# On Building An Interpolation System Using Decision Trees

Amy Briggs abbr5@mst.edu

Andrew Fallgren ajffk6@mst.edu

George Rush gdr34b@mst.edu

## **ABSTRACT**

One class of data is measured or simulated data with error estimation. This data can consist of many continuous dimensions for which values are available only at discrete points. Increasing the number of discrete points at which the data is available can be expensive or even impossible to obtain, but it can still be useful for predicting data trends. Unfortunately, this is difficult when the various dimensions do not follow the same type of fit (linear, logarithmic, polynomial, etc.). Our approach focuses on building decision trees and using them to interpolate new data points that follow existing trends. This is in contrast to previous methods which focused on extrapolating data for specific applications or using purely numerical regression models. By using this approach, sparse data sets or those that exhibit unusual patterns can be analyzed effectively.

## **Categories and Subject Descriptors**

H.2.8 [Database Management]: Database Applications—data mining

#### **General Terms**

Algorithms

#### **Keywords**

data mining, sparse data, interpolation

# 1. INTRODUCTION

There are times when the only class of data available for analysis is measured or simulated data. Collecting such data can be time and labor intensive, and the data points may be sparse at best. As such, it can be worthwhile to use techniques that increase the utility of data points which have already been collected. Interpolation in particular can help to clarify trends within the data by generating new points according to the patterns established by existing data.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

Copyright 20XX ACM X-XXXXX-XX-X/XX/XX ...\$15.00.

Prior methods of interpolation all work by utilizing different mathematical functions to fit patterns of values. This process can be ineffective if different dimensions or classifications of the data do not fit to a single formula, such as linear, logarithmic, polynomial, et cetera, as then certain formulas would only perform accurately for specific segments of the data.

Our proposed solution first requires modelling the data using decision trees. By doing this, it is possible to group data points by their classification and similar characteristics at various leaf nodes. We then interpolate new data instances based upon the distribution of instances within each of the leaf nodes. This produces interpolated data with a similar distribution and attribute values when compared to the original data set. Also, the new instances retain roughly the same proportion as the original data regarding the number of instances per leaf node.

By using this method against multiple well known test data sets, we demonstrate a proof of concept for its applicability across multiple problem domains.

#### 2. RELATED WORK

Early works on the interpolation of scattered data evaluate a variety of different computational methods that focus on obtaining a smooth function F(x,y) to follow the data set. They utilize numerous different mathematical methods including the following: inverse distance weighted method, rectangle based blending method, triangle based blending method, finite element based method, Foley's method, global basis function type method, and modified Maude method [3]. All of these techniques focus only on developing a function in order to interpolate scattered data sets.

Later works build off of that approach, by taking classical radial basis functions, such as Duchon's thin plate splines and Hardy's multiquadratics, and compressing them in order to shorten the excessive computation times that result from applying these functions to large data sets, while trying to maintain a smooth data fitting [2].

There are reapplications of some of these interpolation functions to generate continuous surfaces from irregularly distributed data, in attempts to analyze which function best for spatial analysis. The methods include: inverse square distance method, Kriging method, tension finite difference method, and Hardy's multiquadric method [1].

Several cases can be found in which these interpolation functions are modified to more accurately apply to specific data sets. One example is the use of a combination of the thin plate smoothing spline and Kringing method in spatial interpolation in order to create a more comprehensive archive of Australian climate data [4]. Another uses spatial interpolation in improving the MODIS global data sets for terrestrial gross and net primary production [5].

Although more applications of function-based interpolation can be found, we could not locate any use of data mining classification models for interpolation purposes.

# 3. METHODOLOGY

## 3.1 Decision Tree Interpolation

Decision Tree Interpolation follows this process:

- 1. Build a decision tree from the original data.
- 2. For each leaf node in the tree:
  - (a) Obtain all attribute values for associated data instances.
  - (b) Define ranges for attribute values.
    - For numeric attributes, define the range using minimum and maximum values.
    - For discrete attributes, define the range as all distinct values.
  - (c) Calculate the distribution of all associated data instances
  - (d) Create new data points within the ranges that match the statistical distribution.

