

Short packets over a massive random-access channel

Giuseppe Durisi

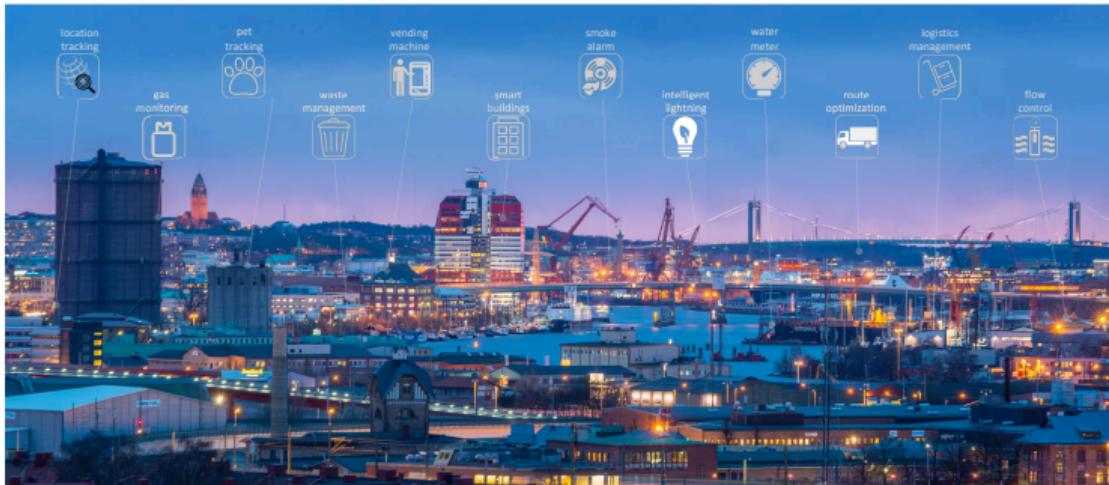
Chalmers, Sweden

LSIT, May, 2023



Joint work with H. K. Ngo, A. Lancho, A. Graell i Amat, P. Popovski, A. Kalør,
B. Soretz

Wireless connectivity enables new services



source: IoTpool

Challenges

- 🔧 Collect data from a **massive number** of **low-cost sensors**
- 🔧 Communicate **reliably critical** information

Massive and critical wireless connectivity

massive machine-type comm. (**mMTC**)

- Uplink mostly
- High energy efficiency
- Great commercial interest
- LPWAN, satellite

ultra-reliable low-latency comm. (**URLLC**)

- Bidirectional
- Low latency, high reliability
- Limited commercial interest (so far)
- Private 5G network

Some characteristics

mMTC

- Small information payload (100 bits)
- High user density (10^7 devices/Km²)
- Sporadic TX (less than 1 per minute)
⇒ 120 dof per user at $B = 20$ MHz

URLLC

- Small information payload (100 bits)
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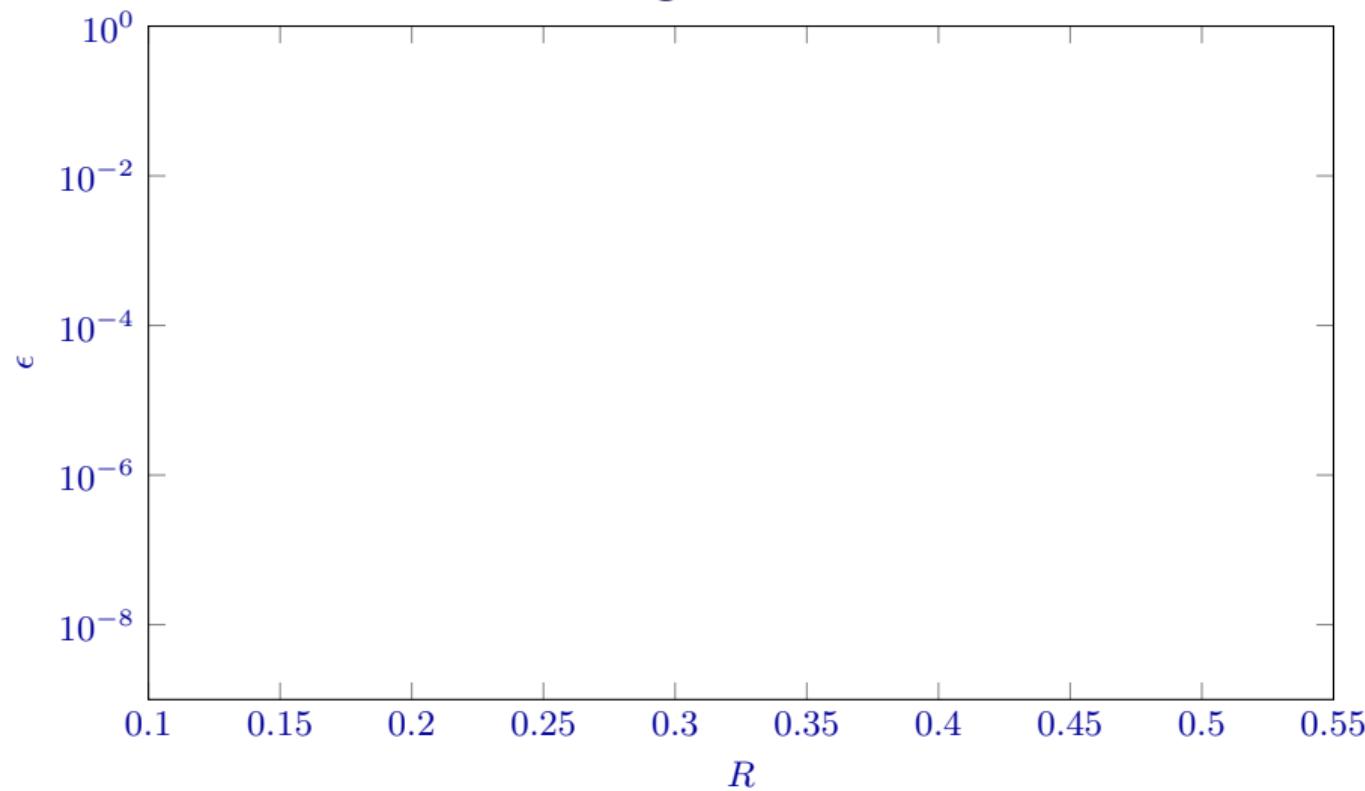
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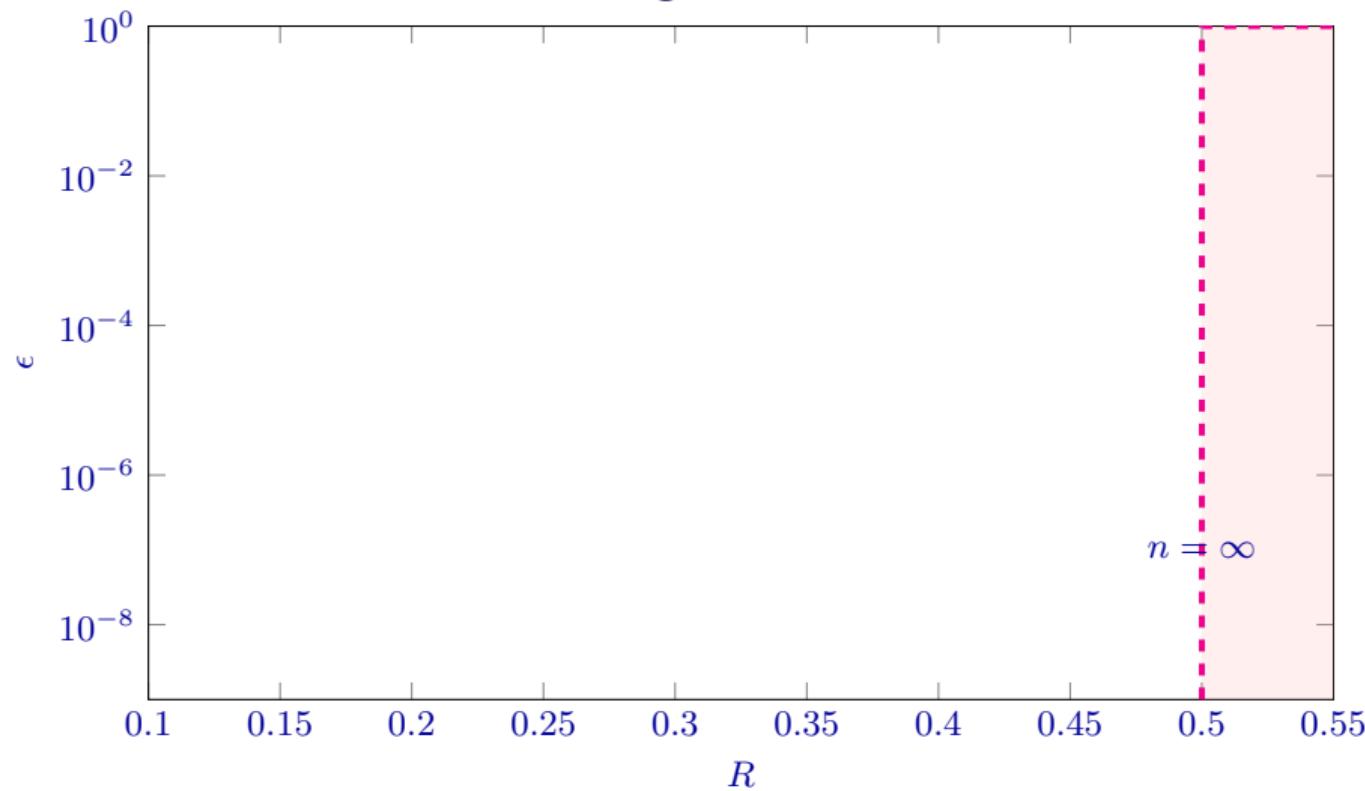
Key design tool

