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Smile Interpolation and Extrapolation

Yukinori Iwashita
yuki@opengamma.com

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Abstract

In this note we review techniques for smile interpolation and extrapolation, and compare numerical results across the different methodologies. Examples of the smile interpolation include spline interpolations, mixed lognormal model and SABR model together with various volatility approximation formulas. For smile extrapolation we introduce two methods: shifted lognormal model and elementary-function-based approach.

Contents

1 Introduction

Construction of a reliable volatility smile is a key role for the risk management of financial derivatives. In order to achieve this, it is necessarily to apply robust interpolation methods to a set of discrete volatility data. There are primarily two approaches to the smile interpolation. The first strategy is to calibrate a smile model including mixed lognormal model [Whi12], stochastic volatility inspired (SVI) parametrisation [Gat04], and stochastic volatility models such as Heston model [Hes93] and stochastic alpha beta rho (SABR) model [HKLW02]. Alternatively one can interpolate the volatility data based on a specific functional form. Its examples are spline interpolations (see [Iwa13] and references therein).

In certain situations it is also required to estimate volatility values outside of the range of the market-available strikes, that is, to extrapolate the smile outside of the data range. For example we encounter the following integrals for pricing in-arrears products [Hen11b],

$$\int_{K^*}^{\infty} F(\sigma(K), K) dK \,, \tag{1.1}$$

$$\int_{0}^{K^*} F(\sigma(K), K) dK \,, \tag{1.2}$$

where $F(\sigma(K), K)$ is a function of option strike K and implied volatility $\sigma(K)$, and K^* is inside the data range.

For the former integral (1.1) large contribution from the extrapolated part is expected. Thus conducting the integral requires the entire smile structure in $K^* \leq K < \infty$. On the other hand, the latter (1.2) should be evaluated with an extrapolation such that the volatility smile is well-behaved in the $K \to 0$ limit.

In this note we review techniques for smile interpolation and extrapolation, and compare their numerical results. Comprehensive survey on the volatility smile construction is given in [Hom11].

In the next section we start a discussion on the smile interpolation. The simplest approach without any specific model is to use spline interpolations. We discuss double quadratic interpolator for the smile spline interpolation in 2.1. The SABR model is reviewed in 2.2 with various approximation formulas for the implied volatility. As an alternative smile model the mixed lognormal model is briefly discussed in 2.3, where our argument is based on another OpenGamma Quantitative Documentation [Whi12]. The extrapolation techniques are reviewed in section 3. Benaim-Dodgson-Kainth extrapolation in 3.1 assumes a specific functional form such that option prices converge to zero at $K = 0, \infty$ and C^2 continuity is held everywhere [BDK08]. Another example of the smile extrapolation discussed in 3.2 is the shifted lognormal model. All the figures are attached at the end of the note.

2 Interpolation

This section is devoted to discussing smile interpolations. Given a volatility data set (K_i, σ_i) $i = 1, 2, \ldots, n$ with $K_1 < K_2 < \cdots < K_n$ for time to expiry T, the smile interpolation problem is to find a functional defined in $K_- \leq K \leq K_+$. Although some interpolations are designed to produce a sensible curve beyond K_1, K_n, and reliable outside of the data range, let us here consider interpolations strictly apply to the interval $K_1 \leq K \leq K_n$. Thus we have $K_- = K_1$ and $K_+ = K_n$.

An example of the smile parametrisation is spline interpolation discussed in 2.1. In 2.2 we review the SABR model. A variety of approximation formulas of the implied volatility are summarised in appendix C. It is well known that the volatility formulas of the SABR model can produce arbitrage option prices in certain situations. Thus the mixed lognormal model is introduced as an arbitrage-free way of smile interpolation in 2.3.

2.1 Spline Interpolation

An approach to the volatility smile construction is to apply spline interpolation techniques (e.g., [Iwa13]) to the market data. In this note we shall consider OpenGamma's double quadratic interpolation. The double quadratic interpolator is reviewed in appendix B.

Figure 1 shows the resulting curve of the double quadratic interpolation on the sample data set in (A.1). While the double quadratic interpolation is a semi-local interpolation in the sense that one interpolant is determined by the information on four data points, it is not shape-preserving, then it may suffer from unnatural bumping and wiggling. In the present case, however, the resulting curve is almost piecewise monotone, and a similar curve is obtained even if we use shape-preserving interpolators, e.g., the monotonicity preserving Hermite interpolation in [Iwa13].