Note that the number of data points created per leaf node is proportional to the number of data points already classified by that leaf node. This ensures that any interpolated data will follow the overall data distribution, at least relative to the data density per leaf node.

# 3.2 Interpolated Data Validation

All interpolated data is validated through this process:

- 1. A new decision tree is built based on the interpolated data. Note that the original data is *not* included here.
- 2. Both the new and original decision trees are compared for accuracy against the new and original data sets.

Note that any decision tree with an arbitrarily large maximum depth can classify data with perfect accuracy. Defining a low maximum depth means that classification is imperfect, and it is under these conditions that differences in the quality of different decision trees become apparent.

#### 3.3 Experiment Parameters

We completed 30 experiments based on two variables: data source and maximum depth of the decision tree. The maximum depth ranged from one to five, and there were six data sources pulled from Orange's documentation data sets.

#### 4. RESULTS

Result data is shown in Table 1. The first two columns list the data source and the maximum depth for generated decision trees. Note that two decision trees are generated per row, one for the original data and one for the new (interpolated) data. The last four columns list the accuracy of both the original and new decision trees against the original

and new data sets. For example, the column labeled "OT -> ND" shows the accuracy of the original decision tree (OT) when used to classify instances in the new data set (ND). Results and outliers for each of the data sets will be explored further here.

## 4.1 adult\_sample

The only significant outlier in this group was for a max tree depth of two. The "NT -> OD" column shows an accuracy of 0.2497 while the other accuracy values ranged from 0.8085 to 0.8135. Ignoring the outlier, accuracy ranged from 0.7807 to 0.8522 for this data source.

#### 4.2 car

Results in this section largely followed expected trends. Accuracy ranged from 0.7002 to 0.9823 for this data source, and accuracy increased in each column as the max tree depth increased.

# **4.3** iris

Results in this section followed expectations, but classification started out relatively inaccurate. At a max tree depth of 1, accuracy ranged from 0.6667 to 0.6933. At a max tree depth of 2, accuracy increased rapidly to range from 0.9467 to 0.9867. Accuracy remained above 0.9467 for all other entries.

# 4.4 lung-cancer

Accuracy had enormous variance here, ranging from 0.375 to 1.0 across all categories. While both decision trees increased in accuracy as max depth increased for their own data sets, they did not consistently increase in accuracy for the other data set.

# 4.5 tic\_tac\_toe

Results in this section were mostly consistent, and accuracy ranged from 0.6827 to 0.9338. Accuracy tended to increase as max tree depth increased, with only slight exceptions.

# 4.6 voting

While there was no clear trend in the results for this data source, accuracy was consistently greater than or equal to 0.8661 for all entries. All but three values were above 0.9, so accuracy was generally high across all instances.

#### 5. DISCUSSION

In general, points generated by our algorithm were classified with similar accuracy to the original data. In cases where the tree generated from the original data classified that data poorly, the newly generated data had the same behavior. In cases where the original data was classified with high reliability, the same was true for the interpolated points. This is a positive outcome as the goal is to generate new instances of data that behave similarly to the old instances, as opposed to generating instances which would be predicted with any particular accuracy by the decision tree.

There are two main cases for which the generated data does not produce a tree which provides similar classification to the original tree: the adult\_sample data set with a maximum depth of two and the lung-cancer data set in general.

In the original set of experiments, 0.5 data points were generated for each original data point assigned to a leaf.

