Math 120 B Midterm 1 (80 minutes) April 20, 2017

Name: Key

Directions:

- You have 80 minutes to complete this exam.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

Question	Points	Score
1	15	
2	15	
3	20	
4	10	
Total:	60	

1. A parabola contains the three points, $(0,-5), (1,1)$, and $(2,11)$.

   (a) (5 points) Write the equation of the parabola in standard form $f(x) = ax^2 + bx + c$.

System

(i) $a \cdot 0^2 + b \cdot 0 + C = -5$ +1

(ii) $a + b + c = 1$

(iii) $4a + 2b + C = 11$

(i) $\Rightarrow c = -5$ +1 Rewriting

(ii) $a + b = 6$

(iii) $4a + 2b = 16$

(iii) − 2(ii):

$2a = 4$

$a = 2$ +1

Plug $a = 2$ into (ii)

$2 + b = 6$

$b = 4$ +1

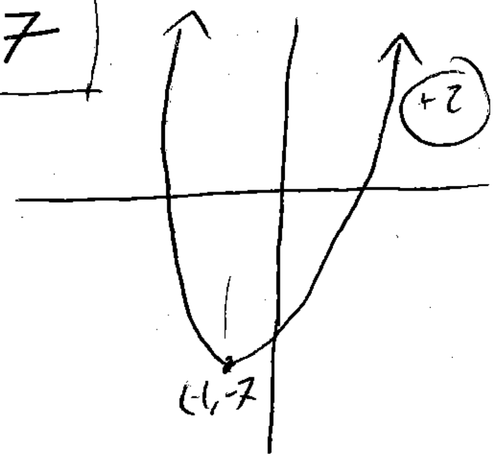
$f(x) = 2x^2 + 4x - 5$ +1

(b) (5 points) Convert $f(x)$ into vertex form $f(x) = a(x-h)^2 + k$ and then sketch a graph of $y = f(x)$, labelling the vertex

$h = \frac{-b}{2a} = \frac{-4}{4} = -1$ +1

$k = f(-1) = 2 - 4 - 5 = -7$ +1

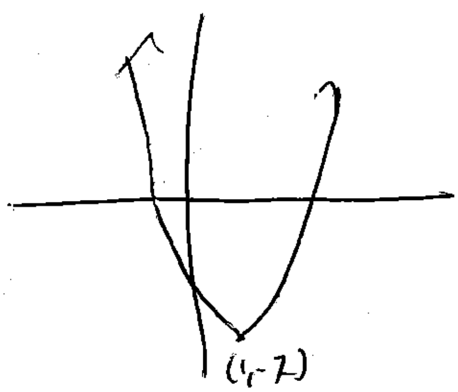
So $\boxed{f(x) = 2(x+1)^2 - 7}$ +1



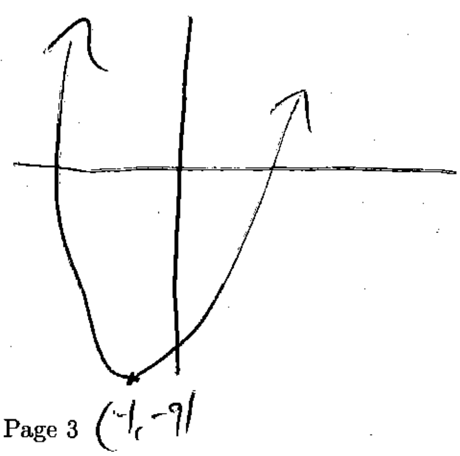
(c) (5 points) Let $g(x) = x - 2$. Sketch the compositions $f \circ g(x)$ and $g \circ f(x)$. (HINT: Can you express what is happening in terms of shifts?).