2.2 SABR Model

The SABR model was introduced in [HKLW02] as a simple class of stochastic volatility processes for the underlying. CEV (constant elasticity of variance) dynamics for the forward F_t and lognormal dynamics for its volatility σ_t are assumed in the model,

$$dF_t = \alpha_t\, F_t^{\beta} dW_t^1\,, \qquad d\alpha_t = \nu \alpha_t dW_t^2\,, \tag{2.1}$$

where the forward and volatility are correlated, $dW_t^1\, dW_t^2 = \rho\, dt$. Given the initial value of the forward, F_0, the model is governed by 4 parameters, α_0, β, ν, ρ.[1]

Although no closed form of option valuation is known, initiated by the original paper [HKLW02] various approximation formulas for the Black implied volatility have been proposed. In appendix C we review a couple of approximation formulas frequently used in the SABR model calibration. The optimal model parameter set α, β, ν, ρ, generally depends on the choice of the formulas.

As mentioned in [HKLW02] and demonstrated in [GR09] the value of the β parameter can be fixed by ATM volatility data or implication from other smile models. Once β is fixed, the three parameters, α, ν, ρ, remain to be determined. Here let us discuss two ways to calibrate the SABR model to market volatility data. The first methodology is to find the three parameters of a single SABR model for the all of the given data (global fit). Figure 2 shows the calibration of the SABR model to the sample data (A.1) by using Hagan et al. in (C.1) (black line), Berestycki et al. in (C.4) (dashed line), Johnson-Nonas (C.6) (red line) and Paulot (C.8) (blue line). The resulting curve obtained by the Paulot formula is distinguished form the others. This is considered to be a consequence of the fact that the formula (C.8) is a valid approximation for any strike value.

It is seen from Figure 2 that several market volatility values are not recovered by the global fit reflecting the fact that the degrees of freedom in the SABR model are smaller than the number of the data points. This can be circumvented with multiple SABR models: one calibrates a SABR model to consecutive three data points and takes a weighted average of the two SABR models

[1] Hereafter we will use simple notations, F for the current forward value and α for the current value of the volatility α_0.

in each interval (local fit).[2] The comparison between the global fit and local fit is illustrated in Figure 3. As expected the curve obtained by the local fit nicely reproduces the market volatility values. In addition to the computational effort the local fit involves potential instability if the market data are very close to each other, $K_i \simeq K_{i+1}$. In the context of the caplet volatility smile this may happen when the ATM is close to one of other market available strikes.

2.3 Mixed Lognormal Model

An alternative smile model is the mixed lognormal model which offers a simple closed form for option price and produces an arbitrage free implied volatility surface. Here we shall quickly review the model and show numerical results. See [Whi12] for the detailed discussion on the model and its calibration.

The model assumes logarithm of the underlying follows the process which is a mixture of independent normal distributions, ϕ,

$$\rho(x) = \sum_{i=1}^{N} w_i \phi\left[x; \left(\mu_i - \frac{\sigma_i^2}{2}\right) T, \sigma_i^2 T\right], \tag{2.2}$$

such that

$$\sum_{i=1}^{N} w_i = 1\,, \qquad \sum_{i=1}^{N} w_i e^{\mu_i T} = 1\,, \qquad w_i \geq 0 \ \ \forall i\,. \tag{2.3}$$

With this setup the option prices are given by a weighted sum of Black prices.

Let us count the degrees of freedom in the mixed lognormal model. The individual normal distributions involve two parameters, μ_i, σ_i, and they are summed up with weights w_i. The two constraints (2.3) are independently imposed. Hence $3N-2$ parameters need to be calibrated for a mixture of N normal distributions.

Figure 4 shows the result of calibrating the mixed lognormal model with $N = 2$ to the sample data (A.1). In the present case the resulting curve is quite similar to the spline interpolation in Figure 1.

