Level Set Topology Optimization for Problems in Solid & Fluid Mechanics

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Engineering & Applied Science

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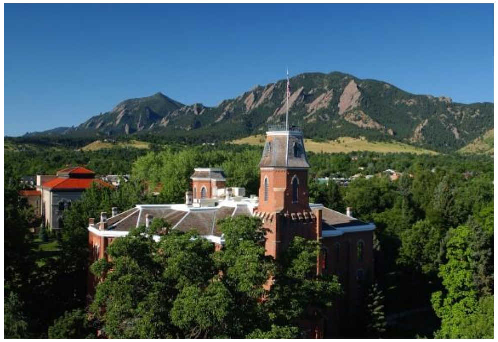

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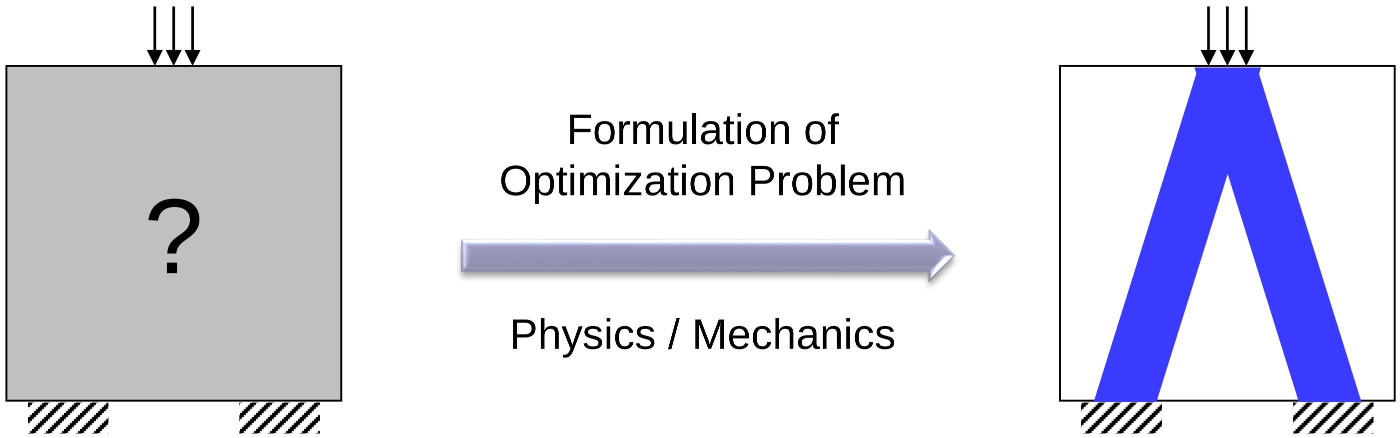
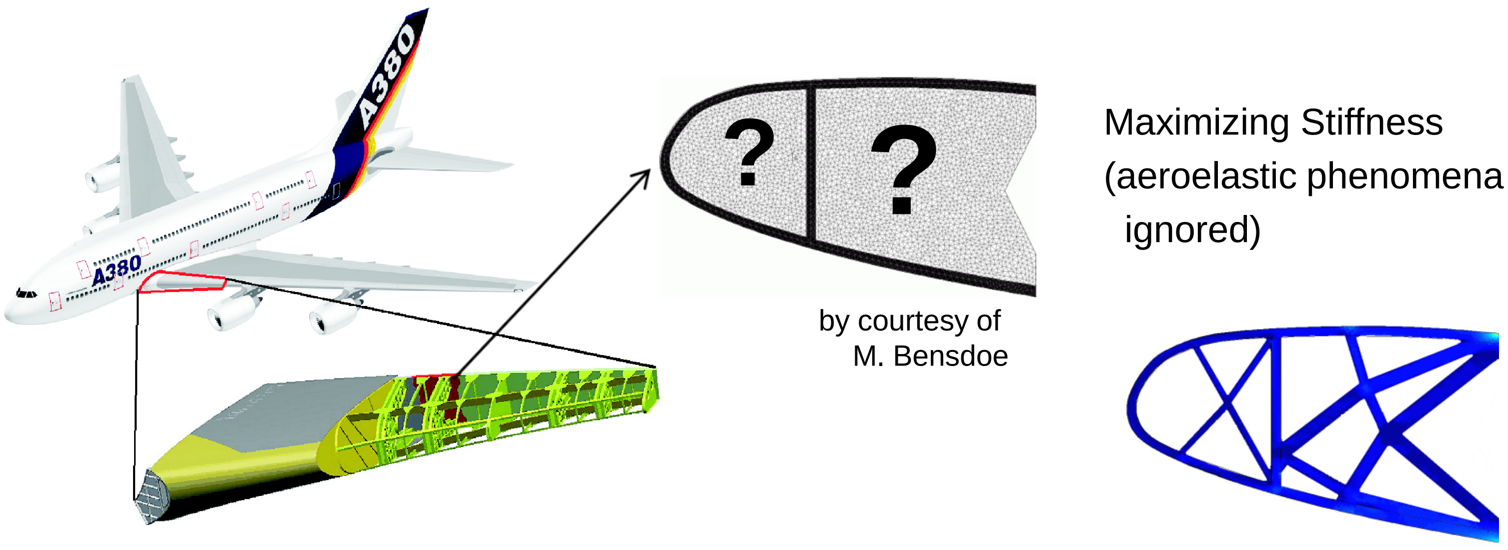
Topology Optimization



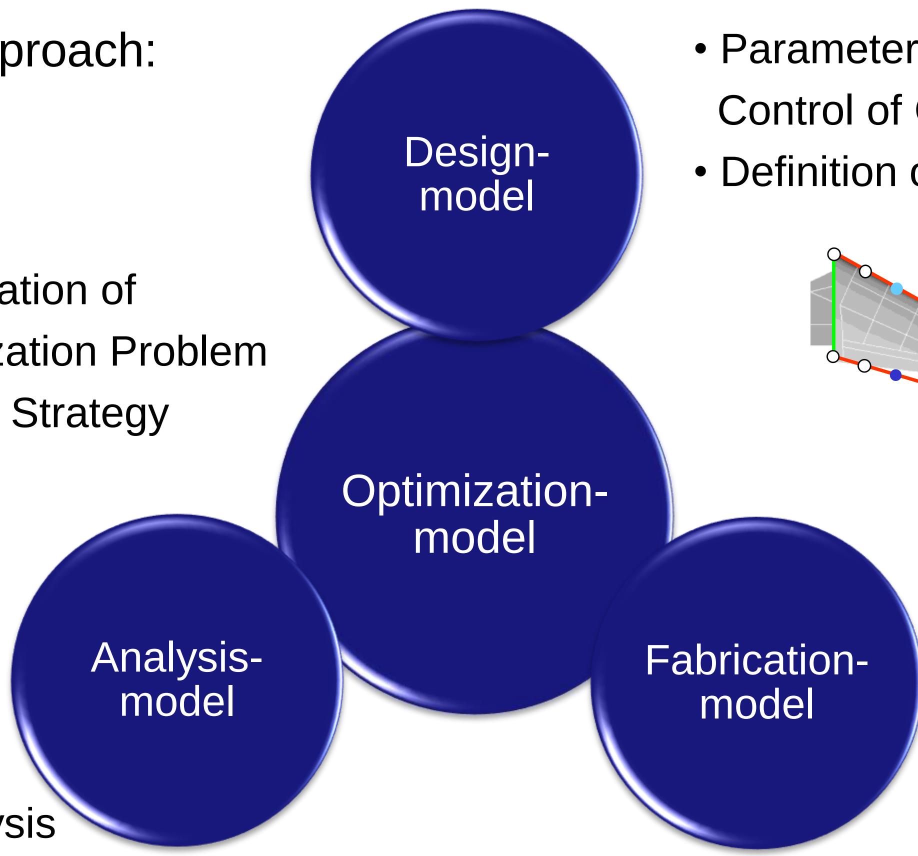


Maximizing Stiffness
(aeroelastic phenomena
ignored)

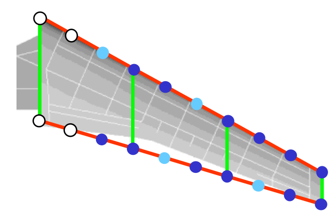
Design Optimization

Inverse Problem: *Find a Realizable Arrangement and Geometry of Materials and Components with desired Functionality*

Computational Approach:



- Parameterization & Control of Geometry
- Definition of Design Variables



- Formulation of Optimization Problem
- Search Strategy

- Multiscale- / Multiphysics analysis

- Fabrication Constraints/Costs

Overview

- Classical Methods in Topology Optimization
- Level-Set based Topology Optimization
    - Parameterization of Level-Set Field
    - Update Schemes
    - Numerical Modeling of Physical Response
    - Regularization Schemes
- Case Studies
    - Structural Mechanics
    - Conjugated Heat Transfer
- Conclusions

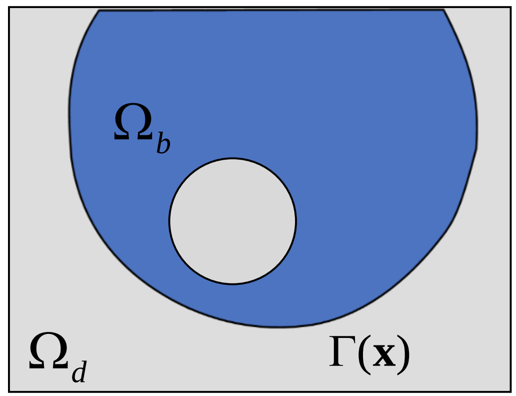
Topology Optimization

Geometry of a body in design domain Ω_d

Definition:

interior / exterior

& boundary $\Gamma(\mathbf{x})$



Material? yes / no

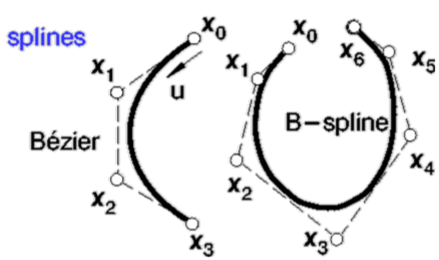
$$\chi(\mathbf{x}) = \begin{cases} 0 & \forall\ \mathbf{x} \in \Omega_b \\ 1 & \forall\ \mathbf{x} \in \Omega_d / \Omega_b \end{cases}$$

Parameterization

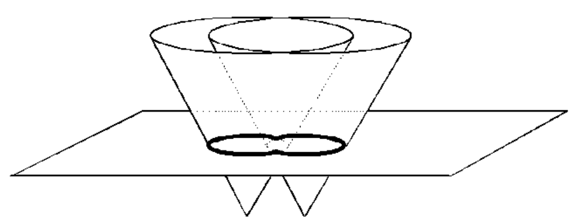
$$\Gamma(\mathbf{x}) \rightarrow \Gamma^h(\mathbf{p}, \mathbf{x})$$

Parameterization

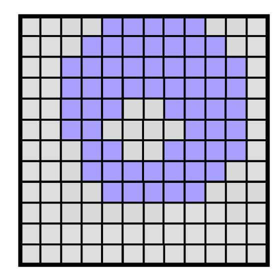
$$\chi(\mathbf{x}) \rightarrow \chi^h(\mathbf{p}, \mathbf{x})$$



CAD, CAD-free



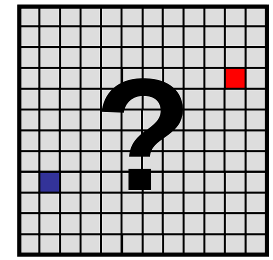
implicit function (level set)



element wise Ansatz functions

Material based Topology Optimization

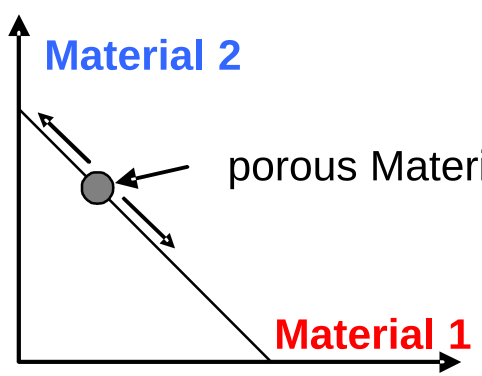
$$\chi(\mathbf{x}) = \begin{cases} 0 & \forall\ \mathbf{x} \in \Omega_b \\ 1 & \forall\ \mathbf{x} \in \Omega_d / \Omega_b \end{cases}$$



Relaxation of Integer Problems: $\chi \rightarrow \left\{ \rho \in \mathbb{R} \,\middle|\, 0 \leq \rho \leq \rho_0 \right\}$

Interpolation of Material Properties: Homogenization, SIMP, RAMP, …
& Penalty & Projection

Search Strategies / Algorithms: NLP or Phase-field Methods &
adjoint Sensitivity Analysis



$$E(p) = E_1 + \left(E_2 - E_1\right) p^{\beta}$$

$$\rho(p) = \rho_1 + \left(\rho_2 - \rho_1\right) p$$

$$p \geq 0\, ;\, \beta \geq 1$$

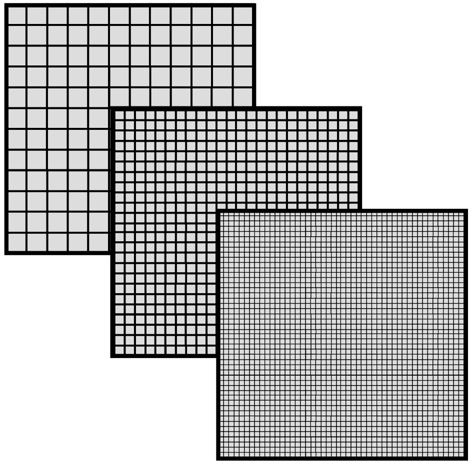
"SIMP"

Solid Isotropic Material with Penalization

Existence of Solution

Spaces: $\chi(\mathbf{x}), \rho(\mathbf{x}) \in L_\infty(\Omega_d)$

$\mathbf{u}(\mathbf{x}) \in H_1(\Omega_d)$



Convergence depends on Physical Model:

E.g. Elasticity $\frac{d}{dx}\left(\rho^\beta\ EA\frac{du}{dx}\right) + b = 0$ No convergence

E.g. Navier-Stokes & Brinkman $F_{NS}(p, v) + \frac{1}{1-\rho}v = 0$ Convergence

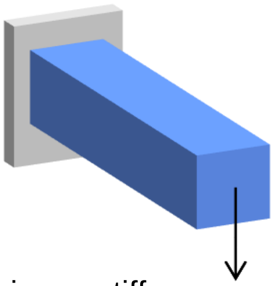
Regularization $\chi(\mathbf{x}) \to H_1(\Omega_d)$

- Filter (density, sensitivities*)
- Constraints $\int_\Omega \left|\nabla\rho(\mathbf{x})\right|^n d\Omega_d \le \varepsilon$

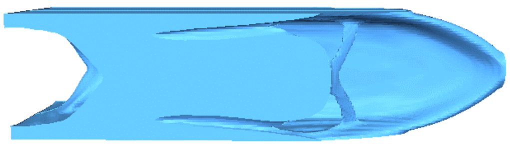
*Analogy to Elasticity with non-local Strains, Sigmund & Maute 2012

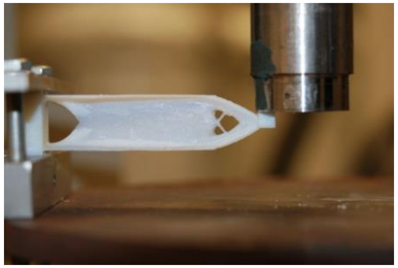
Material based Topology Optimization

Linear / Nonlinear Elasticity



maximum stiffness
for given mass

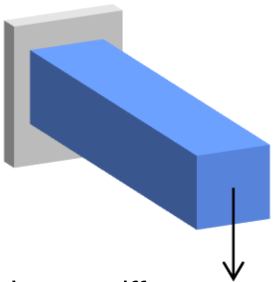




3-D Printer & Test

Material based Topology Optimization

Linear / Nonlinear Elasticity



maximum stiffness
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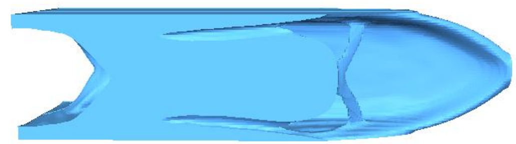
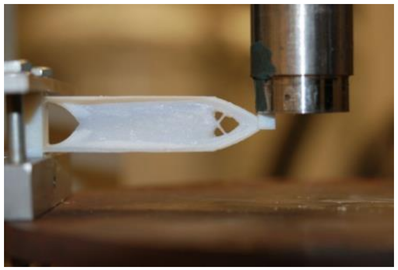
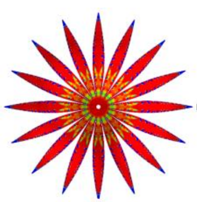


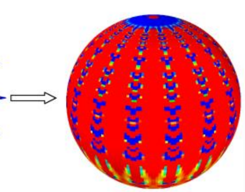


3-D Printer & Test

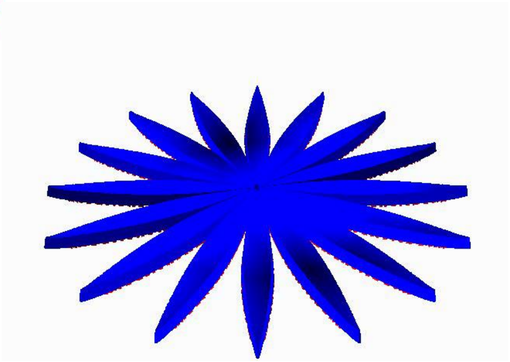
Photo-Active Polymers



flat



spherical

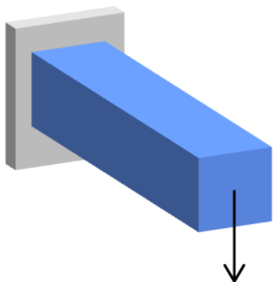


substrate coated with
photo-active polymer

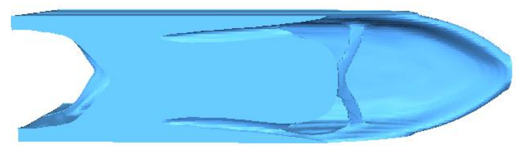
active passive

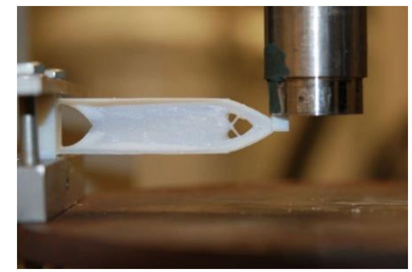
Material based Topology Optimization

Linear / Nonlinear Elasticity



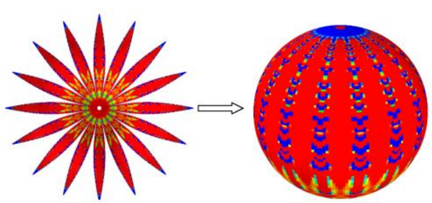
maximum stiffness
for given mass





3-D Printer & Test

Photo-Active Polymers

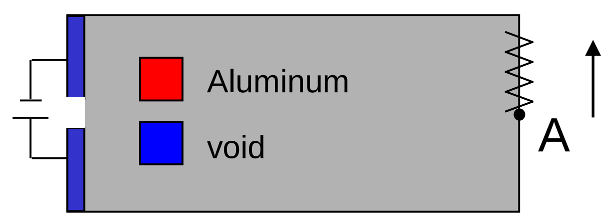


flat spherical

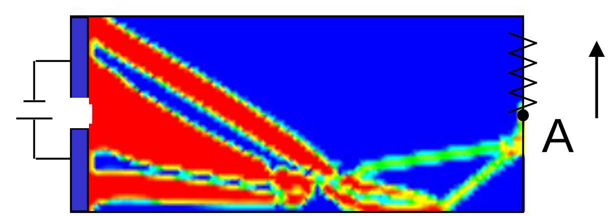
substrate coated with
photo-active polymer

active passive

MEMS Actuator



electro-thermo-mechanical coupling



Material based Topology Optimization

Optimization variables p_n $\rightarrow$ material parameters

Well suited for problems where physical response can be "manipulated" via constitutive behavior.

