ChemE Visitor Scheduling 2

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1 Introduction

This document describes a mathematical model for solving the ChemE Visitor Scheduling problem.

My understanding of the problem is as follows:

- Each visitor submits a rank-ordered list of the faculty they'd like to meet.
- Each faculty member submits their availability.
- Visitors are assigned to meet with faculty during specific periods.
- We want to maximize the overall happiness of the visitors.
- We want to maximize the minimum level of happiness.

2 Definitions

2.1 Sets

- Let $V = \{1, ..., n_V\}$ denote the set of visitors.
- Let $F = \{1, ..., n_F\}$ denote the set of faculty.
- Let $T = \{1, ..., n_T\}$ denote the set of time periods.

2.2 Problem Data

- Let $A \in \{0,1\}^{n_F \times n_T}$ be a matrix describing the availability of the faculty. In particular, if faculty f is available for a meeting during period t, then $A(f,t) = 1$. Otherwise, $A(f,t) = 0$.
- Let $P \in \{0, \ldots, 10\}^{n_V \times n_F}$ be a matrix describing the visitors' preferences regarding who they'd like to meet. In particular, if visitor v really wants to meet faculty f (that is, if faculty f is their first choice), then $P(v,f) = 10$. If faculty f is their second choice, $P(v,f) = 9$, and so on, down to their tenth choice, for which $P(v,f) = 1$. For a faculty member f not appearing on visitor v's preference list, $P(v,f) = 0$.

2.3 Decision Variables

- Let $x \in \{0,1\}^{n_V \times n_F \times n_T}$ be a matrix describing decisions to schedule meetings between visitors and faculty. In particular, if visitor v is assigned to meet with faculty f during period t, then $x(v,f,t) = 1$. Otherwise, $x(v,f,t) = 0$.

- Let $y \in \{0,1\}^{n_V \times n_T}$ be a matrix describing decisions to schedule the visitors for free time. In particular, if visitor v is scheduled for free time during period t, then $y(v,t) = 1$. Otherwise, $y(v,t) = 0$.

- Let $h \in \mathbb{R}$ be the minimum level of happiness. This definition will be enforced via a combination of the objective function and constraints.

3 Constraints

1. Each visitor must be assigned to either a faculty meeting or free time for each period:

$$\sum_{f \in F} x(v,f,t) + y(v,t) = 1, \quad \forall v \in V, \forall t \in T. \tag{1}$$

2. Each faculty can meet with at most one student during a given period:

$$\sum_{v \in V} x(v,f,t) \leq 1, \quad \forall f \in F, \forall t \in T. \tag{2}$$

3. Visitors can only meet with faculty when the faculty are available:

$$\sum_{v \in V} x(v,f,t) \leq A(f,t), \quad \forall f \in F, \forall t \in T. \tag{3}$$

4. Each visitor-faculty pair can only meet at most once

$$\sum_{t \in T} x(v,f,t) \leq 1, \quad \forall v \in V, \forall f \in F. \tag{4}$$

5. (Optional) Each visitor must have at least n^L periods of free time:

$$\sum_{t \in T} y(v,t) \geq n^L, \quad \forall v \in V. \tag{5}$$

6. (Optional) Each visitor can have at most n^U periods of free time:

$$\sum_{t \in T} y(v,t) \leq n^U, \quad \forall v \in V. \tag{6}$$

7. Each visitor's happiness level must be at least h:

$$\sum_{t \in T} \sum_{f \in F} x(v,f,t) P(v,f) \geq h, \quad \forall v \in V. \tag{7}$$

4 Objective

The objective function (to be maximized) will have three terms:

$$c_1 \cdot x + c_2 \cdot x + c_3 h \tag{8}$$

The rationale for each of these terms is described below.

- Give the visitors their preferences:

$$c_1(v, f, t) = P(v, f), \quad \forall v \in V, \forall f \in F, \forall t \in T, \tag{9}$$

- It's better to have a meeting than no meeting at all:

$$c_2(v, f, t) = w_{\text{meet}}, \quad \forall v \in V, \forall f \in F, \forall t \in T, \tag{10}$$

  The weighting factor $w_{\text{meet}} \geq 0$ determines how important this objective is. I recommend that its value be kept small, for example, $w_{\text{meet}} = 1$.

- Maximize the minimum level of happiness:

$$c_3 = w_{\min}. \tag{11}$$

  The weighting factor $w_{\min} \geq 0$ determines how important this objective is. I recommend that its value be kept small, for example, $w_{\min} = 1$.

5 The Full Problem

The full optimization problem is:

$$\begin{aligned}
\max_{x,y} \quad & c_1 \cdot x + c_2 \cdot x + c_3 h \\
\text{s.t} \quad & x \in \{0,1\}^{n_V \times n_F \times n_T}, \\
& y \in \{0,1\}^{n_V \times n_T}, \\
& \sum_{f \in F} x(v, f, t) + y(v, t) = 1, \quad \forall v \in V, \forall t \in T, \\
& \sum_{v \in V} x(v, f, t) \leq 1, \quad \forall f \in F, \forall t \in T, \\
& \sum_{v \in V} x(v, f, t) \leq A_V(f, t), \quad \forall f \in F, \forall t \in T, \\
& \sum_{t \in T} x(v, f, t) \leq 1, \quad \forall v \in V, \forall f \in F, \\
& \sum_{t \in T} y(v, t) \geq n^L, \quad \forall v \in V, \\
& \sum_{t \in T} y(v, t) \leq n^U, \quad \forall v \in V, \\
& \sum_{t \in T} \sum_{f \in F} x(v, f, t) P(v, f) \geq h, \quad \forall v \in V.
\end{aligned} \tag{12}$$