Finite-blocklength information theory

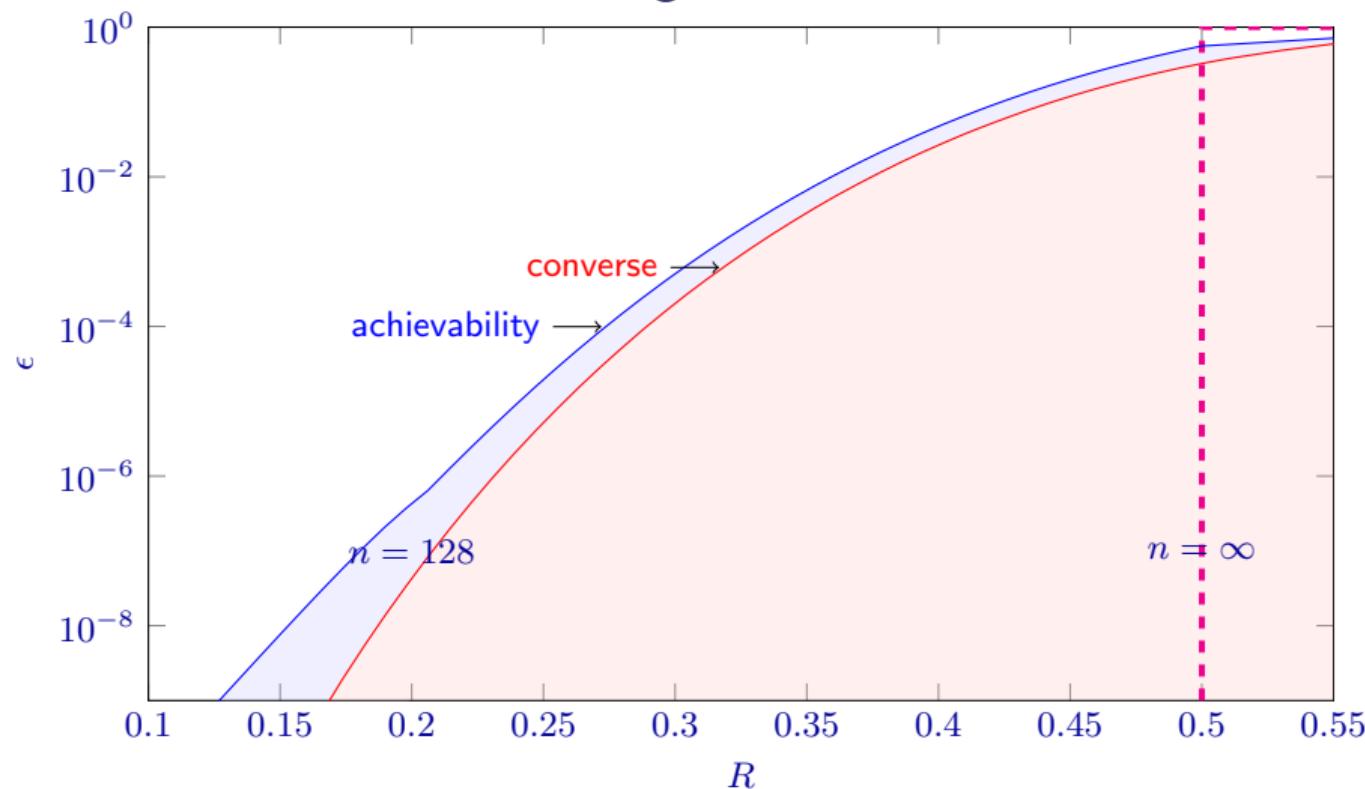
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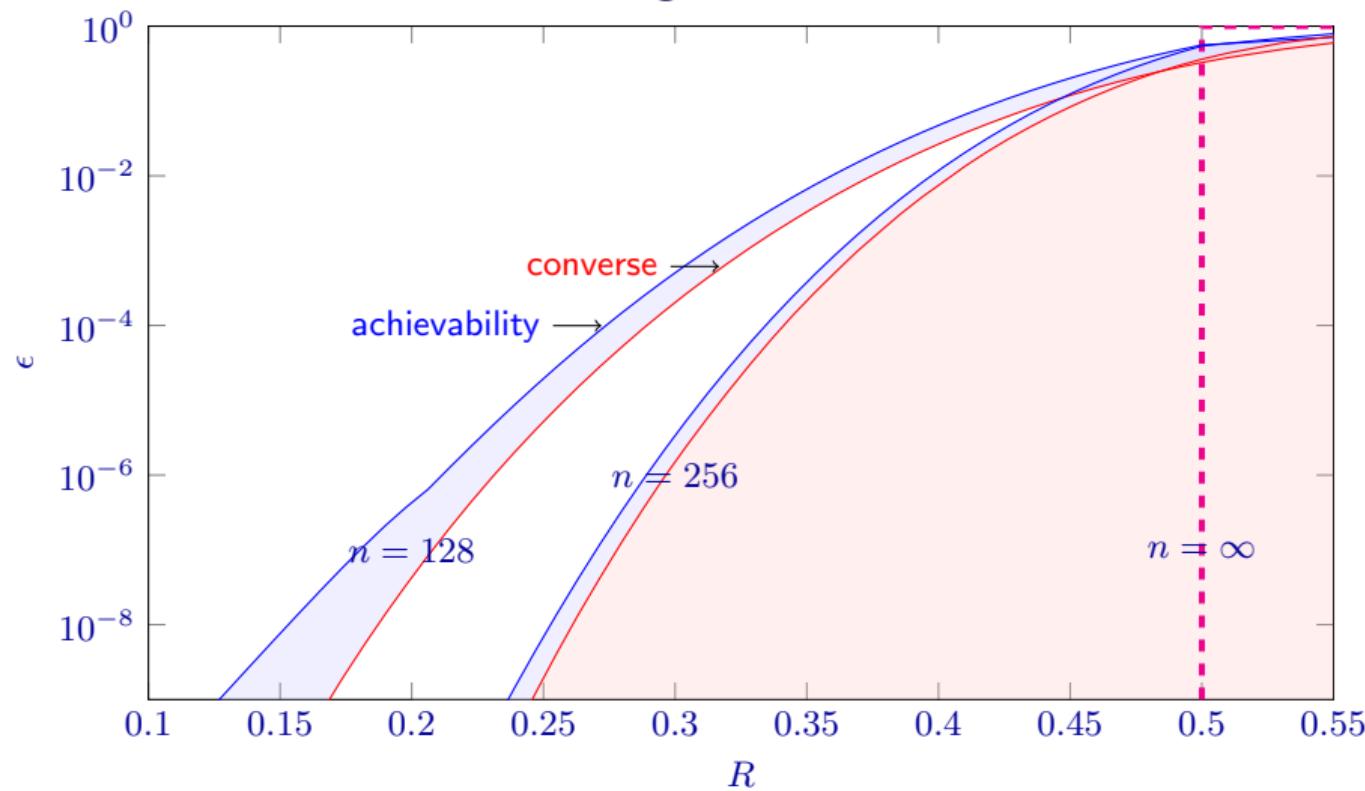
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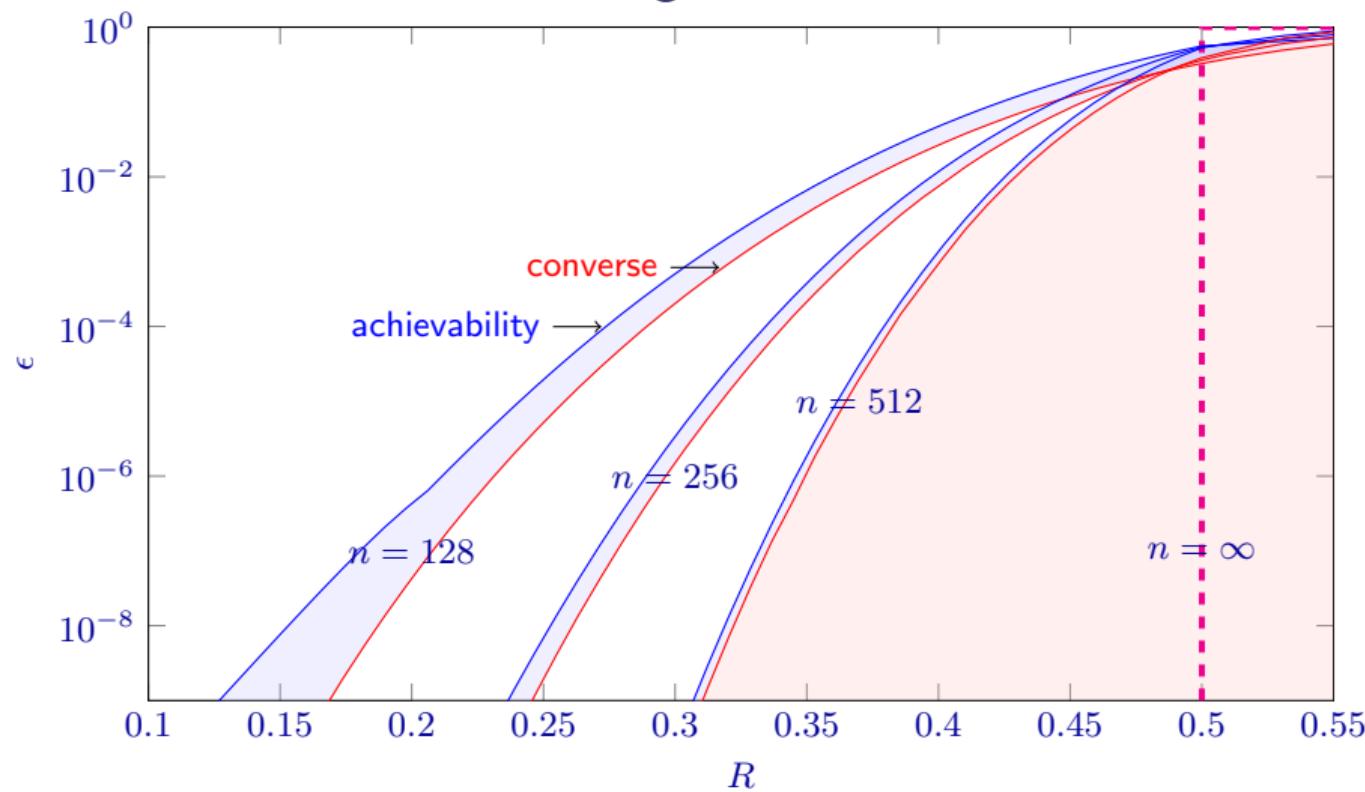
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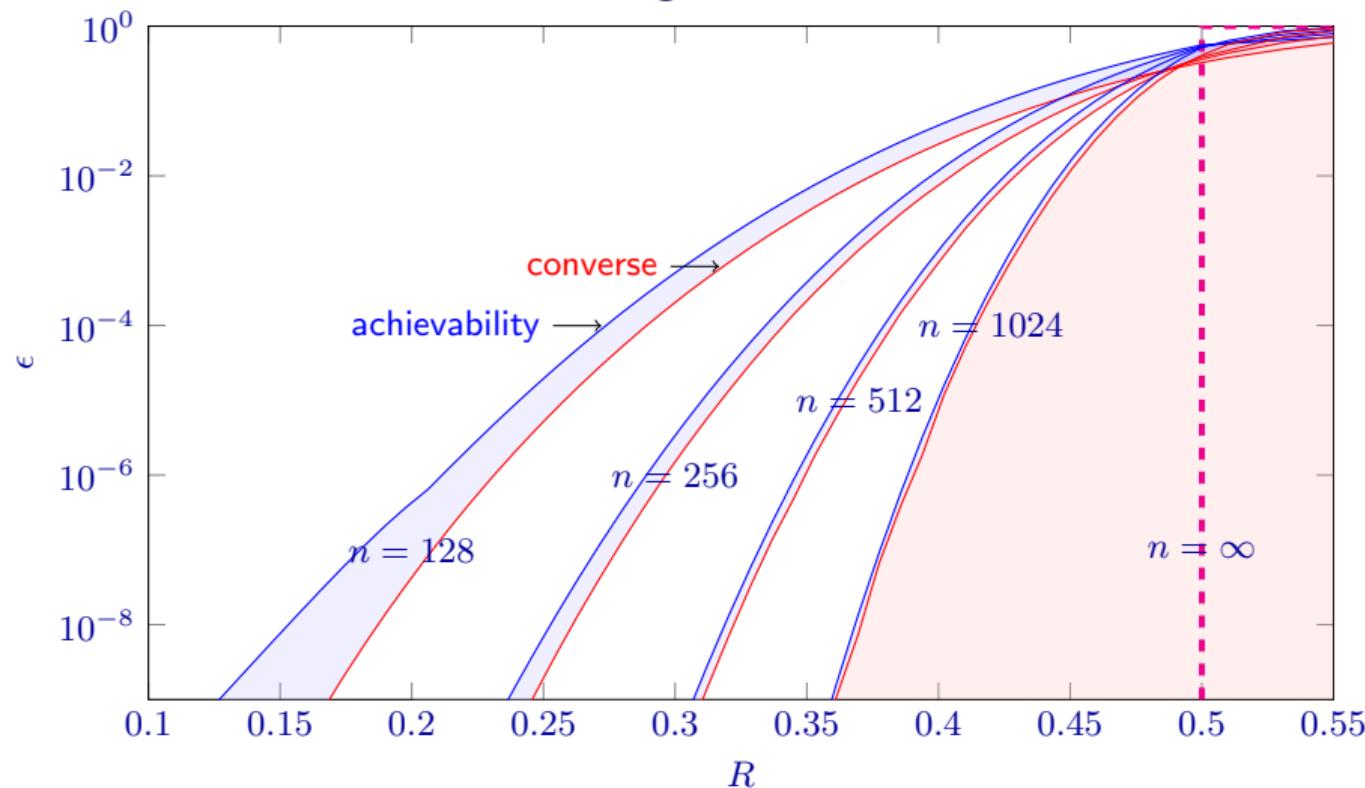