Multi-physics example: Thermo-Elastic Coupling

thermal flux: $$J_j = K_{ij} \cdot T_{,i}$$

elastic stress: $$\sigma_{ij} = C_{ijkl} \left(\varepsilon_{kl} - \alpha_{kl} \ \Delta T \right)$$

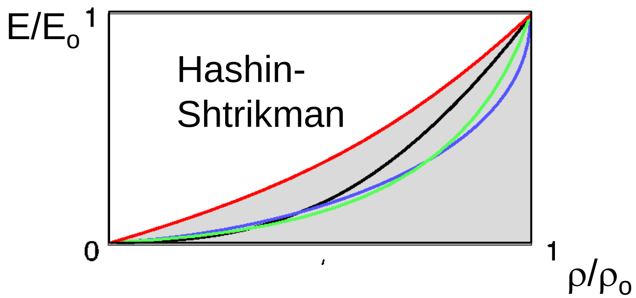
Design $\rightarrow$ Physical Model:

$$k_{ij} = k_{ij}\left(p_n \right)$$
$$C_{ijkl} = C_{ijkl}\left(p_n \right)$$
$$\alpha_{ij} = \alpha_{ij}\left(p_n \right)$$

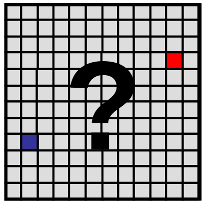
Interpolation functions chosen such that resulting design problem is smooth and converges to "0-1" solution

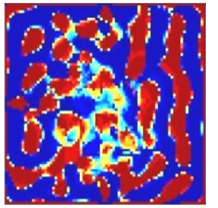
Convergence toward "0-1" Density Distribution

SIMP-type Methods

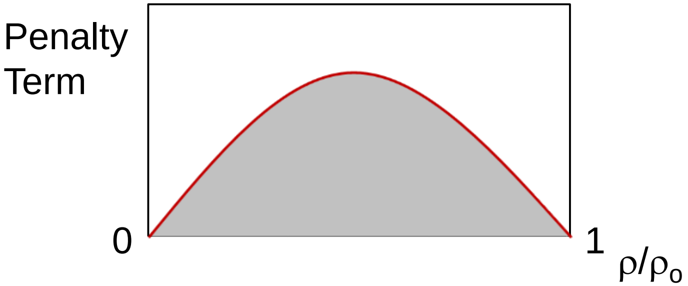


Implicit Penalization of Porosity for Stiffness / Mass Problems



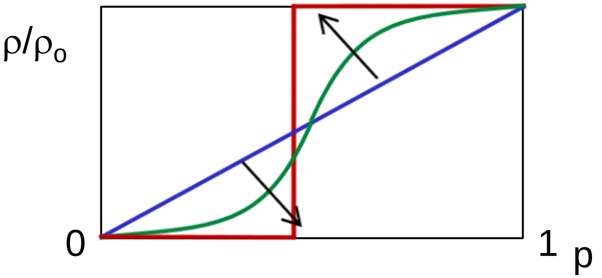


Explicit Methods



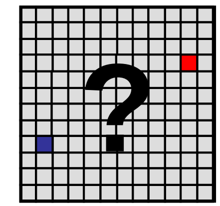
Strong Dependence of Results on Control Penalty Term (continuation methods)

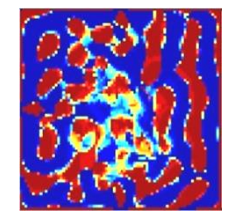
Projection Methods



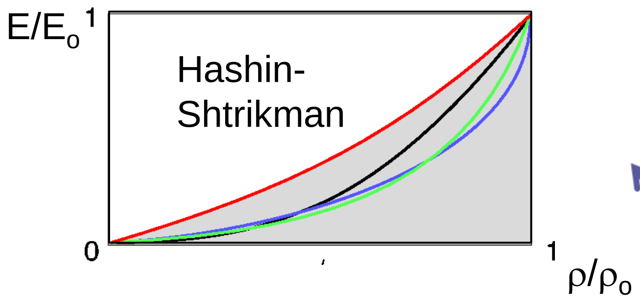
Only enhancement of methods above to mitigate blurring due to filters (continuation methods)

Convergence toward "0-1" Density Distribution

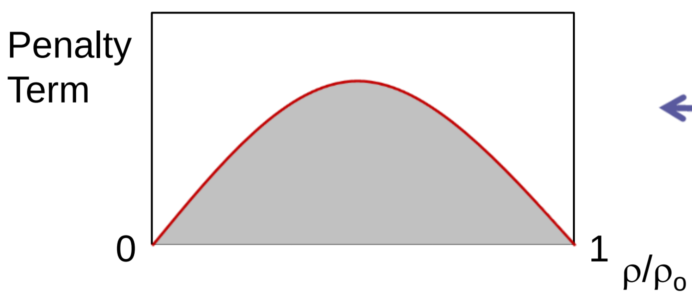




SIMP-type Methods



Explicit Methods

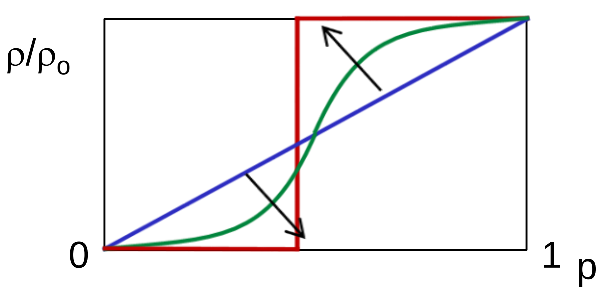


Non-convex,

Non-linear

Optimization Problems

Projection Methods



Unresolved Issues

- Dilemma: Convergence “0-1” vs. local minima

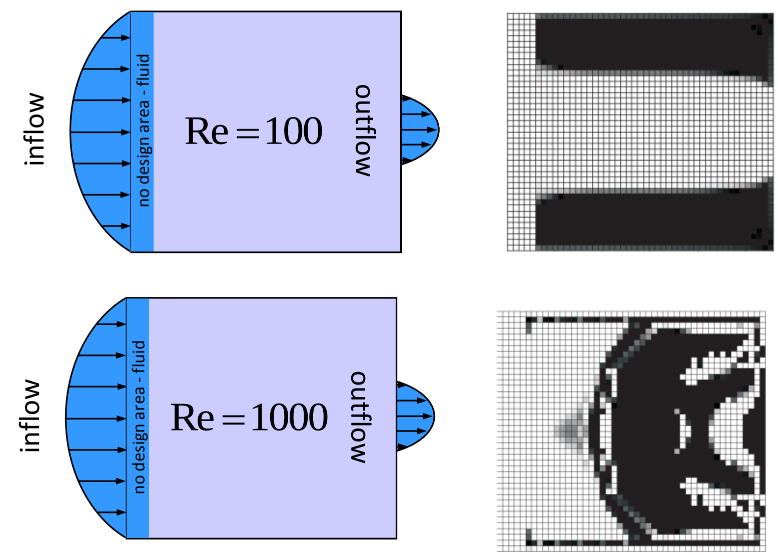
- Enforcing Boundary- and Interface-Conditions:

Kreissl & Maute 2011

e.g.: incompressible Navier-Stokes with Brinkman Model:

$$\min F = \int_{\Gamma_{in}} P_{inlet}^{tot}\, d\Gamma - \int_{\Gamma_{out}} P_{outlet}^{tot}\, d\Gamma$$

$$\text{s.t.} \quad g = \int_{\Omega_d} p\, d\Omega - V_{fluid} \leq 0$$



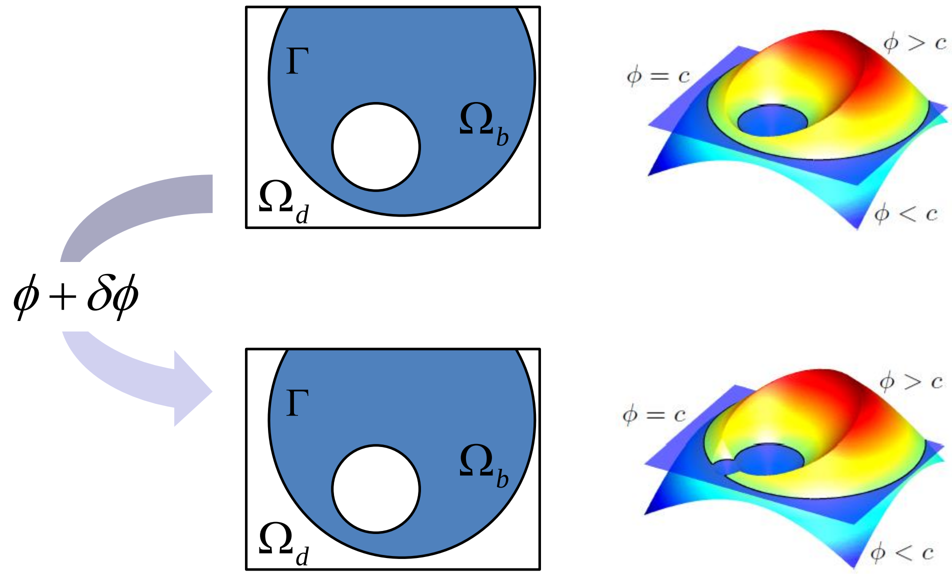
Other examples: non-diffuse transport, high-frequency wave propagation, surface reactions, sub-continuum interface-phenomena (phonon & photon scattering, surface stress), …

Overview

- Classical Methods in Topology Optimization
- **Level-Set based Topology Optimization**
  - Parameterization of Level-Set Field
  - Update Schemes
  - Numerical Modeling of Physical Response
  - Regularization Schemes
- Case Studies
  - Structural Mechanics
  - Conjugated Heat Transfer
- Conclusions

Level-Set based Topology Optimization

$$\phi(\mathbf{x}) = \begin{cases} > c & \forall\ \mathbf{x} \in \Omega_b \\ = c & \forall\ \mathbf{x} \in \Gamma \\ < c & \forall\ \mathbf{x} \in \Omega_d / \Omega_b \end{cases}$$

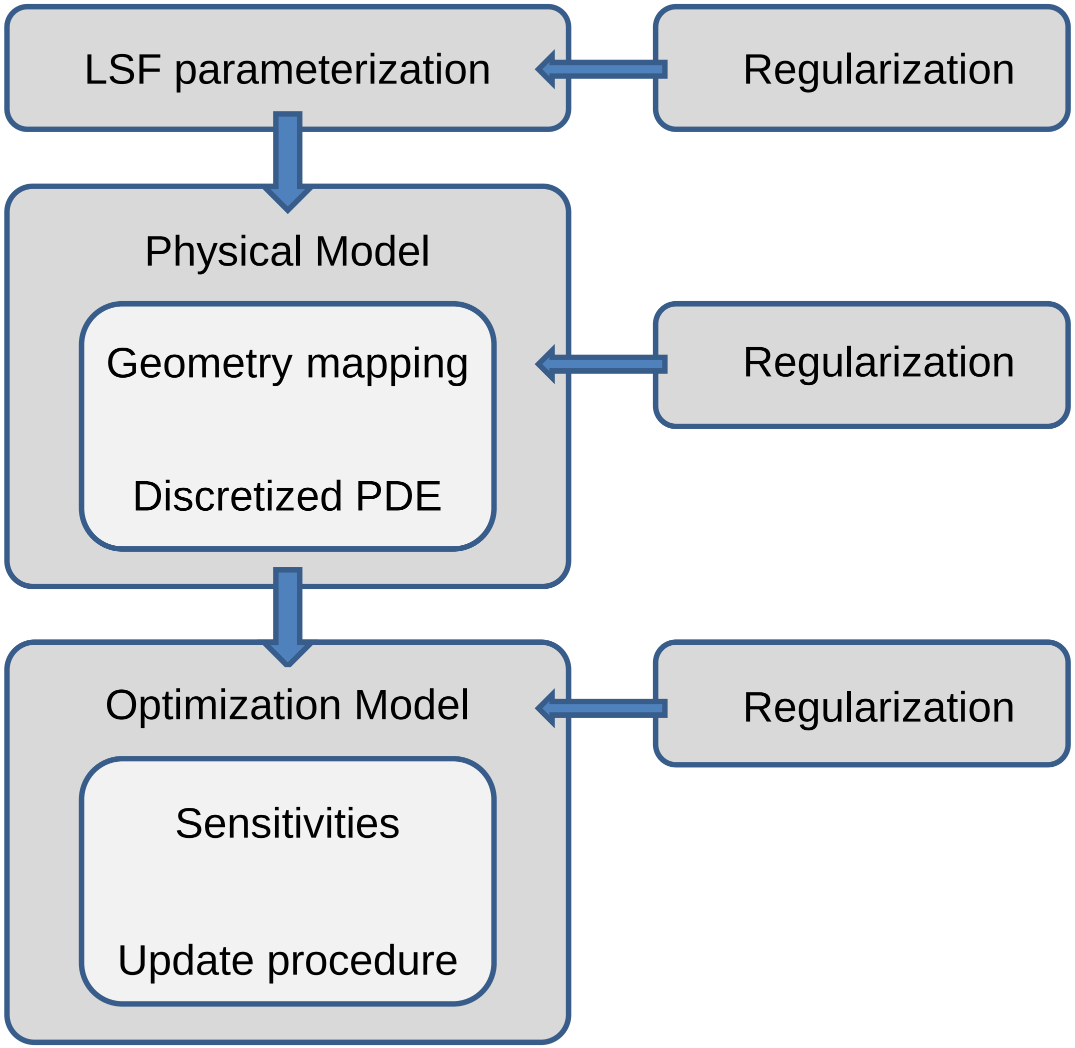


Remarks:

- continues change in level-set field leads to change in topology (geometry)
- geometry evolves through shape changes driven by shape sensitivities
- to seed new “holes” additional information needs to be provides
  (e.g. topological derivatives)

Level-Set based Topology Optimization

Common Framework:



Parameterization of Level Set Function

optimization variables

$$\mathbf{p}$$

↓

filter

e.g. $\varphi(\mathbf{x}) = N_{\phi}(\mathbf{x})\,\mathbf{p}$

↓

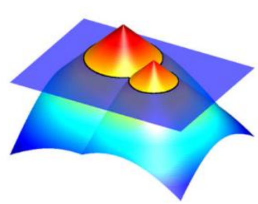
discretized LSF

e.g. $\phi_k = N_{\phi}(\mathbf{x})\,\varphi(\mathbf{x})$

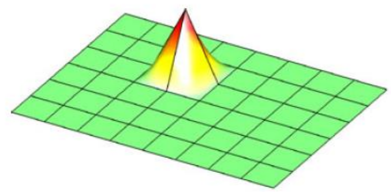
↓

Combination of LSFs

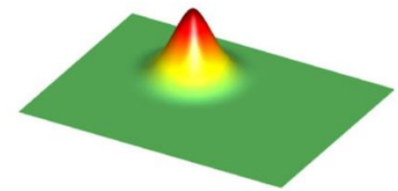
$$\phi = \mathfrak{M}(\phi_1, \phi_2, \ldots)$$



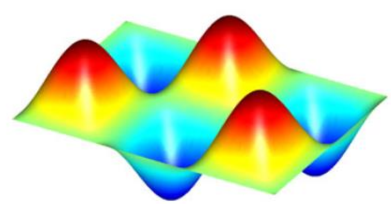
Common discretization schemes of LSFs



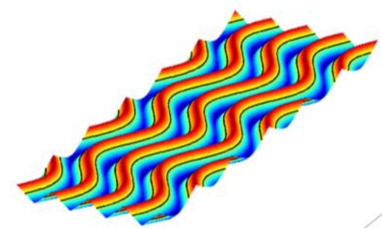
bilinear basis function



radial basis function



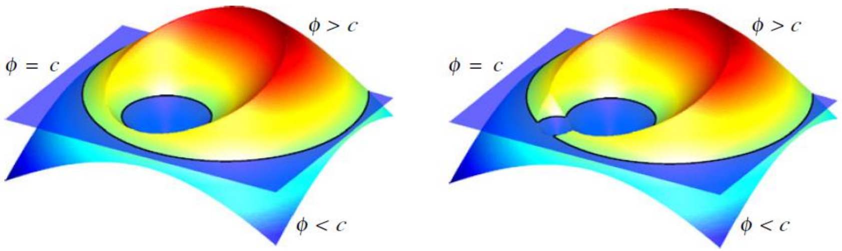
spectral basis function



geometric primitives

Update Schemes

continuous $\phi(\mathbf{x})^{(\mathbf{n})} \rightarrow \phi(\mathbf{x})^{(\mathbf{n}+1)}$



parameterized $\phi\left(\mathbf{p}^{(\mathbf{n})}\right) \rightarrow \phi\left(\mathbf{p}^{(\mathbf{n}+1)}\right)$

Update scheme in level set methods include:

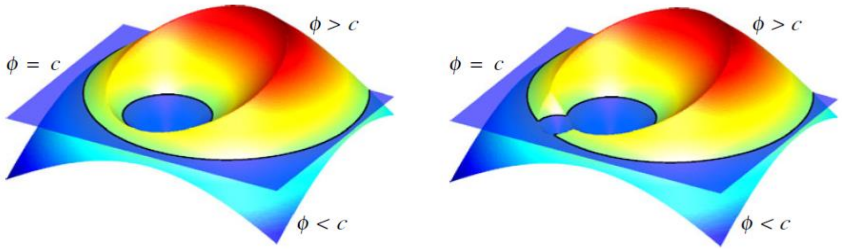
- Parameter optimization problems solved by Nonlinear Programming Methods
- Pseudo-time integration of (generalized) Hamilton-Jacobi equation

$$\frac{d\phi}{dt} - v_n \left|\nabla \phi\right| - \mathbb{D} - \mathbb{R} = 0$$

- Seeding of new “holes” via e.g. topological derivatives

Update Schemes

Nonlinear Programming



$$\mathbf{p}^{(\mathbf{n})} \quad \rightarrow \quad \mathbf{p}^{(\mathbf{n}+1)}$$

$$\min_{\mathbf{p},\mathbf{u}} \; F(\mathbf{p},\mathbf{u})$$

$$s.t. \quad h_j(\mathbf{p},\mathbf{u}) = 0 \qquad j = 1 \ldots N_h$$

$$g_j(\mathbf{p},\mathbf{u}) \geq 0 \qquad j = 1 \ldots N_g$$

$$\hat{\mathbf{R}}(\mathbf{p},\mathbf{u}) = \mathbf{0}$$

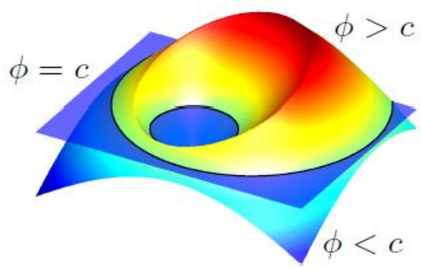
$$\mathbf{p} \in \prod = \left\{ \Re^{N_p} \mid \mathbf{p}_L \leq \mathbf{p} \leq \mathbf{p}_U \right\}$$

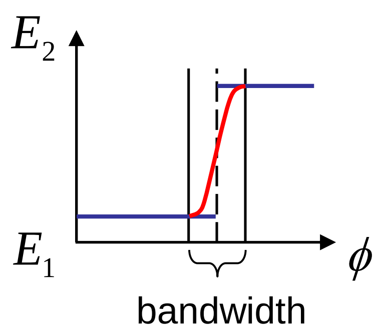
$$\mathbf{u} \in \Re^{N_u}$$

- augmented by regularization techniques
- solved by NLP (e.g. GCMMA)

Geometry → Physics Model

Geometry Model

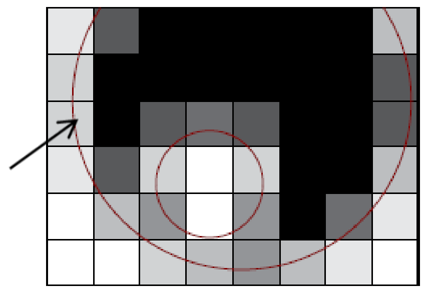




*Ersatz Material**

Physics Model

Porous Material along Interface



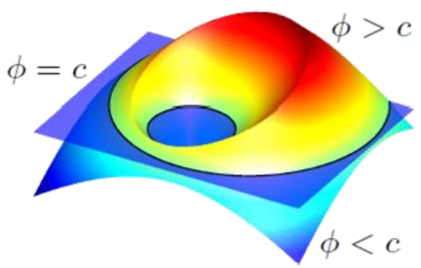
Imprecise Enforcement of Boundary- & Interface-Conditions

* close relation to projection methods in material-based approaches (van Dijk et al. 2013, Sigmund & Maute, 2013)

XFEM for Level-Set based Optimization

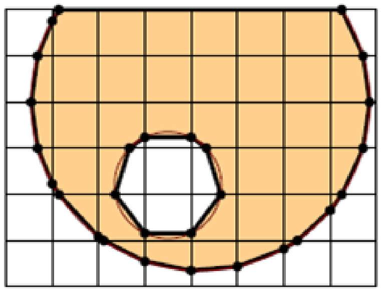
Geometry Model

(Level-set)