Table 1: Experiment Result Summary

Data Set	Max Tree Depth	OT -> OD	OT -> ND	NT -> OD	NT -> ND
adult_sample	1	0.805527123849	0.780737704918	0.804503582395	0.782786885246
adult_sample	2	0.808597748209	0.809426229508	0.249744114637	0.813524590164
adult_sample	3	0.816786079836	0.850102669405	0.801432958035	0.852156057495
adult_sample	4	0.822927328557	0.794661190965	0.787103377687	0.784394250513
$adult\_sample$	5	0.822927328557	0.84052532833	0.792221084954	0.84052532833
car	1	0.700231481481	0.710648148148	0.700231481481	0.710648148148
car	2	0.77777777778	0.783564814815	0.774305555556	0.789351851852
car	3	0.824074074074	0.809027777778	0.824074074074	0.815972222222
car	4	0.894097222222	0.903935185185	0.889467592593	0.915509259259
car	5	0.96412037037	0.966981132075	0.938078703704	0.982311320755
iris	1	0.666666666667	0.6933333333333	0.666666666667	0.6933333333333
iris	2	0.96	0.9733333333333	0.946666666667	0.986666666667
iris	3	0.9733333333333	0.959459459459	0.9533333333333	0.972972972973
iris	4	0.98	0.959459459459	0.946666666667	1.0
iris	5	1.0	1.0	0.966666666667	1.0
lung-cancer	1	0.59375	0.6	0.375	0.666666666667
lung-cancer	2	0.625	0.571428571429	0.4375	0.714285714286
lung-cancer	3	0.625	0.642857142857	0.5625	1.0
lung-cancer	4	0.6875	0.428571428571	0.53125	1.0
lung-cancer	5	0.78125	0.538461538462	0.53125	1.0
tic_tac_toe	1	0.699373695198	0.68267223382	0.699373695198	0.68267223382
tic_tac_toe	2	0.705636743215	0.690376569038	0.703549060543	0.696652719665
tic_tac_toe	3	0.769311064718	0.779874213836	0.755741127349	0.758909853249
tic_tac_toe	4	0.831941544885	0.82264957265	0.745302713987	0.856837606838
tic_tac_toe	5	0.918580375783	0.907284768212	0.83611691023	0.933774834437
voting	1	0.95632183908	0.923766816143	0.95632183908	0.923766816143
voting	2	0.95632183908	0.956896551724	0.95632183908	0.956896551724
voting	3	0.963218390805	0.913357400722	0.95632183908	0.927797833935
voting	4	0.963218390805	0.892156862745	0.928735632184	0.90522875817
voting	5	0.972413793103	0.866071428571	0.937931034483	0.8958333333333

In exploring possible reasons for the discrepancies in how closely the generated data matched the original, this parameter was increased. As it was increased, the old data was classified by the new decision tree with similar accuracy to that of the new data and to the accuracy of the original tree. This suggests that false trends were being created by statistical noise when not enough new points were generated. In the case of the adult\_sample data set with depth two, it is purely an artifact of using random selection and a particular combination of random seed and data. For the lung cancer data, this type of statistical noise can be expected to be a larger problem as the data set has a large number of fields, but only a few possible values for each of those fields.

#### 6. CONCLUSION AND FUTURE WORK

This initial investigation showed that it is possible to use our method to generate new data points that behave similarly to the original data regarding their classification. As the selection of values is done in a relatively simple way, a large number of new data points can be generated quickly. This generated data can then be used as a substitute for the original data without many of the privacy concerns or other issues that might restrict the use of such data. This approach worked on multiple types of data, both continuous and discrete values, and on varying sized data sets.

Future work in this area would be to more fully explore the optimum selection of parameters used in the algorithm. In the case of this paper, continuous data was binned into 20 discrete bins, and values were chosen from within a bin with equal probability. This level of discretization worked for the data sets examined, but there is no reason to expect this would always be the case. Another parameter that was shown to have impact is the number of new data points to generate. A larger set of generated points seems to produce better solutions, but this correlation has not been adequately explored to make definitive statements.

For any decision node in our algorithm, there is also no consideration of the possible correlation between fields. We implicitly assume that said correlation is unnecessary for our method of interpolation. This assumption works well if the original decision tree has already captured these correlations in resulting model, but in the cases where that is not true, expansion of the algorithm to include correlation between fields may increase the usefulness of the generated points.

#### 7. REFERENCES

- C. Caruso and F. Quarta. Interpolation methods comparison. Computers & Mathematics with Applications, 35(12):109–126, 1998.
- [2] M. S. Floater and A. Iske. Multistep scattered data interpolation using compactly supported radial basis functions. *Journal of Computational and Applied Mathematics*, 73(1):65–78, 1996.
- [3] R. Franke. Scattered data interpolation: Tests of some methods. *Mathematics of computation*, 38(157):181–200, 1982.