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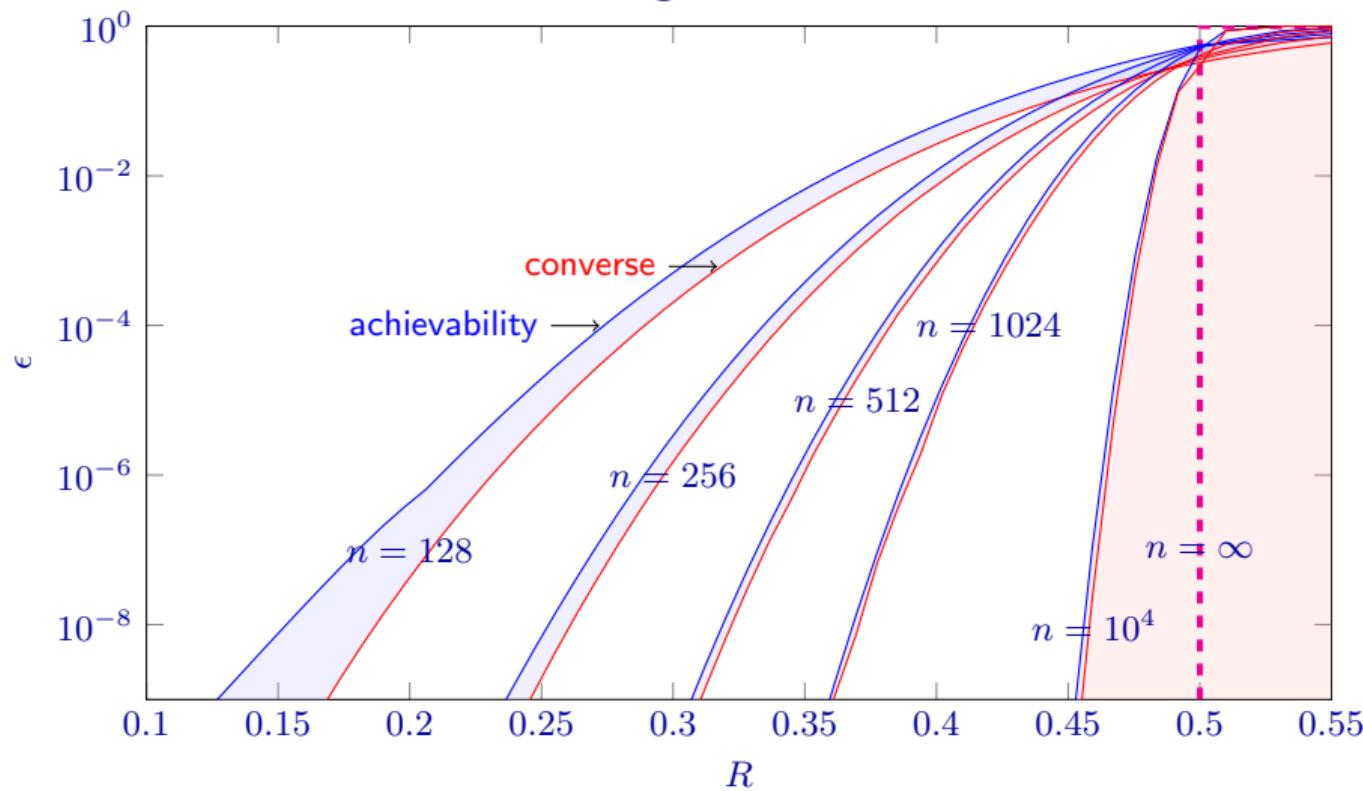