Mechanics Model

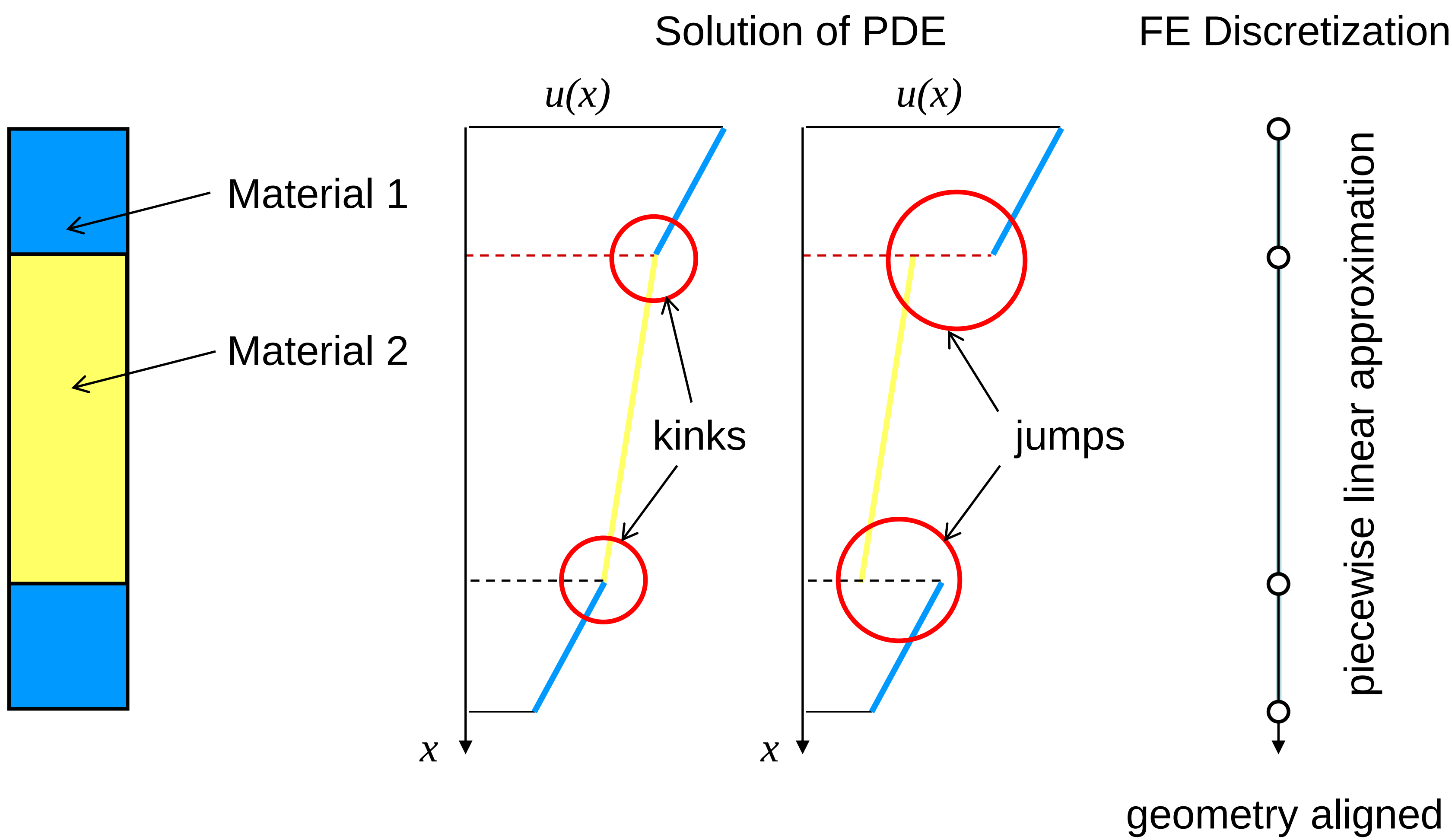
eXtended Finite Element Method (XFEM)



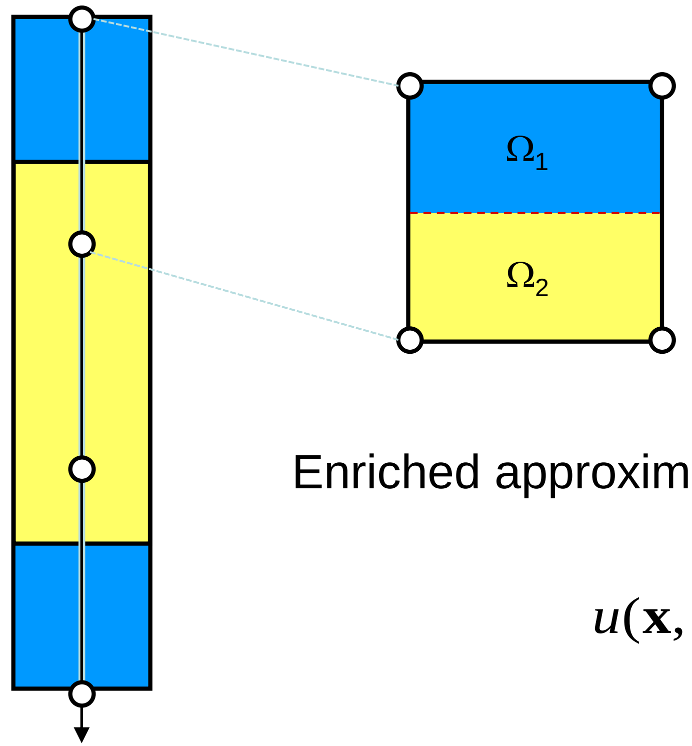
- Alternative to “Ersatz Material” Method
- Preserves Crispness of Level-set based Geometry Description
- “Standard” Methods for Boundary- und Interface-Constraints
- Robust Analysis of Complex Configurations

XFEM 101

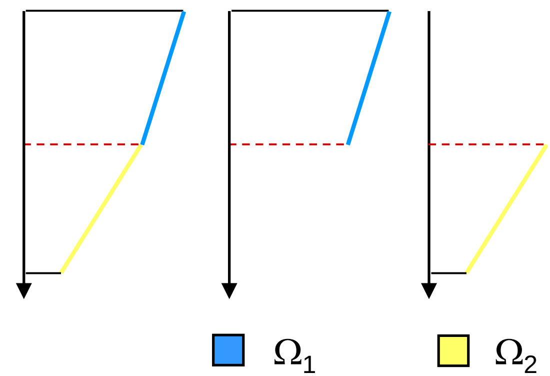
Model problem: $\nabla \cdot \left(k \, \nabla u\right) = q \quad in \, \Omega_b$



XFEM 101



solution across interface

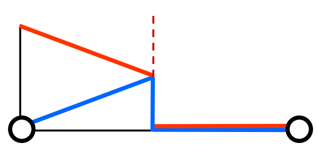


Enriched approximation:

$$u(\mathbf{x},t)=\sum_{i=1}^{n}N_i(\mathbf{x})\cdot\left[u_i\left(t\right)+\sum_{j=1}^{n_{rich}}G_j(\mathbf{x},t)a_{ij}(t)\right]$$

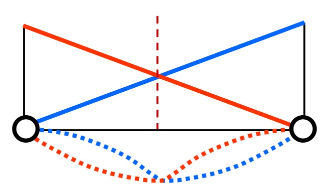
C^0-discontinuity:

$$G_j=\begin{cases}1 \;\forall\; \phi(x,t)\geq 0\\ 0 \;\forall\; \phi(x,t)<0\end{cases}$$



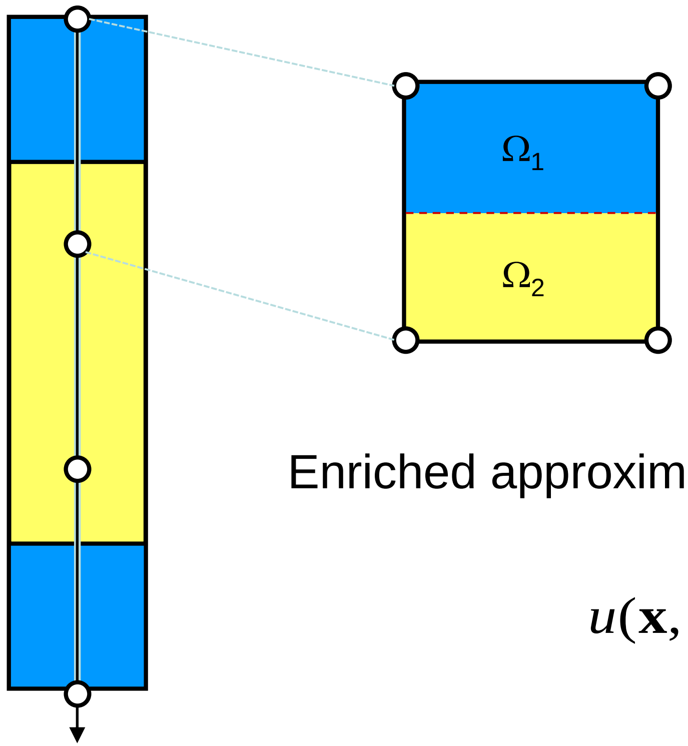
C^1-discontinuity:

$$G_j=\left|\phi(x,t)\right|-\left|\phi_j(t)\right|$$

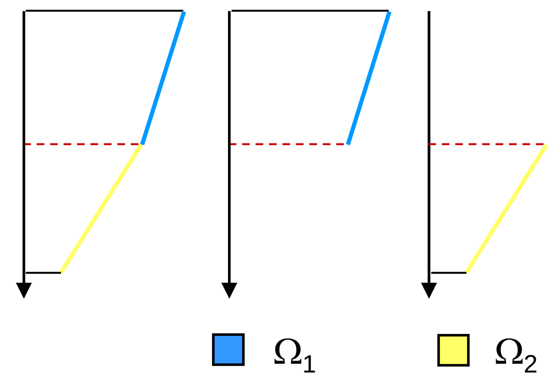


Interface definition via Level-set function φ

XFEM 101



solution across interface



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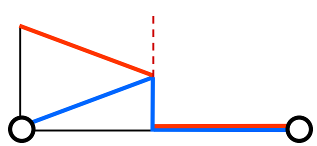


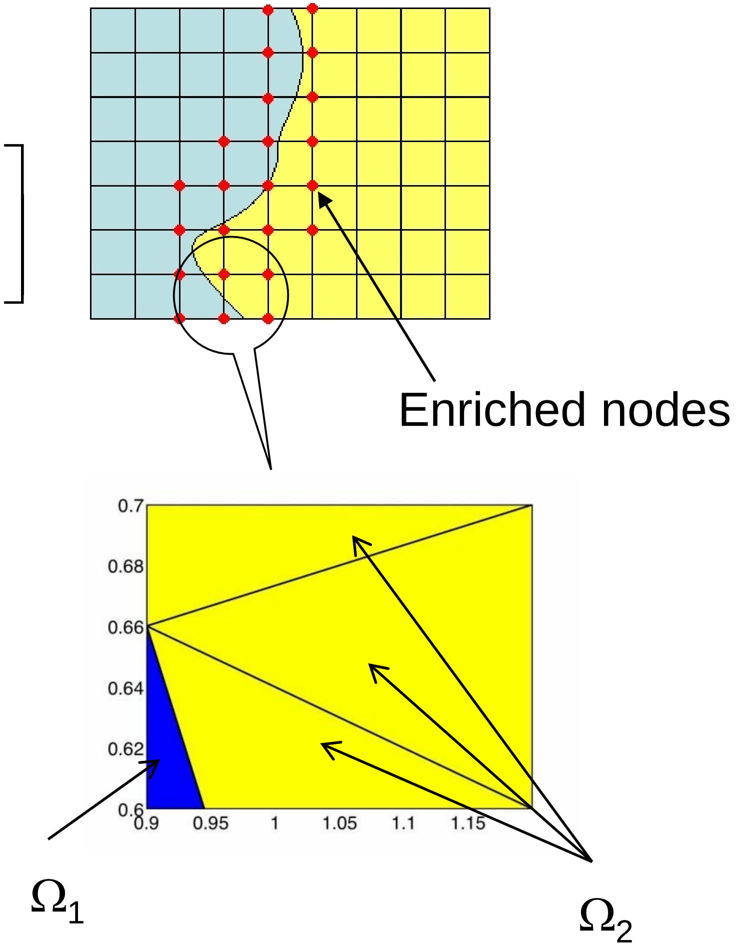
Note: N_i represents any type of shape function

XFEM 101

$$\int_{\Omega} \delta u \left(\nabla \cdot \left(k \, \nabla u \right) - q \right) d\Omega = 0$$

$$u(\mathbf{x},t) = \sum_{i=1}^{n} N_i(\mathbf{x}) \cdot \left[u_i\left(t\right) + \sum_{j=1}^{n_{rich}} G_j(\mathbf{x},t) a_{ij}(t) \right]$$

- Numerical integration via triangulization of intersected element



XFEM 101

$$\int_{\Omega} \delta u \left(\nabla \cdot \left(k \, \nabla u \right) - q \right) d\Omega = 0$$

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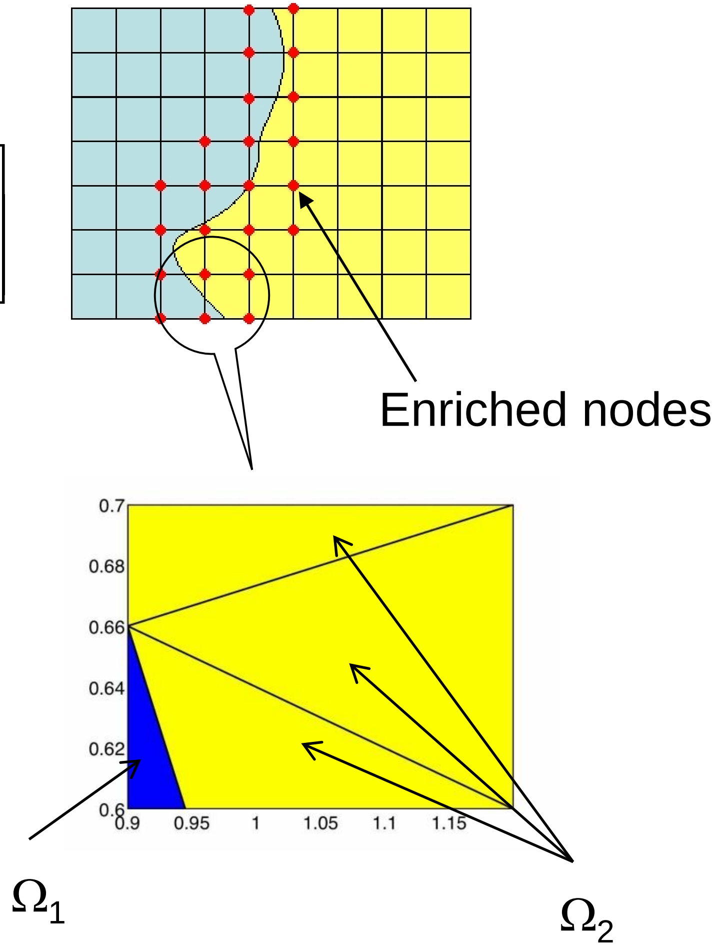

XFEM 101

$$\int_{\Omega} \delta u \left(\nabla \cdot \left(k\, \nabla u \right) - q \right) d\Omega = 0$$

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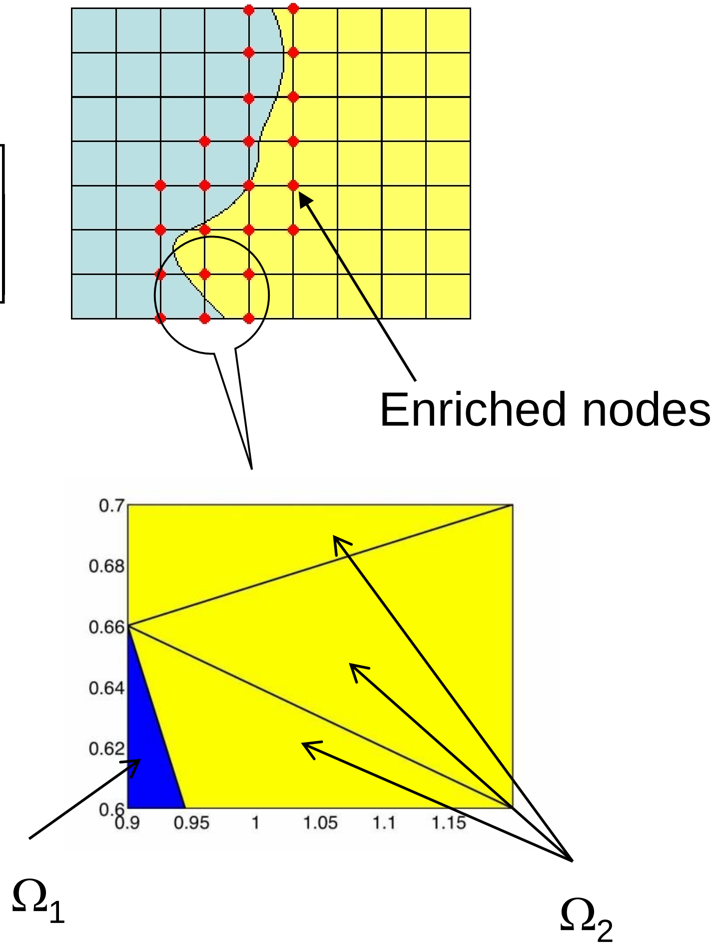
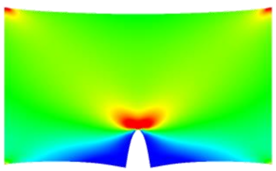


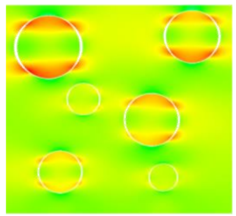
- Numerical integration via triangulization of intersected element
- Enforcement of interface conditions*:
  - Penalty method 💣
  - Lagrange multipliers 💣
  - Stabilized Lagrange multipliers
  - Nitsche method

* proper treatment of interface conditions is key to obtaining accurate solution gradient along interface

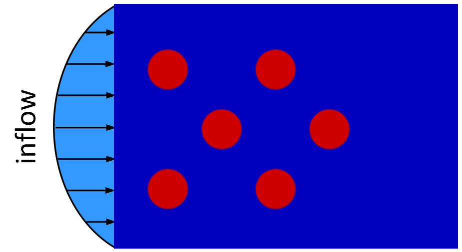
Application Examples of XFEM

Fracture-Mechanics

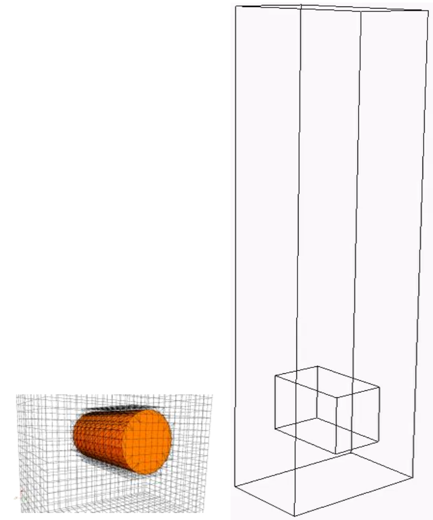

Crack Propagation


Delamination

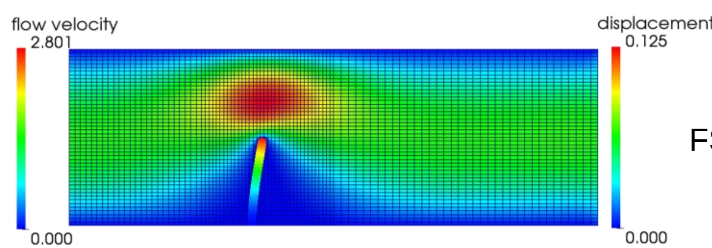
Fluid Mechanics



Forces Convection
w. hydrodynamic BTG

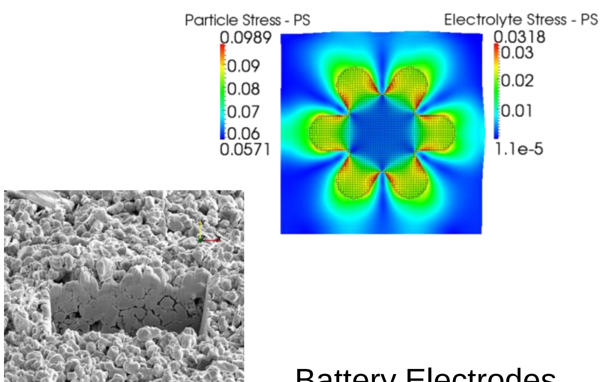

free Convection
with incompressible N.-S.


