.

to imagine in four dimensions (which is what it takes to conceptualize a hypersphere). In particular, a nineteenth-century mathematician named Georg Riemann developed a "non-euclidean geometry" with which one could theorize the curvature of space and which, right from the start, began to suggest plausible solutions to ancient conundrums, such as the problem of the edge of the universe. If the universe is finite rather than infinite, as most ancient philosophers apart from the Epicureans believed it to be, what would occur when one arrived at its edge? What would lie on the other side, and how could that other side not be itself part of the universe?

> Riemann's model resolved that paradox, which is rooted in the assumption that the universe is flat, or euclidean. If, instead, it is positively curved and Riemannian, then it can be finite in extent and still not have any "edge" or "boundary." In Riemann's model, every part of the universe looks like every other part, as far as shapes and measurements go.[4]

Acknowledging that it is one thing to explain an abstract mathematical model and quite another to have a reader or a listener understand and imagine this model, Osserman turns from mathematical theory to poetry, namely, to Dante's *Paradiso,* in which he claims,

> the poet arrived at a view of the universe with striking similarities to that of Riemann. In the *Divine Comedy*, Dante describes the universe as consisting of two parts. One part has its center at the earth, surrounded by larger and larger spheres on which move the moon, the sun, successive planets, and the fixed stars. The outer sphere, bounding all the visible universe, is called the *Primum Mobile*. What lies beyond this is the "Empyrean," which Dante pictures as another sphere, with various orders of angels circling on concentric spheres about a center where a point of light radiates with almost blinding intensity.[5]

For Osserman this image is a startlingly accurate rendition of the retro-verse, in which the point of light the poet depicts at the center of the universe—representing, of course, God—coincides perfectly with the Big Bang in contemporary cosmology.[6]

It is perhaps best to view this passage from *Paradiso* in the context of a larger cluster of verses detailing Dante's approach to the place of God at the center of the Empyrean. The first pertinent moment occurs in canto XXVII, in

[4] *Ibid.*, 88.
[5] *Ibid.*, 89.
[6] *Ibid.*, 118.

which Dante and his guide, the beloved Beatrice, step into the outermost sphere from a terrestrial perspective, the Primum Mobile, an invisible sphere beyond the heaven of the fixed stars that accounts for the diurnal rotation of the heavens. Upon entering this sphere, Dante notes that it is everywhere uniform,

> [l]e parti sue vivissime ed eccelse
> sì uniforme son, ch'i' non so dire
> qual Bëatrice per loco mi scelse.[7]
>
> [Its parts were all so equally alive and excellent, that I cannot say which place Beatrice selected for my entry.]

The fact that Dante stresses this aspect of the Primum Mobile suggests another parallel between his vision and that of modern cosmology: it should not matter what path Beatrice chooses to enter the Empyrean, for as long as one continues "up," one will eventually arrive at the same point, in this case the seat of God. The "uniformity" of the Primum Mobile shows that the Empyrean is not just perched on top of the mundane cosmos in one specific place, as it necessarily appears in any two or three-dimensional rendition. Rather, the edges of the Empyrean are everywhere around us and, like the contemporary cosmos, homogeneous. Having been informed of its uniformity, Dante immediately wonders about its location or, better, its locatability. As she is wont to do, Beatrice answers his as-yet-unasked question as follows:

> The nature of the universe, which holds the center still and moves all else around it, begins here as if from its turning post. This heaven has no other *where* than this: the mind of God, in which are kindled both the love that turns it and the force it rains. As in a circle, light and love enclose it, as it surrounds the rest—and that enclosing, only He who encloses understands. No other heaven measures this sphere's motion, but it serves as the measure for the rest, even as half and fifth determine ten; and now it can be evident to you how time has roots within this vessel and, within the other vessels, has its leaves.[8]

The place of the Primum Mobile, which as we shall see was a central concern for cosmological speculation in Dante's time, is attributed to God's mind. This, however, does not detract from its reality. On the contrary, time, motion, and the physical existence of the lower spheres are utterly dependent on this place, and God is seen as encircling it, providing both its force and the incentive—love—of its motion, and hence the motion of the entire universe. Now, if there

[7] Dante Alighieri, *Paradiso*, tr. Allen Mandelbaum (New York, 1984), XXVII, 100-102.
[8] Alighieri, *Paradiso*, XXVII, 106-20.

is no physical location for the Primum Mobile, what is the nature, or form, of its spiritual location in the mind of God? As it turns out, the Primum Mobile forms the outer boundary of a new set of concentric spheres, toward the center of which Dante directs his gaze:

> ... I saw a point that sent forth so acute a light, that anyone who faced the force with which it blazed would have to shut his eyes, and any star that, seen from earth, would seem to be the smallest, set beside that point, as star conjoined with star, would seem a moon. Around that point a ring of fire wheeled, a ring perhaps as far from that point as a halo from the star that colors it when mist that forms the halo is most thick. It wheeled so quickly that it would outstrip the motion that most swiftly girds the world. That ring was circled by a second ring, the second by a third, third by a fourth, fourth by a fifth, and fifth ring by a sixth.[9]

And thus it continues until he observes nine spheres mirroring the nine spheres on the other side of the Primum Mobile except that these increase in speed and perfection as their size decreases and they approach the absolute center, which is God.[10]

Dante's description of this absolute center as the origin of all movement and force in the universe is indeed, as Osserman's puts it, an "eerie"[11] likeness of what he terms the retro-verse, through which an imagined journey would require "ascending" away from the central point of the earth until at some moment the traveler would find himself or herself "descending" toward the fixed point, the origin of the universe. Naturally, we must stress the imaginary nature of such a journey, since our traveler would have to be outside of space as well as time in order to "see" the convergence of space-time to a single, infinitely dense point. The fact that Dante imagined something so similar is extraordinary, but does it make sense to attribute this similarity to anything more than "a rare confluence of the poetic and mathematical imagination," as Osserman reasonably assumes?[12] Rather than as a mere coincidence this convergence should perhaps be understood as an indication of a deep-seated epistemological difference between Dante's world and our own, as a sign of some possibility of knowledge which, while incapable of generating the abstract theories of Riemann's geometry, could still imagine a world where space was curved, and where the universe could be at once finite and boundless.