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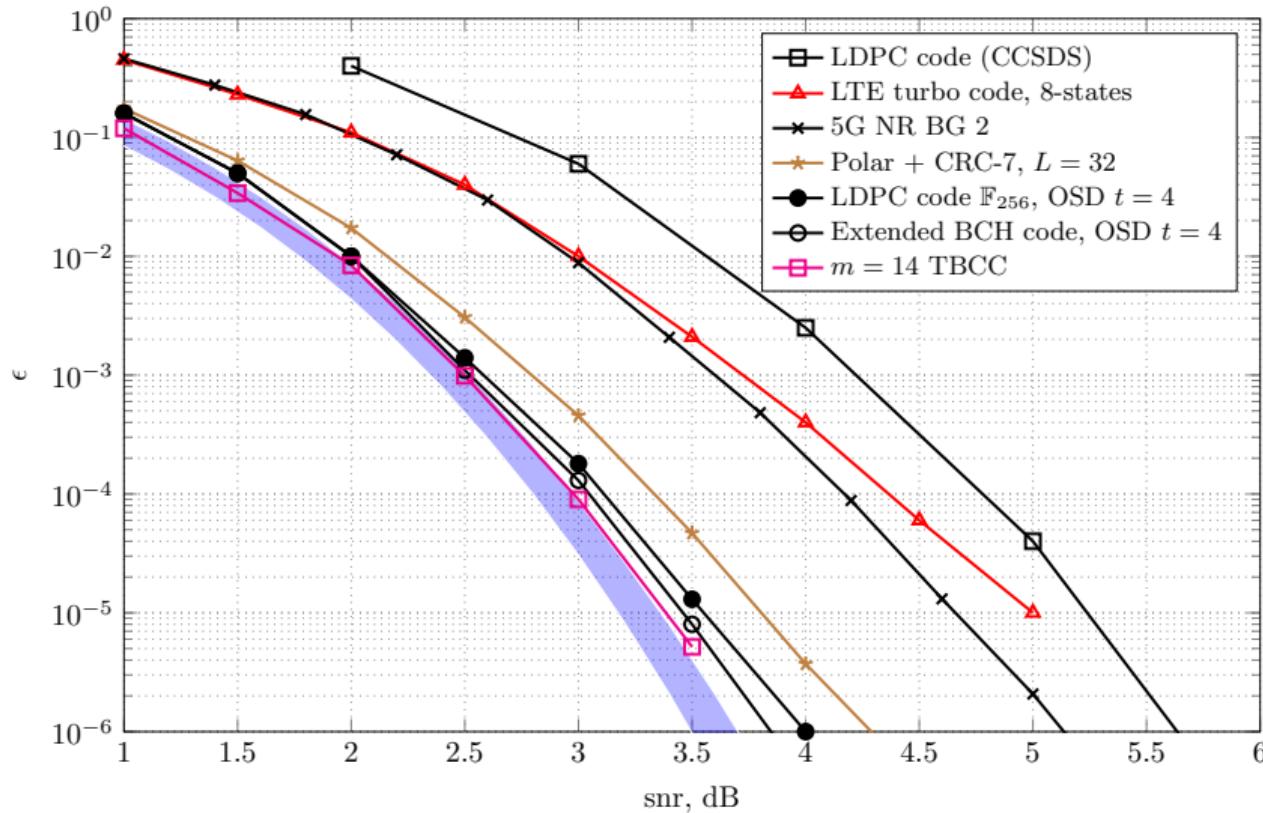
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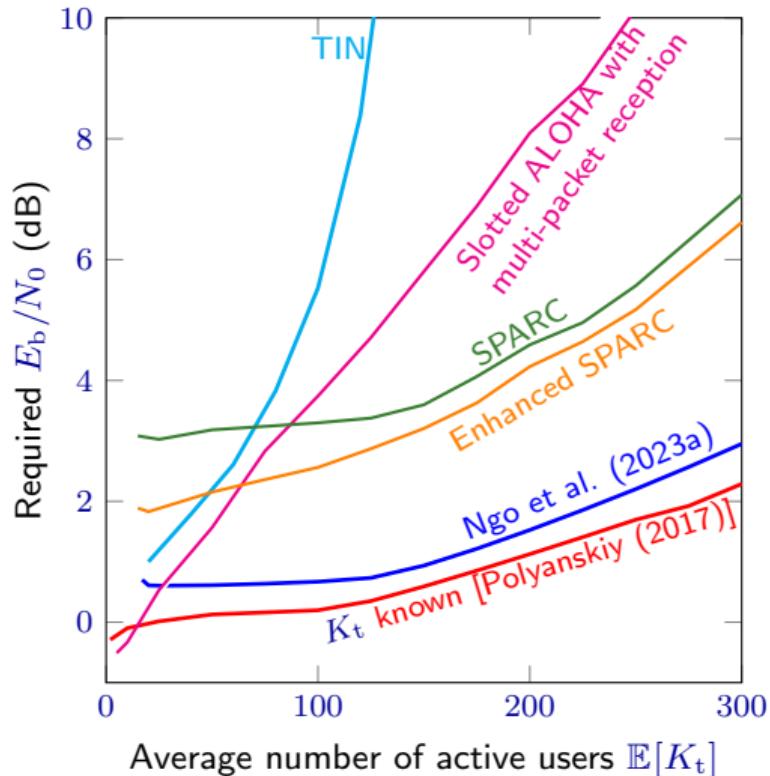
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Guidelines for optimal design ($R = 1/2$ bit/channel use)



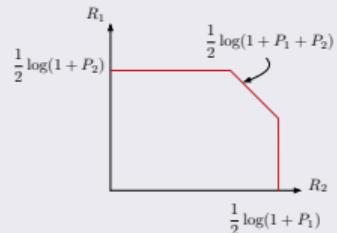
This talk

- FBL-IT bounds for mMTC [Polyanskiy '17]
- Coding schemes approaching the bound
- Extension to
 - Unknown number of active users [Ngo et al. 2023a]
 - Heterogeneous traffic [Ngo et al. 2023b]
- Further extensions and open problems



Traditional multiple access models and their limitations [Gallager '85]

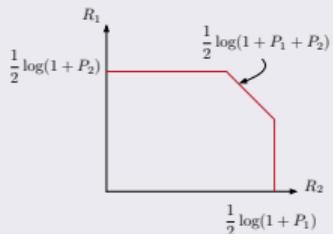
Multiaccess IT [Cover '75, Wyner '74]



- ✖ All users **active** (no sporadicity)
 - Each user is given a **different codebook**
- ✖ Not feasible for mMTC (**overhead** too large)

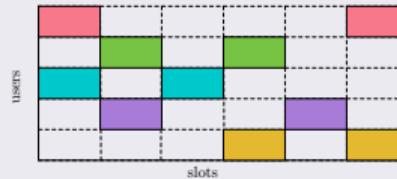
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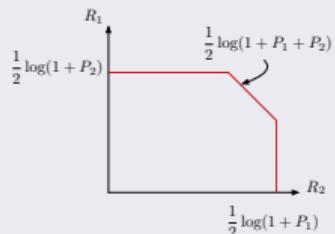
Collision resolution [Abramson '70, Roberts '72, Liva '11]



- ✓ Infinitely many, **sporadically** active users
- ✗ Crude modeling of communication aspects
 - De-facto **standard** for mMTC