Rigid body motion



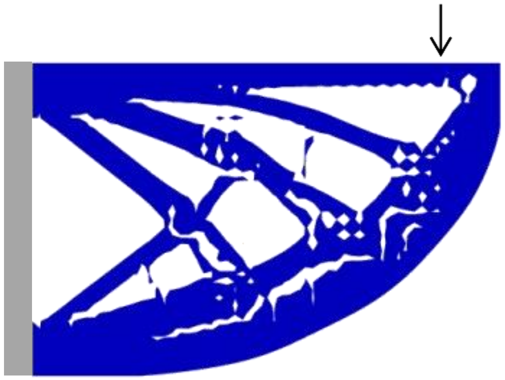
FSI via XFEM²

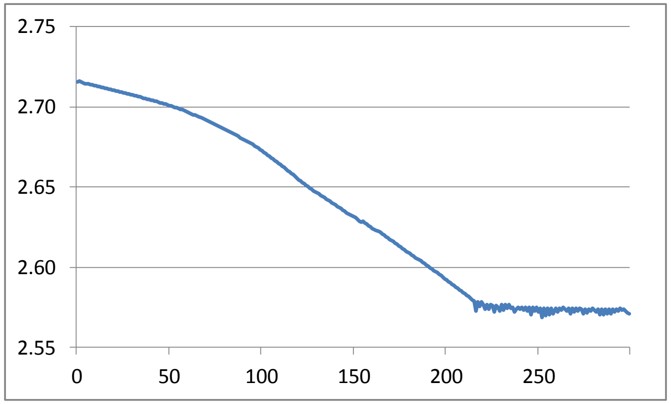
Multiscale Problems



Battery Electrodes

XFEM for Level-Set based Optimization

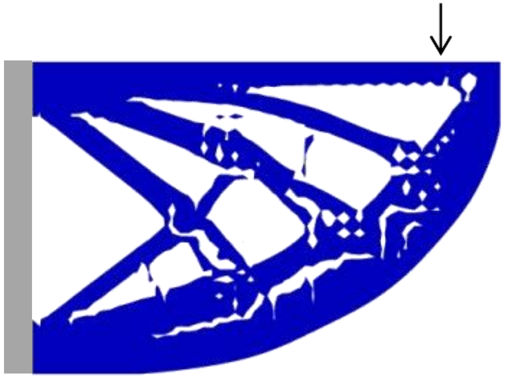


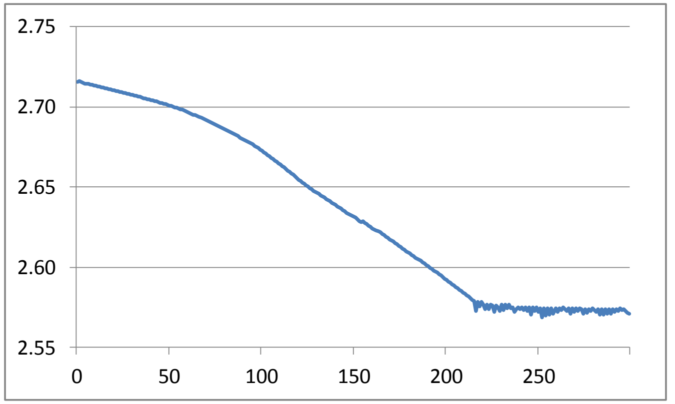


Issues:

- Ill-conditioned Analysis Problems
- Emergence of numerical geometric Artifacts
- Discontinuities as Topology changes
- Slow convergence of Optimization Process

XFEM for Level-Set based Optimization





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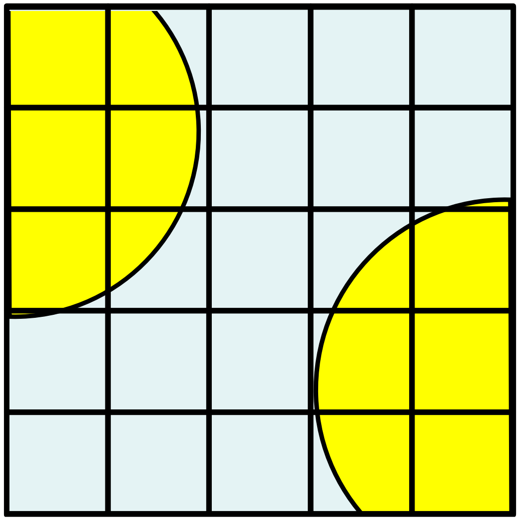
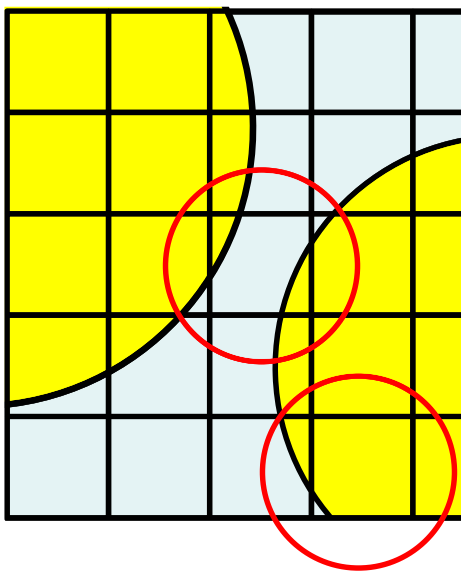
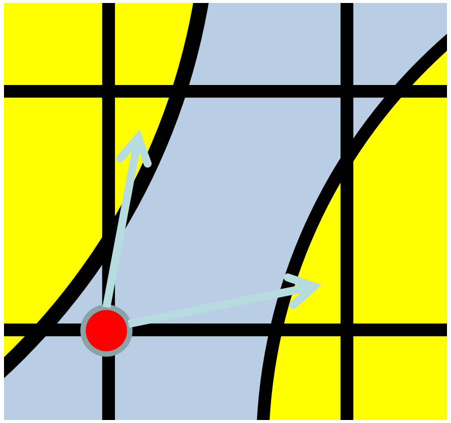
XFEM for Level-Set based Optimization

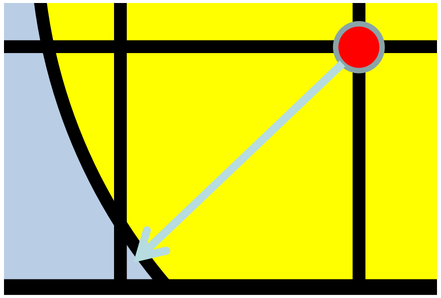


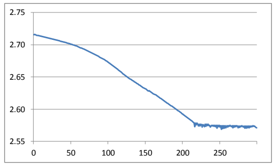
XFEM for Level-Set based Optimization





- Interpolation in spatially disconnected areas of same phase*

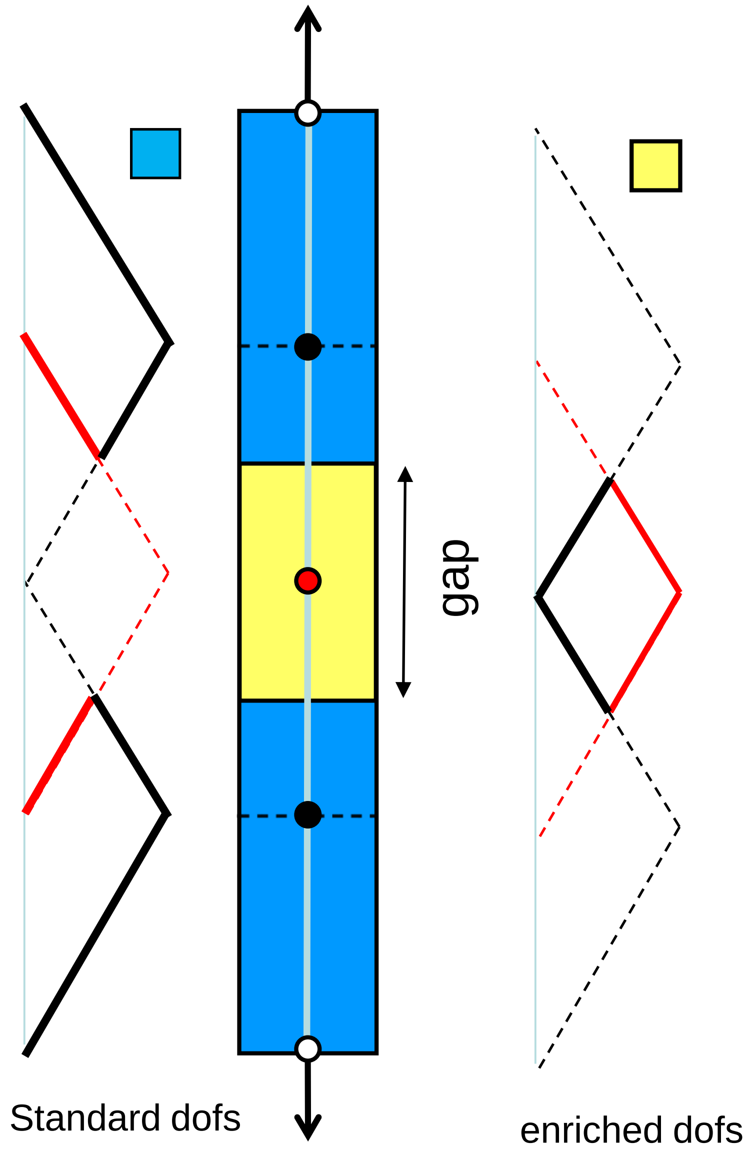


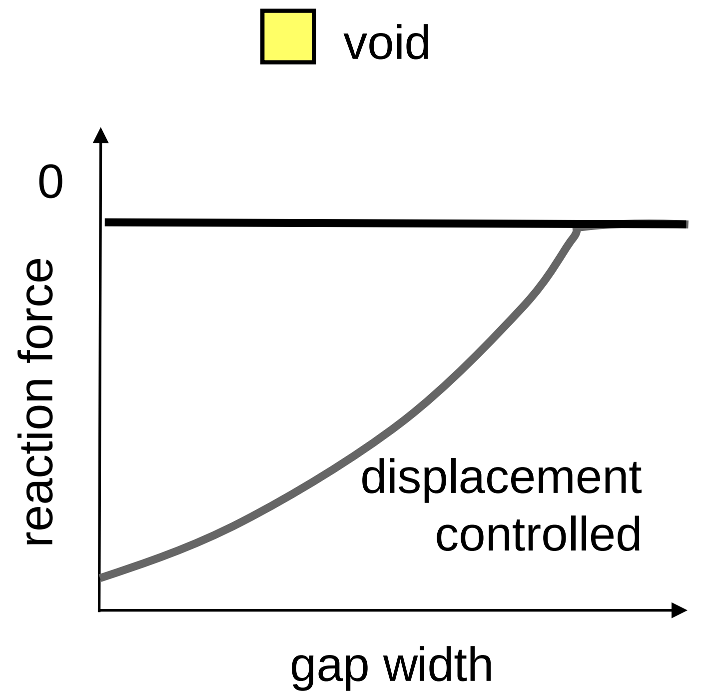


- vanishing zone of influence

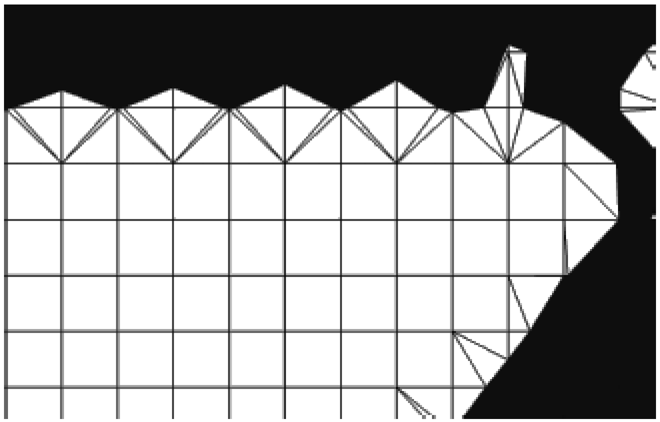
* Issues typically mitigated via locale mesh refinement, not suited for topology optimization.

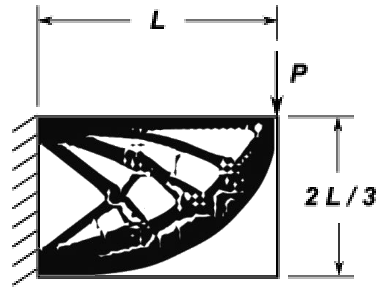
Checker-boarding



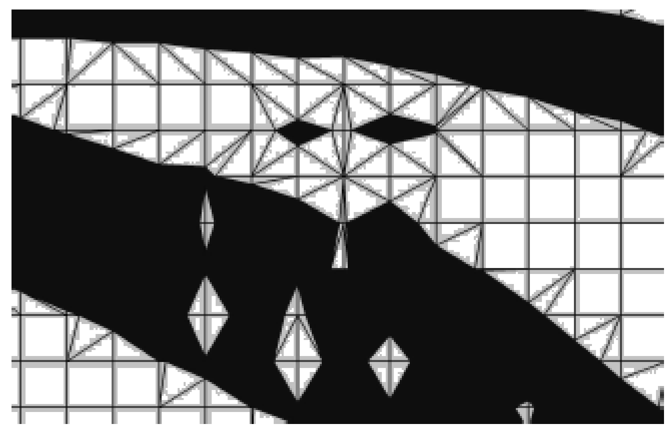


Checker-boarding



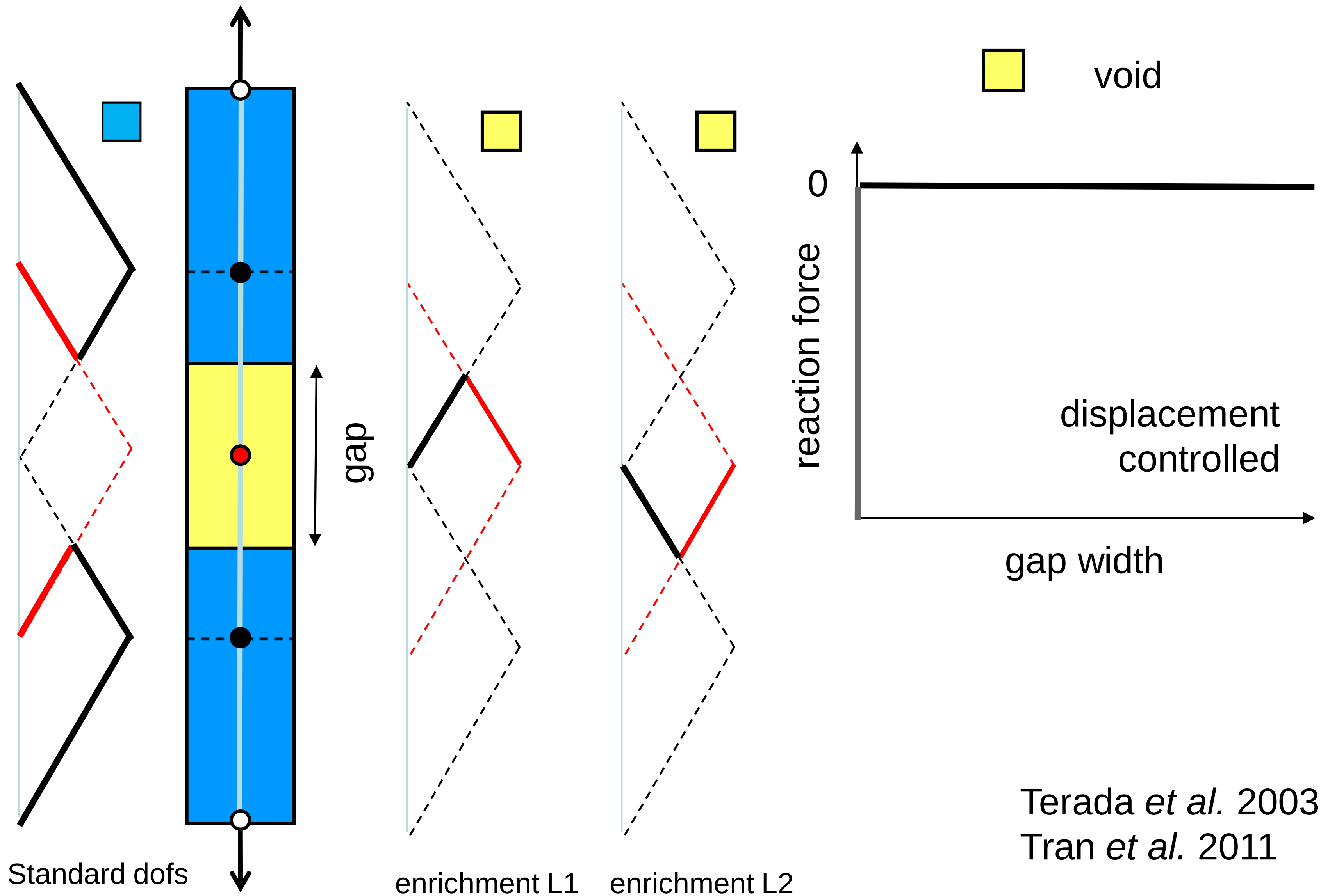


artificial stiffness across
void domains



“free-floating” material

Checker-boarding



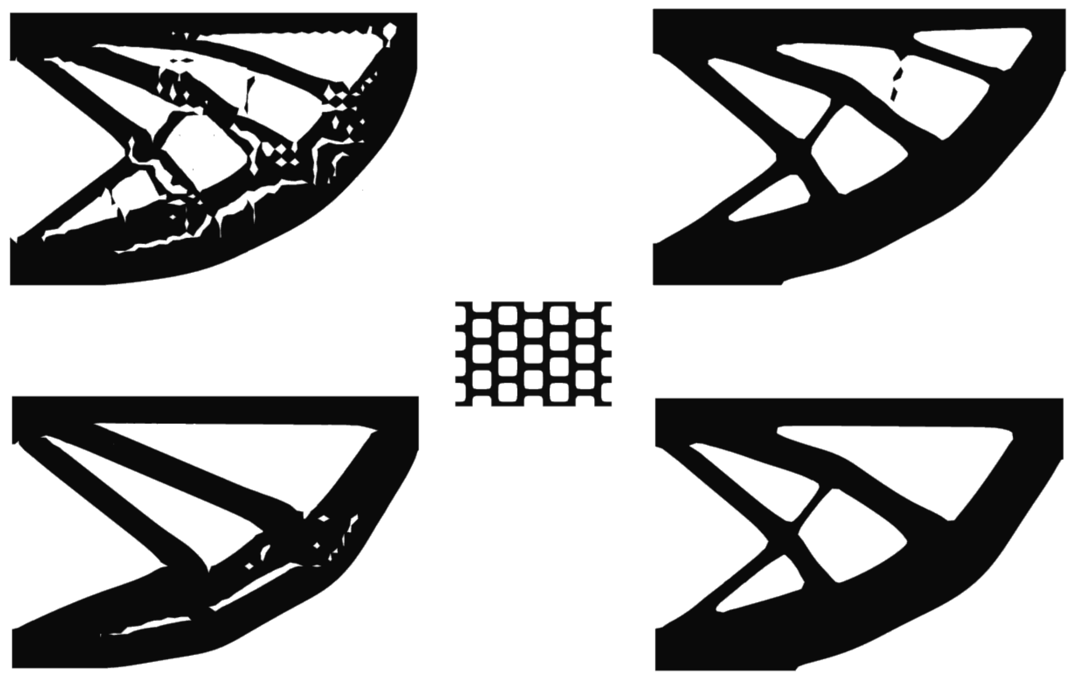
Terada *et al.* 2003
Tran *et al.* 2011

Checker-boarding

without perimeter constraint

Standard enrichment

Generalized enrichment

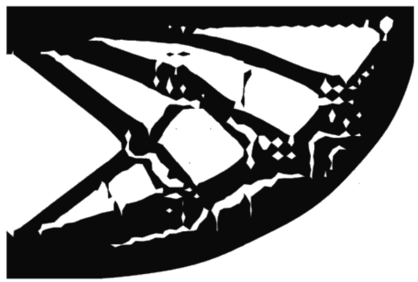


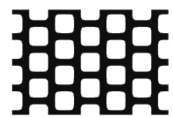
with perimeter constraint

Checker-boarding

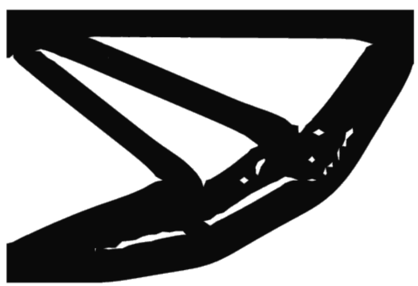
without perimeter constraint

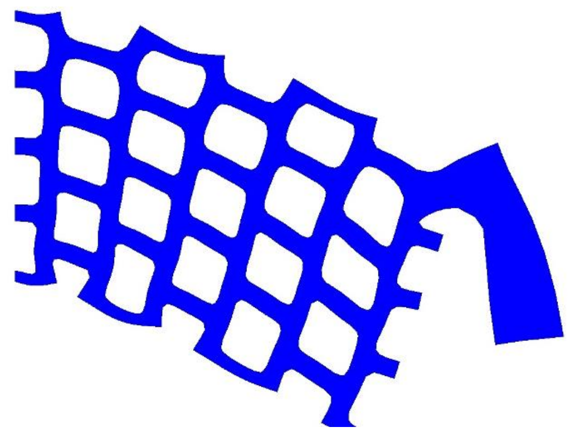
Standard enrichment



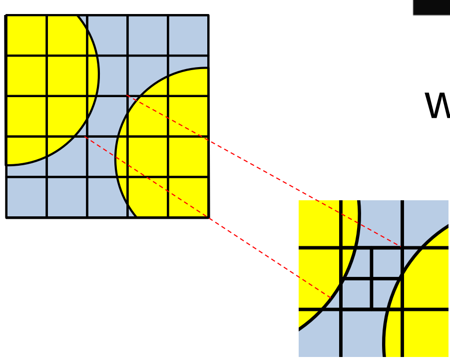


Generalized enrichment



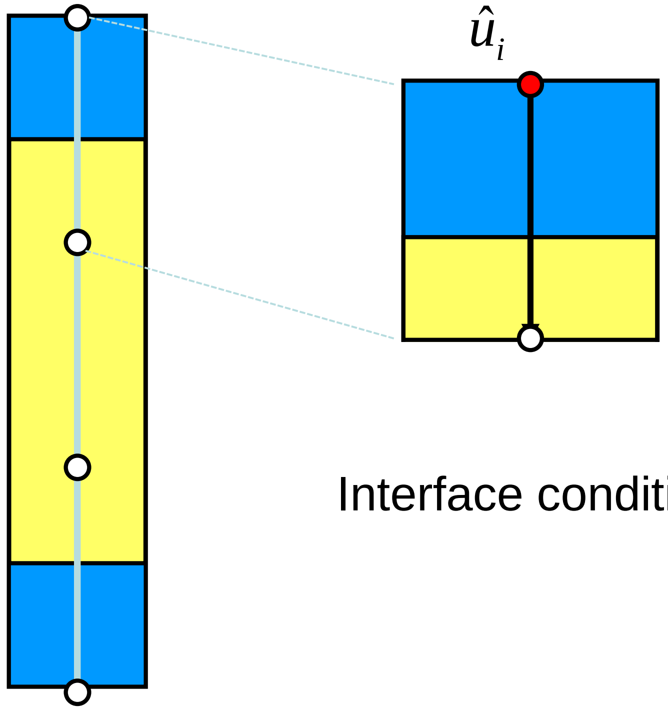


with perimeter constraint



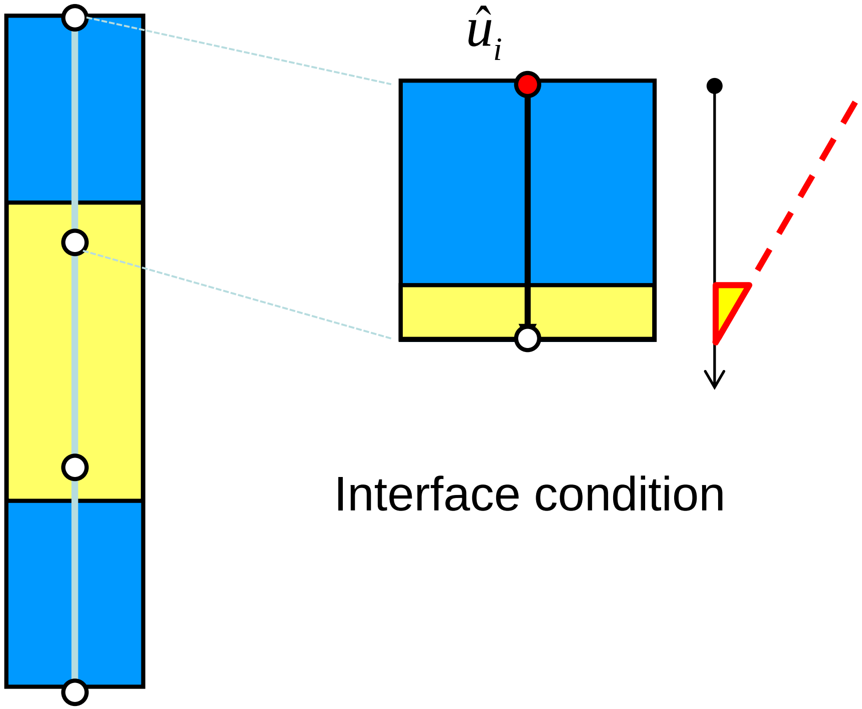
Note: adaptive mesh refinement also resolves artificial stiffening issue but is less attractive for topology optimization