[9] *Ibid.*, XXVIII, 16-30.
[10] See M. Peterson, "Dante and the 3-sphere," *American Journal of Physics*, 47.12.
[11] Osserman, *Poetry of the Universe*, 118.
[12] *Ibid.*, 89.

Natural Philosophy in Dante's World

The principle tension characterizing all knowledge in the Middle Ages was that between Natural Philosophy—the study of nature by way of reason, observation, and, most important, the study and commentary of texts by Aristotle—and Christian Theology. "In the Middle Ages," according to Edward Grant, "the structure of the world was never conceived solely in physical and metaphysical terms, but had to be made compatible with a variety of theological concepts which, in the end, transformed the Aristotelian cosmos into a Christian universe."[13] Many of the texts of the pagan philosophers had been a recent discovery for scholars of the Christian Occident, and while the power of the Aristotelian system was particularly attractive, it became rapidly evident that much work was required in order for this new knowledge-base to be assimilated into a theologically correct world-view.[14] In fact it is fair to say that in many areas, such as cosmology, no satisfactory synthesis was ever developed.[15] This is of particular interest when we consider that, in many ways, such a synthesis is what the representation of the cosmos in *Paradiso* was intended to produce.

By the beginning of the thirteenth century, scholars in Europe had access to Latin translations of most of Aristotle's works, and there was no doubt that his was the dominant intellectual system by the second half of the century.[16] Nevertheless, Aristotle's conquest of the medieval universities did not come without ecclesiastical protest. Partial bans were placed on the teaching of his works on several occasions, but their effectiveness faded over time, and in 1255 the faculty at Paris made the teaching of Aristotle compulsory at all levels.[17] This hegemony went unopposed for another twenty years until in 1277 the Bishop of Paris published a list of specific points of Aristotelian doctrine, the teaching of which would be considered an excommunicable offense. Paradoxically, this act of censorship on the part of the Church had the effect of opening certain avenues of thought, in that it was no longer permissible to deny God the power to do anything, and thus almost any fantasy could be treated as a viable hypothesis, no matter how ludicrous it may have appeared in terms of Aristotelian assumptions. Even these condemnations, however, were eventually repealed (largely as a result of Thomas Aquinas's work in assimilating scholastic and Christian doctrine), but not before they had their effect, and not before Dante Alighieri had finished his *Divine Comedy*.

[13] Edward Grant, "Cosmology," *Science in the Middle Ages*, ed. David C. Lindberg (Chicago, 1978), 266.

[14] David C. Lindberg, *The Beginnings of Western Science* (Chicago, 1992), 215.

[15] Grant, "Cosmology," 267.

[16] Lindberg, *Beginnings*, 212, 216.

[17] *Ibid.*, 218.

The Problem of Place

Speaking again to his Oxford students (and to later readers), C. S. Lewis compares the way the night sky appears to a modern with how it might have been perceived by a medieval walker:

> Whatever else a modern feels when he looks at the night sky, he certainly feels that he is looking out—like one looking out from the saloon entrance on to the dark Atlantic or from the lighted porch upon dark and lonely moors. But if you accepted the Medieval Model, you would feel like one looking *in*.[18]

In this way, Lewis expresses the paradox of Dante's cosmos: a sphere in which the circumference (what we see out there) coincides exactly with the center. Is this then a common topos in medieval thought? It would seem to be from the prominence that Lewis grants it, and yet his one example is none other than our own:

> But I have already hinted that the intelligible universe reverses it all; there the Earth is the rim, the outside edge where being fades away on the border of nonentity. A few astonishing lines from the Paradiso (XXVIII, 25 sq.) stamp this on the mind forever. Seven [sic] concentric rings of light revolve around that point, and that which is smallest and nearest to it has the swiftest movement.... The universe is thus, when our minds are sufficiently freed from the senses, turned inside out.[19]

Let us not pass over in silence the subordinate clause, "when our minds are sufficiently freed from the senses," for this is precisely the crux of our discussion. For the modern reader it requires a radical wrenching of mind from sense to conceive of a universe turned inside out, and yet Lewis implies that this way of thinking comes naturally to the medieval mind. Perhaps we cannot say with certainty whether such a model came naturally to medieval thinkers. We can say, however, that such a model occurred to Dante (whether naturally or not) and that it was therefore well within the realm of possibilities for a poet of his time to imagine the universe in a way that, while paradoxical to the modern mind, solved certain problems peculiar to the medieval mind. The problem that Dante solved was, of course, "the age-old problem of the edge of the universe" which, according to Osserman, had stumped philosophers until Riemann.[20]

[18] C.S. Lewis, *The Discarded Image: An Introduction to Medieval and Renaissance Literature* (Cambridge, 1964), 119.

