Math 120 KAUST | Midterm 1 (80 minutes) | October 24, 2016

Name:

Directions:

- You have 80 minutes to complete this exam.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet (so two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

Question	Points	Score
1	15	
2	15	
3	20	
4	20	
Total:	70	

1. Let $f(x) = -x^2 + 4x - 6$.

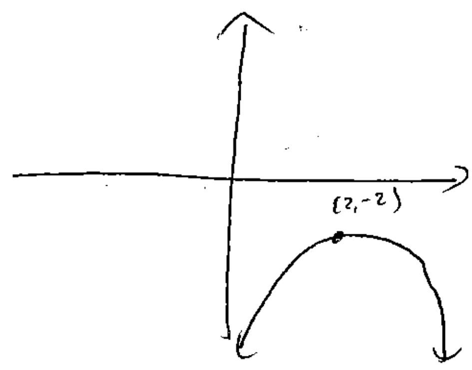
(a) (5 points) Complete the square to write $f(x)$ in vertex form, and then sketch the graph of $y = f(x)$.

$h = \frac{-b}{2a} = 2$

$k = f(2) = -4 + 8 - 6$

$= -2$

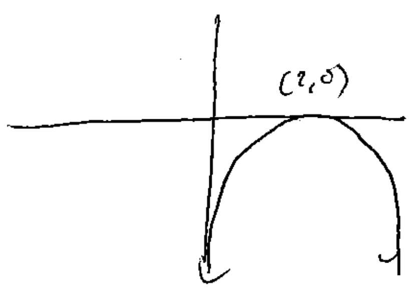
$y = -(x-2)^2 - 2$

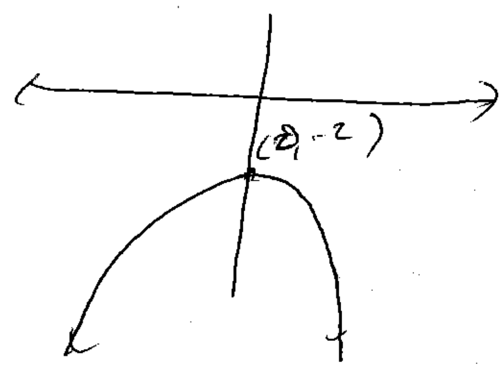


(b) (5 points) Sketch the graphs of $y = f(x) + 2$ and $y = f(x+2)$.

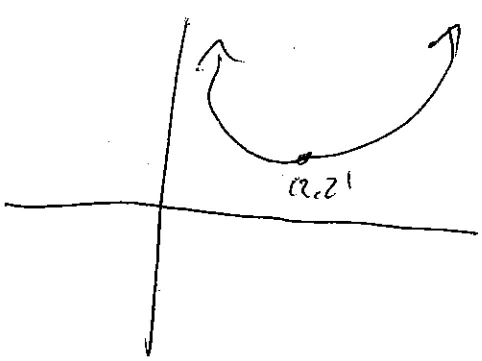
up 2

left 2





(c) (5 points) Sketch the graph of $y = |f(x)|$ and write the associated multipart rule.