3 Extrapolation

Once interpolation is completed for the strike range $K_- \leq K \leq K_+$, the next steps is to extrapolate the smile for $0 < K < K_-$ (left extrapolation) and $K_+ < K$ (right extrapolation). A primary but frequently-used prescription is flat extrapolation where the volatility values at K_- and K_+ are used for all $K < K_-$ and $K_+ < K$, respectively. However, far OTM options tend to be mispriced in this approach, and also, the resulting smile is only C^0 continuous at $K_\pm$. In this section we introduce two extrapolation methods, Benaim-Dodgson-Kainth (BDK) extrapolation in 3.1 and the shifted lognormal model in 3.2.

[2] For example, if the data contains n data points ($n > 3$), they are interpolated in terms of $n-2$ SABR models. Individual intervals, except the rightmost and leftmost ones, are covered by two SABR models, and a common choice for weighting the two models is to employ a linear function or a trigonometric function.

3.1 Benaim-Dodgson-Kainth Extrapolation

An arbitrage-free method was originally proposed in [BDK08] such that the extrapolation is continuous, twice differentiable and option prices converge to 0 as $K \to 0, \infty$. This extrapolation is detailed and analysed focusing on the right extrapolation in [Hen11a].

The smile is extrapolated for low strikes in terms of put prices,

$$P(K) = K^{\mu} \exp\left(a + bK + cK^2\right) . \tag{3.1}$$

While one can realise zero option price at zero strike by imposing $\mu > 0$, the probability density at $K = 0$ is nonzero for $0 < \mu \leq 1$. Thus we will investigate the $\mu > 1$ cases in the numerical examples below.

For the right extrapolation, the functional form for the call option is given by

$$C(K) = K^{-\mu} \exp\left(a + b/K + c/K^2\right) . \tag{3.2}$$

Setting $\mu > 0$ ensures that the call price behaves as $C(K) \to 0$ for $K \to \infty$. A further restriction $\mu > 1$ is needed for a price integral to converge.

Once μ is chosen by the user, the functional involves three free parameters, a, b and c for both the left and right extrapolations. These degrees of freedom are used to achieve C_2 continuity of the option prices at the boundaries $K_{\pm}$. Accordingly the parameters are uniquely determined by solving a linear system.

Figure 5 shows the BDK right extrapolation for $\mu = 0.5$ (solid black line), $\mu = 5.0$ (red line) and $\mu = 10.0$ (blue line) applied to the SABR model interpolation with the Hagan formula (C.1) for the sample data (A.1). It is observed that the resulting curves largely depend on the choice of μ.

In addition to the μ value, the resulting extrapolation also depends on the interpolation. This is illustrated in Figure 6, where we compare the BDK right extrapolations with $\mu = 10.0$ for the interpolations with the Hagan formula (C.1) (black line) and the Paulot formula (C.8) (red line) on the data set (A.1).

The interval, $0 < K < K_{-}$, should be also extrapolated with a sensible curve in order to achieve the robust numerical evaluation of the integral (1.2) because irregular behaviour tends to be observed around $K = 0$ for certain volatility models. This is crucial especially when we use the SABR mode but prefer to avoid arbitrage option prices at small strikes. Figure 7 shows the BDK left extrapolation for $\mu = 1.5$ (solid black line), $\mu = 4.5$ (red line) and $\mu = 7.0$ (blue line) applied to the SABR model interpolation where we use the Hagan formula (C.1). Again changing the value of μ drastically alters the curve profile.

3.2 Shifted Lognormal Model

Noticing that far OTM option prices can be reproduced in the Black model if we shift the forward,

$$F_t \to F_t e^{\alpha} , \tag{3.3}$$

one obtains the smile tail extrapolations by this shifted lognormal model.[3] The model parameters are σ, α. While the Benaim-Dodgson-Kainth method requires one user-input parameter μ, all of

[3] One should not be confused with the displaced diffusion model,

$$dF_t = \sigma_t \left(F_t + \alpha\right) dW_t , \tag{3.4}$$

where the model is characterized by α, σ. Calibration of the displaced diffusion model to volatility data is discussed in [BM02].

model parameters are automatically fixed in the shifted lognormal as described below.

To determine the two parameters we need to provide two independent sets of information at each boundary. If the boundary values of first derivative of the implied volatility with respect to strike, $\partial\sigma_B/\partial K|_{K=K_\pm}$, are obtained from the interpolation, the volatility value and the derivative value are used to determine the shifted lognormal model parameters. If they are not available, another volatility value at the vicinity of the boundary, $\sigma_B|_{K=K_\pm\mp\delta}$, or the value at the next node, σ_2, σ_{n-1}, is passed instead of the derivative value.