[19] *Ibid.*, 116.

[20] Osserman, *The Poetry of the Universe*, 88.

However, while it was still a problem for the middle ages, it would appear that by the time of Newton the universe's edge could not be a problem because the universe was believed to be infinite and thus have no edge. For the best understanding of Dante's solution, then, let us consider how the problem was phrased during his time, in the form of questions concerning the nature of space and the definition of place.

Prior to the Condemnations of 1277, which prohibited those aspects of scholastic doctrine that would restrict divine omnipotence in any way, the Aristotelian cosmos of nesting spheres, while still the most prevalent model, was trapped in a serious logical conundrum. According to book four of the *Physics*, the definition of a thing's place was the "innermost, motionless surface of the containing body in direct contact with the contained body."[21] In addition, place was understood to remain necessarily immobile with respect to the body it contained because that body's motion could only be perceived and measured with respect to its place.[22] However, since Aristotle also insisted on the finite nature of the cosmos and the absolute non-existence of void space, the cosmos almost by definition, could not have a place of its own.[23] But if the cosmos did not have a place, it could not move; and this conclusion, although rigorously drawn, was in blatant contradiction with observed reality.[24]

Throughout the late Middle Ages, various natural philosophers and commentators of Aristotle's works tried to grapple with this problem. First came the solutions offered by the Islamic commentators, such as Avicenna (Ibn-Sinâ):

> According to Ibn-Sinâ, the revolution of a sphere upon itself is not the movement from one place to another; it is movement in place; for a body to be animated by such a movement in place, it is not necessary for it to be in a place; the eighth heaven, then, is in a place neither *per se* nor *per accidens*; yet it can, nevertheless, turn upon itself.[25]

Averroës did not find this answer satisfactory; even if the entirety of a sphere can be said not to require a place, the sphere can easily be divided into parts, each of which will require a place in order to be said to be moving as the sphere rotates.[26] But if, by virtue of its perceived motion, the cosmos must have a place, there is no requirement that it be a place *per se*. Rather, Averroës argues, the outermost sphere of the cosmos has a place *per accidens*, because one of its

[21] Grant, "Cosmology," 272.

[22] Pierre Duhem, *Le Système du monde: Histoire de doctrines cosmologiques de Platon à Copernic* (Paris, 1913-59), VII, 158.

[23] Grant, "Cosmology," 272.

[24] Duhem, *Système du monde*, 158.

[25] *Ibid.*, 160.

[26] *Ibid.*

parts, its center, is a place *per se*.[27] This was also the eventual conclusion of other noted scholastics such as Roger Bacon and Albert the Great.

Dissent arrives with Thomas Aquinas, who finds ridiculous the idea that heaven has its place by virtue of its center, in that it would thereby require a center that moves and exists independently of the whole. He opts rather for Themistius's answer, that place is determined accidentally by virtue of the place of its various parts.[28] In addition, he develops a new distinction: two kinds of place must exist, mobile and immobile place. On the one hand the immediately contiguous environment can change; this is the body's place, which is mobile and relative. On the other hand there is place as determined by the relationship between the outer limits of the body and the borders of the universe, which cannot change. This is rational, or absolute, place: "The rational place of whatever container comes, therefore, from the first container, the first lodging place, that is, heaven."[29]

Avempace proposes another theory, also based on Themistius, which is, according to Pierre Duhem, the most "curious" of all. His solution is of particular interest here because it requires one to think of the cosmos of nesting spheres as inverted in some way, such that the last sphere, containing all the others, is in turn contained by the sphere immediately interior to it.[30] Unlike the motion of the sublunar realm, which is imperfect and hence linear and therefore needs to be limited by an outside, the spheres are perfect and circular and hence may be contained by their inside: "The celestial spheres have no need whatsoever to be lodged in such a way; neither are they lodged by the outside, but rather by the inside; each one of them has as a place the convex surface of the sphere that it encloses and around which it turns."[31] In Avempace's solution, therefore, the last sphere may have nothing outside it and still be in a place, because it is "enclosed" by the convex surface of the very sphere it encloses—in this case, the sphere of Saturn.[32]

Although Aristotle's two-dimensional container model of place was taken very seriously by medieval thinkers, maintaining its prominence well into the fourteenth century, it did not represent the exclusive possibility. While they would eventually reject it on logical grounds, Thomas Aquinas and other scholastics played with a definition of place that was much closer to what a modern mind would consider common-sensical: the three-dimensional space that a body occupies. Aristotle himself had considered this possibility and rejected it, and

[27] *Ibid.*, 166, and Grant, "Cosmology," 272-73.
[28] Duhem, *op. cit.*, 176.
[29] *Ibid.*, 177.
[30] *Ibid.*, 160.
[31] *Ibid.*, 161.
[32] Grant, "Cosmology," 274.

his medieval followers for the most part thought of it as a vulgar notion.[33] The ultimate rejection of this position depended on its involving a choice between two fundamentally untenable options: either the space that the body in question occupied was itself substantial, in which case there would be an impossible co-penetration of substances; or it was entirely insubstantial, i.e., nothing, and therefore did not exist and could not be said to be a place.[34] What this reasoning shows is that, while dimensional nothingness was a possibility of the imagination, it was not a logical possibility. Empty space was simply an empty logical category that could not be reconciled with the physical world. Dimensionality was a predicate of substance, not its precondition. In fact,

> not until the sixteenth century, perhaps first by Francesco Patrizi, was the nexus between dimensionality and corporeality broken by assigning tridimensionality to place essentially and to body only accidentally. For Patrizi, place is prior to, and wholly separate from, body, whose primary property is resistance.[35]

If modern space is predicated on the notion of nothing as something, a positive negativity that is itself dependent on a subjective projection, a "filling out" of something that is phenomenally just not-there with positive content, the medieval imagination could still countenance the exact opposite of this notion. Medieval space opted for a more phenomenally "correct" conception of space, as that which could be perceived as "being there." *Nothing* was exactly that; *it* did not and could not exist. All of this was in perfect accordance with Aristotle's orthodoxy that there could be no void space in nature.

