

# Homework9ImplementationSolutions

December 7, 2021

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[3]: ##### Preamble:
def fastPowerSmall(g,A,N):
    a = g
    b = 1
    while A>0:
        if A % 2 == 1:
            b = b * a % N
        A = A//2
        a = a*a % N
    return b

def extendedEuclideanAlgorithm(a,b):
    u = 1
    g = a
    x = 0
    y = b
    while True:
        if y == 0:
            v = (g-a*u)/b
            return [g,u,v]
        t = g%y
        q = (g-t)/y
        s = u-q*x
        u = x
        g = y
        x = s
        y = t

def findInverse(a,p):
    inverse = extendedEuclideanAlgorithm(a,p)[1] % p
    return inverse

def textToInt(words):
    number = 0
    i = 0
    for letter in words:
        number += ord(letter)*(256**i)
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        i+=1
    return number

def intToText(number):
    words = ""
    while number>0:
        nextLetter = number % 256
        words += chr(nextLetter)
        number = (number-nextLetter)/256
    return words

def millerRabin(a,n):

    #first throw out the obvious cases
    if n%2 == 0 or extendedEuclideanAlgorithm(a,n)[0]!=1:
        return True

    #Next factor n-1 as 2^k m
    m = n-1
    k = 0
    while m%2 == 0 and m != 0:
        m = m//2
        k = k+1

    #Now do the test:
    a = fastPowerSmall(a,m,n)
    if a == 1:
        return False

    for i in range(0,k):
        if (a + 1) % n == 0:
            return False
        a = (a*a) % n

    #If we got this far a is not a witness
    return True

# This function runs the Miller-Rubin test on 20 random numbers between 2 and
↪ p-1. If it returns true there is a probability of (1/4)^20 that p is prime.
def probablyPrime(p):
    for i in range(0,20):
        a = ZZ.random_element(2,p-1)
        if millerRabin(a,p):
            return False
    return True

def findPrime(lowerBound,upperBound):
    while True:

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        candidate = ZZ.random_element(lowerBound,upperBound)
        if probablyPrime(candidate):
            return candidate

#####We will also need Legendre Symbols
def legendreSymbol(a,p):
    #Make sure the base is even
    if p == 2:
        print("The base of a Legendre symbol must be odd!")
        return

    #Then use Euler's criterion
    else:
        a = a % p
        m = (p-1)//2
        return fastPowerSmall(a,m,p)

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[4]: ##### Problem 1

#####Part (a)

#I'm initializing p and q to -1 here so that we have the option of generating a
↳ GM key with chosen primes Notice if you choose primes the choice of b
↳ doesn't matter
def generateGMKey(b, p=-1, q=-1):
    if p==-1:
        p = findPrime(2**(b-1),2**b)
        q = findPrime(2**(b-1),2**b)
    N = p*q

    #Find an a which is not a square is
    while True:
        a = ZZ.random_element(2,N)
        if (legendreSymbol(a,p) == p-1) and (legendreSymbol(a,q)==q-1):
            break

    publicKey = [N,a]
    privateKey = p
    return [publicKey,privateKey]

#####Part (b)
def GMEncrypt(publicKey,m):
    N = publicKey[0]
    a = publicKey[1]

    r = ZZ.random_element(isqrt(N)+1,N)
    if m:

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        return (a*r*r) % N
    else:
        return (r*r) % N

def GMDecrypt(privateKey,c):
    #The private key is just a prime p.
    if legendreSymbol(c,privateKey) == 1:
        return 0
    else:
        return 1

##### Part (c)
keys = generateGMKey(16)
m0 = 0
m1 = 1
c0 = GMEncrypt(keys[0],m0)
c1 = GMEncrypt(keys[0],m1)
print("ciphers are",c0,"and",c1)
mm0 = GMDecrypt(keys[1],c0)
mm1 = GMDecrypt(keys[1],c1)

print("0 decrpyted to",mm0,"and 1 decrypted to",mm1)

##### Part(d)
keys = generateGMKey(0,151,233)
cipher = 33482
message = GMDecrypt(keys[1],cipher)
print("The message in part (d) decrypted to",message)

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ciphers are 1686539232 and 31043263  
 0 decrpyted to 0 and 1 decrypted to 1  
 The message in part (d) decrypted to 1

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##### Problem 2
#####Part (a)
def generateRSAKey(b):