We depict the right extrapolation by calibrating the shifted lognormal model to the sample data set in Figure 8. Here the Hagan formula (C.1) is used for the interpolation on the data (A.1). The solid line represents the shifted lognormal extrapolation, whereas the dashed line is the SABR model with no extrapolation method. In this case the two curves are distinct but very close.

Figure 9 shows the left extrapolation with the shifted lognormal model where the interpolation of the data set (A.1) is first completed with the SABR model using the Hagan formula (C.1), and compares the resulting curve of the shifted lognormal extrapolation (solid line) with the SABR model with no extrapolation method (dashed line).

As seen for the BDK extrapolation in Figure 6 the resulting curve differs if we change the interpolation method for the range $K_- \leq K \leq K_+$ as different derivative values are obtained at the boundaries. Another point to note is that the shifted lognormal model can be calibrated when arbitrage is absent at $K_\pm$. This is encoded into the constraint that $\partial\sigma_B/\partial K|_{K=K_\pm}$ should be in a range specific to the values of F, T and $K_\pm$. Even if the calibration fails, one can continue to use the shifted lognormal model, e.g., for the right extrapolation, by setting $K_+ = K_+ - \delta$ and finding the minimum δ such that the gradient on the boundary becomes in the range, or alternatively, by modifying the derivative value $\partial\sigma_B/\partial K|_{K=K_+}$ with the cutoff K_+ the same.

A Volatility Data

The "market data" used in the numerical examples in this papar are shown below. These are a form of inferred caplet volatilities for a specific time to expiry derived from cap market prices via caplet stripping. See [WI14] for detailed discussion on the caplet stripping.

Strikes (%)	Volatilities (%)
0.5	60.58
1.0	52.23
1.5	46.38
2.0	42.24
2.5	40.71
3.0	40.45
3.5	40.75
4.0	41.28
4.5	41.83
5.0	42.42
5.5	43.00
6.0	43.54
7.0	44.54
8.0	45.41
9.0	46.17
10.0	46.85
11.0	47.45
12.0	47.98
ATM	46.57

$$\begin{aligned} &\text{Forward (\%)}: \ 1.4845 \\ &\text{Expiry } (yrs): \ 9.4934 \end{aligned} \tag{A.1}$$

B Double Quadratic Interpolation

Let us briefly review the double quadratic interpolation used in 2.1. While the interpolation is not shape-preserving, it is widely used since it is semi-local and the interpolant is determined analytically.

The interpolation procedure is divided into two steps. Given a data set (x_i, y_i) three consecutive data points are first interpolated by a quadratic polynomial, i.e., finding $f_i(x) = a_i + b_i x + c_i x^2$ such that $f_i(x_i) = y_i$, $f_i(x_{i+1}) = y_{i+1}$ and $f_i(x_{i+2}) = y_{i+2}$ are satisfied. Then the interpolant for $x_i < x < x_{i+1}$ is constructed as a linearly weighted sum of f_{i-1} and f_i,

$$F_i(x) = \frac{x_{i+1} - x}{x_{i+1} - x_i} f_{i-1} + \frac{x - x_i}{x_{i+1} - x_i} f_i \, . \tag{B.1}$$

Note that the interpolants for the leftmost and rightmost intervals are single quadratic polynomials.

In general C^2 smoothness of the quadratic polynomials is smeared by the weighting procedure. Then the interpolation is C^1 continuous everywhere.

C Implied Volatility Formulas for SABR Model

In this appendix we shall introduce a couple of approximation formulas frequently used in the SABR model calibration.

The formula shown in the pioneering work [HKLW02] is most widely used, where the closed expression is derived approximately by singular perturbation,

$$\begin{aligned}\sigma_{B} = &\frac{\alpha z}{\chi(z)\,(FK)^{(1-\beta)/2}}\left(1+\frac{(1-\beta)^2\,(\ln F/K)^2}{24}+\frac{(1-\beta)^4\,(\ln F/K)^4}{1920}\right)^{-1}\\ &\times\left(1+\left(\frac{(1-\beta)^2\,\alpha^2}{24\,(FK)^{1-\beta}}+\frac{\alpha\beta\nu\rho}{4\,(FK)^{(1-\beta)/2}}+\frac{\left(2-3\rho^2\right)\nu^2}{24}\right)T\right)\,,\end{aligned} \tag{C.1}$$

where

$$z=\frac{\nu\ln F/K}{\alpha}\,(FK)^{(1-\beta)/2}\,,\qquad \chi\,(z)=\ln\left[\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right]\,. \tag{C.2}$$

Here we have used simple notations, F for the current forward value and α for the current value of the volatility α_0.