The Problem of Empty Space

Even in Aristotle's time there was no total consensus on the impossibility of a void. The stoic philosophers resisted this notion by way of the following thought experiment, available to medieval thinkers through Simplicius's sixth-century commentary on *De Caelo*, as translated by William of Moerbeke:

> [T]he Stoics, however, thinking that there is a vacuum beyond the sky, prove it by this kind of assumption: let it be assumed that someone

[33] Edward Grant, "Place and Space in Medieval Physical Thought," *Motion and Time, Space and Matter: Interrelations in the History of Philosophy and Science*, eds. Peter K. Machamer and Robert G. Turnbull (Columbus, 1976), 138; repr. in Grant, *Studies in Medieval Science and Natural Philosophy* (London, 1987).

[34] In the words of Pseudo-Siger of Brabant, if "such dimensions are assumed to be a place and since such dimensions are nothing, it follows that place is nothing, which is impossible." In Grant, "Place and Space," 140.

[35] Grant, "Place and Space," 140.

> standing motionless at the extremity [of the world] extends his hand upward. Now if his hand does extend, they take it that there is something beyond the sky to which the hand extends. But if the arm could not be extended, then something will exist outside that prevents the extension of the hand; but if he then stands at the extremity of this [obstacle that prevents the extension of his hand] and extends his hand, the same question as before [is asked], since something could be shown to exist beyond that being.[36]

This is, of course, the medieval formulation of what Osserman called the age-old problem of the edge of the universe, a problem that, as we can see in this context, had everything to do with a particular notion of space. In fact, for the Stoics themselves, this edge was not a problem, in the same way that it ceased being a problem for the modern mind: their answer was that the edge did not exist, for the known cosmos was itself suspended in an infinite void space. Such a conception, however, was not widely accepted in the Middle Ages, because, as we have seen, medieval thinkers could see little purpose in "a mere privation made three-dimensional."[37]

What was not a problem for the Stoics became a real conundrum for the scholastics, one intimately related to the problem of the place and subsequent motion that had to be attributed in some way to the cosmos. In attempting to deal with what lay outside the universe, some thinkers, such as the thirteenth-century Henry of Ghent, were willing to advance the notion of a vacuum that would qualify as something by the mere fact that it was dimensional, but they insisted on distinguishing this space from a pure nothing without dimensions, which they argued could not exist.[38] Thus dimensionality would necessarily remain an attribute of *something*; even if this something were void space, it would still *exist* in the sense of being substantial. But a majority of thinkers maintained their opposition to the existence of extra-cosmic space despite the paradoxes it seemed to spawn. This was the case throughout the fourteenth and fifteenth centuries,[39] although, for reasons we will now consider, thought experiments concerning such space became increasingly common.

Place and Space after the Condemnation of 1277

"As a consequence of three articles in the Condemnation of 1277," says Grant, "it became an excommunicable offense to deny God's power to create an accident without a substance or to deny the possibility of the separate exist-

[36] Edward Grant, "Medieval and Seventeenth-Century Conceptions of an Infinite Void Space Beyond the Cosmos," *ISIS*, 60 (1969), 41; repr. in Grant, *Studies in Medieval Science.*

[37] Grant, "Infinite Void," 42.

[38] Grant, "Place and Space," 151.

[39] *Ibid.*, 150.

ence of a quantity or dimension on grounds that such an entity would be a substance."[40] That is, as a consequence of establishing the theological doctrine of God's omnipotence, the Church created an institutional space in which the problems of space and place could be rethought and new presuppositions tested. Earlier thinkers had balked at the seeming absurdity of attributing dimensionality, an obvious attribute of things that exist, to pure, empty space, but now such a hypothesis would be supported by the authority of the Church. Equally, while scholastic philosophy and theology had always agreed that, de facto, God had created only one world, the Condemnation made clear that no one could deny the possibility that He had made others, or had the power to do so.[41] Indeed, in the wake of the Condemnation, "[a]ppeals to God's absolute power were made in order to justify formulation of a host of thought experiments that were, in one way or another, contrary to Aristotelian physics and cosmology."[42]

Despite this new freedom to hypothesize, most thinkers still desired to reconcile the dictates of the Philosopher to theological orthodoxy rather than to relinquish them entirely. Consequently, attempts were made to think of multiple worlds in a way that would not necessitate the intermediation of void space. The new problems tended to turn on the question of distance: if another world were to exist somewhere outside of our own, and yet if there were nothing actually separating the worlds, how would one measure distances between them? Or, more generally, if one acknowledged the existence of a void, could the void itself be measurable, or would this require the mediation of some form of matter? In the view of Marsilius of Inghen,

> God could cause a stone to separate from the convex surface of the last celestial sphere only if He also created a body or bodies between the celestial surface and the stone. Otherwise, says Marsilius, a void space would intervene without the capacity to function as a corporeal interval of separation. In another example, Marsilius argues that if God created three spherical worlds in mutual contact, no measurable distances could be said to intervene in the vacua that lie between their convex surfaces. In such a configuration of worlds, distances could only be measured curvilinearly between the points of any particular surface, since these would be separated by the continuous matter of the surface.[43]

Space could not sustain measurement because, despite the Condemnation, it was still preferable to medieval thinkers to avoid attributing dimensionality to something that was, by definition, the absolute absence of substance.