Label the vertex of each

$f \circ g(x) = f(x-2)$ shift right 2

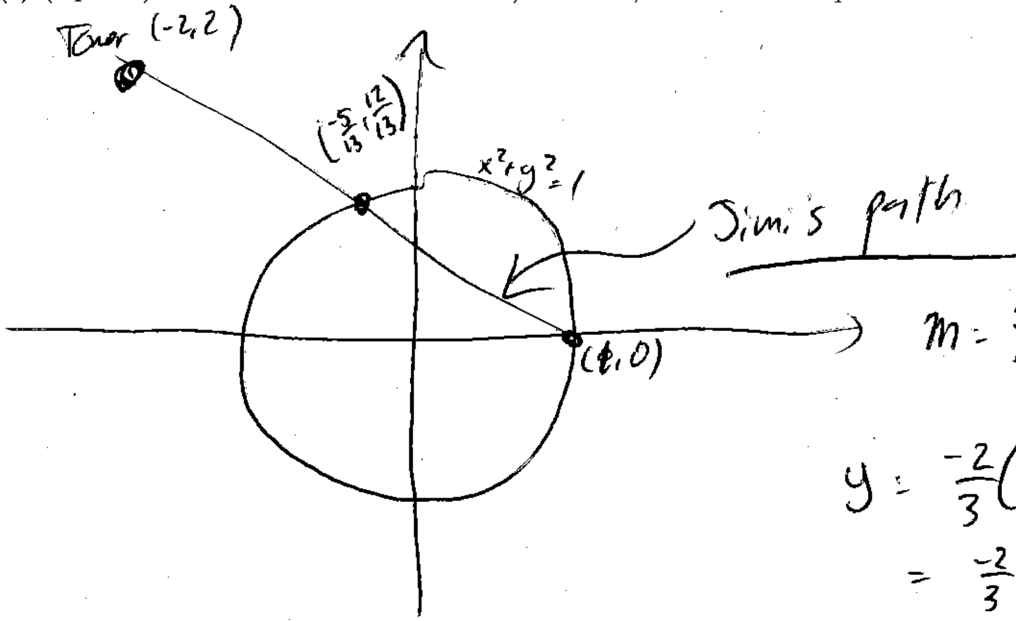


$g \circ f(x) = f(x) - 2$
shift down 2



2. There is a perfectly circular lake with a 1 km radius. Jimi sits on the eastern shore when he spots a tower across the lake. He immediately begins swimming towards it a rate of 2 m/s. Suppose the tower is exactly 2 miles west and 2 miles north of the center of the lake.

(a) (4 points) Set coordinates. Draw the lake, the tower, and label Jimi's path.



$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{3}$$

$$y = \frac{-2}{3}(x-1)$$
$$= \frac{-2}{3}x + \frac{2}{3}$$

(b) (3 points) Where does Jimi exit the lake?

Intersect $y = \frac{-2}{3}x + \frac{2}{3}$

& $x^2+y^2=1$

$$x^2 + \left(\frac{-2}{3}x + \frac{2}{3}\right)^2 = 1$$

$$x^2 + \frac{4}{9}x^2 - \frac{8}{9}x + \frac{4}{9} = 1$$

$$\frac{13}{9}x^2 - \frac{8}{9}x - \frac{5}{9} = 0$$

$$13x^2 - 8x - 5 = 0$$

$$(x-1)(13x+5) = 0$$

$x = 1$ or

$$x = \frac{-5}{13}$$

$$y = \frac{-2}{3}\left(\frac{-5}{13} - 1\right)$$

$$= \frac{-2}{\cancel{3}}\left(\frac{-\cancel{18}^{\,6}}{13}\right) = \frac{12}{13}$$

$$\left(\frac{-5}{13}, \frac{12}{13}\right) \approx (-.385, .923)$$

(c) (3 points) How long is Jimi swimming?