Noticing that the expression (C.1) is in the form,

$$\sigma_{B}\,(F,K,T)=\sigma_0\,(F,K)\Big(1+\sigma_1\,(F,K)\,T\Big)+O\left(T^2\right)\,, \tag{C.3}$$

the approximation can be regarded as small T expansion of the implied volatility, and then, the approximation becomes poor for long time to maturity. In fact probability density implied by the Hagan volatility formula can be negative for low strikes and long maturities. Thus the formula is not arbitrage free.

In [BBF04] (see also [HL05]) methodologies of computing short maturity asymptotics for general stochastic volatility models are proposed and it is proved that the Black implied volatility allows for well-defined expansion in terms of small T as in (C.3). Especially for SABR it turns out that the equation (C.1) should be modified as

$$\sigma_{B}=\frac{\nu\ln F/K}{\chi(\zeta)}\left(1+\left(\frac{(1-\beta)^2\,\alpha^2}{24\,(FK)^{1-\beta}}+\frac{\alpha\beta\nu\rho}{4\,(FK)^{(1-\beta)/2}}+\frac{\left(2-3\rho^2\right)\nu^2}{24}\right)T\right)\,, \tag{C.4}$$

where the function $\chi(\cdot)$ is given in (C.2) and

$$\zeta=\frac{\nu}{\alpha}\frac{F^{1-\beta}-K^{1-\beta}}{1-\beta}\,. \tag{C.5}$$

The difference between the original volatility formula (C.1) and the modified formula (C.4) is investigated in [Obl08].

Aiming at the construction of an approximation formula applicable for longer maturities, Johnson and Nonas truncate the lognrmal SABR model, i.e., $\beta=1$ and extend it to $0<\beta<1$ such that the formula recovers the implied volatility for the CEV model in the $\nu\to 0$ limit [JN09]. The possibility to produce arbitrage option price is reduced by using the following formula,

$$\sigma_{B}=\frac{F^{1-\beta}}{\beta}\left(\frac{w(T)}{\sigma_{\ln}(\xi)}+\frac{1-w(T)}{\sigma_{\rm tr}(\xi)}\right)^{-1}\left(1+\left(\frac{\alpha\beta\nu\rho}{4F^{1-\beta}}+\frac{\left(2-3\rho^2\right)\nu^2}{24}\right)T\right)\,, \tag{C.6}$$

where $w(T) = \max(1/(\nu\sqrt{T}), 1)$ and

$$\begin{gathered}\sigma_{\text{tr}}(\xi) = \frac{\alpha\beta}{F^{1-\beta}} \frac{\xi}{\chi(\xi)}, \quad \sigma_{\ln}(\xi) = \frac{\alpha\beta}{F^{1-\beta}} \left(1 - 4\rho\xi + \left(\frac{4}{3} + 5\rho^2\right)\xi\right)^{1/8}, \\ \xi = \frac{\nu F^{1-\beta}}{\alpha\beta} \ln \frac{F+\triangle}{K+\triangle}, \qquad \triangle = \frac{1-\beta}{\beta} F.\end{gathered} \tag{C.7}$$

Again we have used the same definition for $\chi(\cdot)$ in (C.2). It should be noted that the derivation of this approximation is not mathematically rigorous but heuristic, and accordingly, the resulting volatility values are not improved in terms of accuracy.