ILL-Conditioning



Interface condition

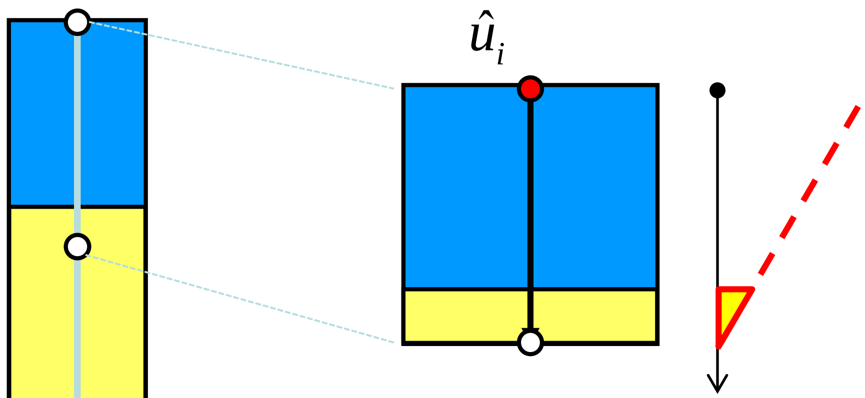
ILL-Conditioning



Interface condition

$$\int_{\Omega} \delta \hat{u}_i \left(\nabla \cdot \left(k \, \nabla u \right) - q \right) d\Omega \rightarrow 0$$

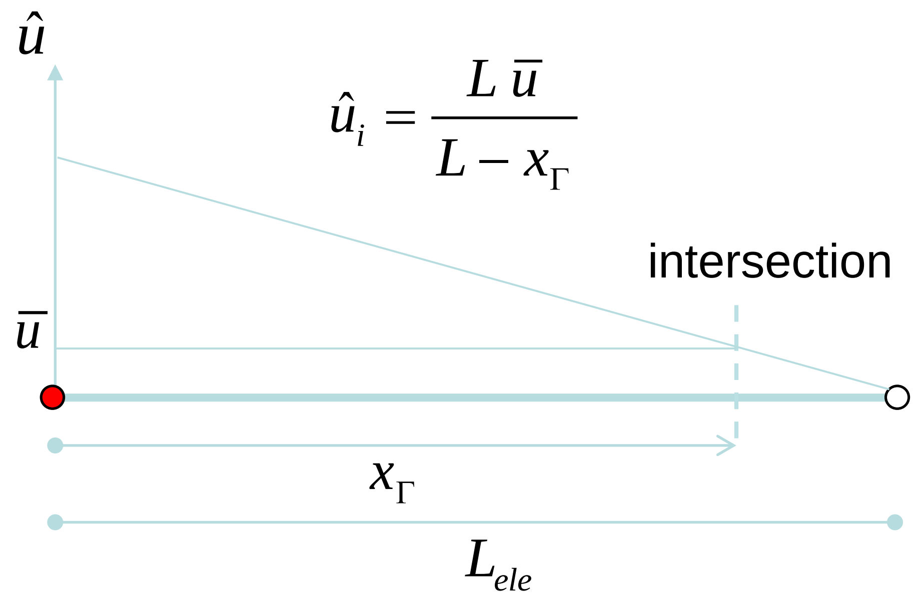
ILL-Conditioning



Interface condition

$$\int_{\Omega} \delta \hat{u}_i \left(\nabla \cdot \left(k \, \nabla u \right) - q \right) d\Omega \rightarrow 0$$

$$\hat{u}_i = \frac{L \, \bar{u}}{L - x_\Gamma}$$



Loss of influence of $\hat{u}_i$

ILL-Conditioning

Moving Nodes

$\rightarrow$ *efficiency, robustness, “moving mesh”*

Modify Vibrational Form of Governing Equations

$\rightarrow$ *mostly limited to linear elliptic problems*

Pre-conditioning (standard or tailored techniques)

$\rightarrow$ *linear system only, effectiveness*

Modifying Enrichment Functions

$\rightarrow$ *affect XFEM solution, complexity*

Dropping DOFs with smaller Domain of Influence

$\rightarrow$ *strong influence of threshold on effectiveness versus accuracy*

ILL-Conditioning

$$\hat{\mathbf{u}} = \mathbf{T}\,\tilde{\mathbf{u}}$$

Projected Space $\tilde{\mathbf{u}}$

FEM Space $\hat{\mathbf{u}}$

solve (non)linear system

evaluate discretized governing equations

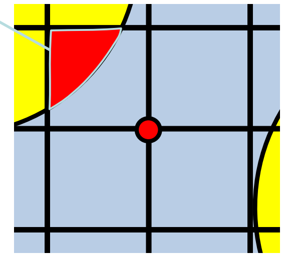
$$\tilde{\mathbf{R}} = \mathbf{T}\,\hat{\mathbf{R}}$$

$$\tilde{\mathbf{J}} = \mathbf{T}^{T}\,\hat{\mathbf{J}}\,\mathbf{T}$$

$$\mathbf{T} = \mathbf{T}(\phi): \ \text{projection matrix}$$

ILL-Conditioning

Diagonal Projection Operator



$$T_{ii,m}^{I} = \max_{e\in E_i}\left(\frac{\int_{\Omega_e^I} N_i(x)\,dx}{\int_{\Omega e} N_i(x)\,dx}\right)^{\left(-\frac{1}{2}\right)}$$

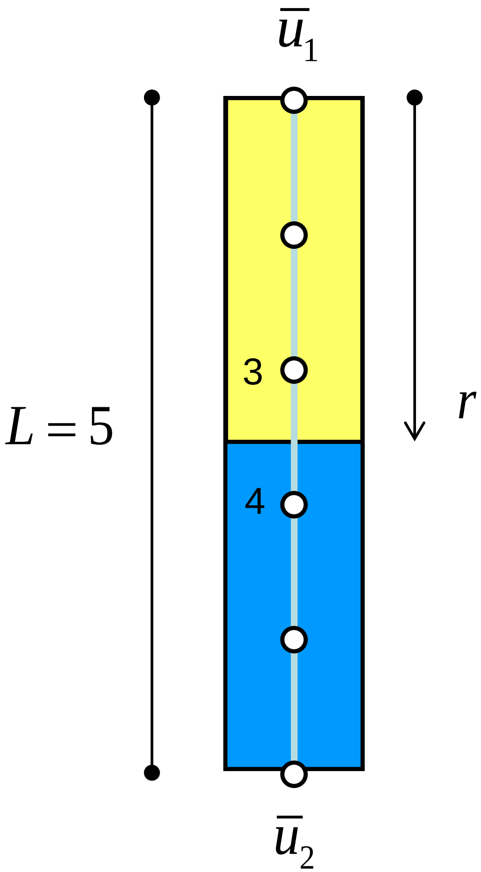
$$T_{ii,m}^{I} = \max_{e\in E_i}\left(\frac{\int_{\Omega_e^I} \nabla N_i(x)\cdot\nabla N_i(x)\,dx}{\int_{\Omega e} \nabla N_i(x)\cdot\nabla N_i(x)\,dx}\right)^{\left(-\frac{1}{2}\right)}$$

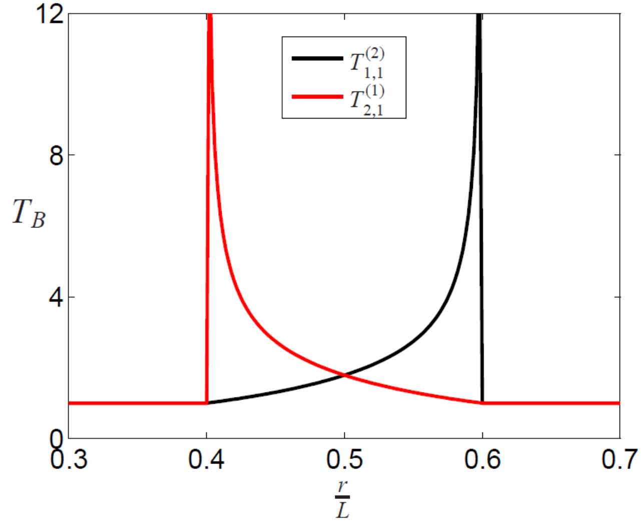
non-local measure

Constrain Degrees of Freedom

$$T_{ii,m}^{I} > T_{tol}$$

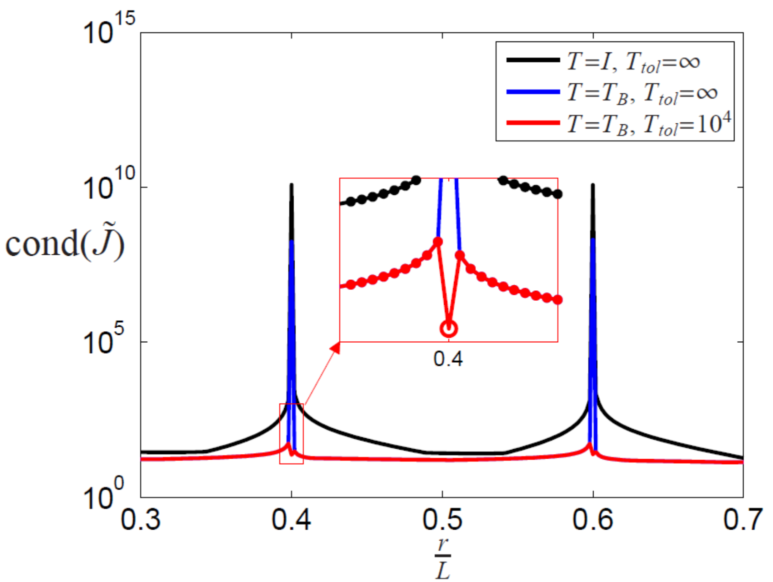
ILL-Conditioning





gradient-based formulation

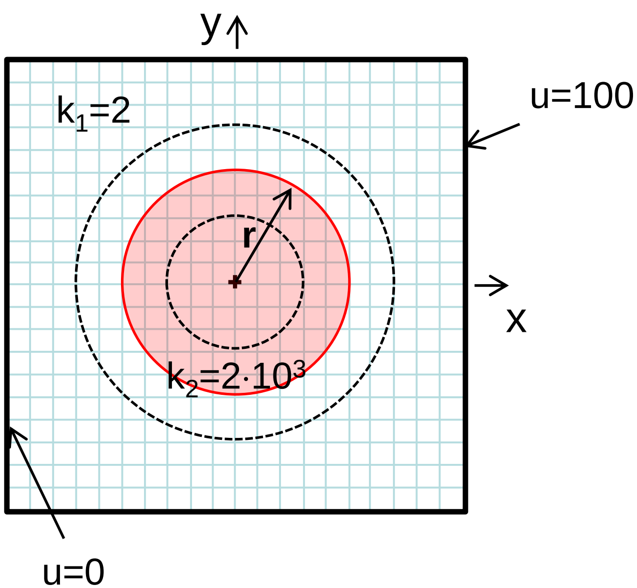
condition number

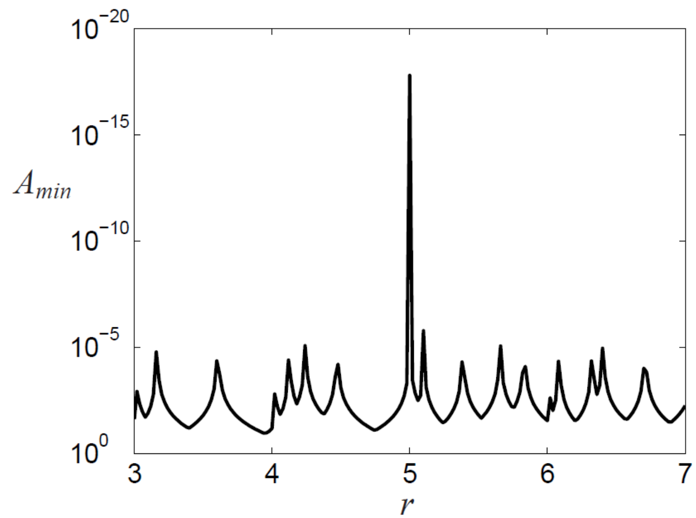


stabilized Lagrange multiplier formulation

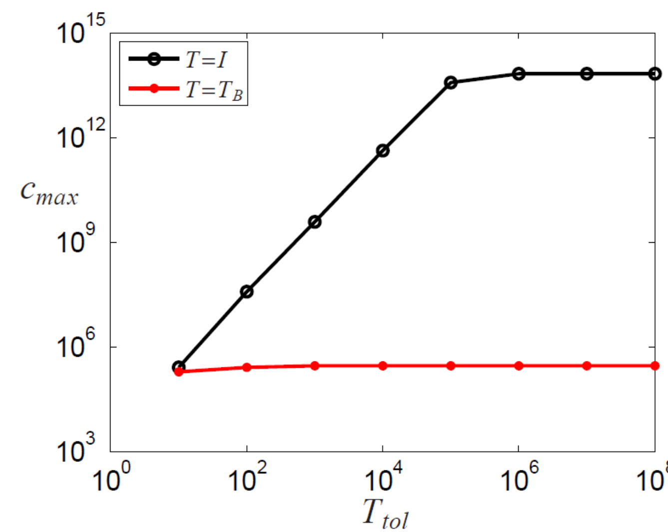
ILL-Conditioning

stabilized Lagrange multiplier formulation



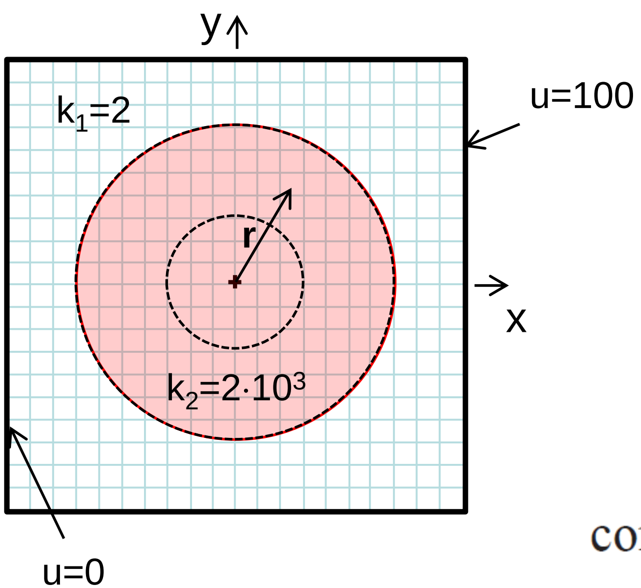


maximum condition number encountered during sweep as function of drop tolerance

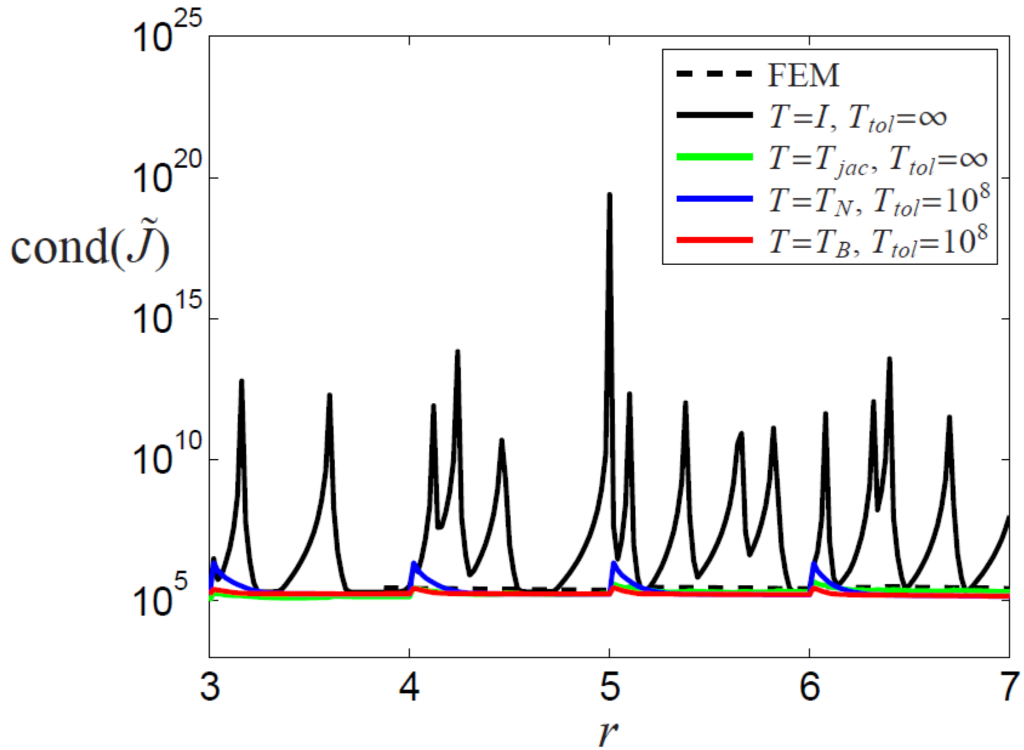


ILL-Conditioning

stabilized Lagrange multiplier formulation



condition number



Regularization

Issues:

- Checker-boarding for low-order elements
- “Irregular shapes” (i.e. wiggly boundaries)
- Many || small features
- Level set function too flat or too step
- Slow convergence of optimization process

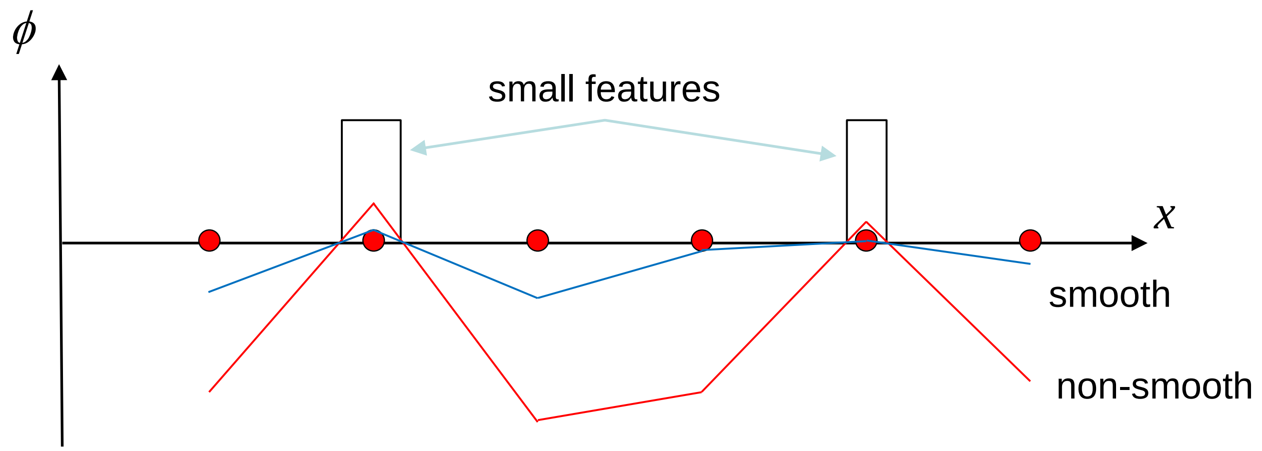
Common concepts:

- Interpolation of level set function
- Filtering of sensitivities
- Re-initialization of level set function
- Adding constraints or penalty terms (e.g. perimeter, curvature)

Note: several of the above concepts are “built-in” into augmented formulations and/or time integration schemes of the HJ equation

Regularization

Smooth level set function (e.g. by interpolation or filtering):



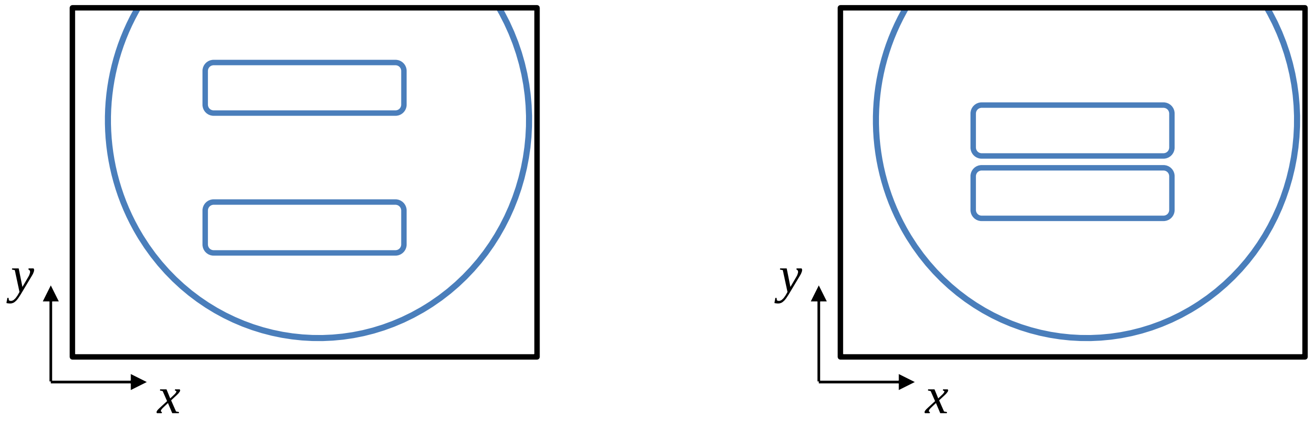
- smoothing the level set function does not suppress small features
- promotes flat level-set field

Regularization

Perimeter Constraint / Penalty:

$$P = \int_{\Gamma} dS \leq P_{\max}$$

- smoothens boundary globally; i.e. reduces wiggles and sharp corners.
- does not control local shape, e.g. thickness of members.



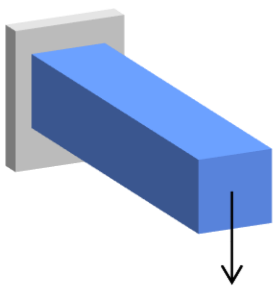
both designs have the same perimeter.

Overview

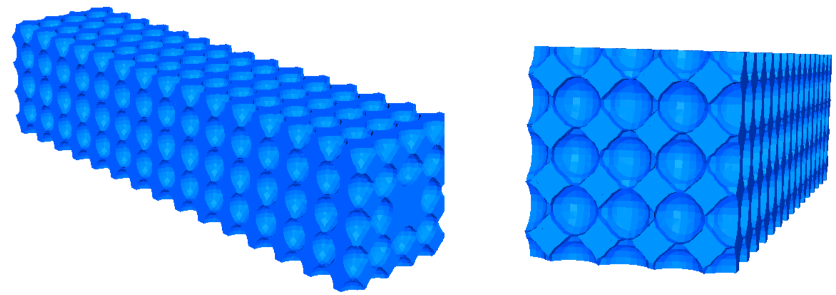
- Classical Methods in Topology Optimization
- Level-Set based Topology Optimization
  - Parameterization of Level-Set Field
  - Update Schemes
  - Numerical Modeling of Physical Response
  - Regularization Schemes
- **Case Studies**
  - Structural Mechanics
  - Conjugated Heat Transfer
- Conclusions

LSM & XFEM – Structural Design

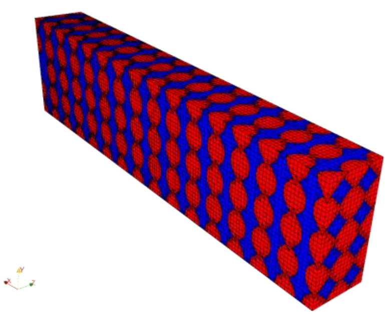
Linear Elasticity



Maximum Stiffness
for given Mass (50%)



Initial design – solid Phase



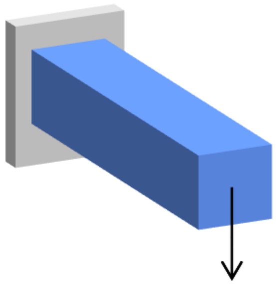
symmetry

324,448 elements

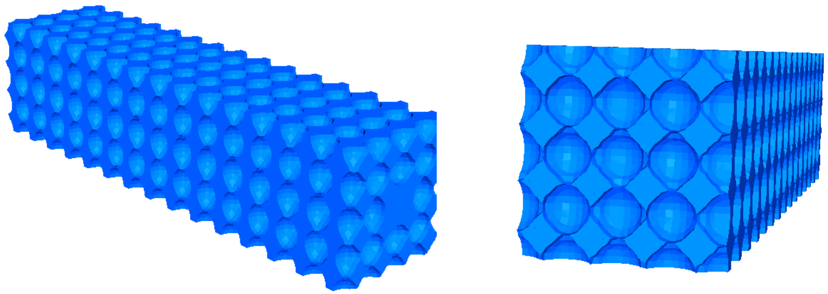
GCMMA & adjoint SA

LSM & XFEM – Structural Design

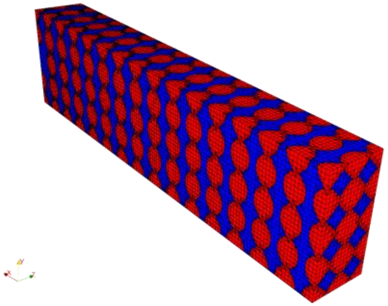
Linear Elasticity



Maximum Stiffness
for given Mass (50%)



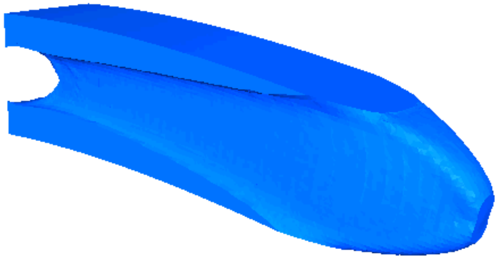
Initial design – solid Phase



symmetry

324,448 elements

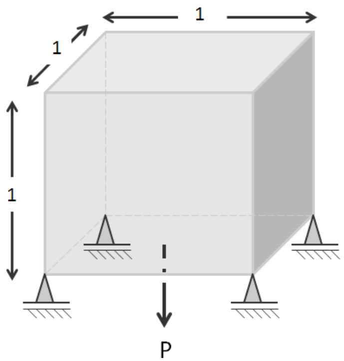
GCMMA & adjoint SA



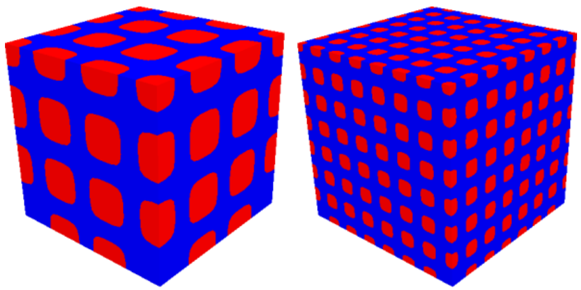
Optimized Design (deformed)

LSM & XFEM – Structural Design

Idea: from Optimization Process directly to 3D Printing

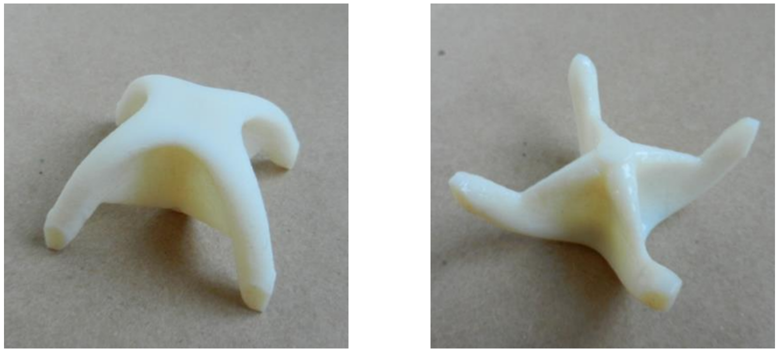


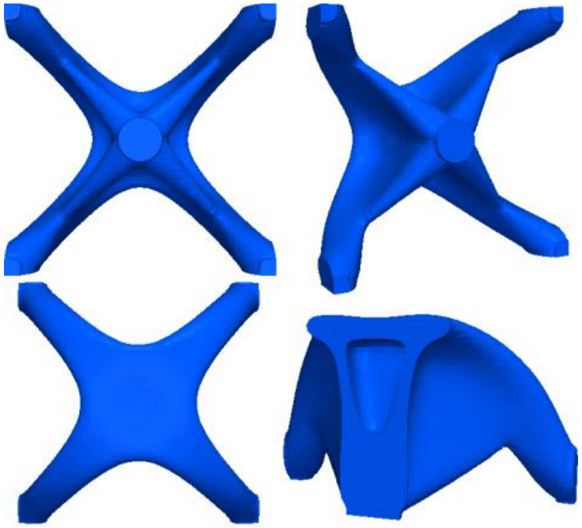
Maximum Stiffness
for given Mass (10%)



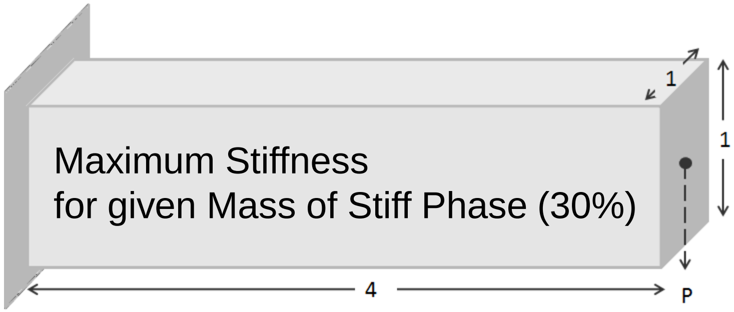
3-D Printed Structure

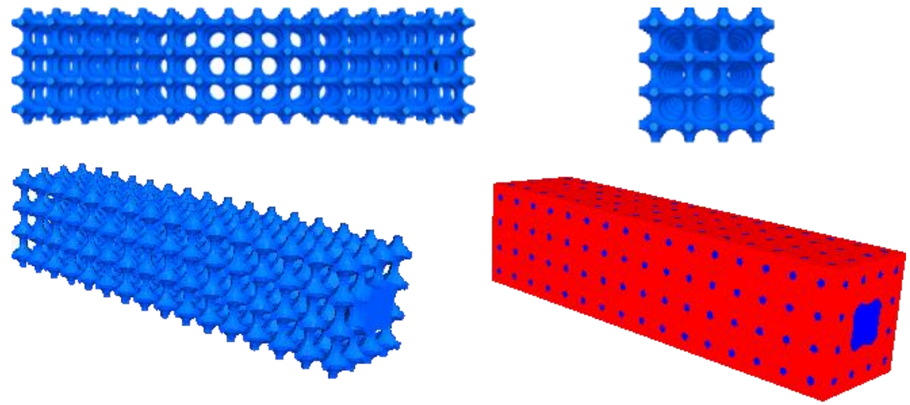
LS&XFEM

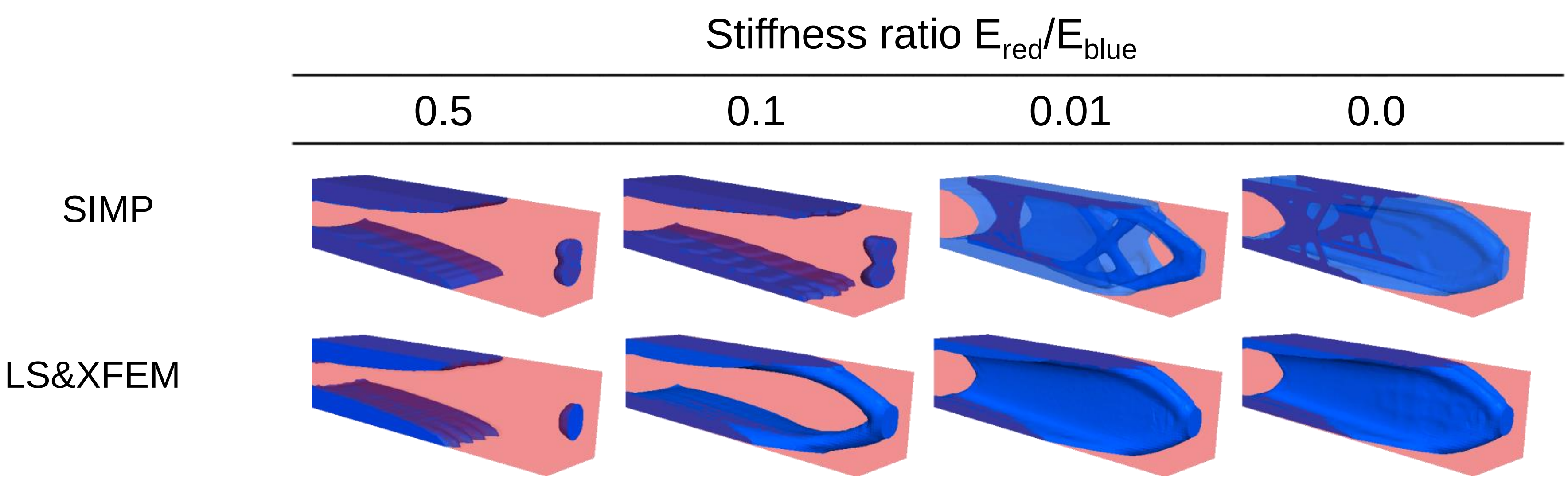




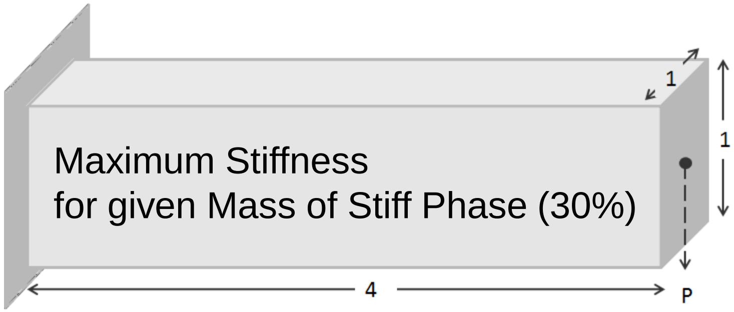
LSM & XFEM – Bi-Material Structural Design

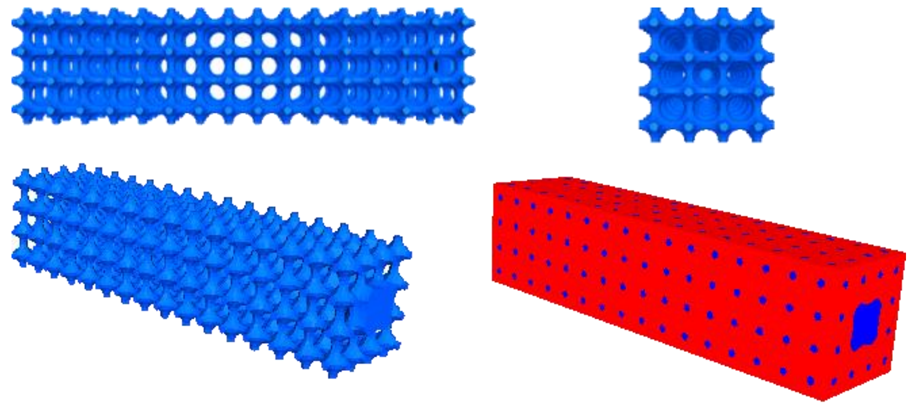




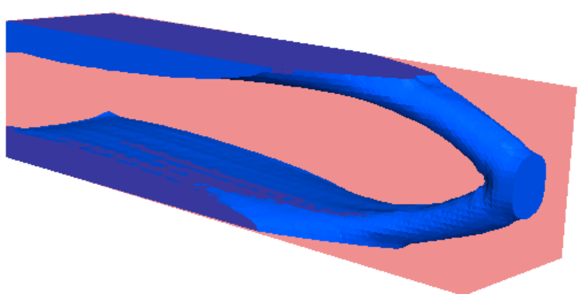


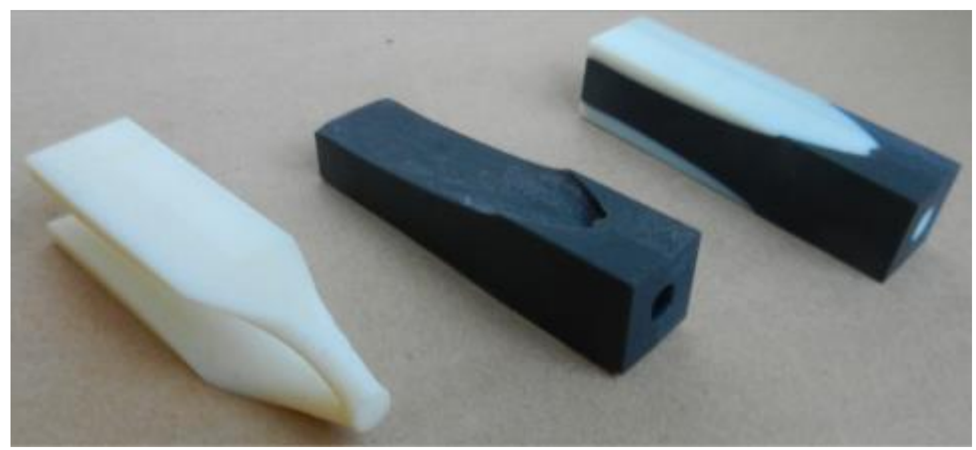
LSM & XFEM – Bi-Material Structural Design



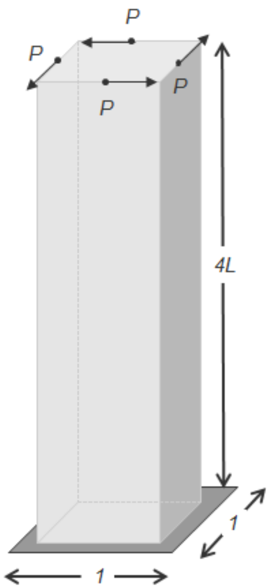


Stiffness ratio: E1/E2 = 0.1



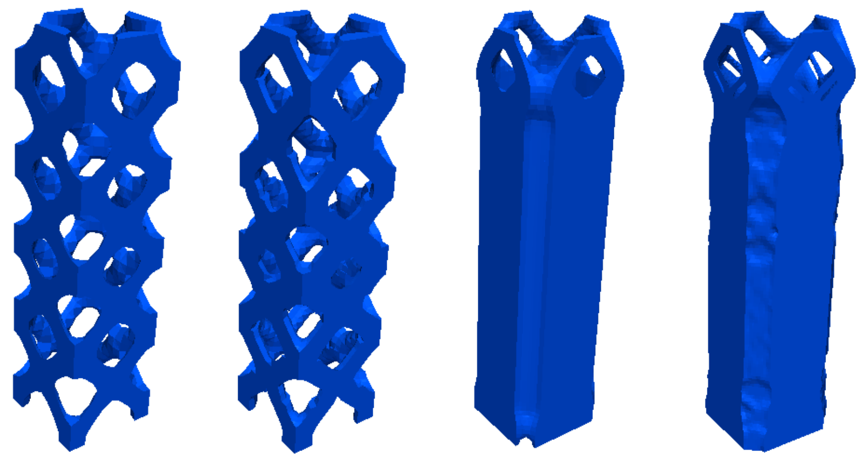


Comparison of SIMP vs. LSM & XFEM



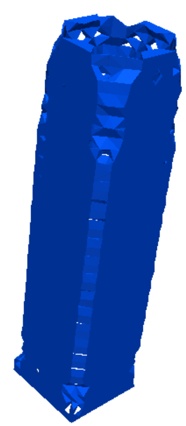
Maximum Stiffness
for given Mass (10%)

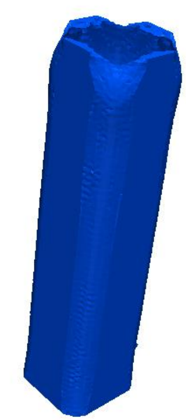
SIMP



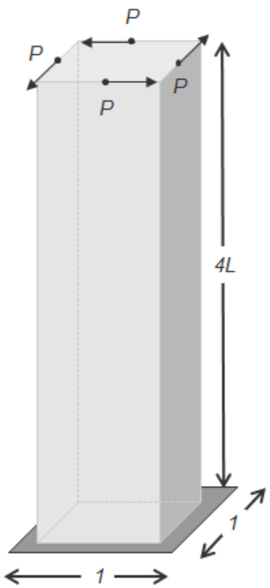
mesh refinement

LS&XFEM



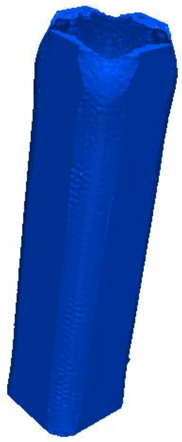


Comparison of SIMP vs. LSM & XFEM

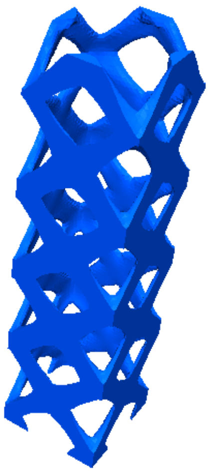


Maximum Stiffness
for given Mass (10%)