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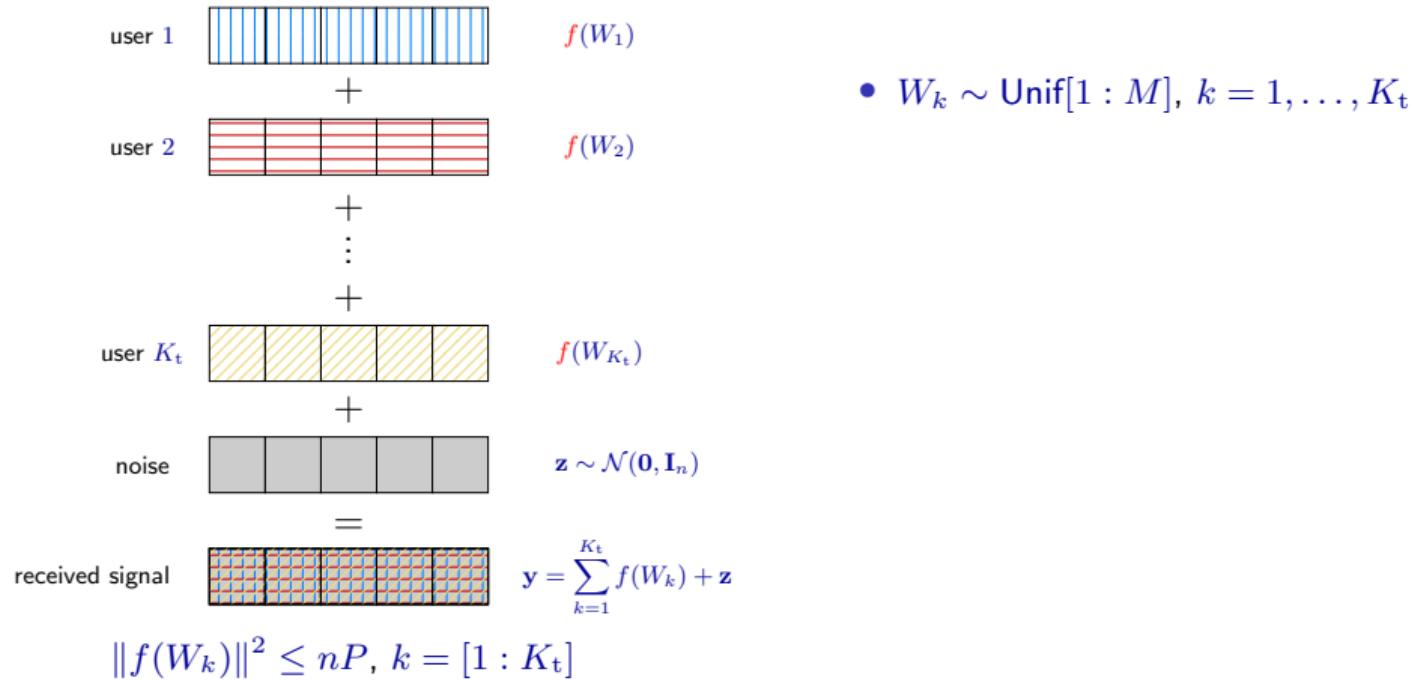
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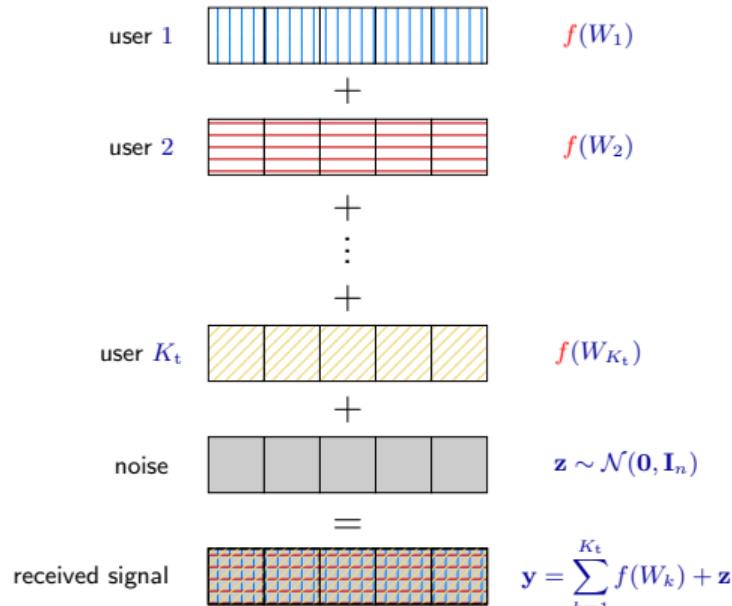
Addressing these limitations

- Noiseless adder channel (e.g., [Bar-David et al., '97])
- More general information-theoretic perspective [Polyanskiy '17]

Unsourced GMAC model [Polyanskiy '17]



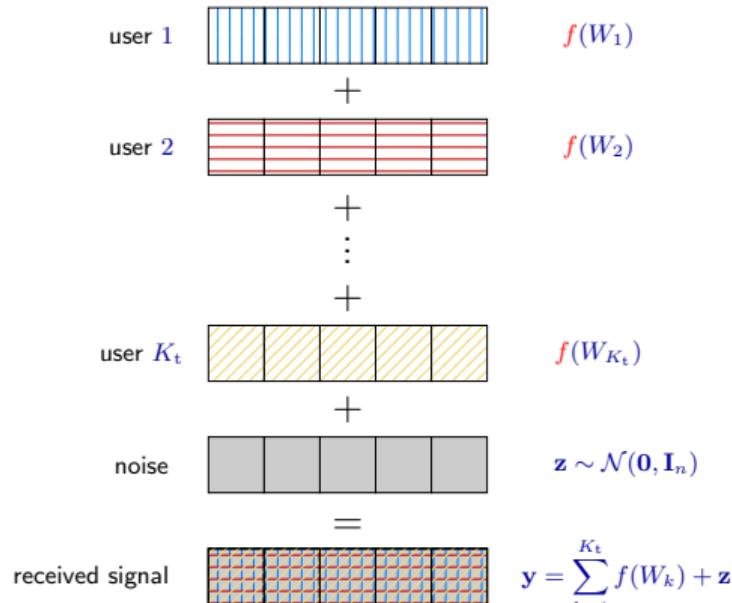
Unsourced GMAC model [Polyanskiy '17]



$$\|f(W_k)\|^2 \leq nP, \quad k = [1 : K_t]$$

- $W_k \sim \text{Unif}[1 : M], k = 1, \dots, K_t$
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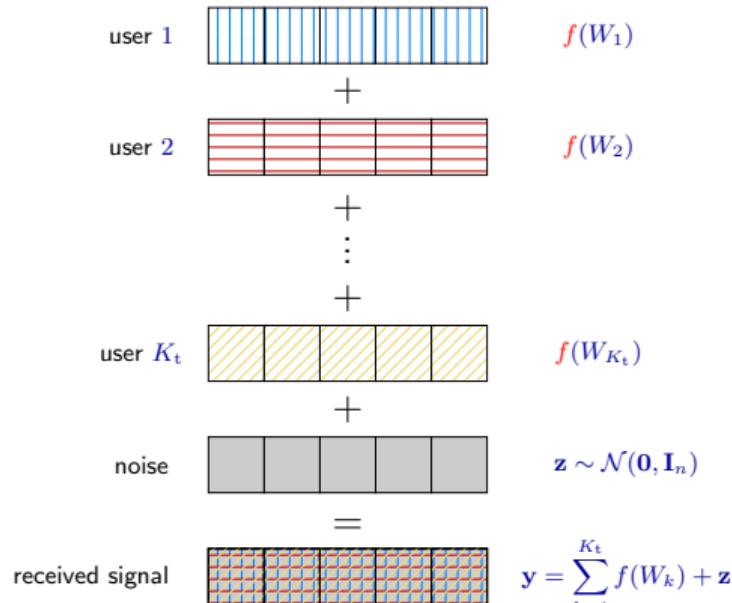
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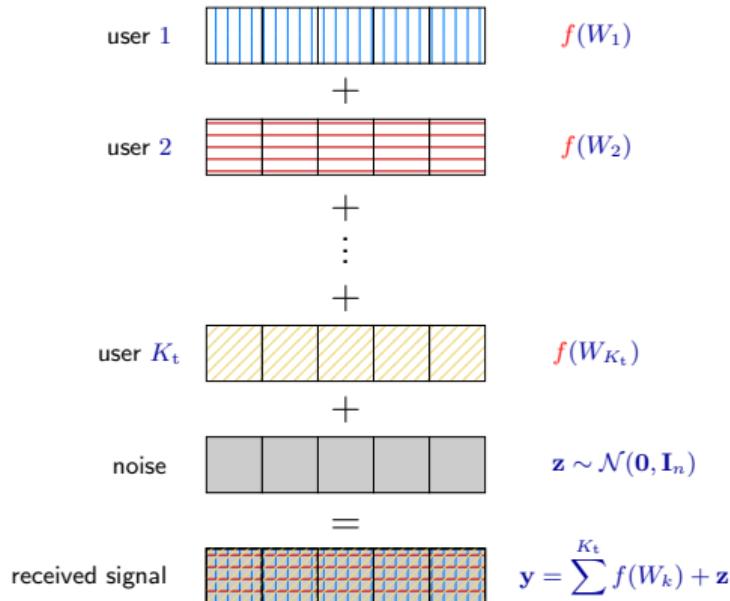
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- ! Per-user error-probability

$$P_e = \frac{1}{K_t} \sum_{k=1}^{K_t} \mathbb{P}\left[W_k \notin \widehat{\mathcal{W}}\right]$$

Random coding achievability bound

(M, n, ϵ) code for K_t -user unsourced GMAC with power constraint P

It consists of a pair of possibly randomized encoder and decoder satisfying $P_e \leq \epsilon$

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Random-coding achievability bound [Polyanskiy '17]

For every $P' < P$, there exists an (M, n, ϵ) code for the K_t -user unsourced GMAC with power constraint P satisfying

$$\epsilon \leq \sum_{k=1}^{K_t} \frac{k}{K_t} \min\{p_k, q_k\} + p_0, \quad \text{where}$$

$$p_0 = \frac{\binom{K_t}{2}}{M} + K_t \mathbb{P} \left[\frac{1}{n} \sum_{j=1}^n z_j^2 > \frac{P}{P'} \right]$$

$$p_k = e^{-E(t)}$$

$$E(t) = \max_{0 \leq \rho_1, \rho_2 \leq 1} -\rho_1 \rho_2 k R_1 - \rho_2 R_2 + E_0(\rho_1, \rho_2)$$

$E_0(\rho_1, \rho_2)$: complicated expression in ρ_1, ρ_2, k, P'

$$q_k = \inf_{\gamma} \mathbb{P}[I_k \leq \gamma] + e^{n(kR_1 + R_2) - \gamma}$$

I_k : related to inf. dens.

$$R_1 = \frac{1}{n} \log M - \frac{1}{nk} \log k!$$

$$R_2 = \frac{1}{n} \log \binom{K_t}{k}$$

Key ideas and steps in the proof

Random codebook generation and encoder

- **Gaussian codebook:** fix $P' < P$; generate M codewords $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P' \mathbf{I}_n)$
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Decoder

