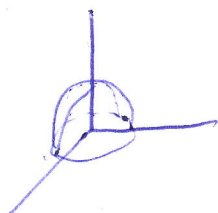Name:

> Answer the questions in the spaces provided. If you run out of room for an answer,
> continue on the back of the page. Leave your answers in *exact form* instead of decimal
> approximations.

1. (5 points) Let H be the upper hemisphere of radius 1 centered at the origen. Compute:

$$\iiint_H (x^2 + y^2 + z^2)^{3/2} dV.$$

$H = \{(\rho, \theta, \phi) \mid 0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi/2,\ 0 \le \rho \le 1\}.$

$\displaystyle \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 (\rho^2)^{3/2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$
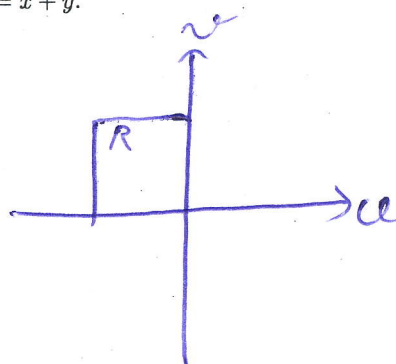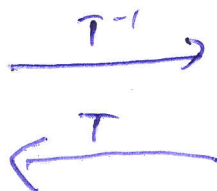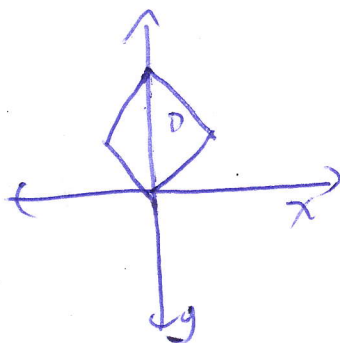
$= \displaystyle \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho^5 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \dfrac{\pi}{3}$

2. (5 points) Let D be the diamond shaped region with vertices $(0,0), (1,1), (-1,1)$, and $(0,2)$. Compute the following integral:

$$\iint_D (x-y)\cos(x^2-y^2) \, dA.$$

Hint: Try a change of coordinates with $u = x - y$ and $v = x + y$.

T is

$x = \frac{1}{2}(u+v)$

$y = \frac{1}{2}(-u+v)$

$\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{vmatrix} = \dfrac{1}{2}$

$\Rightarrow \dfrac{1}{2} \displaystyle \int_{-2}^{0} \int_0^2 u\cos(uv) \, dv \, du$

$= \dfrac{\cos(-4) - \cos 0}{4} = \boxed{\dfrac{\cos(-4) - 1}{4}}$