## Homework2

## September 22, 2021

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[2]: ######## Problem 1
     def slowPower(g,A,N):
         x = 1
         for i in range(0,A):
             x = x*g % N
         return x
     print("2^5 mod 10 =",slowPower(2,5,10))
    2^5 \mod 10 = 2
[1]: ######## Problem 2
     def getBinary(A):
         binaryList = []
         while A>0:
             if A\%2 == 0:
                 binaryList.append(0)
             else:
                 binaryList.append(1)
             A = A//2 #This does the same operation as math.floor(A/2), but without
      →ever converting to a float, and therefore avoiding the lack of precision for
      →values of A that are larger than 64 bits.
         return binaryList
     print("7 in binary is",getBinary(7))
     print("The list is [A_0, A_1, ..., A_r] is reversed from traditional binary, as
      \hookrightarrow A\_i corresponds to the coefficient of 2^i, so read backwards if you want to_\sqcup
      ⇔write in binary")
    7 in binary is [1, 1, 1]
    The list is [A_0,A_1,...,A_r] is reversed from traditional binary, as A_i
    corresponds to the coefficient of 2^i, so read backwards if you want to write in
    binary
[3]: ######## Problem 3(a)
     def fastPower(g,A,N):
         binaryList = getBinary(A)
         powersOfG = [g \% N] #initiate a list of the powers of g with g^1
         for i in range(0,len(binaryList)):
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newPower = powersOfG[-1]**2 % N #square the last element of the list

→ and add it

powersOfG.append(newPower)

output = 1

for i in range(0,len(binaryList)):

if binaryList[i]==1:

output = output * powersOfG[i] % N #multiply all the ones

→ corresponding to nonzero binary coefficients

return output

print("2^5 mod 10 =",fastPower(2,5,10))
```

 $2^5 \mod 10 = 2$ 

```
[4]: ######## Problem 3(b)
def fastPowerSmall(g,A,N):
    a = g
    b = 1
    while A>0:
        if A % 2 == 1:
            b = b * a % N
        A = A//2
        a = a*a % N
    return b
print("2~5 mod 10 =",fastPowerSmall(2,5,10))
```

 $2^5 \mod 10 = 2$ 

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[5]: ####### Problem 4
     ##### part(a)
     print("17^183 mod 256:")
     print("slow:",slowPower(17,183,256))
     print("fast:",fastPower(17,183,256))
     print("fastSmall:",fastPowerSmall(17,183,256))
     ##### part(b)
     print("11^507 mod 1273:")
     print("slow:",slowPower(11,507,1237))
     print("fast:",fastPower(11,507,1237))
     print("fastSmall:",fastPowerSmall(11,507,1237))
     ##### part(c)
     print("2^123456789 mod 987654321:")
     #print("slow:",slowPower(2,123456789,987654321)) TOO SLOW!
     print("fast:",fastPower(2,123456789,987654321))
     print("fastSmall:",fastPowerSmall(2,123456789,987654321))
     ##### part(d)
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```
print("5^100000000000 mod 10000:")
     #print("slow:",slowPower(5,100000000000,10000)) TOO SLOW!
     print("fast:",fastPower(5,100000000000,10000))
     print("fastSmall:",fastPowerSmall(5,100000000000,10000))
    17<sup>183</sup> mod 256:
    slow: 113
    fast: 113
    fastSmall: 113
    11<sup>507</sup> mod 1273:
    slow: 322
    fast: 322
    fastSmall: 322
    2^123456789 mod 987654321:
    fast: 804307517
    fastSmall: 804307517
    5^100000000000 mod 10000:
    fast: 625
    fastSmall: 625
[0]:
```