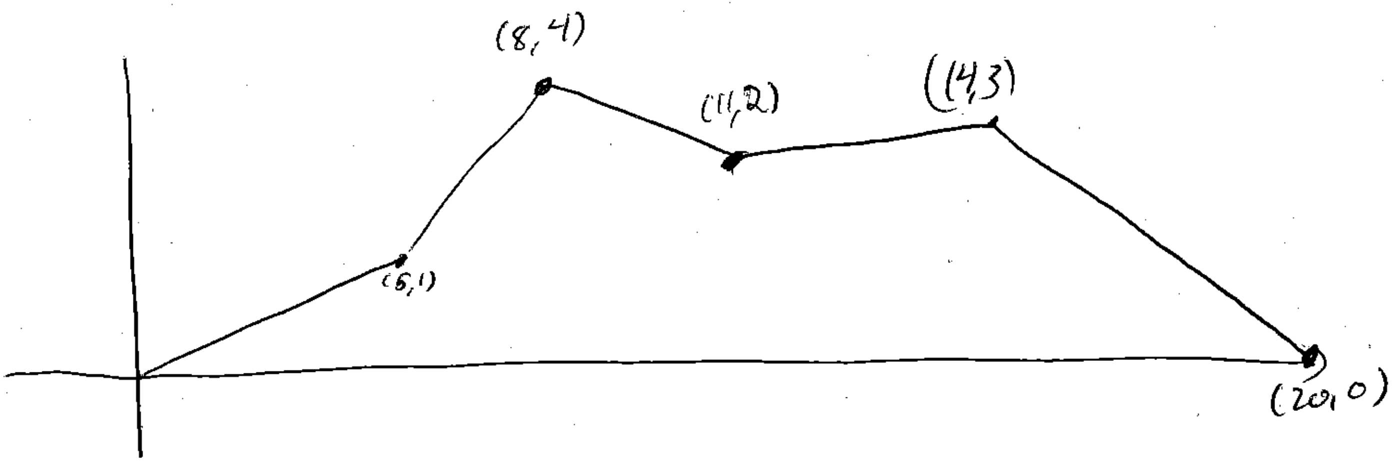
$|f(x)| = -f(x)$

$= x^2 - 4x + 6$.

2. Danielle decides to go on a 20 mile hike around Mount Ranier. It has a lot of (very steep) ups and downs. Assume that they are all linear.

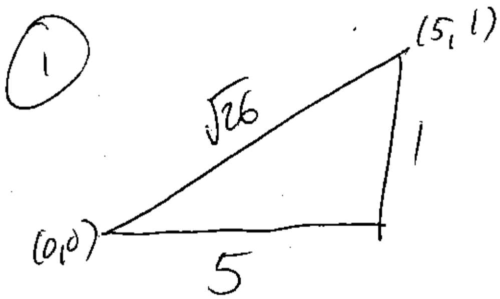
   - During the first 5 horizontal miles, the trail ascends 1 mile vertically.
   - It then gets much steeper, ascending 1 mile vertically for each horizontal mile. It continues this way for 3 horizontal miles.
   - It then descends sharply descending 2 miles vertically in the next 3 horizontal miles.
   - Next it climbs once more, ascending 1 more mile over the next 3 horizontal miles.
   - The last 6 horizontal miles descend at a steady rate to the starting height.

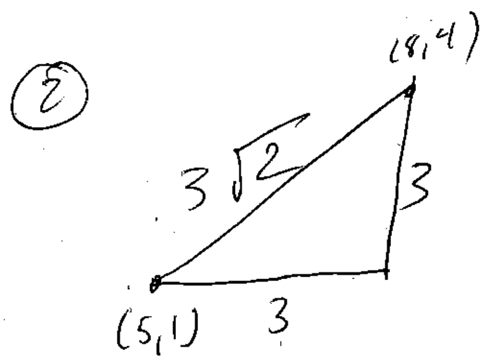
   (a) (10 points) Sketch a graph of $y = h(x)$ where x is the horizontal distance traversed, and $h(x)$ is the height at x. Write a multi-part rule for $h(x)$.

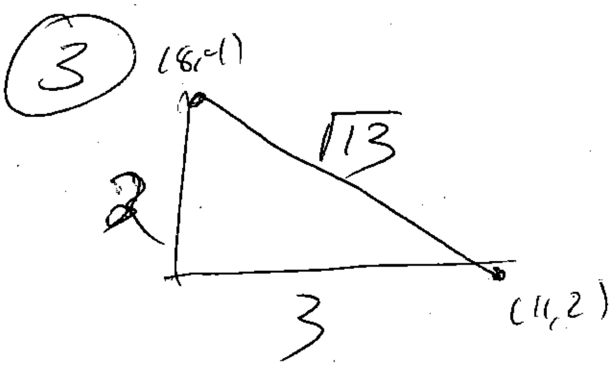


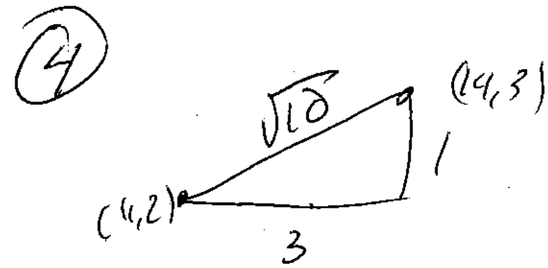
$$y = \begin{cases} \frac{1}{5}x & 0 \le x \le 5 \\ (x-5)+1 & 5 \le x \le 8 \\ -\frac{2}{3}(x-8)+4 & 8 \le x \le 11 \\ \frac{1}{3}(x-11)+2 & 11 \le x \le 14 \\ -\frac{1}{2}(x-2) & 14 \le x \le 20 \end{cases}$$

