Assume $p^f << K_s$

$$\frac{D\Phi^f}{Dt} = \frac{D\alpha}{Dt}\left(\log J + \frac{p^f}{K_s}\right) + \frac{\alpha}{J}\frac{DJ}{Dt} + \frac{\alpha - \Phi^f}{K_s}\frac{Dp^f}{Dt}$$

$$\rho_f\left(\frac{D\alpha}{Dt}\left(\log J + \frac{p^f}{K_s}\right) + \frac{\alpha}{J}\frac{DJ}{Dt} + \frac{1}{M}\frac{Dp^f}{Dt}\right) = -\nabla_X \cdot \vec{W}$$

where

$$M = \frac{K_s K_f}{K_f(\alpha - \Phi^f) + K_s \Phi^f}$$ is Biot's Modulus

Use Darcy's law

$$\vec{v}^f - \vec{v}^s = \frac{1}{\cancel{\phi^f}\rho_f}\vec{w} = \frac{\bar{\bar{k}}}{\mu}\cdot\left[-\nabla_x \cdot p^f + \phi^f \rho_f \vec{g}\right]$$ ← Eulerian

$$\frac{1}{\cancel{\Phi^f}\rho_f}\vec{W} = \frac{\bar{\bar{K}}}{\mu}\cdot\left[-\nabla_X \cdot p^f + \Phi^f \rho_f F^T \vec{g}\right]$$ ← Lagrangian

$$\bar{\bar{K}} = J\, F^{-1} \cdot \bar{\bar{k}} \cdot F^{-T}$$ $\bar{\bar{k}}$ → permeability tensor