Paulot investigated higher order expansions in [Pau09] and derived the exact first order and second order corrections for any strike value.

$$\sigma_B(F, K, T) = \sigma_0\Big(1 + \sigma_1 T + \sigma_2 T^2\Big), \tag{C.8}$$

with

$$\begin{aligned}\sigma_0 &= \frac{\nu}{\psi(\zeta)} \ln \frac{K}{F}, \\ \sigma_1 &= \frac{-\nu^2}{\psi(\zeta)^2} \left(\frac{1}{2} \ln \frac{\sigma_0^2 (FK)^{1-\beta}}{\alpha\nu^2\sqrt{\alpha^2 + 2\alpha\nu\rho\zeta + \nu^2\zeta^2}} - \frac{\rho\beta\Big(G(x_2) - G(x_1)\Big)}{(1-\beta)\sqrt{1-\beta}}\right).\end{aligned} \tag{C.9}$$

The variable ζ is defined in (C.5) and we have introduced the function $\psi(\zeta)$ by

$$\psi(\zeta) = \ln\left[\frac{\sqrt{\alpha^2 + 2\alpha\nu\rho\zeta + \nu^2\zeta^2} + \alpha\rho + \nu\zeta}{(1+\rho)\,\alpha}\right], \tag{C.10}$$

and the function $G(x)$ by

$$\begin{aligned}G(x) = {} & \tan^{-1}\sqrt{\tfrac{R-x-X}{R+x-X}} \\ & + \begin{cases} -\frac{a+bX}{\sqrt{(a+bX)^2-(1-\beta^2)R^2}} \tan^{-1} \frac{cR+(a+b(X-R))\sqrt{\frac{R-x-X}{R+x-X}}}{\sqrt{(a+bX)^2-(1-\beta^2)R^2}} & (a+bX)^2-(1-\beta^2)R^2>0 \\ \frac{a+bX}{cR+(a+b(X-R))\sqrt{\frac{R-x-X}{R+x-X}}} & (a+bX)^2-(1-\beta^2)R^2=0 \\ \frac{a+bX}{\sqrt{(1-\beta^2)R^2-(a+bX)^2}} \widehat{\tanh}^{-1} \frac{cR+(a+b(X-R))\sqrt{\frac{R-x-X}{R+x-X}}}{\sqrt{(1-\beta^2)R^2-(a+bX)^2}} & (a+bX)^2-(1-\beta^2)R^2<0 \end{cases}\end{aligned} \tag{C.11}$$

together with

$$X = \tfrac{x_2^2 - x_1^2 + y_2^2 - y_1^2}{2(x_2 - x_1)}, \qquad R = \sqrt{y_1^2 + (x_1 - X)^2}, \qquad \widehat{\tanh}^{-1}(z) = \tfrac{1}{2} \ln\left|\tfrac{1+z}{1-z}\right|. \tag{C.12}$$

The variables a, b, c, x_i, y_i are related to the SABR model parameters,

$$\begin{gathered}a = F^{1-\beta}, \qquad b = (1-\beta)\sqrt{1-\rho^2}, \qquad c = (1-\beta)\,\rho, \\ x_1 = \frac{-\alpha\rho}{\nu\sqrt{1-\rho}}, \quad x_2 = \frac{\nu\zeta - \rho\sqrt{\alpha^2 + 2\alpha\nu\rho\zeta + \nu^2\zeta^2}}{\nu\sqrt{1-\rho^2}}, \quad y_1 = \frac{\alpha}{\nu}, \quad y_2 = \frac{\sqrt{\alpha^2 + 2\alpha\nu\rho\zeta + \nu^2\zeta^2}}{\nu}.\end{gathered} \tag{C.13}$$

The second order term σ_2 involves integrals which can not be evaluated analytically. Thus one should rely on a numerical integration technique to compute σ_2 (See the original paper [Pau09]). Due to this increased numerical effort, Paulot's formula is frequently used with the subleading correction omitted. Still it is worth noting that the formula without σ_2 should be distinguished from others as it offers a valid approximation for any strikes.

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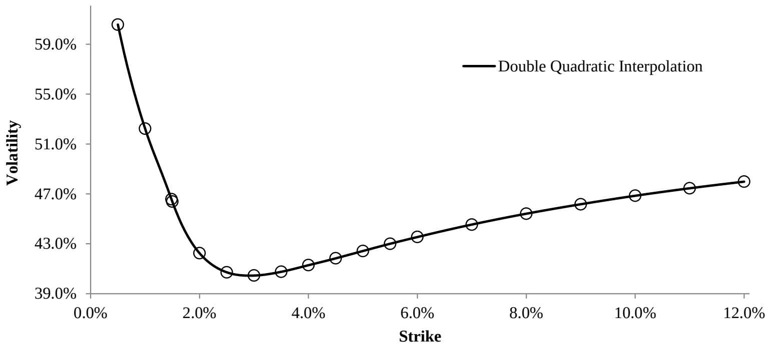

Figure 1: Smile interpolation by using the double quadratic interpolator.