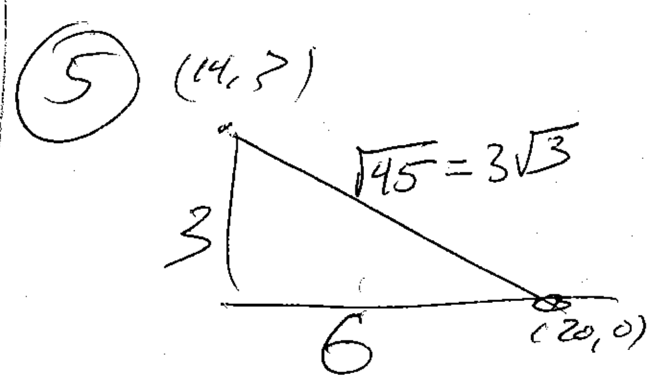
(b) (5 points) 20 horizontal miles were traveled. Use the pythagorean theorem (perhaps more than once) to find the actual hiking distance. (Note, you hike along the slopes, and the x-coordinate measures just horizontal distance.)











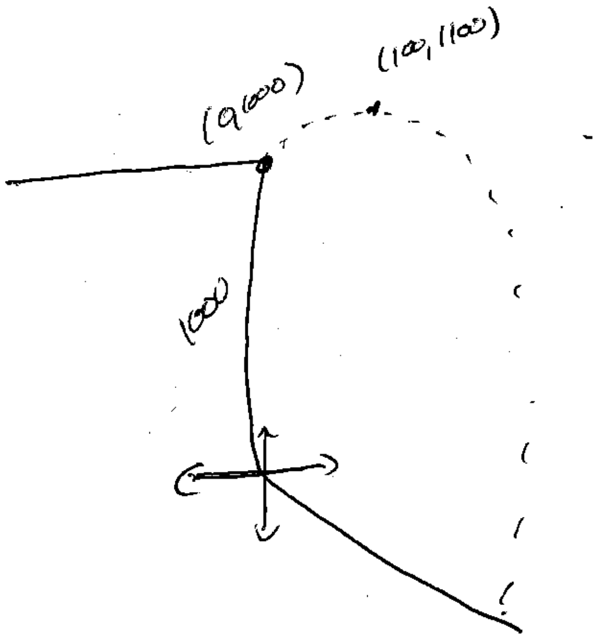
Total

$$= \sqrt{26} + 3\sqrt{2} + \sqrt{13} + \sqrt{10} + 3$$

$$\boxed{21.3}$$

3. You are standing at the edge of a 1000 ft cliff. At the base of the cliff, a hill a descending at a rate of one vertical foot for each horizontal foot, continues away. You throw a rock off of this cliff, and it follows a parabolic trajectory.

(a) (5 points) Using the base of the cliff as the origin, draw a coordinate plane and label where the rock is thrown from. Sketch the trajectory of the rock and draw a line representing the hill at the base of the cliff sloping downwards.



(b) (5 points) The rock reaches the maximum height 100 feet out, going up 100 feet up from where it was thrown. Write a function $h(x)$, for the path of the rock. That is, x is horizontal distance and $h(x)$ is the elevation of the rock above the base of the cliff. (Do not worry about stating the domain.)

$$y = a(x-100)^2 + 1100$$

$$1000 = a(0-100)^2 + 1100$$

$$-100 = 10000a$$

$$a = \frac{-1}{100}$$

$$y = \frac{-1}{100}(x-100)^2 + 1100$$

(c) (5 points) Write an equation $d(x)$, for the vertical distance between the rock and the ground. Where does the rock land?

grand has equaton $y = x$

So $d(x) = \frac{-1}{100}(x-100)^2 + 1100 - x$

$= \frac{-1}{100}(x^2 - 200x + 10000) - x + 1100$

$= \frac{-1}{100}x^2 + 2x - 100 + 1100 - x$

$= \frac{-1}{100}x^2 + x + 1000$

$d(x) = 0$ @ $x = \frac{-1 \pm \sqrt{1 - 4(+1000)(\frac{-1}{100})}}{-2/100}$ $= 370.150$

positive root.

Coords = $(370.156, -370.156$

(d) (5 points) What is the maximum distance of the rock from the ground? Where does it achieve this height.

Maximize $d(x)$

$x = \frac{-b}{2a} = \frac{-1}{-1/50} = 50$

$d(50) = 1025$ ← max distance.

$h(50) = d(50) + 50 = 1075$

Coords are $(50, 1075)$

4. The following problem has a fixed coordinate plane ruled in millimeters. Particle A starts at the origin and moves in uniform linear motion towards the point $(5, -12)$, reaching it in 6.5 seconds. At the same time, a second particle (call it particle B), leaves from $(-5, 0)$ and moves towards the point $(-2, 4)$ traveling at 10 millimeters per second.

