EE 5239 Optimization Homework 2 Cover Sheet

Instructor name: Mingyi Hong   Student name: ____________________

- Date assigned: Thursday 9/19/2019
- Date due: Sunday 9/29/2019 11:59pm
- This cover sheet must be signed and submitted along with the homework answers on additional sheets.
- By submitting this homework with my name affixed above,
  - I understand that late homework will not be accepted,
  - I acknowledge that I am aware of the University's policy concerning academic misconduct (appended below),
  - I attest that the work I am submitting for this homework assignment is solely my own, and
  - I understand that suspiciously similar homework submitted by multiple individuals will be reported to the Dean of Students Office for investigation.
- Academic Misconduct in any form is in violation of the University's Student Disciplinary Regulations and will not be tolerated. This includes, but is not limited to: copying or sharing answers on tests or assignments, plagiarism, having someone else do your academic work or working with someone on homework when not permitted to do so by the instructor. Depending on the act, a student could receive an F grade on the test/assignment, F grade for the course, and could be suspended or expelled from the University.

1 Reading

- Reading: Textbook Section 1.1 - 1.2
- Appendix A.

2 Problems

1. Exercise 1.2.1, 1.2.2, 1.2.3, 1.2.7 in the textbook (version 2)
2. Suppose that $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is convex. Please show the following properties
   (a) $f(\mathbf{x}) - f(\mathbf{y}) \geq \langle \nabla f(\mathbf{y}), \mathbf{x} - \mathbf{y} \rangle, \ \forall \ \mathbf{x}, \mathbf{y} \in \mathbb{R}^n$
   (b) If $g(y) : \mathbb{R} \to \mathbb{R}$ is a monotonically non-decreasing **convex** function, then $g(f(\mathbf{x}))$ is also a convex function.
   (c) If $g(y) : \mathbb{R} \to \mathbb{R}$ is a decreasing **convex** function, then provide an example showing that $g(f(\mathbf{x}))$ may not be a convex function.
3. Let $\mathbf{A} \neq 0$ be a rank one square matrix, i.e., $\mathbf{A} = \mathbf{x}_0 \mathbf{y}_0^T \neq 0$ with $\mathbf{x}_0, \mathbf{y}_0 \in \mathbb{R}^n$. Consider the following optimization problem

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y}} \quad & \frac{1}{2} \|\mathbf{A} - \mathbf{x}\mathbf{y}^T\|_F^2 \\ \text{s.t.} \quad & \mathbf{x}, \mathbf{y} \in \mathbb{R}^n \end{aligned} \tag{1}$$

   Please answer the following questions.
   - Show that it is **not** a convex optimization problem.
   - Find the set of stationary points.
   - **(bonus)** Show that every local optimum of the problem is a global optimum

3 Programming Assignment

In this assignment, you are asked to experiment with

1. Steepest descent method with Armijo step size rule
2. Diagonally scaled gradient method (using the Hessian diagonals), constant step size rule
3. Conjugate gradient method, exact minimization step size rule

to solve the following convex quadratic minimization problem:

$$\begin{aligned} \min \quad & \frac{1}{2} x^T Q x + b^T x \\ \text{s.t.} \quad & x \in \mathbb{R}^n \end{aligned} \tag{2}$$

Always use the starting point $x = (1; 1; \cdots, 1)^T$. Please see below for a Matlab script that you should use to generate your data sets.

What you need to turn in:

```
% problem dimension (you can change this)
n=50;

% condition number (you can change this)
c=1000;

% Generating data matrix Q
A=randn(n);
[V,D]=svd(A);
alpha=(c*D(n,n)/D(1,1))^(1/(n-1));
for i = 1:n; a(i) = alpha^(n-i); end
D=D*diag(a);
Q=V'*D*V;

% Generating vector b
b=randn(n,1);
```

1. Well documented MATLAB codes (or any other codes of a scientific computing language).
2. Plots of CPU time and progression of objective values (relative to the minimum) for various choices of step size rules, algorithms, problem sizes ($n = 100, 500$), and condition numbers ($c = 10, 5000$).
3. Brief discussions of your findings relative to the convergence theory.