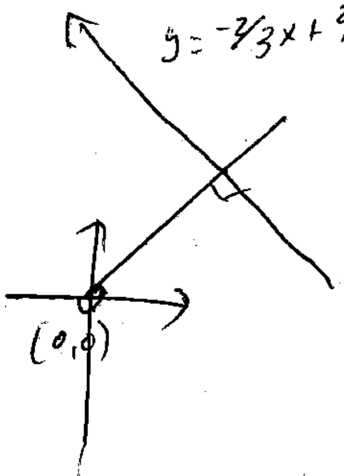
Compute distance swimming

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{\left(\frac{18}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = \frac{1}{13}\sqrt{18^2 + 12^2} = \frac{6}{13}\sqrt{13} \approx 1.66$$

$$d = rt \implies t = \frac{d}{r} = \frac{\frac{6}{13}\sqrt{13}}{2} \times \frac{1000\,\text{m}}{1\,\text{km}} = \frac{3000\sqrt{13}}{13} \approx 832.05$$

(d) (5 points) What is the closest distance Jimi comes to the center of the lake.

↳ (0,0)



orthogonal line is $y = \frac{3}{2}x$

intersect

$$-\frac{2}{3}x + \frac{2}{3} = \frac{3}{2}x$$

$$\frac{2}{3} = \frac{3}{2}x + \frac{2}{3}x$$

$$\frac{2}{3} = \frac{13}{6}x$$

$$x = \frac{12}{39} = \frac{4}{13}$$

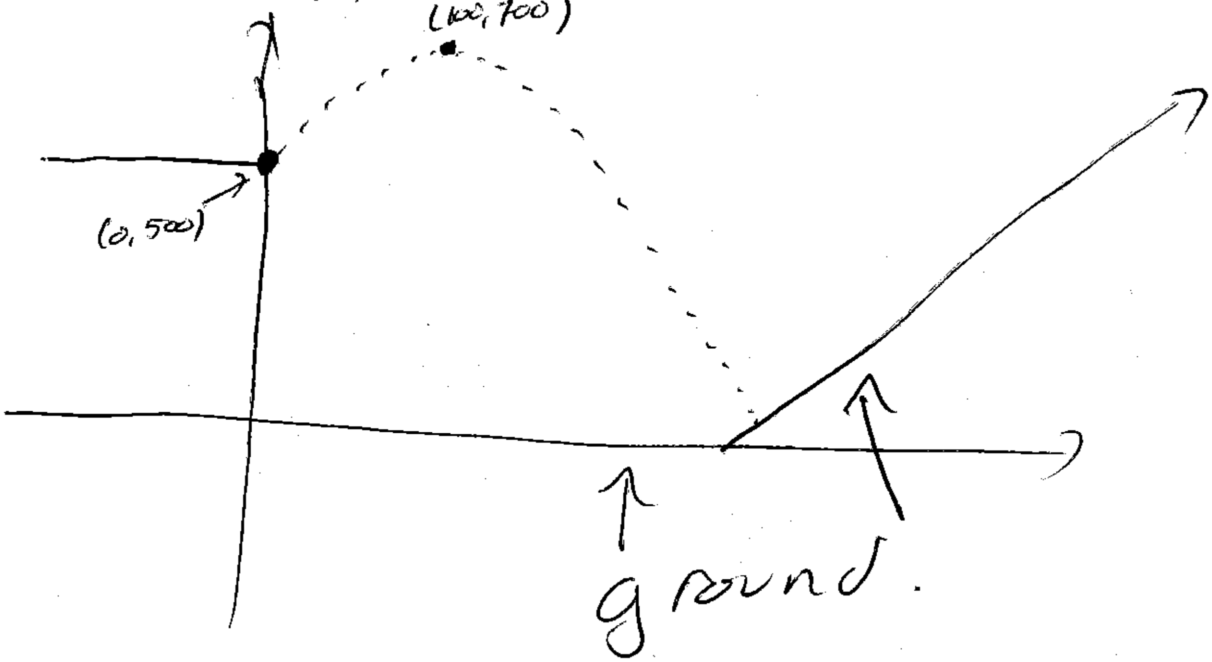
$$y = \frac{3}{2}\left(\frac{4}{13}\right) = \frac{6}{13}$$

$$d = \sqrt{\left(\frac{4}{13}\right)^2 + \left(\frac{6}{13}\right)^2} = \frac{1}{13}\sqrt{16+36} = \frac{1}{13}\sqrt{52} = \frac{2\sqrt{13}}{13}$$

0.5547 km

3. I stand on the top of a 500 foot cliff, and launch a stone in the air with a powerful slingshot. 100 feet out, it reaches its peak 200 feet above where I had shot it. Down below, the ground is flat for 250 feet, and then begins an upward slope of 1 vertical foot for every 2 horizontal.