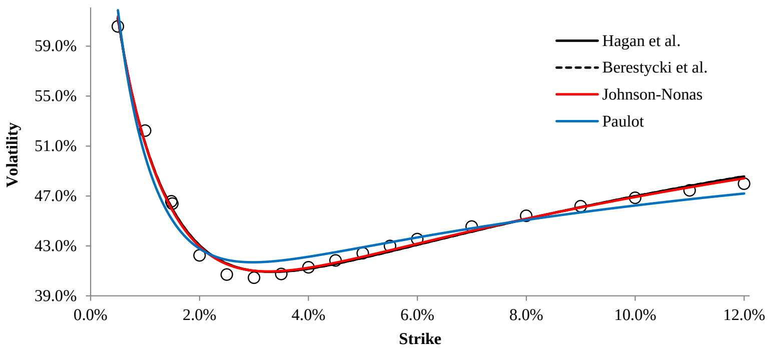

Figure 2: Volatility smile obtained by calibrating the SABR model with Hagan et al. in (C.1) (black line), Berestycki et al. in (C.4) (dashed line), Johnson-Nonas (C.6) (red line) and Paulot (C.8) (blue line). The circles represent the caplet volatility data.

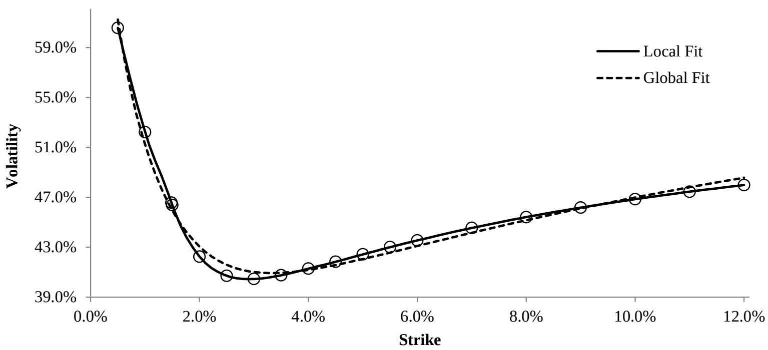

Figure 3: Comparison between the global fit (dashed line) and local fit (solid line) of the SABR model.

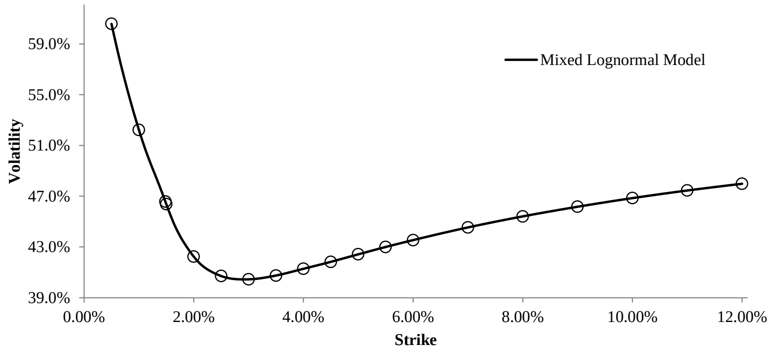

Figure 4: Volatility smile obtained by calibrating the mixed lognormal model with $N = 2$ to the sample data set.

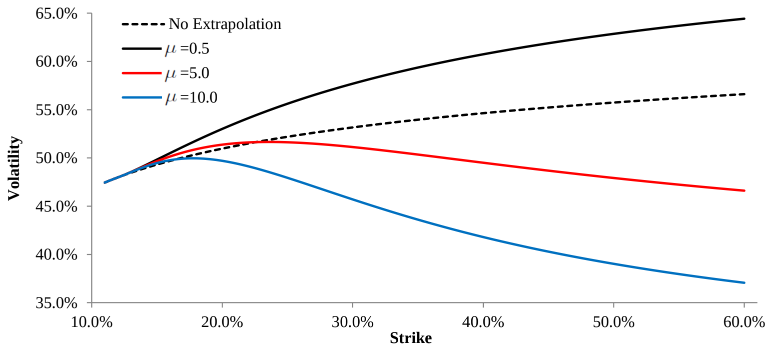

Figure 5: BDK right extrapolation for $\mu = 0.5$ (solid black line), $\mu = 5.0$ (red line) and $\mu = 10.0$ (blue line) applied to the SABR model interpolation. The dashed black line represents the SABR model without using any extrapolation technique.