Unordered list $\widehat{\mathcal{W}}$ of decoded messages obtained by solving

$$\widehat{\mathcal{W}} = \arg \min_{\mathcal{W}' \subset [1:M], |\mathcal{W}'| = K_t} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|, \quad \text{with} \quad \mathbf{c}(\mathcal{W}') = \sum_{w \in \mathcal{W}'} \mathbf{c}_w$$

Analysis of per-user error probability

$$P_e = \frac{1}{K_t} \mathbb{E}_P \left[\sum_{k=1}^{K_t} \mathbb{1} \left\{ W_k \notin \widehat{\mathcal{W}} \right\} \right]$$

P : probability measure on noise, uniform messages, and conditionally Gaussian codewords given that the power constraint is satisfied

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Change of measure

- Change measure from P to Q for which: messages are distinct and codewords are i.i.d. Gaussian
- For every event \mathcal{E} , $\mathbb{E}_P[\mathcal{E}] \leq \mathbb{E}_Q[\mathcal{E}] + d_{\text{TV}}(P, Q)$

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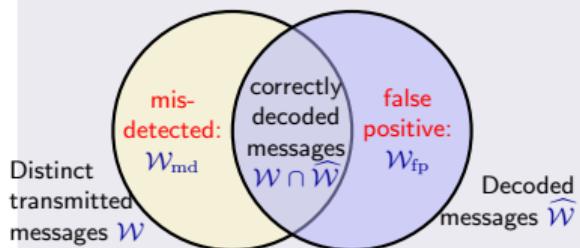
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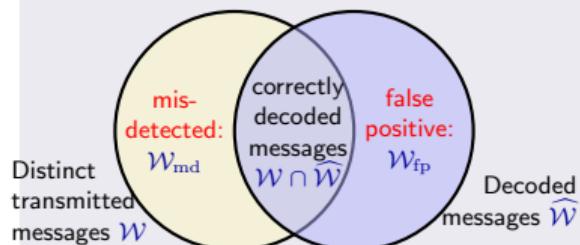
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$$P_e \leq \sum_{k=1}^{K_t} \frac{k}{K_t} \mathbb{P}[|\mathcal{W}_{\text{md}}| = |\mathcal{W}_{\text{fp}}| = k] + p_0$$

$$\mathbb{P}[|\mathcal{W}_{\text{md}}| = |\mathcal{W}_{\text{fp}}| = k] = \mathbb{P} \left[\bigcup_{\substack{\mathcal{W}_{\text{md}} \subset \mathcal{W} \\ |\mathcal{W}_{\text{md}}|=k}} \bigcup_{\substack{\mathcal{W}_{\text{fp}} \subset [1:M] \setminus \mathcal{W} \\ |\mathcal{W}_{\text{fp}}|=k}} \right]$$

$$\left. \|\mathbf{z} + \mathbf{c}(\mathcal{W}_{\text{md}}) - \mathbf{c}(\mathcal{W}_{\text{fp}})\| \leq \|\mathbf{z}\| \right]$$

Three tools and their applications

Chernoff: for every random \mathbf{u} and every $\lambda > 0$

$$\mathbb{P}[\|\mathbf{z} + \mathbf{u}\| \leq v] \leq e^{\lambda v^2} \mathbb{E}_{\mathbf{z}}[e^{-\lambda \|\mathbf{z} + \mathbf{u}\|^2}]$$

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- Assume that $\mathbb{P}[\mathcal{A}_j | \mathbf{z}] \leq e^{-nE(\mathbf{z})}, j = 1, \dots, m$
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2. Gallager's trick on $\bigcup_{\mathcal{W}_{\text{fp}}}$

Three tools and their applications

Chernoff: for every random \mathbf{u} and every $\lambda > 0$

$$\mathbb{P}[\|\mathbf{z} + \mathbf{u}\| \leq v] \leq e^{\lambda v^2} \mathbb{E}_{\mathbf{z}}[e^{-\lambda \|\mathbf{z} + \mathbf{u}\|^2}]$$

MGF of Gaussian vector: if $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$

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Gallager's trick to tighten the union bound

- Assume that $\mathbb{P}[\mathcal{A}_j | \mathbf{z}] \leq e^{-nE(\mathbf{z})}, j = 1, \dots, m$
- Then for all $0 \leq \rho \leq 1$

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4. Gallager's trick on $\bigcup_{\mathcal{W}_{\text{md}}}$
5. MGF to compute expectation over \mathbf{z}

Random coding achievability bound [Polyanskiy '17]

For every $P' < P$, there exists an (M, n, ϵ) code for the K_t -user unsourced GMAC with power constraint P satisfying

$$\epsilon \leq \sum_{k=1}^{K_t} \frac{k}{K_t} \min\{p_k, q_k\} + p_0, \quad \text{where}$$

$$p_0 = \frac{\binom{K_t}{2}}{M} + K_t \mathbb{P}\left[\frac{1}{n} \sum_{j=1}^n z_j^2 > \frac{P}{P'}\right]$$

$$p_k = e^{-E(t)}$$

$$E(t) = \max_{0 \leq \rho_1, \rho_2 \leq 1} -\rho_1 \rho_2 k R_1 - \rho_2 R_2 + E_0(\rho_1, \rho_2)$$

$E_0(\rho_1, \rho_2)$: complicated expression in ρ_1, ρ_2, k, P'

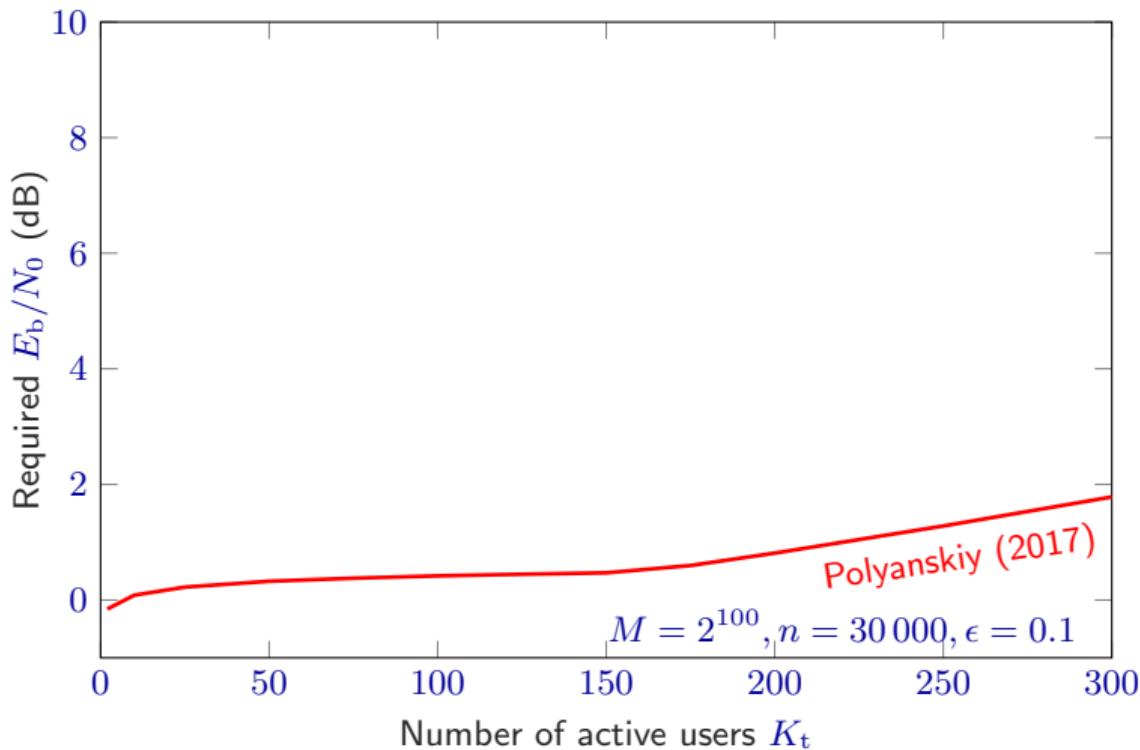
$$q_k = \inf_{\gamma} \mathbb{P}[I_k \leq \gamma] + e^{n(kR_1 + R_2) - \gamma}$$

I_k : related to inf. dens.