(a) (5 points) Draw a picture of the situation and set coordinates. Draw the cliff, the ground, and the path of the stone. (Do not worry about whether the stone lands on the flat or the hill, we do not know this yet!).



(b) (5 points) Write a function $h(x)$ for the height of the stone.

$$h(x) = a(x-100)^2 + 700$$

$$h(0) = 500 = a(-100)^2 + 700$$

$$-200 = 10{,}000a$$

$$a = \frac{-1}{50}$$

$$h(x) = \frac{-1}{50}(x-100)^2 + 700.$$

(c) (5 points) Write a function $g(x)$ for the height of the ground. Notice that it should be a multipart function. Use this to write a new function $d(x)$ which gives the distance of the rock from the ground (this should also be a multipart function).

If $0 \le x \le 250$

$g(x) = 0$

Else:

line of slope $\frac{1}{2}$

w/ $(250, 0)$

$y = \frac{1}{2}(x - 250)$

$= \frac{1}{2}x - 125$

$$g(x) = \begin{cases} 0 & 0 \le x \le 250 \\ \frac{1}{2}x - 125 & x \ge 250 \end{cases}$$

$d(x) = f(x) - g(x)$

$$= \begin{cases} \frac{-1}{50}(x-100)^2 + 700 & 0 \le x \le 250 \\ \frac{-1}{50}(x-100)^2 + 825 - \frac{x}{2} & x \ge 250 \end{cases}$$

(d) (5 points) Does the stone land on flat ground or on the hill? What are the coordinates of the landing spot?

If on ground

$\frac{-1}{50}(x-100)^2 + 700 = 0$

$(x-100)^2 = 35000$

$x = 100 + \sqrt{35000}$

$\approx 287.1 > 250$

So

$\frac{-1}{50}(x-100)^2 + 825 - \frac{x}{2} = 0$

$\frac{-1}{50}(x^2 - 200x + 10{,}000) + 825 - \frac{x}{2} = 0$

$\frac{-1}{50}x^2 + \frac{7}{2}x + 625 = 0$

so $x = \dfrac{-7/2 \pm \sqrt{\frac{49}{4} - 4(\frac{-1}{50})(625)}}{-2/50}$

$x \approx 284.75$ ✓

So on hill

coords

$(284.75, 17.375)$

4. (a) (5 points) A particle is exhibiting uniform linear motion on the plane. After 1 second you measure it at the coordinates $(2,3)$. 7 seconds later is is at the coordinates $(7,-4)$. Write parametric equations of motion for this particle.

$t_1 = 1, (2,3)$

$t_2 = 8, (7,-4)$

$$\boxed{\begin{aligned} x(t) &= \frac{5}{7}(t-1) + 2 \\ y(t) &= -1(t-1) + 3 \end{aligned}}$$

(b) (5 points) If $f(x) = 2x^2 + 5$ and $g(x) = \sqrt{x-1}$, give formulas for $f \circ g(x)$ and $g \circ f(x)$.

$f(g(x)) = 2(x-1) + 5 = 2x - 2 + 5 = 2x + 3$

$g(f(x)) = \sqrt{2x^2 + 5 - 1} = \sqrt{2x^2 + 4}$

(c) **(BONUS 3 Points)** Are there any concerns about the domain of either of the functions you wrote down in the previous part? Explain.

we'd worry if $f(x) \le 0$ ever -1

but $f(x) - 1 = 2x^2 + 4 > 0$ $\forall x$ +2

So no worrys