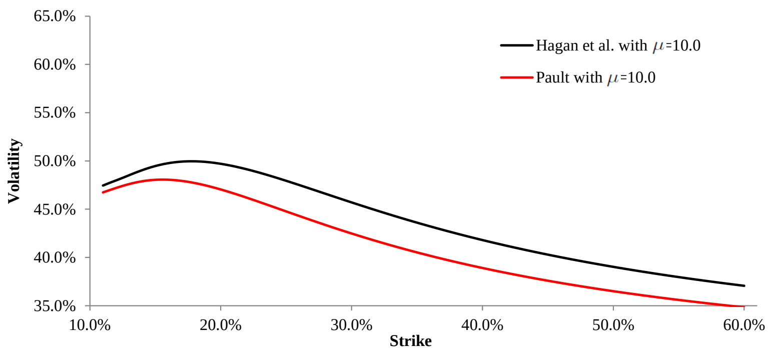

Figure 6: BDK right extrapolation with $\mu = 10.0$ for the interpolations with the Hagan formula (C.1) (black line) and the Paulot formula (C.8) (red line)

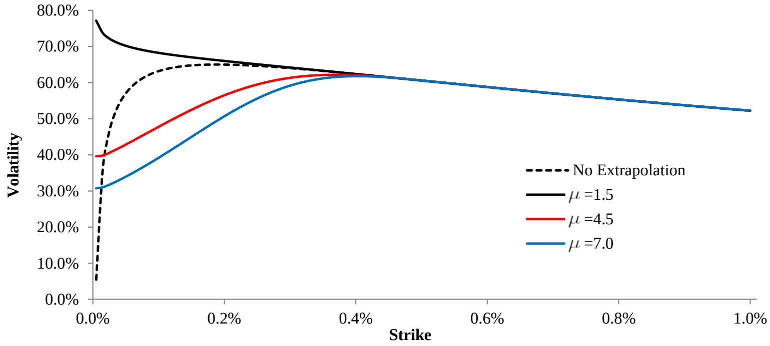

Figure 7: BDK left extrapolation for $\mu = 1.5$ (solid black line), $\mu = 4.5$ (red line) and $\mu = 7.0$ (blue line) applied to the SABR model interpolation. The dashed black line represents the SABR model without any extrapolation technique.

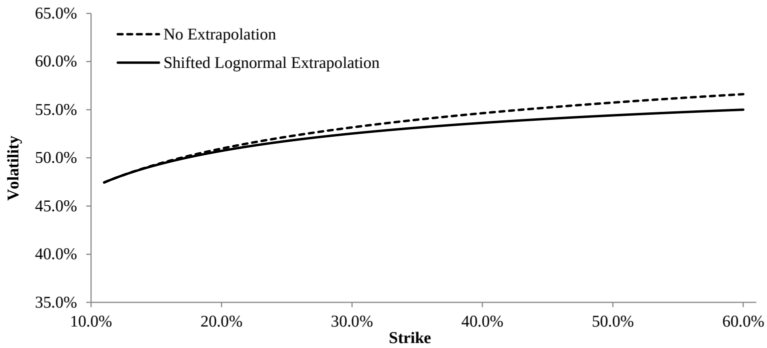

Figure 8: Right extrapolation obtained by calibrating the shifted lognormal model to the sample data set (solid line). It is compared with the SABR model with no extrapolation technique (dashed line).

Figure 9: Left extrapolation obtained by calibrating the shifted lognormal model to the sample data set (solid line). It is compared with the SABR model without using an extrapolation technique (dashed line).

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Released under the open source Apache License 2.0, the OpenGamma Platform covers a range of asset classes and provides a comprehensive set of analytic measures and numerical techniques.

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Europe
OpenGamma
185 Park Street
London SE1 9BL
United Kingdom

North America
OpenGamma
125 Park Avenue
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New York, NY 10017
United States of America

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