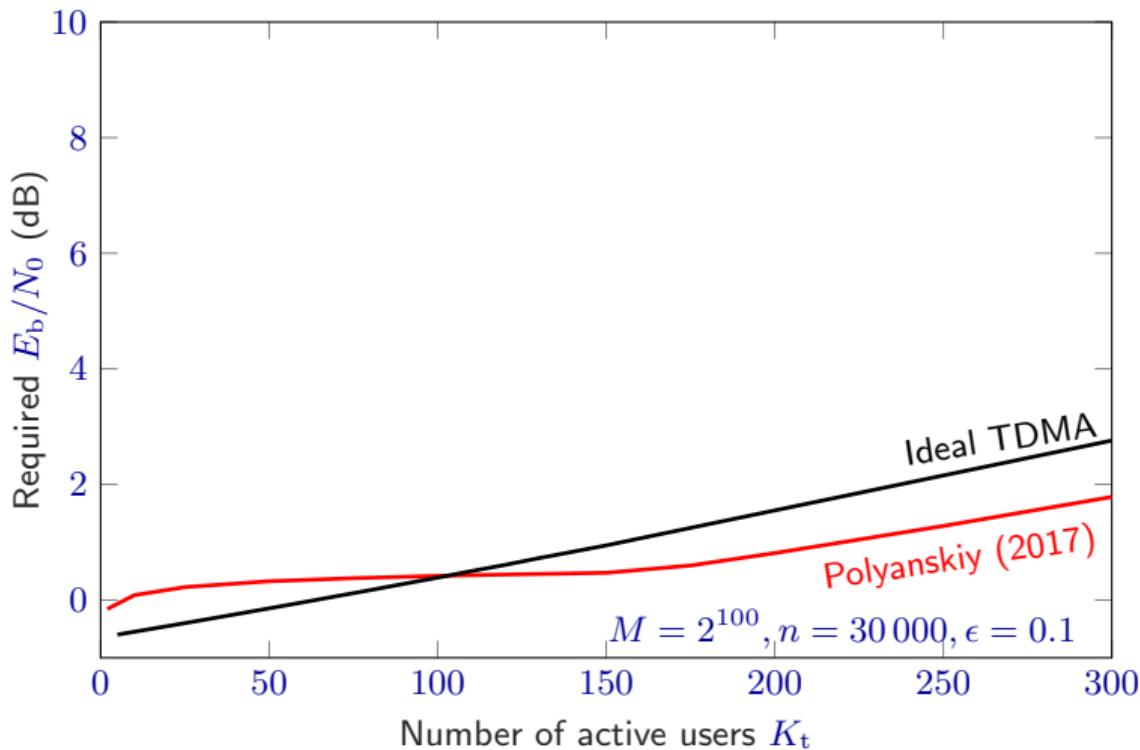
$$R_1 = \frac{1}{n} \log M - \frac{1}{nk} \log k!$$