- [4] S. J. Jeffrey, J. O. Carter, K. B. Moodie, and A. R. Beswick. Using spatial interpolation to construct a comprehensive archive of australian climate data. Environmental Modelling & Software, 16(4):309–330, 2001.
- [5] M. Zhao, F. A. Heinsch, R. R. Nemani, and S. W. Running. Improvements of the modis terrestrial gross and net primary production global data set. *Remote* sensing of Environment, 95(2):164–176, 2005.

#### **APPENDIX**

# A. EXAMPLE DECISION TREES

In the following pages, we display some of the decision trees from our experiments. Each tree is labeled with the corresponding data set name, whether it was generated from the original or interpolated data, and the maximum depth. For each data set, we chose to display decision trees at the highest maximum depth that was viewable in this format. This is because some decision trees at depth five were impossible to examine when resized to fit on one page. This section is meant to demonstrate how the complexity of decision trees can vary depending on the data set, but it also provides a visual contrast of the decision tree structure for original versus interpolated data.

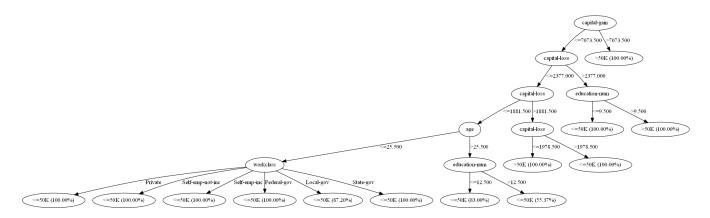


Figure 1: adult\_sample - original - max depth 5

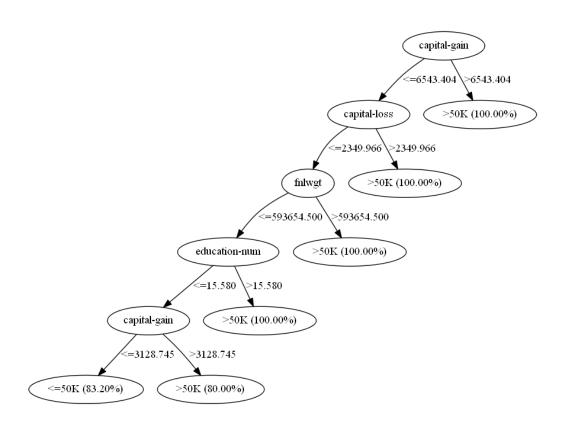


Figure 2: adult\_sample - interpolated - max depth 5

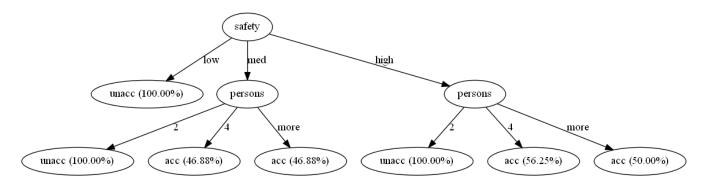


Figure 3: car - original - max depth 2

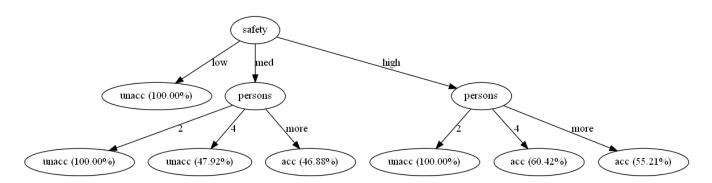


Figure 4: car - interpolated - max depth 2

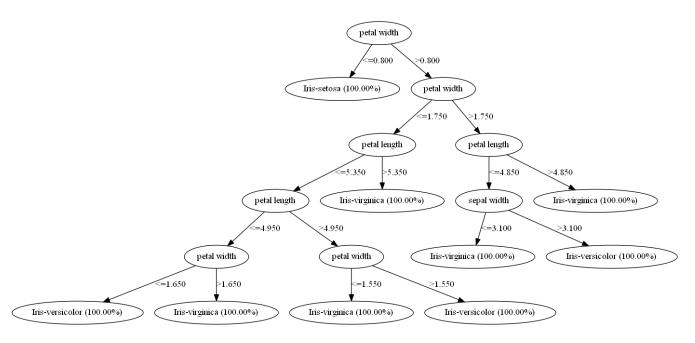


Figure 5: iris - original -  $\max$  depth 5

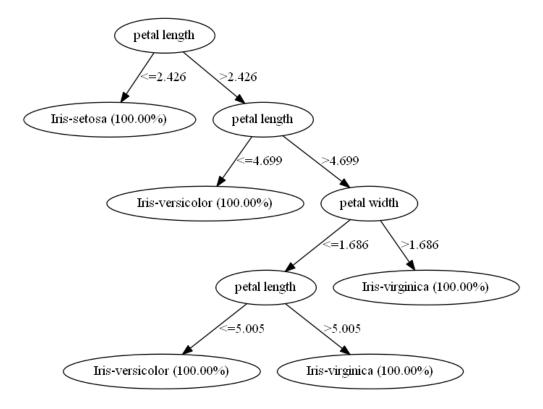


Figure 6: iris - interpolated -  $\max$  depth 5

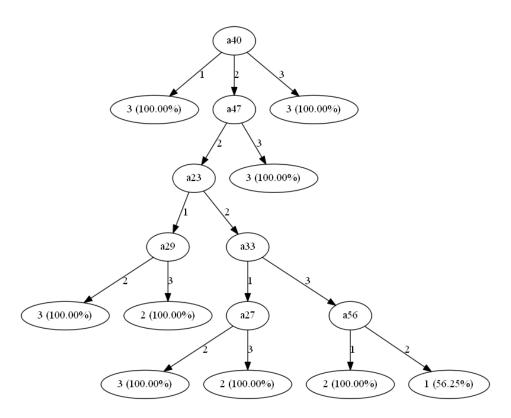


Figure 7: lung-cancer - original -  $\max$  depth 5

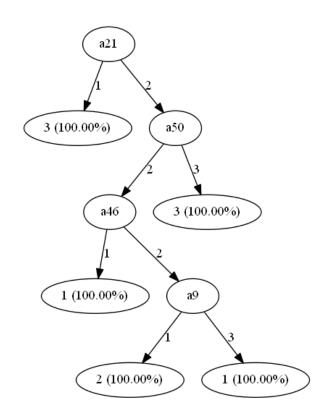


Figure 8: lung-cancer - interpolated - max depth 5

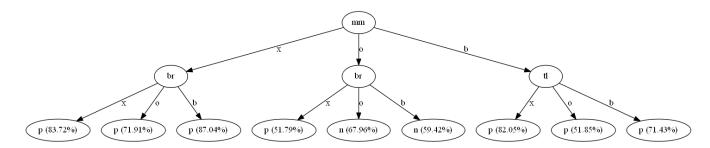


Figure 9: tic\_tac\_toe - original - max depth 2  $\,$ 

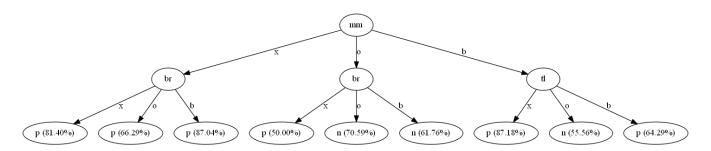


Figure 10: tic\_tac\_toe - interpolated - max depth 2  $\,$ 

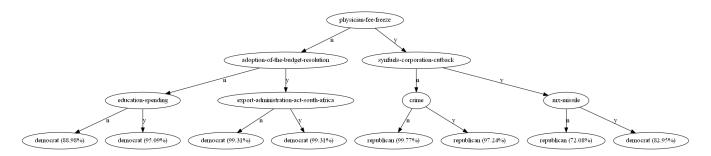


Figure 11: voting - original -  $\max$  depth 3

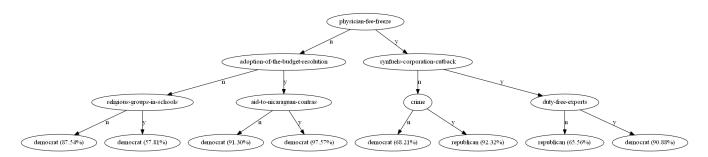


Figure 12: voting - interpolated -  $\max$  depth 3