[40] Grant, "Place and Space," 142.
[41] Grant, "Cosmology," 270.
[42] Grant, "Place and Space," 142.
[43] *Ibid.*, 151.

Jean Buridan took a decisive step toward the modern view of space as pure dimensionality but stopped short of the notion that such a space could ever exist separately from matter. Considering the Stoic paradox, Buridan insists that there is nothing stopping the hypothetical man from thrusting his hand through the last barrier of the universe because, while there is no space there into which his hand may extend, the hand itself provides its own space, since "that space is nothing but the dimension of your body."[44] Space is still not capable of independent existence, as in the case of a vacuum, because "this space would not be a substance; and if it were an accident, then it would be an accident without a subject which is not naturally possible."[45] Nevertheless, it does exist as an abstract dimensionality, the extension of matter itself.

Nicole Oresme entertained the notion of void space as really existing outside the boundaries of the universe, as well as with the possible existence of multiple worlds, each with its own center and directional system.[46] His notion of void space, however, is also completely identified with God, who may occupy any space, but is himself dimensionless, which suggests that his infinite and void space is equally dimensionless.[47] Neither Buridan nor Oresme accepts the notion of an infinite void space extended in three dimensions, but each does take a fundamental step in that direction: Oresme in his allowance of the notion of a real but dimensionless void space and Buridan in his conception of a real, existing, three-dimensional space, which is nevertheless inseparable from matter and hence abstract. In this sense both can be seen (and often have been, though for different reasons)[48] as precursors of the Scientific Revolution of the seventeenth century.

Beside Oresme, only two other thinkers before the sixteenth century argued for the existence of an infinite, extra-cosmic void: Jean de Ripa and Thomas Bradwardine. Not one of the three believed that the void could have dimensions. Rather, as best exemplified in the work of Bradwardine, the infinite void was associated with God himself, who was defined as infinite but lacking dimension or extension.[49] Nevertheless, it was such theologically inspired specu-

[44] Edward Grant, "Jean Buridan: A Fourteenth Century Cartesian," *Archives internationales d'histoire des science*, 64 (1963), 252; repr. in Grant, *Studies in Medieval Science.*

[45] *Ibid.*, 253.

[46] Grant, "Cosmology," 270.

[47] Grant, "Infinite Void," 48.

[48] See, e. g., Richard C. Dales, *The Scientific Achievement of the Middle Ages* (Philadelphia, 1973).

[49] Edward Grant, *Much Ado About Nothing: Theories of Space and Vacuum from the Middle Ages to the Scientific Revolution* (Cambridge, 1981), 135-47. Bradwardine actually denied that God could make an infinite void that would be independent of Himself, as such a space would therefore be co-terminus with God or contain him, an impossibility since God contained all and could not be contained or equalled. Therefore the infinite void was God and hence could not have extension or dimension. See E. Grant, *Planets, Stars, and Orbs: The Medieval Cosmos, 1200-1687* (Cambridge, 1994), 176.

lation about the possibility of extra-cosmic space that paved the way for the infinite, geometrical space of the seventeenth century.

A far more radical departure from the medieval model of space does not occur until the sixteenth century when there emerges a conception of space as both dimensional in Buridan's abstract sense and independent in Oresme's sense of an external void. While even then one could still find the occasional fanatical Aristotelian who denied the reality of external space, that Aristotelian would nevertheless grant the validity of abstract three-dimensional space as a distinct entity from the matter that inhabits it. One such sixteenth-century voice is that of the Aristotelian Toletus, who distinguishes between

> imaginary space as a fiction of the mind, and as an abstraction, which is nonetheless entirely true. Extracosmic spaces and voids are wholly fictional, whereas the abstraction of space, what we might call mathematical space, is perfectly valid: "by abstracting from this or that space of particular bodies, [we arrive at] the space common to the whole world in which there are only bodies, by abstracting, I say, from this or that body; and this consideration is not a fiction."[50]

This "abstract world space" is common to all things and completely immobile and provides the foundation for one of the fundamental epistemological foundations of the Scientific Revolution: the abstract, universal, and, most importantly, *flat* [51] space in which things interact according to universal laws and upon which things can be adequately measured in their interaction.

The next step in our narrative, then, is to show that this abstract space indeed became real and that the Aristotelian final and desperate distinction between the abstract and the imaginary was, at the height of the seventeenth century, rejected for good. In short, *nothing* became *something*. In the words of Otto von Guericke, from his *New Magdeburg Experiments on Void Space* (1672): "the nothing [*nihil*] beyond the world and space [*spatium*] are one and the same; and so-called imaginary space is true space, for imaginary space [in the common opinion of philosophers] is nothing and nothing is space, and the space which they call imaginary space is true space [*spatium verum*]."[52] Not only is the imaginary nothing to be animated as something, it will become the most important something of them all, the *sine qua non* of all existence:

> For the "Uncreated" is that whose beginning does not pre-exist; and Nothing, we say, is that whose beginning does not pre-exist. Nothing contains all things. It is more precious than gold, free of origin and

[50] Grant, "Place and Space," 157.