(a) (5 points) Write parametric equations for each particle.

Partical a.

$t=0: \quad (0,0)$

$t=6.5: \quad (5,-12)$

$$x_A(t) = \frac{\Delta x}{\Delta t}(t-t_0) + x_0$$

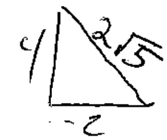
$$= \frac{5}{6.5}t = \frac{10}{13}t$$

$$y_A(t) = \frac{-12}{6.5}t = \frac{-24}{13}t$$

Particle B

$t=0 \quad (-5,0)$

$t=\sqrt{5}/5 \quad (-2,4)$



$$\frac{2\sqrt{5}\text{ mm}}{10\text{ mm/sec}} = \frac{\sqrt{5}}{5}$$

$$x_B(t) = \frac{3}{\sqrt{5}/5}(t) - 5$$

$$= \frac{15}{\sqrt{5}}t - 5$$

$$y_B(t) = \frac{20}{\sqrt{5}}t$$

(b) (5 points) Write a function $d(t)$, where t is time in seconds, and $d(t)$ is distance between the two particles, in millimeters.

$$d(t) = \sqrt{\left(\frac{10}{13}t - \frac{15}{\sqrt{5}}t + 5\right)^2 + \left(\frac{-24}{13}t - \frac{20}{\sqrt{5}}t\right)^2}$$

$$= \sqrt{\frac{1}{13}\left[(132\sqrt{5} + 1667)t^2 + (100-390\sqrt{5})t + 325\right]}$$

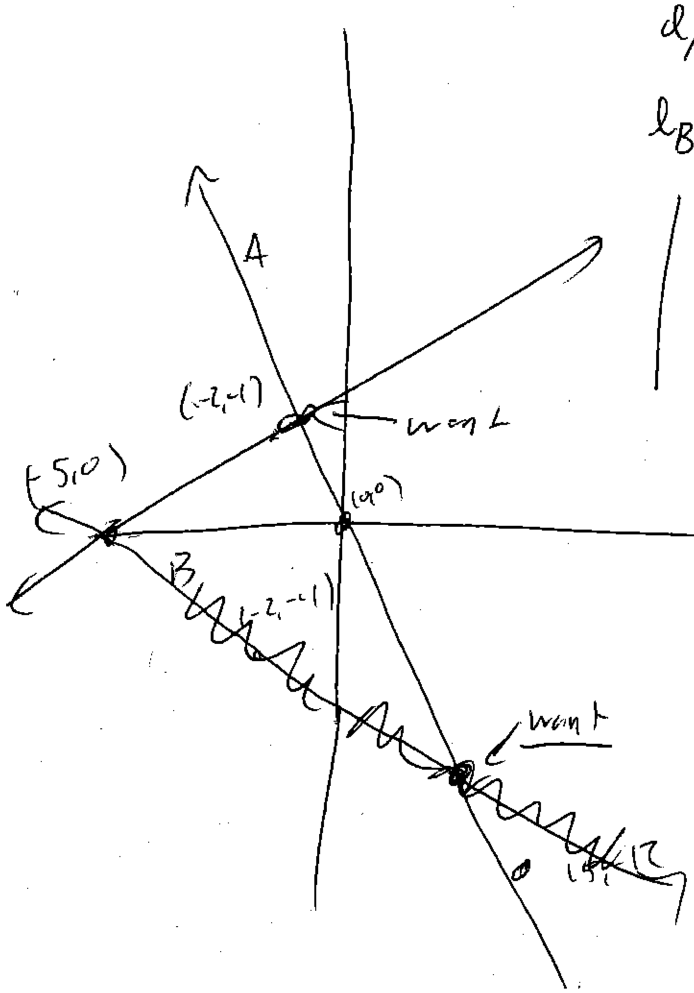
$$\approx \sqrt{150.935t^2 - 59.390t + 96.429}$$

(c) (5 points) What is the minimum distance between the two particles?

Max @ $t = \frac{-b}{2a} = \frac{59.390}{301.870} = .197$

$d(.197) = 4.380$

(d) (5 points) The two paths cross at some point. Which point is this?



$l_A: y = \frac{-12}{5}x$

$l_B: y = \frac{4}{3}(x+5)$

Substitute $\frac{-12}{5}x = \frac{4}{3}x + \frac{20}{3}$

$\frac{-56}{15}x = 20/3$

$x = \frac{-300}{168} \approx -1.786$

$y = \frac{-12}{5}(x) \approx 4.286$

$(-1.786, 4.286)$