$\LaTeX$ CSC 208 Homework Week 4 **Feb 13th 2017 Monday**

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Problem 1:
datatype option = NONE | SOME of int;
fun optionMax(NONE, NONE) = NONE | optionMax(SOME(m),NONE) = SOME(m) | optionMax(NONE, SOME(m)) = SOME(m) | optionMax(SOME(m), SOME(n)) = if m >= n then SOME(m) else SOME(n);

Problem 2:
fun setAddElem(nil, y) = [y] | setAddElem(x::xs, y) = if y ¡ x then y::x::xs else if y = x then x::xs else x::setAddElem(xs,y);

fun setUnion(nil, ys) = ys | setUnion(xs, nil) = xs | setUnion(x::xs,y::ys) = if x < y then x::setUnion(xs,y::ys) else if x = y then x::setUnion(xs, ys) else y::setUnion(x::xs,ys);

fun setIntersection(nil, ys) = nil | setIntersection(xs,nil)=nil | setIntersection(x::xs, y::ys) = if x = y then x::setIntersection(xs,ys) else if x < y then setIntersection(xs,y::ys) else setIntersection(x::xs,ys);

fun setMax(xs) = List.last(xs);

Problem 3:
a. $\{b\}\cup$ A basically gives us all the elements in A plus b since b is not an element of A. Adding one more element into set A means that the powerset of A expands by inserting b in every set in the powerset of A (including inserting b into the empty set that results in $\{b\}$). Therefore, if there are x elements in powerset A, there will be an additional x elements in powerset of union A and the set containing b. Therefore, cardinality of powerset of union A and the set containing b is two times of the cardinality of powerset A.

b. If we start from the beginning that A is an empty set, $|A| = 0$, $\mathcal{P}$(A) = $\{\emptyset\}$, $\mid \mathcal{P}(A) \mid = 1$. Then if we start adding one element into A, according to part a., we know that the cardinality of the new powerset of A will be twice the cardinality of previous powerset of A, which is 1*2 = 2. Then whenever we add one additional element into A, the cardinality of the new powerset of A doubles. When |A| = n, which means we have added nth element to A, cardinality of the new powerset of A is doubled n times, which will be 2^n.

Problem 4:
a. $\mathcal{P}(\mathcal{P}(\{1,2\})) = \mathcal{P}(\{\{\emptyset\},\{1\},\{2\},\{1,2\}\}) = \{\emptyset,\{\emptyset\},\{1\},\{2\},\{1,2\},\{\{\emptyset\},\{1\}\},\{\{\emptyset\},\{2\}\},$
$\{\{\emptyset\},\{1,2\}\},\{\{1\},\{2\}\},\{\{1\},\{1,2\}\},\{\{2\},\{1,2\}\},\{\{\emptyset\},\{1\},\{2\}\},\{\{\emptyset\},\{1\},\{1,2\}\},$
$\{\{\emptyset\},\{2\},\{1,2\}\},\{\{1\},\{2\},\{1,2\}\},\{\{\emptyset\},\{1\},\{2\},\{1,2\}\}\}$
$\mid \mathcal{P}(\mathcal{P}(\{1,2\})) \mid = 16$.
(Citation: Only with Maddie Kirwin's advice of replacing each element with letter A,B,C,D, I could possibly with)

b. $\mathbb{Z}$ is the set of all integers. Powerset of $\mathbb{R}$ is the set of all possible subsets of $\mathbb{R}$. Since $\mathbb{Z}$ is a subset of $\mathbb{R}$, therefore $\mathbb{Z}$ is an element of the powerset of $\mathbb{R}$. Therefore $\mathbb{Z} \in \mathbb{R}$ but $\mathbb{Z} \not\subset \mathbb{R}$.

c. True. Since powerset of a set X is the set of all possible subsets of X. While A $\cap$ B is the set of all common elements in both A and B. Therefore, powerset of A $\cap$ B is all possible combination of the common elements in both A and B. The $\mathcal{P}(A) \cap \mathcal{P}(B)$ then powerset of inter

d. False. For example, element k is only in set A, and element h is only in set B. Both elements will appear in A $\cup$ B, and thus $\{k, h\}$ will be in $\mathbb{P}$(A$\cup$B). However, since k is only in A and h is only in B, $\{k, h\}$ will not be in either $\mathbb{P}$(A) or $\mathbb{P}$(B). Thus, $\{k, h\}$ will not be in $\mathbb{P}$(A) $\cup$ $\mathbb{P}$(B).
(Citation: discussed with Minh and wrote the answer on my own.)

Problem 5:
a. datatype natural = Zero | Successor of natural;
fun isEven(Zero) = true | isEven(Successor(n)) = not(isEven(n));

b.fun plus(Zero,m) = m | plus(m, Zero) = m | plus(Successor(m), Successor(n)) = Successor(Successor(plus(m,n)));

fun times(Zero, m) = Zero | times(Successor(Zero),m) = m | times(Successor(n),m) = plus(m,times(n,m));
(Citation: discussed with Minh and wrote the answer on my own.)