[51] "Flat" is to be understood here and for the remainder of the essay as rectilinear.

[52] Grant, "Infinite Void," 55.

> distinction, more joyous than the appearance of beautiful light, more noble than the blood of kings, comparable to the heavens, higher than the stars, more powerful than a stroke of lightening, perfect and blessed in every part. Nothing always inspires.[53]

In the passage of time between the thirteenth and the seventeenth centuries, then, we have moved from an epistemological configuration in which it was difficult to imagine space as an independent entity and certainly not as one with the attribute of dimensionality, to one in which such a concept with such an attribute is the necessary condition for all being. It is important to stress two fundamental aspects of this new conception of space: first, that it was flat, the embodiment of an abstract, geometrical (Euclidean) system, and second, that it was real, the ultimate hypostasis of what had once been merely a thought experiment. As Koyré puts it, "the replacement of the Aristotelian conception of space—a differentiated set of innerworldy places, by that of Euclidean geometry—an essentially infinite and homogeneous extension—from now on [was] considered as identical with the real space of the world."[54] According to Grant, it is this entity that allowed for the fusion of Theology and Natural Philosophy that had never been possible during the Middle Ages:

> In the seventeenth century, the secular and theological currents would merge for the first time, when God was deemed omnipresent in an infinite, three dimensional, immobile, homogeneous void space. It was this God-filled space that would serve as the infinite container for the motions of bodies whose lawful relationships were described by Sir Isaac Newton in the Mathematical Principles of Natural Philosophy.[55]

This story of the emergence and eventual triumph of a concept of space as flat, three-dimensional, and infinitely extended, which doubles as an abstract, mathematical space that was among the great enabling forces behind the development of modern science, differs little from its narration by such canonized historians of science as Grant and Koyré. What I want to emphasize, however, is a certain foreclosure of possibilities that accompanied this progress. Let us take a look at a passage in which Grant compares scholastic and modern cosmological assumptions:

> The Scholastics, for whom the issue of infinite extramundane void space was divorced from their physics, were content to assume the

[53] *Ibid.*, 57.

[54] Alexandre Koyré, *From the Closed World to the Infinite Universe* (Baltimore, 1957), viii; also Jacques Merleau-Ponty and Bruno Morando, *The Rebirth of Cosmology*, tr. Helen Weaver (New York, 1976).

[55] Grant, "Place and Space," 161.

> reality of extramundane void space—after all, God could create a body there—but deny it dimensionality, choosing to take refuge in transcendent meanings of terms such as "extension" and "dimension" when applied to God. Authors in the seventeenth century had no such options and devices.[56]

I agree with Grant that these authors had no such options—that these "devices" had, in fact, disappeared in the face of the "rigor" of the scientific method. And it is almost certainly in this sense that Grant intends his observation: modern science does not have the luxury of dispensing with rigor, of taking "refuge in transcendent meanings." However, the results of this progress in thought were not always "right" ways of thinking; it is the teleological historicist who sees in every turn the past took the straightest route to present knowledge. The great irony of much of the story we have been telling is that the "right" way that emerged in the seventeenth century, regardless of the specific brilliance of Newton's or anyone else's contributions, may be farther from our own view in its most fundamental presuppositions than the systems of thought it replaced. When we look back at Newtonian physics in order to judge it in terms of where physics is today, we are in fact inscribing it in a story about how physics became what it is today, which is not saying the same thing. When we tell a story leading to the present, the mentality of Newton's age is the rock we stand on; when we spread out those presuppositions like cards on a table, we see how medieval we are, and how much we have forgotten.

The Regime of Abstraction

As we have seen, scholastic dogmatism received a blow from the Condemnation of 1277; and while the reputation of Aristotle had fully recovered by fifty years later, the prohibitions had opened certain speculative possibilities that would never again disappear. The example of greatest interest to the present study is the existence of an outside to the cosmos, an extra-cosmic space, perhaps void, providing the cosmos with a place that, in turn, could explain its movement. As we have also seen, one philosopher of the fourteenth century, Nicole Oresme, argued for the possible existence of a multitude of worlds separated by an intervening space that, while necessarily infinite, appeared to lack dimensionality. However, if this multitude of worlds is only a possibility—and one, in fact, which Oresme rejects in actuality—it appears that the concept of an outside is already becoming an exclusive necessity: "Human understanding consents naturally that beyond the heavens and world, which is not infinite, there is some space, whatever it may be; and one could not readily conceive the contrary."[57] Such a belief, that one cannot imagine the non-existence of an

[56] Grant, "Infinite Void," 60.
[57] Grant, "Place and Space," 144.

outside—a non-existence that was itself a point of doctrine only a short time before—indicates that already in the fourteenth century a process of foreclosure was beginning in which the abstraction of infinitude would finish by colonizing completely the imagination of reality.

It is during this same time frame that, across the sciences, abstraction begins to become an intrinsic element of analytic methodology. Oresme himself adapted some of the insights of a group of thinkers associated with Merton College, Oxford, who had developed a way of thinking about an object's qualities, such as motion, temperature, and weight, as abstract, i.e., separately analyzable, entities.[58] Oresme's particular contribution was to adapt and improve upon a geometrical system of representation of qualities originally developed by Giovanni di Casali in 1351.[59] Oresme's idea was that changes in a particular quality could be mapped in time, if the subject of the quality in question were abstracted, represented in two-dimensions as a line. In this way changes in quality could be measured quantitatively, in that a particular geometric shape could be taken as an adequate substitution for what had previously been largely unrepresentable.