    #Generate some primes
    p = findPrime(2^(b-1),2^b)
    q = findPrime(2^(b-1),2^b)
    N = p*q
    M = (p-1)*(q-1)
    #next lets find an encryption exponenet
    while True:
        e = ZZ.random_element(2,M-1)
        gcd = extendedEuclideanAlgorithm(e,M)

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        if gcd[0]==1:
            d = gcd[1] % M
            break
    publicKey = [N,e]
    privateKey = [N,d]
    return [publicKey,privateKey]

#####Part (b)

def RSASign(signingKey,document):
    N = signingKey[0]
    d = signingKey[1]
    signedDocument = fastPowerSmall(document,d,N)
    return signedDocument

def RSAVerify(verificationKey,document,signedDocument):
    N = verificationKey[0]
    e = verificationKey[1]
    if fastPowerSmall(signedDocument,e,N) == document:
        return True
    else:
        return False

#####Part (c)
print("Part (c)")

keys = generateRSAKey(16)
print("Here are the generated RSA Keys:",keys)
document = 314159
signedDocument = RSASign(keys[1],document)
v1 = RSAVerify(keys[0],document,document)
v2 = RSAVerify(keys[0],document,signedDocument)

print("Verifying the unsigned document:",v1)
print("Verifying the signed document:",v2)

#####Part (d)
print("Part (d)")
#RSA Verification Key
N = 1052213111141408392014271339505076961744243365769525514653731189643431114436152105697372008
e = 2101683628702998674772375980077410183528127585487318016624554305852588739519396307766860254

verificationKey = [N,e]
#Recieved Documents

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D = 44591585690519734445193105605299933531568892342090748601970008137
D1 = 
→ 3373771309292600271199796593868678142398519834098542398067874066854083792016571430875959360

#Let's read them:
DText = intToText(D)
D1Text = intToText(D1)
print("The document D said:",DText)
print("The document D' said:",D1Text)

#Here they are (potentially) signed:
DSig = 
→ 8426665688163375964593143441458964637347414029969631856884655887135468399266491115775348447
D1Sig = 
→ 9059280931350999147776789854356125281873028523394670827691831663199736194473764728663367105

#Let's verify the signatures:
V = RSAVerify(verificationKey,D,DSig)
V1 = RSAVerify(verificationKey,D1,D1Sig)

print("The signature for D is:",V)
print("The signature for D' is:",V1)

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Part (c)

Here are the generated RSA Keys: `[[3478047059, 1546029863], [3478047059, 2945978327]]`

Verifying the unsigned document: False

Verifying the signed document: True

Part (d)

The document D said: I am your Professor Gabriel

The document D' said: Ignore the other message, I am the real professor!

The signature for D is: False

The signature for D' is: True

[6]: `##### Problem 4`

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#####Part (a)
def generateElgamalKey(p,g):
    a = ZZ.random_element(2,p-1)
    #a = 15140
    A = fastPowerSmall(g,a,p)
    signingKey = [a,p,g]
    verificationKey = [A,p,g]
    return [signingKey,verificationKey]

#####Part (b)
def elgamalSign(signingKey,document):

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a = signingKey[0]
p = signingKey[1]
g = signingKey[2]

#k = 10727
while True:
    k = ZZ.random_element(1,p-1)
    if extendedEuclideanAlgorithm(k,p-1)[0]==1:
        break
    kinv = findInverse(k,p-1)
    S1 = fastPowerSmall(g,k,p)
    S2 = ((document - a*S1)*kinv) % (p-1)
    signedDocument = [S1,S2]
    return signedDocument

def elgamalVerify(verificationKey,document,signedDocument):
    A = verificationKey[0]
    p = verificationKey[1]
    g = verificationKey[2]

    S1 = signedDocument[0]
    S2 = signedDocument[1]

    V1 = (fastPowerSmall(A,S1,p)*fastPowerSmall(S1,S2,p)) % p
    V2 = fastPowerSmall(g,document,p)
    print(V1,V2)
    if V1==V2:
        return True
    else:
        return False

#####Part (c)

#The following helper function is crucial. It takes as input a,c,N, and
→returns the set of solutions to  $ax = c \pmod N$ . This could have been written
→back in HW2.
def solveLinearCongruence(a,c,N):
    G,u,v = extendedEuclideanAlgorithm(a,N)
    #First check if there are any solutions using HW2#7
    if c%G!=0:
        print("No solutions to",a,"x =",c,"mod",N)
        return -1

