Homework 9 Hints

2. (e)
   - You want to count the elements of that are in either a Sylow 3 or Sylow 2 subgroup. Notice that these elements have order a power of 3 or a power of 2. But what is the order of
$$\begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$$
   - You may find proving the following Lemma useful:

     Lemma 1. *If G is a group of order 24 with 3 Sylow subgroups H_1, H_2, H_3, then $|H_1 \cup H_2 \cup H_3| \geq 16$.*
   - Recall the inclusion exclusion principle:
$$|H_1 \cup H_2 \cup H_3| = |H_1| + |H_2| + |H_3| - |H_1 \cap H_2| - |H_2 \cap H_3| - |H_1 \cap H_3| + |H_1 \cap H_2 \cap H_3|.$$
     It's easy to prove if you think about venn diagrams.

3. (d) The hint I gave here doesn't really help. Instead compute the normalizer of $\overline{T}$ directly and apply Sylow's theorem. Notice that this approach make part (e) kind of trivial.

4. Skip Question 4. We don't know about finite fields of prime power order. It was silly of me to assign it.

5. Notice that parts (a) and (b) are equivalent. At first I thought that part (a) would help with part (b), but there is a very clean solution doing part (b) first to imply part (a), and many ways to do it work hand in hand. So I think it's misleading to have part (a) first and part (b) second. It may be worthwhile to prove the following lemma first:

   Lemma 2. *Let G be a finite group and $N \trianglelefteq G$ a normal subgroup. If $P \leq G$ is Sylow p-subgroup of G, then $P \cap N \leq N$ is a Sylow p-subgroup of N.*