Thomas Kuhn associates the emergence of methods based on abstraction at this time to a renaissance of Neoplatonism, which valorized ideal, immutable forms over empirically-existing objects.[60] The proposition that geometry could be used to predict and standardize observations could only be a sign that the abstract elements of geometry, mathematically perfect forms, were in fact the reality upon which the physical world was based. According to Kuhn, it was just such an epistemological change, a change in the value accorded to forms, that explains the possibility of the Copernican revolution: "No new discoveries, no sort of astronomical observation, persuaded Copernicus of ancient astronomy's inadequacy or of the necessity for change. Until half a century after Copernicus's death no potentially revolutionary changes occurred in the data available to astronomers."[61] If renewed valorization of abstract forms was a major impetus behind the eventual acceptance of a heliocentric model, one of these abstract forms—namely, infinite, three-dimensional, flat space—had become, by the time of Newton's famous treatise in the seventeenth century, an exclusive possibility of thought.

Newton insists upon the importance of recognizing two components of both space and time, the relative and the absolute. This should remind us of Thomas Aquinas, who also insisted on this division with regard to place. For Aquinas, however, absolute place was still place with regard to the walls of a container, whereas, for Newton, "[p]lace is a part of space which a body takes

[58] Lindberg, *Beginnings of Western Science*, 295.
[59] *Ibid.*, 297.
[60] Thomas S. Kuhn, *The Copernican Revolution* (Cambridge, 1957), 128.
[61] *Ibid.*, 132.

up and is, according to the space, either absolute or relative. I say, a part of space; not the situation nor the external surface of the body."[62] In other words the place of a body is indistinguishable from the space it occupies, a space that, if understood relatively, may change with respect to surrounding objects and points of reference but, if understood absolutely, also remains part of an eternal, immutable backdrop, infinitely extended in all directions. The same distinction is true of time: whereas the time of an event may be measured relative to other, longer or shorter durations, its own duration also has an absolute value, with respect to an absolute, utterly abstract yet utterly real temporality: "Absolute, true and mathematical time, of itself and from its own nature, flows equably without regard to anything external."[63] This absolute character of space and time becomes, for Newton as for the modern age, the condition of possibility of knowledge itself, the foundational table upon which the organization of things can be mapped out. In such a universe not only does the "closed world" of Dante's cosmos of nested spheres become foreign to us, the particular problems that his poetry tries to address cease to exist. The arrangement of the heavens beyond the Primum Mobile appears fanciful, almost psychedelic; and if contemporary mathematicians remark upon the similarities between his descriptions and certain four-dimensional figures, we wonder at his ingenuity but refuse to consider a systematic connection—*we* cannot think in four dimensions, we say; how could Dante do so? While Dante made no pretension that there even was such a thing as a fourth dimension (Aristotle's negation of such a possibility in the *Physics* was never questioned), the universe he described was, from a vantage informed by Riemannian mathematics, in fact four-dimensional. It was four-dimensional because the basic notion of space available to the Middle Ages did not exclude curvature; that exclusion, and perhaps our own difficulty in imagining curved space, is a legacy of the modern age. Dante conceived the cosmos as a form in multi-dimensional, curved space because his age had not yet been taught the impossibility of doing so.

On the Privilege of Poetic Scientificity

Dante wrote the *Divine Comedy* between 1306 and 1321, at a mid-point between the Condemnation and its eventual repeal, a time marked by the tension between Aristotelian hegemony and the prohibition against impinging on the omnipotence of God. It seems logical to imagine that such a time would offer exciting possibilities to a poet of Dante's age, especially a poet whose immediate task is to write an imaginary journey from the physical to the theological world. In other words Dante had to make explicit what many thinkers could disregard; he had to build a poetic bridge between worlds that had as yet not been made commensurable.

[62] Koyré, *Closed World*, 163.
[63] *Ibid.*, 161.

Dante was concerned to make a cosmo-theology, a journey from the lowest, basest, and most specific place in the universe to the highest, most exalted, and most universal place. He had to do so, however, as a person, as a point of view. In the course of his journey his point of view would necessarily encounter, in a real sense, some of the most fundamental problems of metaphysical speculation: Does the cosmos have a place? What lies outside the edge of the universe? How can God be both everywhere and at the same time be a single entity? As we have seen, the scholastics dealt with these problems in various ways; but Dante's eventual solution, in which the universe becomes an enormous sphere whose circumference is a singularity lying at the center of space and time, is closest to the notion subscribed to by the commentator Avempace, in which the outside is contained by the outer surface of the sphere that it, in turn, contains. Of course Dante's solution is not identical. His theological concerns make it essential to efface any hint of symmetry between the poles of the universe: God is the center that contains, the earth is the center that is contained. In doing so he foreshadowed a startling revelation of modern, secular physics: the earth is the center of an enormous sphere whose circumference is a singularity at the origin of time and space.

The point is that, while scientific knowledge at the time was sufficiently unconstrained to allow for the possibilities of such apparently paradoxical conclusions, such an exact formulation was enabled precisely by a series of constraints: just as the constraints of the Condemnation permitted the circulation of volatile combinations of conflicting ideas, so the particular constraints of Dante's poetic project—to tell a story across an antagonistic divide, to locate the placeless in language—propelled him to offer a solution to an "age-old" problem. When Riemann offered his solution to the "age-old" problem in the nineteenth century, it was not so much a solution as a reminder that this was a problem. It was a revelation at the time that the flat space of Euclidean geometry might be a tool of knowledge rather than an ontological certainty.