    #Otherwise we find one solution a0
    l = c//G
    a0 = (u*l) % N

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#Now we can iterate through all of them.
solutionsList = [a0]
for i in range(0,G-1):
    a0 = (a0 + N//G) % N
    solutionsList.append(a0)

#now we have our list!
return solutionsList

def stealElgamalSignature(verificationKey,D,Dsig,D1,D1sig):
    #Let's extract all the info we need
    A,p,g = verificationKey
    S1,S2 = Dsig
    S1a,S2a = D1sig

    #Make sure the mistake was made:
    if S1 != S1a:
        print("The same signing exponent wasn't used")
        return -1

    #Then let's find candidates for k inverse
    d = (D - D1) % (p-1)
    s = (S2 - S2a) % (p-1)
    kList = solveLinearCongruence(d,s,p-1)

    #One of these is k inverse
    k = 0
    for kinv in kList:
        if fastPowerSmall(S1,kinv,p)==g:
            k = findInverse(kinv,p-1)
            break

    #Now we've found k, we use a similar philosophy to find a. We will need to
    →set up an equations
    d1 = S1 % (p-1)
    s1 = (D - k*S2) % (p-1)
    aList = solveLinearCongruence(d1,s1,p-1)

    #One of these is a
    for a in aList:
        if fastPowerSmall(g,a,p) == A:
            return a

#####Part (d)
print("Part (d)")
p = 3700273081

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g = 7
keys = generateElgamalKey(p,g)
D = 314159
Dsig = elgamalSign(keys[0],D)
print("Signed document",D,"to recieve",Dsig)
D1 = elgamalVerify(keys[1],D,Dsig)
print("Verification =",D1)

#####Part (e)
print("Part (e)")
verificationKey = [185149, 348149, 113459]
D = 153405
Dsig = [208913,209176]
D1 = 127561
D1sig = [208913,217800]

print("Attempting to extract signing key...")
print("Returns: ",stealElgamalSignature(verificationKey,D,Dsig,D1,D1sig))

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Part (d)  
Signed document 314159 to recieve [2327006849, 1673301500]  
912253726 912253726  
Verification = True  
Part (e)  
Attempting to extract signing key...  
Returns: 72729

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[7]: ##### Problem 5
def generateDSAPKey(p,q,g):
    a = ZZ.random_element(2,q-1)
    A = fastPowerSmall(g,a,p)
    signingKey = [a,p,q,g]
    verificationKey = [A,p,q,g]
    return [signingKey,verificationKey]

def DSASign(signingKey,document):
    a = signingKey[0]
    p = signingKey[1]
    q = signingKey[2]
    g = signingKey[3]
    k = ZZ.random_element(1,q)
    kinv = findInverse(k,q)
    S1 = fastPowerSmall(g,k,p) % q
    S2 = ((document + a*S1)*kinv) % q
    signedDocument = [S1,S2]
    return signedDocument

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def DSAVerify(verificationKey,document,signedDocument):
    A = verificationKey[0]
    p = verificationKey[1]
    q = verificationKey[2]
    g = verificationKey[3]
    S1 = signedDocument[0]
    S2 = signedDocument[1]
    S2inv = findInverse(S2,q)
    V1 = (document*S2inv) % q
    V2 = (S1*S2inv) % q
    W = (fastPowerSmall(g,V1,p)*fastPowerSmall(A,V2,p) % p) % q
    if W == S1:
        return True
    else:
        return False

#####Part (c)
print("Part (c)")
p = 48731
q = 443
gPrimitive = 7
g = fastPowerSmall(g,(p-1)//q,p)
print("By raising the primitive root to the",p-1,"/",q,"we get an_\u2192element",g,"of order",q)

print("Part (d)")
D = 314
keys = generateDSAKey(p,q,g)
Dsig = DSASign(keys[0],D)
print("Signed document is",Dsig)
Verified = DSAVerify(keys[1],D,Dsig)
print("Verifying DSig:",Verified)

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Part (c)

By raising the primitive root to the 48730 / 443 we get an element 5260 of order 443

Part (d)

Signed document is [83, 178]

Verifying DSig: True

[0]: