Boolean Operation

Some operations of ordinary algebra, in particular,multiplication xy, addition x + y, and negation x, have their counterparts in Boolean algebra. Those operations are :-

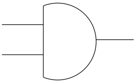
- **AND**
- **OR**
- **NOT**

Basic Boolean operators

- AND - It's symbol is and it is used as multiplication counterpart of linear algebra. It is called conjunction in boolean language.
- OR - It's symbol is and it is used as addition counterpart of linear algebra. It is called disjunction in boolean language.
- NOT - Its symbol is and !and it is used as negation counterpart of linear algebra. It is called negation or complement in boolean language.

AND operator

Figure: The distinctive shape of AND gate



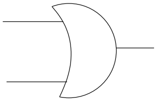
The AND gate is a basic digital logic gate that implements logical conjunction. A HIGH output(1) results only if both the inputs to the AND gate are HIGH (1). If neither or only one input to the AND gate is HIGH, a LOW output results.

input in A	input in B	output(A AND B)
0	0	0
0	1	0
1	0	0
1	1	1

Table: showing the result of AND operator for different values of A and B

OR operator

Figure: The distinctive shape of OR gate



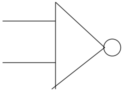
The OR gate is a digital logic gate that implements logical disjunction. A HIGH output (1) results if one or both the inputs to the gate are HIGH (1). If neither input is HIGH, a LOW output (0) results.

input in A	input in B	output(A AND B)
0	0	0
0	1	1
1	0	1
1	1	1

Table: showing the result of OR operator for different values of A and B

NOT operator

Figure: The distinctive shape of NOT gate



A NOT gate is a logic gate which implements logical negation.

input in A	output in A
0	1
1	0

Table: showing the result of OR operator for different values of A and B

De Morgan's Laws in Boolean Algebra

First Law

The negation of a conjunction is the disjunction of the negations.

Second Law

The negation of a disjunction is the conjunction of the negations.

The two laws can be stated in mathematical statement or boolean language for two boolean values P and Q as :-

$\neg(P \wedge Q) \iff (\neg P) \vee (\neg Q)$

$\neg(P \vee Q) \iff (\neg P) \wedge (\neg Q)$

First Law

The negation of a conjunction is the disjunction of the negations.

Proof.

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$\neg(P) \vee \neg(Q)$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

From the table, it can be easily seen that
$\neg(P \wedge Q) \iff (\neg P) \vee (\neg Q)$

$\square$

Second Law

The negation of a disjunction is the conjunction of the negations.

Proof.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

From the table, it can be easily seen that
$\neg(P \vee Q) \iff (\neg P) \wedge (\neg Q)$

$\square$

De Morgan's laws for n variables A_i

For n variables ,De Morgan's laws are genrally written as:
$\overline{\bigcap_{i \in I} A_i} \equiv \bigcup_{i \in I} \overline{A_i}$
$\overline{\bigcup_{i \in I} A_i} \equiv \bigcap_{i \in I} \overline{A_i}$
where I is the set of n variables A_i