When Einstein praised Riemann at the beginning of the twentieth century, he emphasized the dynamism of his new conception of space: "Only the genius of Riemann, solitary and uncomprehended, had already won its way by the middle of the last century to a new conception of space, in which space was deprived of its rigidity, and in which its power to take part in physical events was recognized as possible."[64] It is precisely this lack of rigidity, this involvement of space with physical events that characterizes the space of the medieval imagination. According to Einstein, physicists need to lose their view of space as an empty precondition of the being of matter. Rather, space-time is a result of the very existence of matter, its geometrical properties "are not independent, but they are determined by matter."[65] If medieval thinkers *could* not conceptu-

[64] Osserman, *Poetry of the Universe*, 79.

[65] Albert Einstein, *Relativity: The Special and General Theory*, tr. Robert W. Lawson (New York, 1961), 113.

alize space as a separate entity from matter, Einstein argues that we *should* not: "I wished to show that space-time is not necessarily something to which one can ascribe a separate existence, independently of the actual objects of physical reality. Physical objects are not in space, but these objects are spatially extended."[66] In other words Einstein was concerned with explaining the universe in immanent rather than transcendent terms. Just as Relativity posited space and time as observer-dependent entities, so it required a mode of space-time that was immanent to the objects in the observable world. Of course, the state of knowledge in physics has changed dramatically since Einstein, but current thought has bolstered this conception of space. This conception and the space of Dante's natural philosophy have far more in common than does the space of the intervening five hundred years, which explains why Dante could imagine his cosmos, while lacking the language or tools to call it such, as a hypersphere.

Beyond Continuity vs. Discontinuity; Toward Positive Forgetting

According to David C. Lindberg, one of the most important issues for the history of western science is the extent to which medieval learning influenced, or was "continuous" with, the development of modern science.[67] The opinion that was dominant from the seventeenth century onward was that medieval thought had little if any importance to the progress of science and, in fact, may have been detrimental. In the words of Francis Bacon, "neither the Arabians nor the Schoolmen need be mentioned, who in the intermediate times rather crushed the sciences with a multitude of treatises, than increased their weight."[68] Such a view held sway until the early twentieth century, when a radically different opinion found expression in the work of Pierre Duhem, for whom the advances of modern science would not have been possible were it not for the "multitude of treatises" that Bacon so decried. Duhem offers what has been called the "continuity" thesis, in opposition to the radical discontinuity assumed by earlier historians and scientists, claiming that "the mechanics and physics of which modern times are justifiably proud proceed, by an uninterrupted series of scarcely perceptible movements, from doctrines professed in the heart of the medieval schools."[69] Duhem's claims initiated a debate that has lasted to this day in which historians have alternately claimed, as did Alistair Crombie, that the innovations of the Scientific Revolution were merely repetitions of a revolution in methodology dating from the thirteenth and fourteenth centuries[70] or, in the words of Alexandre Koyré, that the innovations of the seventeenth century were so radical that they required the dissolution of previous world views:

[66] *Ibid.*, vi.
[67] Lindberg, *Beginnings of Western Science*, 355-68.
[68] Ibid., 356.
[69] Duhem, *Système du monde*, 357.
[70] *Ibid.*, 358.

> What the founders of modern science ... had to do, was not to criticize and combat certain faulty theories, and to correct and replace them by better ones. They had to do something quite different. They had to destroy one world and replace it by another. They had to reshape the framework of our intellect itself, to restate and reform its concepts, to evolve a new approach to Being, a new concept of knowledge, a new concept of science.[71]

Lindberg himself adopts a position somewhere in between these two extremes, arguing that while the Middle Ages should not be seen as the Scientific Revolution *avant la lettre*, there can be no doubt that important strides were made that helped prepare the way for the discoveries of the seventeenth century.

All sides in the continuity/discontinuity debate, however, take for granted that, whether the Middle Ages prepared the way for or were largely irrelevant to the Scientific Revolution, the knowledge associated with that "paradigm-shift" was of a purely positive character. In other words, regardless of the side one adopts in this debate, the general assumption is that knowledge is cumulative and that the seventeenth century had more of it than the Middle Ages. I have suggested in this essay that this is not always the case. While civilizations may not lose the effective technology, the specific methodologies, and the compendiums of observations that comprise the existing knowledge of a given time, the fundamental presuppositions of knowledge themselves may change, a culture's framework for viewing the world may be forgotten. With splendid self-assurance the western empirical tradition assumes that such forgetting has only a utilitarian character: one only loses what one no longer needs. But such an assumption rests on a theory of history, implicit though it may be, which exists only to justify the grounds of the assumption itself. We need to recognize that sometimes the very gains that open new possibilities of thought can, at the same time, foreclose other possibilities, ways of thinking, and forms of imagination that future ages will need to rediscover. A debate between continuity and discontinuity would seem to cover the entire horizon of historical options, and yet we must see that the theory of history that this very choice embodies is not all there is; that, in the space of a coincidence, in which an ancient poet seems to wait for us, smiling, on the crest of the modern cosmos, we may glimpse a palimpsest of paths not taken and a hint of the possibilities that lie beyond our own horizons of thought.

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[71] Lindberg, *Beginnings of Western Science*, 359.