& Perimeter Constraint

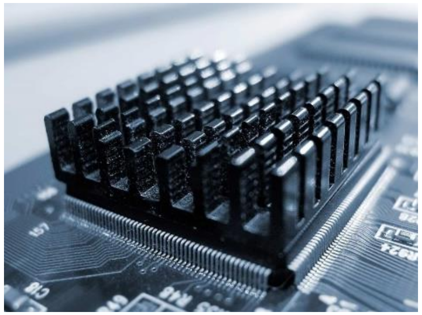


LS&XFEM

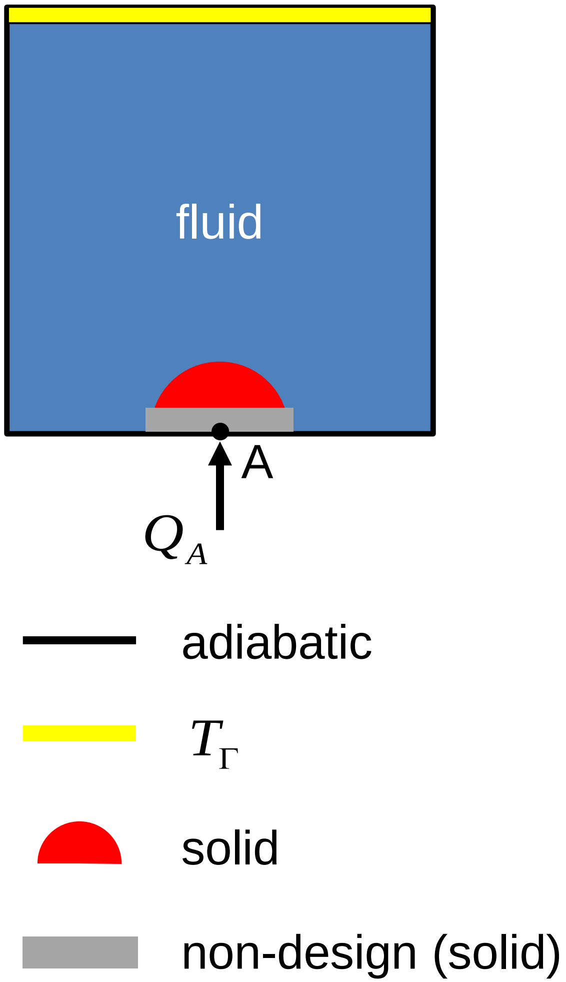


Multi-Physics Design Problems

- **Example: Conjugated Heat Transfer**



Conjugated Heat Transfer – Model Problem



Minimize Temperature T_A

s.t. constraints on solid volume

& perimeter of interface

Relevant physical phenomena:

- Heat conduction in solid
- Convection / diffusion in fluid
- Response potentially unsteady

Conjugated Heat Transfer Modeling Framework

- Diffusive heat conduction in solid

$$\rho_S c\left(\frac{\partial T_S}{\partial t}\right)-\frac{\partial}{\partial x_j}\left(k_S\ \delta_{ij}\ \frac{\partial T_S}{\partial x_i}\right)-q_s=0 \quad in\ \Omega_S$$

$$k_S \delta_{ij}\frac{\partial T_S}{\partial x_j} n_i^{SF}=Q_{SF} \quad on\ \Gamma_{FS} \qquad T=\hat{T} \quad on\ \Gamma_{ST}$$

- Convective heat transfer in fluid

$$\rho_F c_p\left(\frac{\partial T_F}{\partial t}+u_j\frac{\partial T_F}{\partial x_j}\right)-\frac{\partial}{\partial x_j}\left(k_F\ \delta_{ij}\ \frac{\partial T_F}{\partial x_i}\right)-q_F=0 \quad in\ \Omega_S$$

$$k_F \delta_{ij}\frac{\partial T_F}{\partial x_j} n_i^{FS}=Q_{FS} \quad on\ \Gamma_{FS} \qquad T=\hat{T}_F \quad on\ \Gamma_{FT}$$

Conjugated Heat Transfer Modeling Framework

- Flow model: Incompressible Navier- Stokes

$$\rho_F \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial \sigma_{ij}}{\partial x_j} - \rho_F g_i = 0 \qquad \sigma_{ij} = -p\, \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

Boussinesq approximation $\rho_F g_i = \rho_0 \, g \left(1 - \alpha \left(T - T_0\right)\right) n_i$

incompressibility $\rho_F = \rho_0 = const : \quad \frac{\partial u_i}{\partial x_i} = 0$

Conjugated Heat Transfer Modeling Framework

- Interface conditions

Continuity of velocities: $u_i = 0 \quad on \quad \Gamma_{FS}$

Conservation of heat flux: $Q_{FS} + Q_{SF} = 0 \quad on \quad \Gamma_{FS}$

(1) Continuity of temperature: $T_F - T_S = 0 \quad on \quad \Gamma_{FS}$

$$\rightarrow k_F \delta_{ij} \frac{\partial T_F}{\partial x_j} n_i^{FS} + k_S \delta_{ij} \frac{\partial T_S}{\partial x_j} n_i^{SF} = 0$$

(2) Jump in temperatures: $T_F - T_S \neq 0 \quad on \quad \Gamma_{FS}$

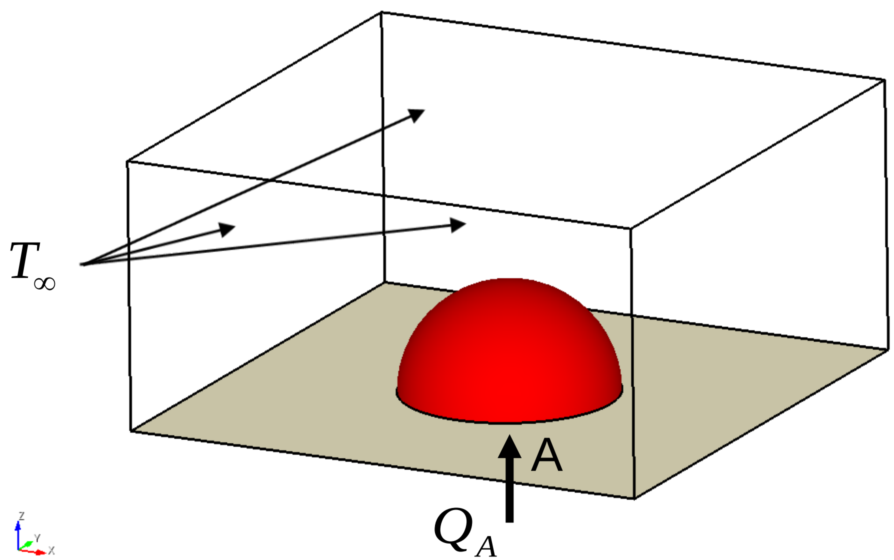
$$\rightarrow k_F \delta_{ij} \frac{\partial T_F}{\partial x_j} n_i^{FS} = h\left(T_S - T_F\right)$$

$$\rightarrow k_S \delta_{ij} \frac{\partial T_S}{\partial x_j} n_i^{SF} = h\left(T_F - T_S\right)$$

Simple Model: Newton's Law of Cooling

$$Q_{SF} = h\left(T_F - T_S\right)$$

- Fluid: isothermal $T_F = T_\infty$ T_∞ : ambient temperature
- Fluid: diffusive medium
  - convection ignored
  - ambient temperature prescribed at design domain boundaries

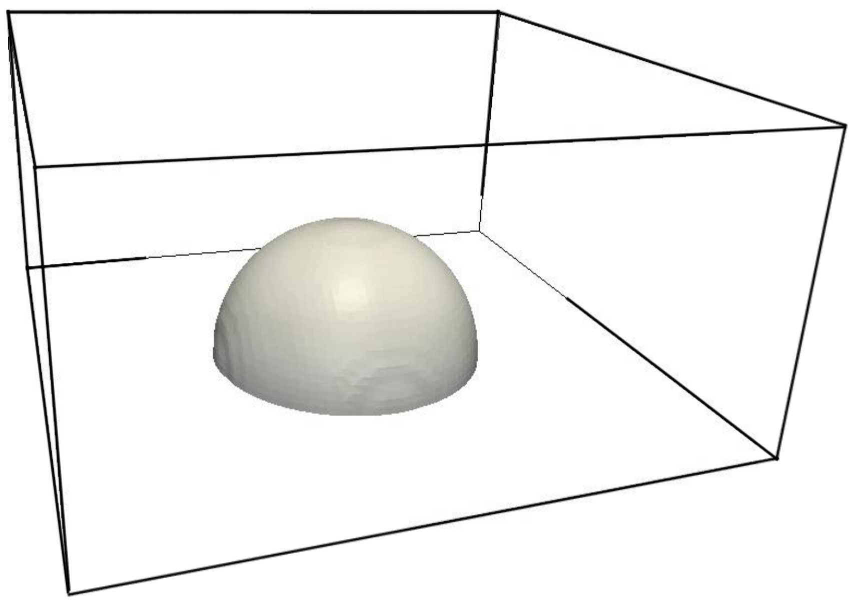


Minimize Temperature T_A

s.t. constraints on solid volume

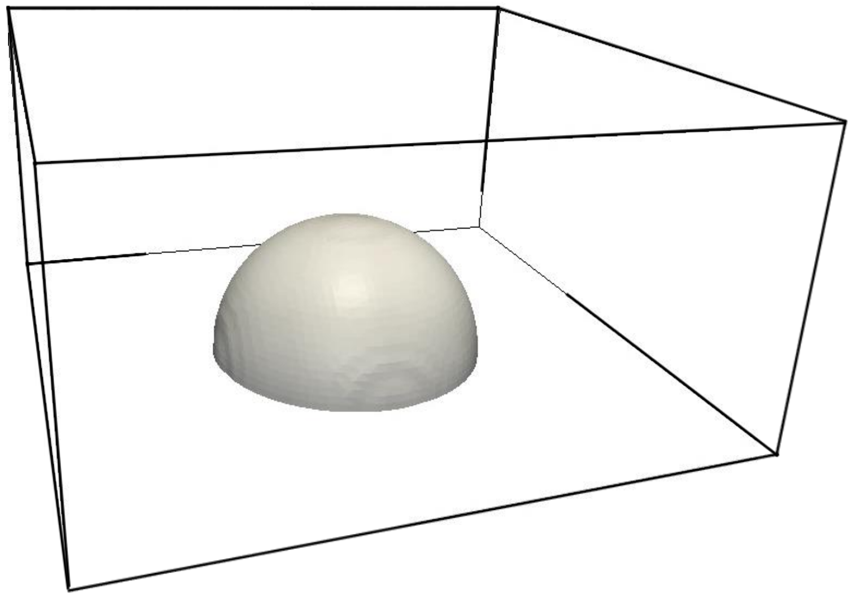
& perimeter of interface

Conjugated Heat Transfer – Simple Model



- Fluid: isothermal

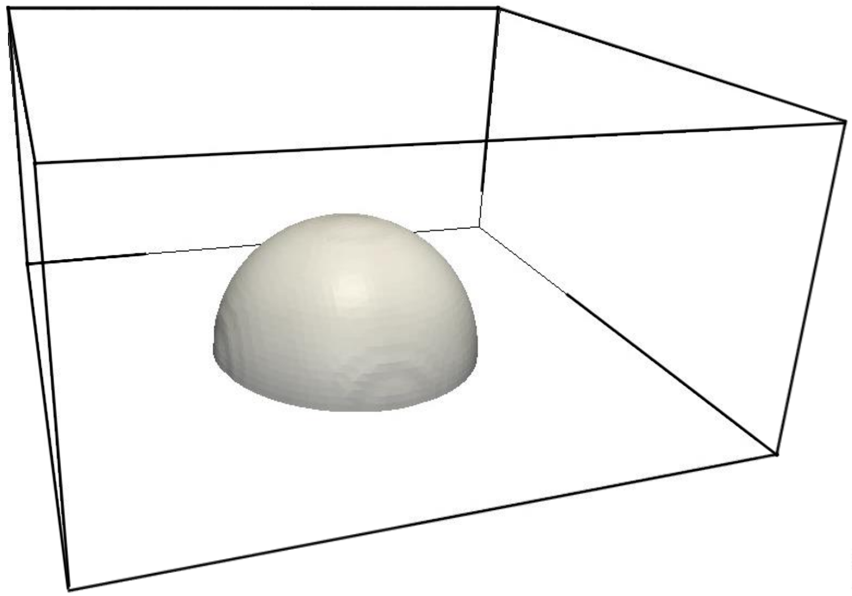
Conjugated Heat Transfer – Simple Model



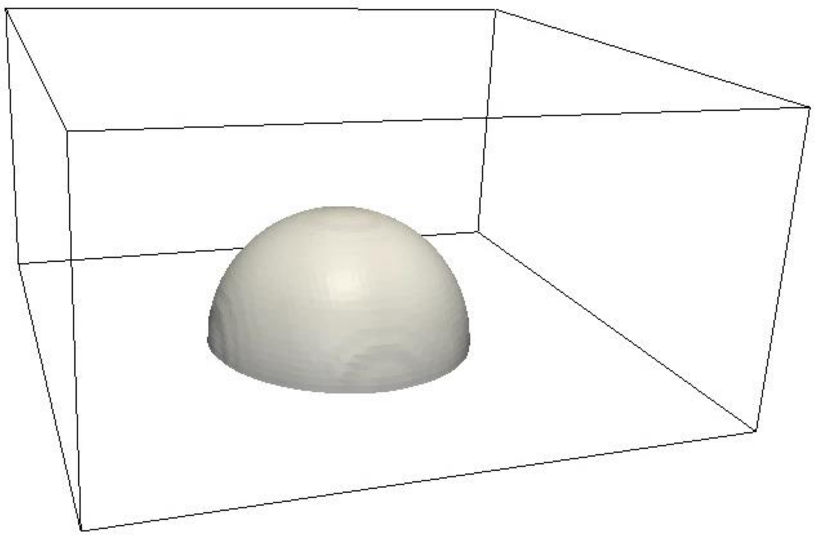
- Fluid: isothermal

slice through final design

Conjugated Heat Transfer – Simple Model



- Fluid: isothermal



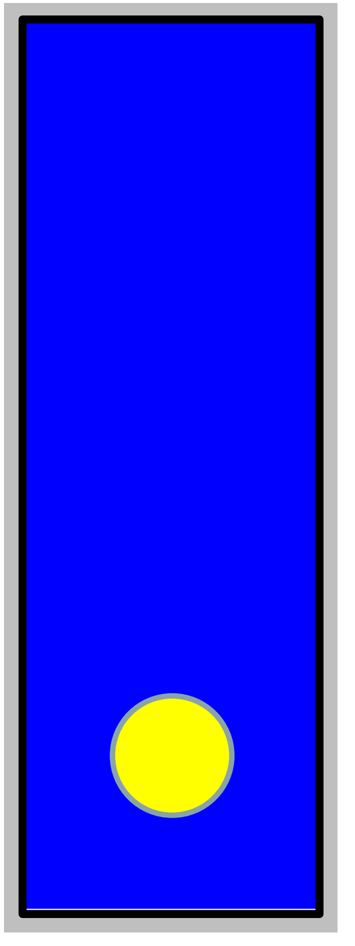
- Fluid: diffusive medium

Conjugated Heat Transfer with Natural Convection Flow

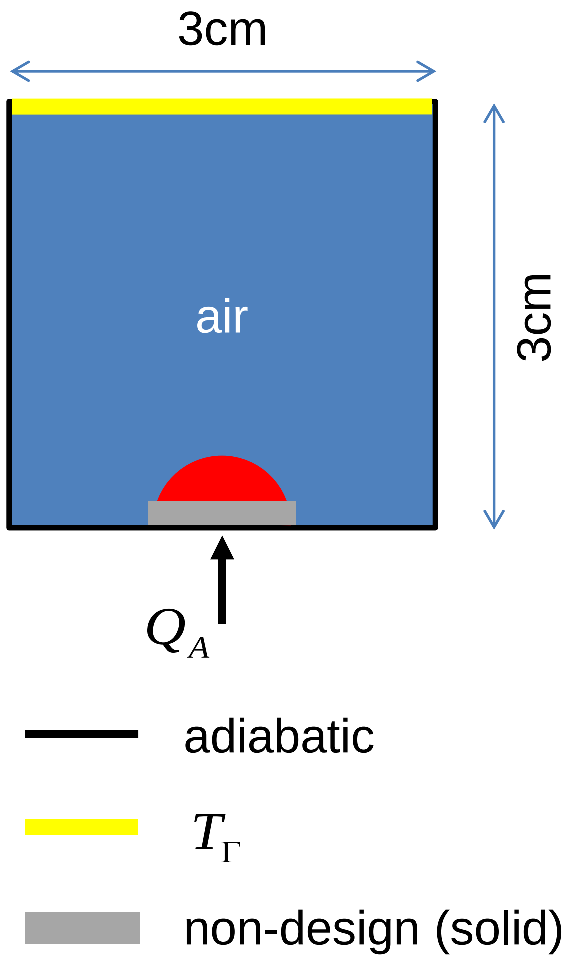
Numerical framework:

- fluid: SUPG/PSPG stabilized formulation
- solid: SUPG stabilized formulation
- generalized Heaviside enrichment along interface
- stabilized Lagrange multiplier interface formulation

  for stick condition and temperature continuity



Natural Convection Example



Minimize Temperature T_A at steady state

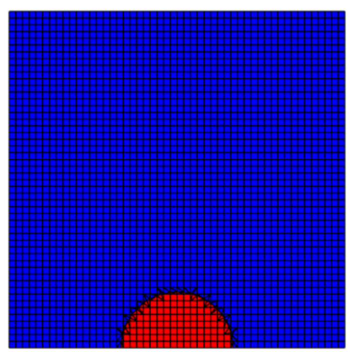
s.t. constraint on solid volume (< initial)

& perimeter (< 2 × initial)

symmetry of design enforced

case 1: $Q_A = 0.05\,W$

case 2: $Q_A = 0.25\,W$



steady-state assumed

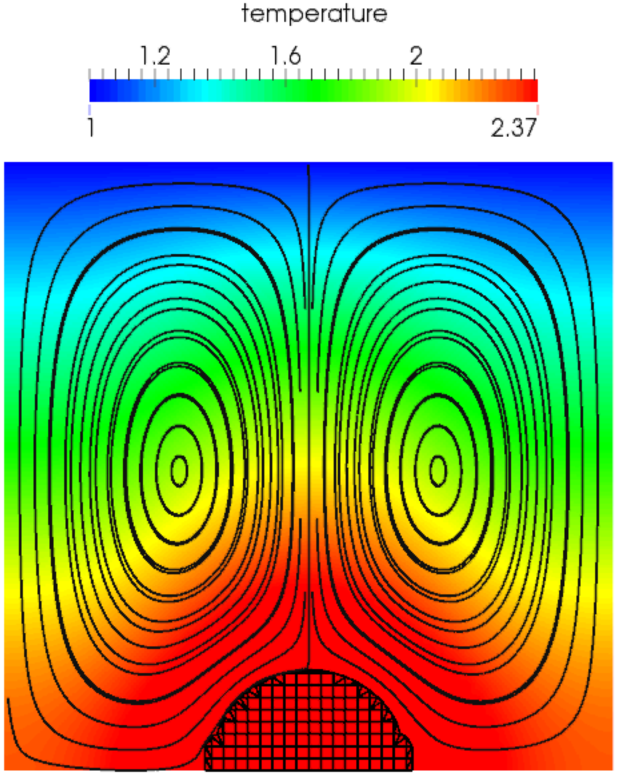
response predicted on full mesh

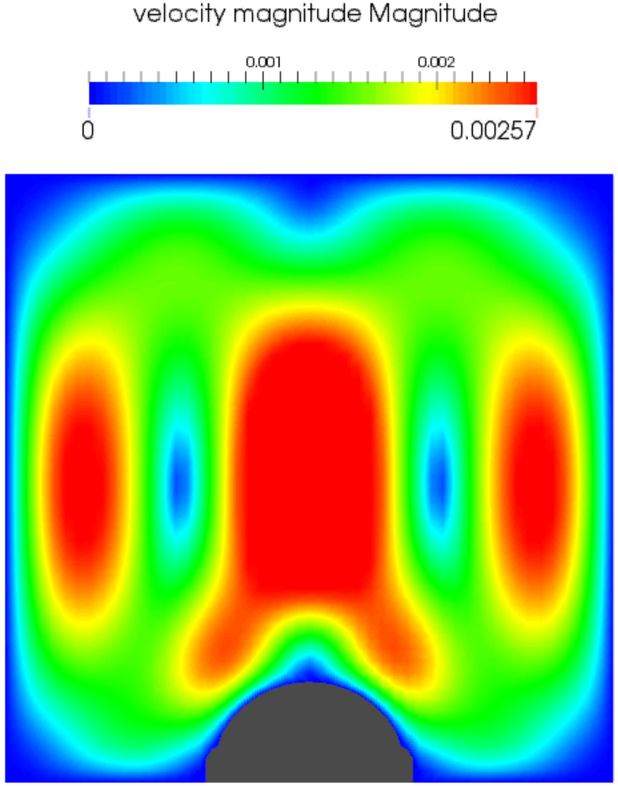
50x50 elements

GCMMA & adjoint SA

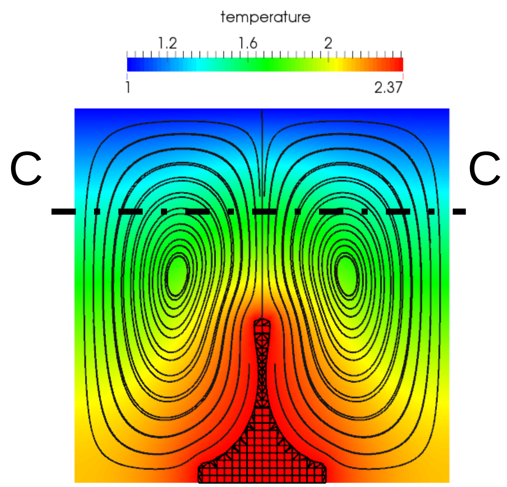
Case 1: Low Heating

$$Q_A = 0.05\,W$$



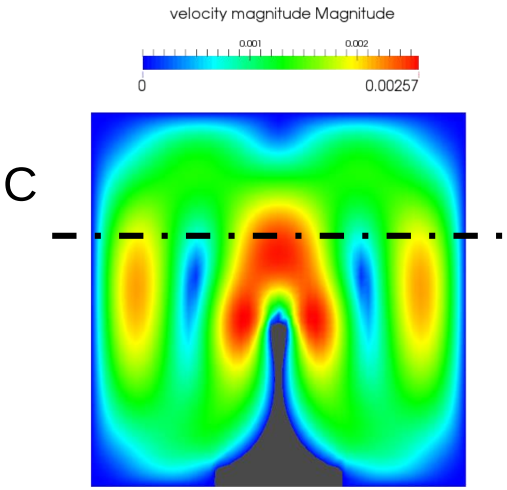


Case 1: Low Heating

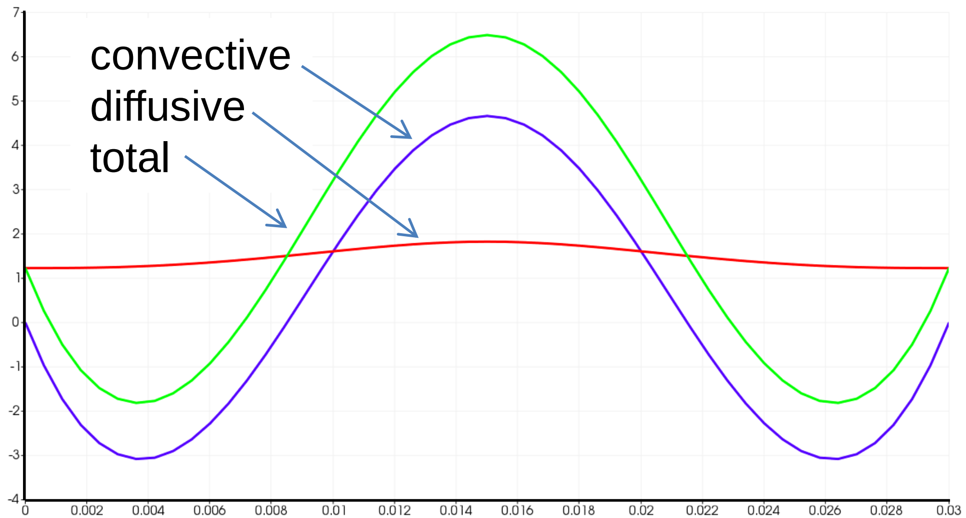


$$Q_A = 0.05\,W$$

$$\boxed{\begin{aligned} \mathrm{Re} &= 6.15 \\ Ra &= 4680 \end{aligned}}$$

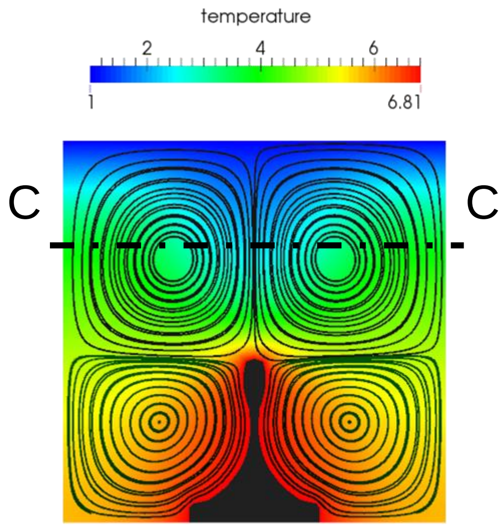


heat flux C-C



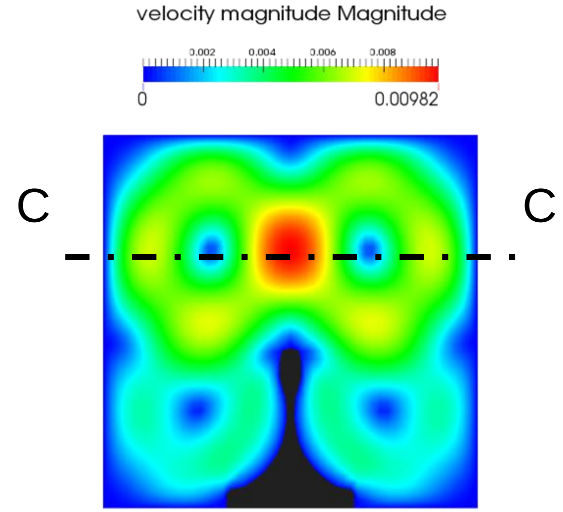
no significant change during optimization

Case 2: Strong Heating

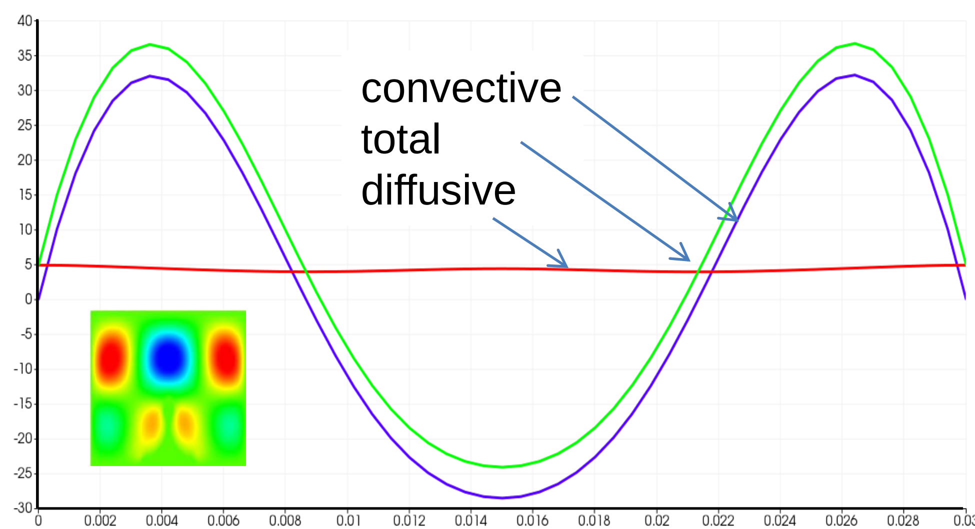


$$Q_A = 0.25\,W$$

$$\boxed{\begin{aligned} \mathrm{Re} &= 23.50 \\ Ra &= 19.840 \end{aligned}}$$

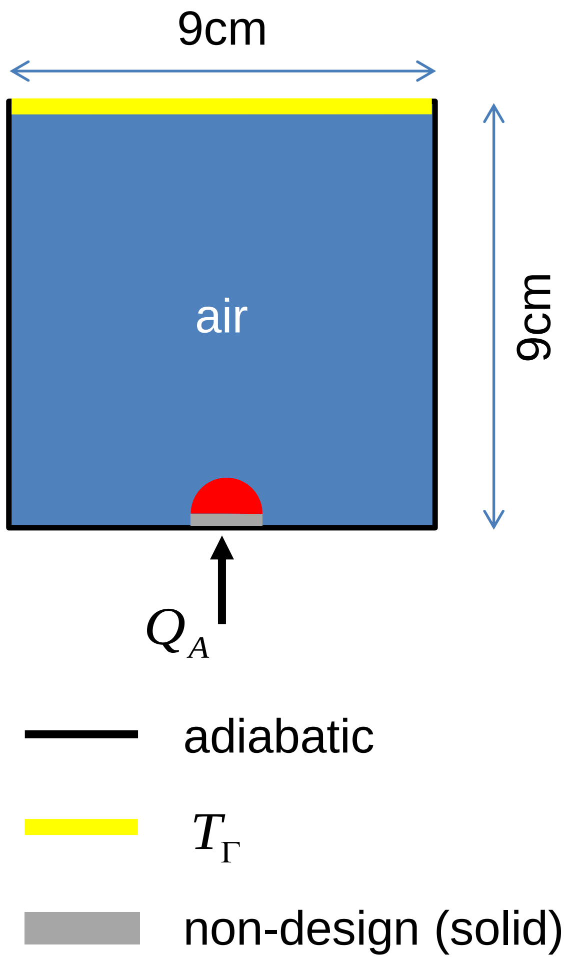


heat flux C-C



higher flux values for optimized design but similar shape

Unsteady Example



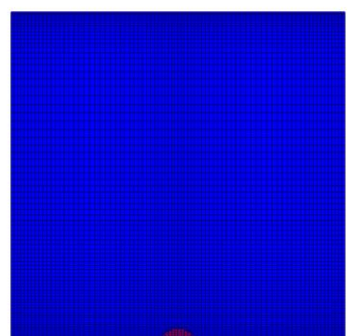
Minimize Mean Temperature T_A after 40 Minutes

s.t. constraint on solid volume (< initial)

& perimeter (< 2 × initial)

symmetry of design enforced

$$Q_A = 50\,W$$



transient response computed

by BDF-2 (2500 time steps)

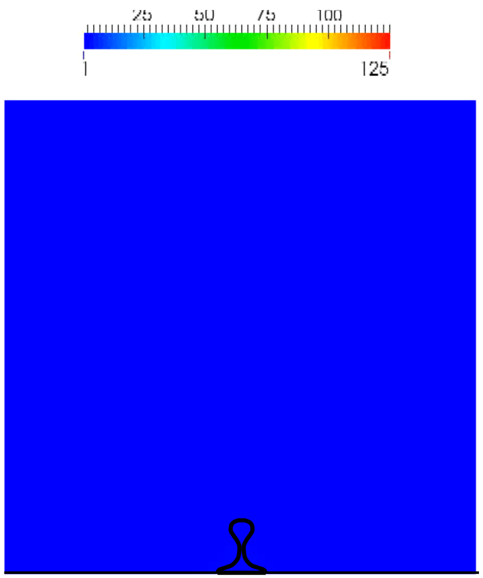
12,896 elements

GCMMA & adjoint SA

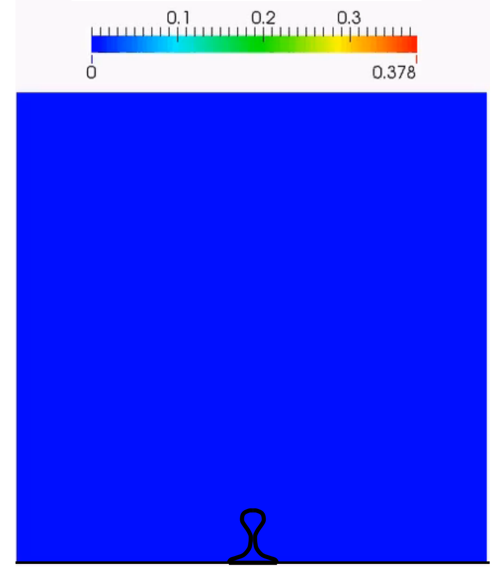
Unsteady Example

Transient Analysis
of
Optimized Design

temperature

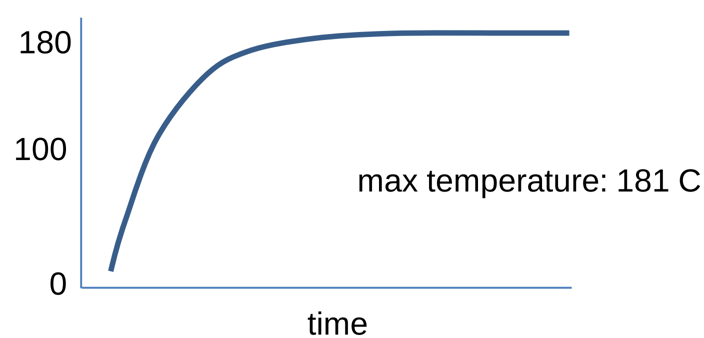


velocity magnitude

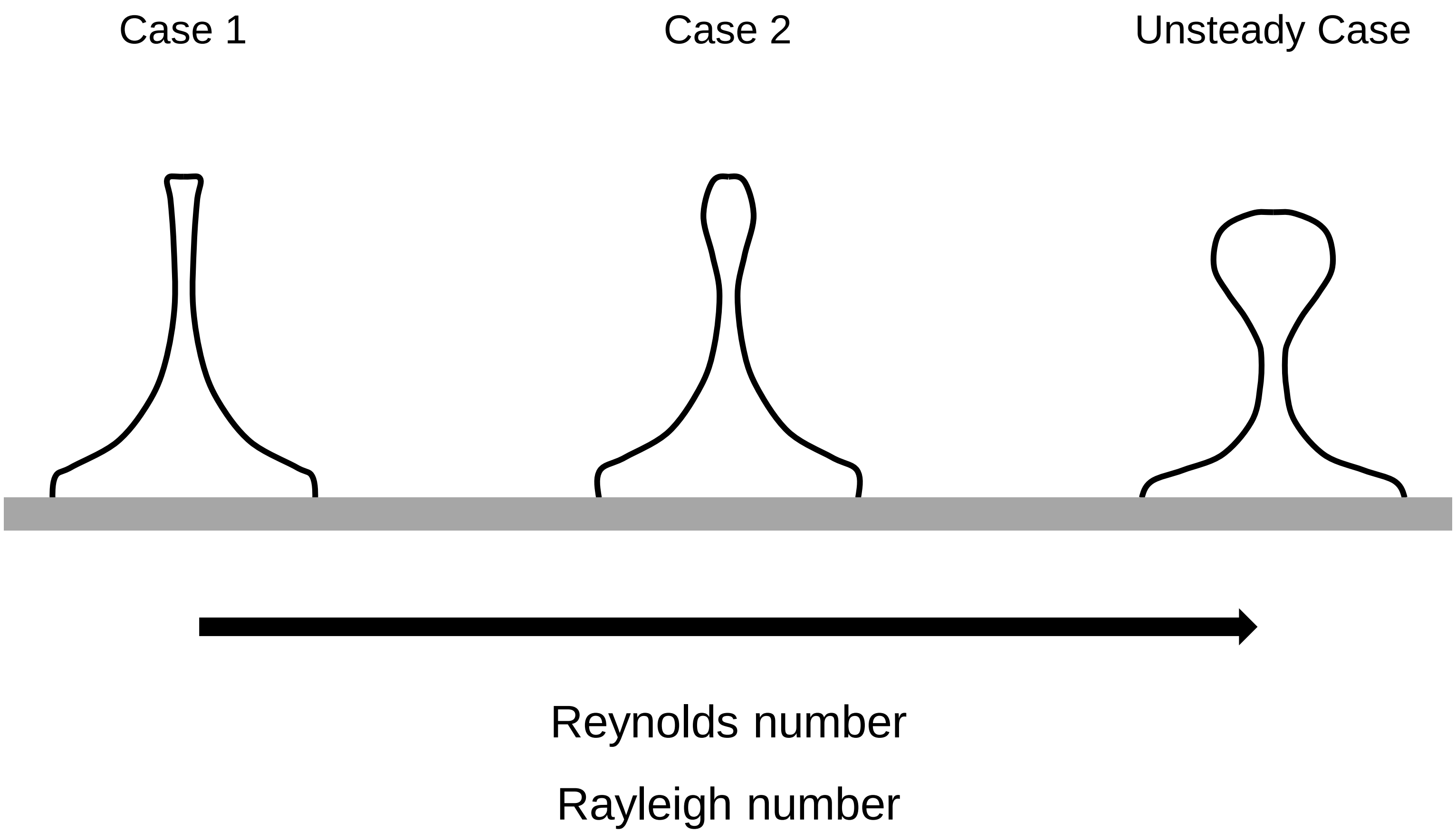


$$\mathrm{Re} = 2,6915$$
$$Ra = 1.67 \cdot 10^7$$

Evolution of temperature at point A

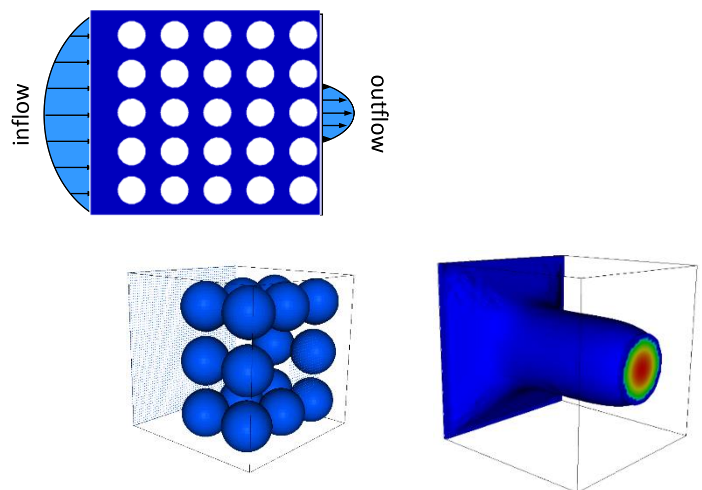


Comparison of Geometries



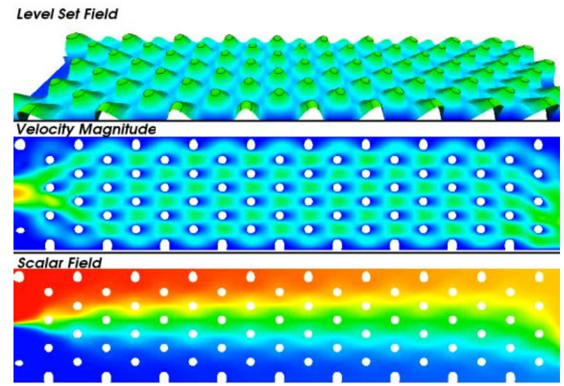
LSM & XFEM Optimization of Flow Problems

Fluid Mechanics:



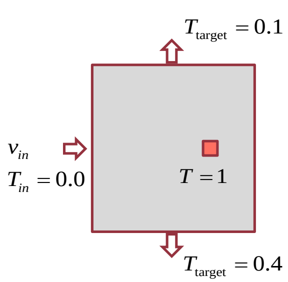
diffuser designs

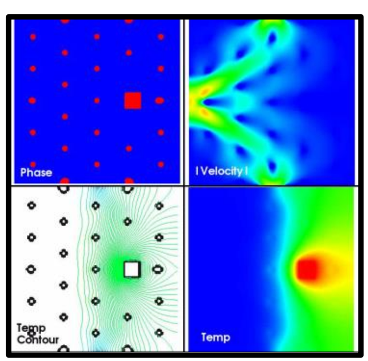
Forced Convection:

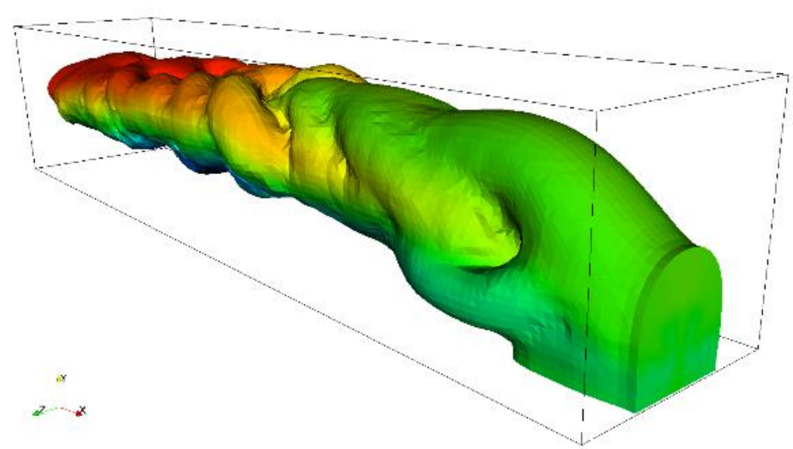


micro-mixer designs

Forced Convection:







Conclusions

LSM & XFEM Topology Optimization:

- a promising alternative to SIMP:
  - no need for material interpolation & penalization
  - higher geometry resolution on given mesh
- requires a robust and versatile XFEM formulation
- features great algorithmic and implementation complexity

Open Issues:

- stable enforcement of boundary conditions for nonlinear, non-elliptic problems
- regularization methods for local shape control
- overcoming discontinuities in shape evolution