$$R_2 = \frac{1}{n} \log \binom{K_t}{k}$$

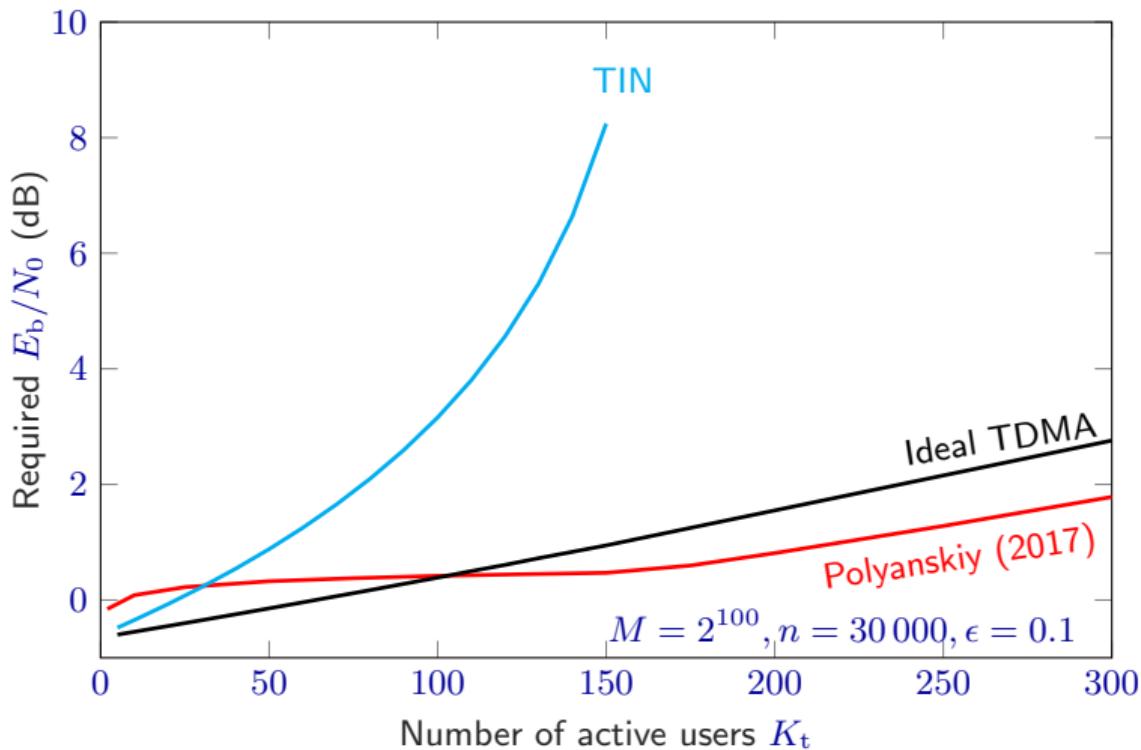
Numerical evaluation of the bound



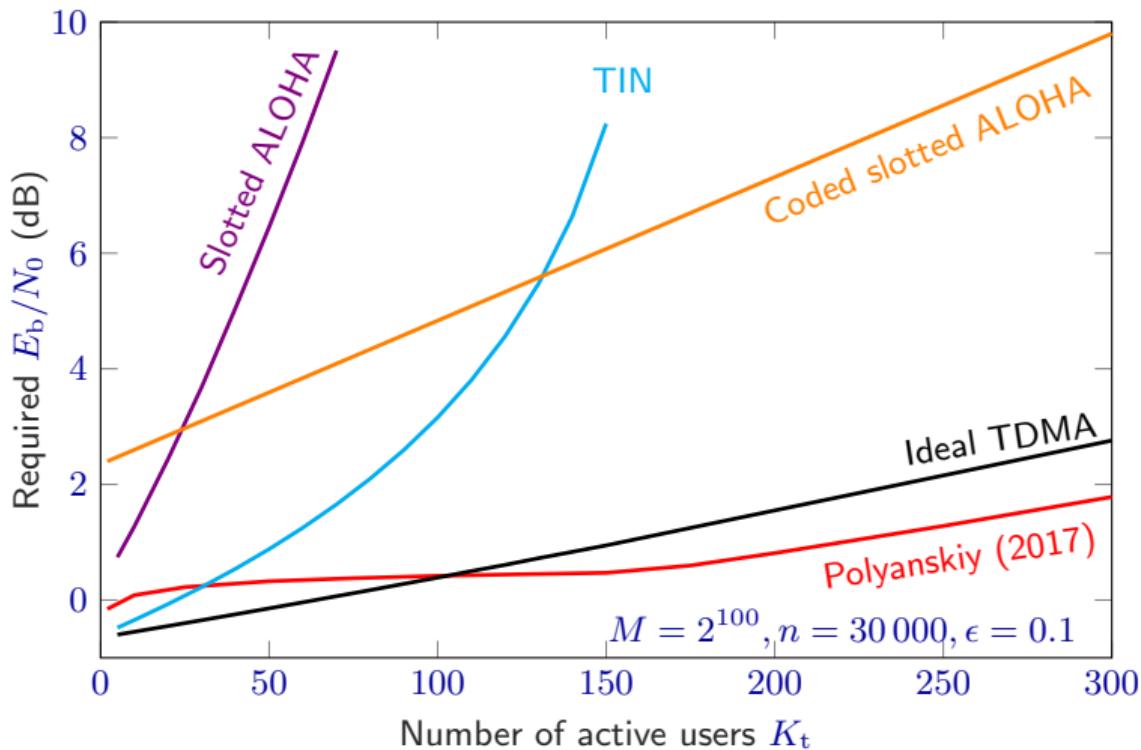
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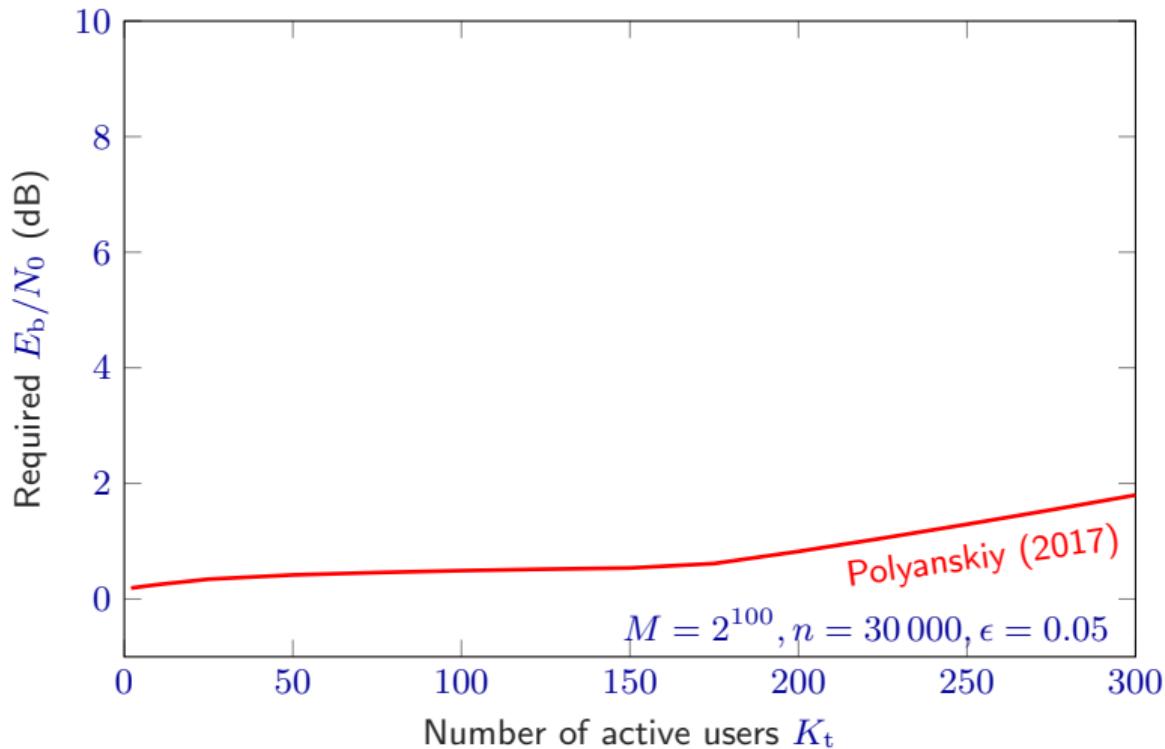
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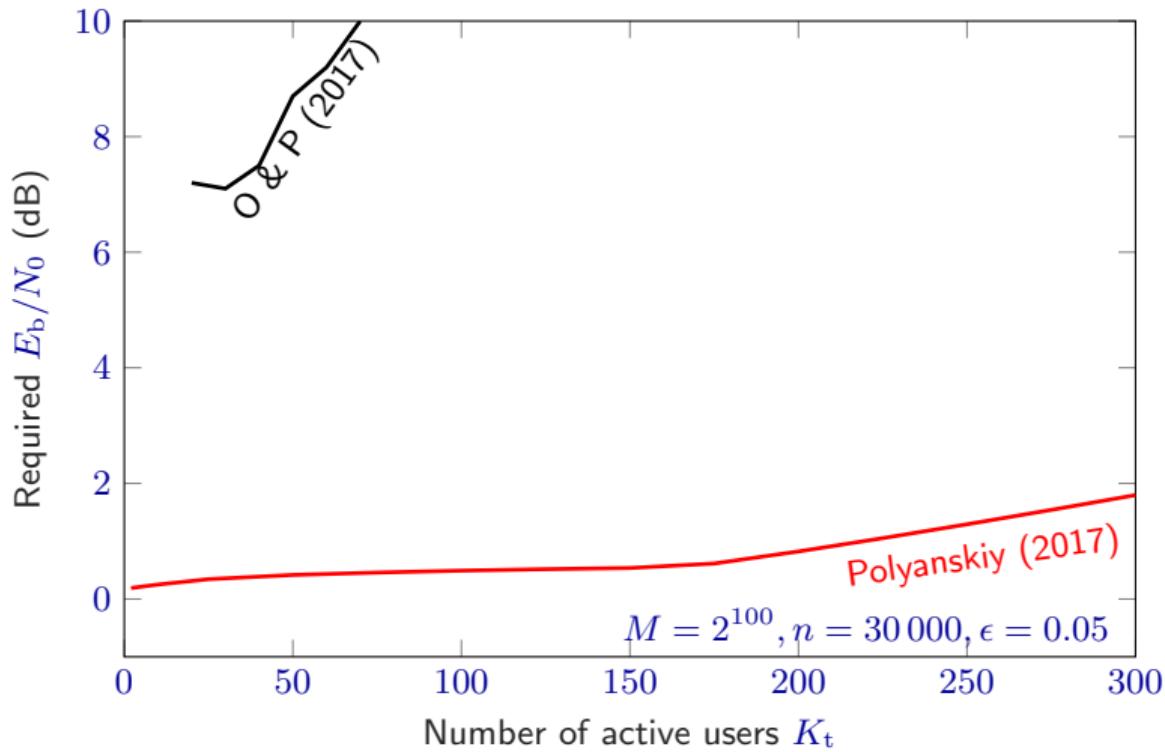
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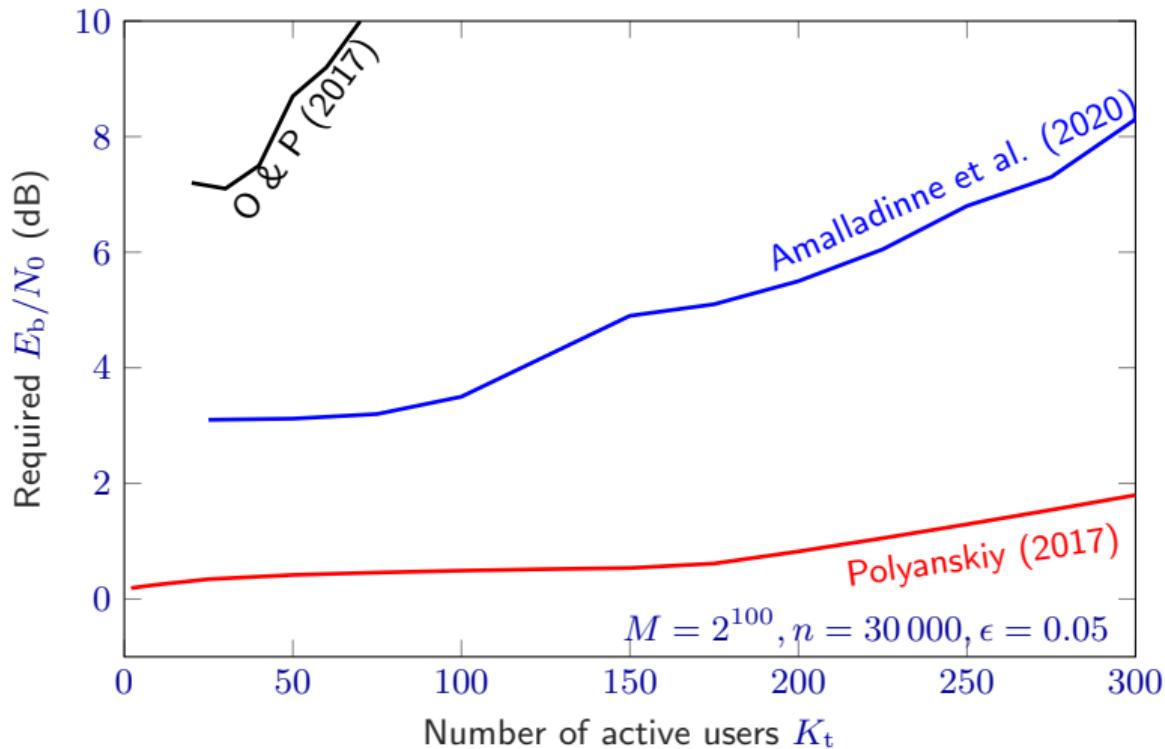
Novel coding schemes (2017–2022)



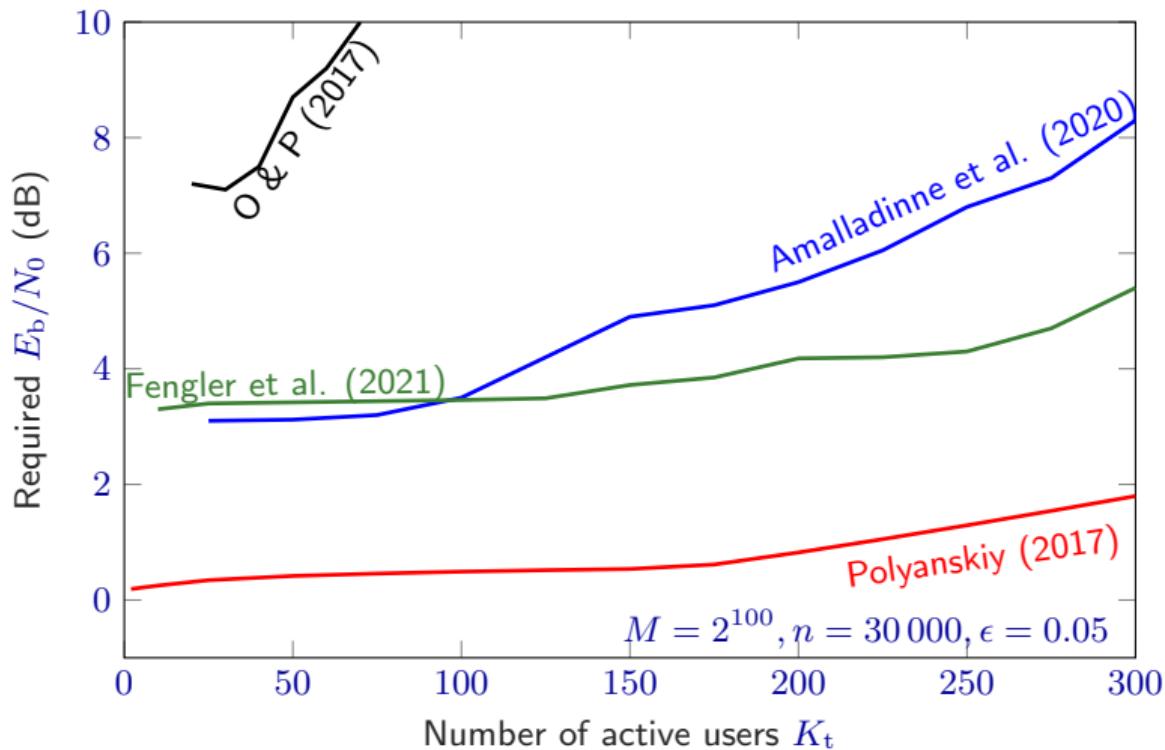
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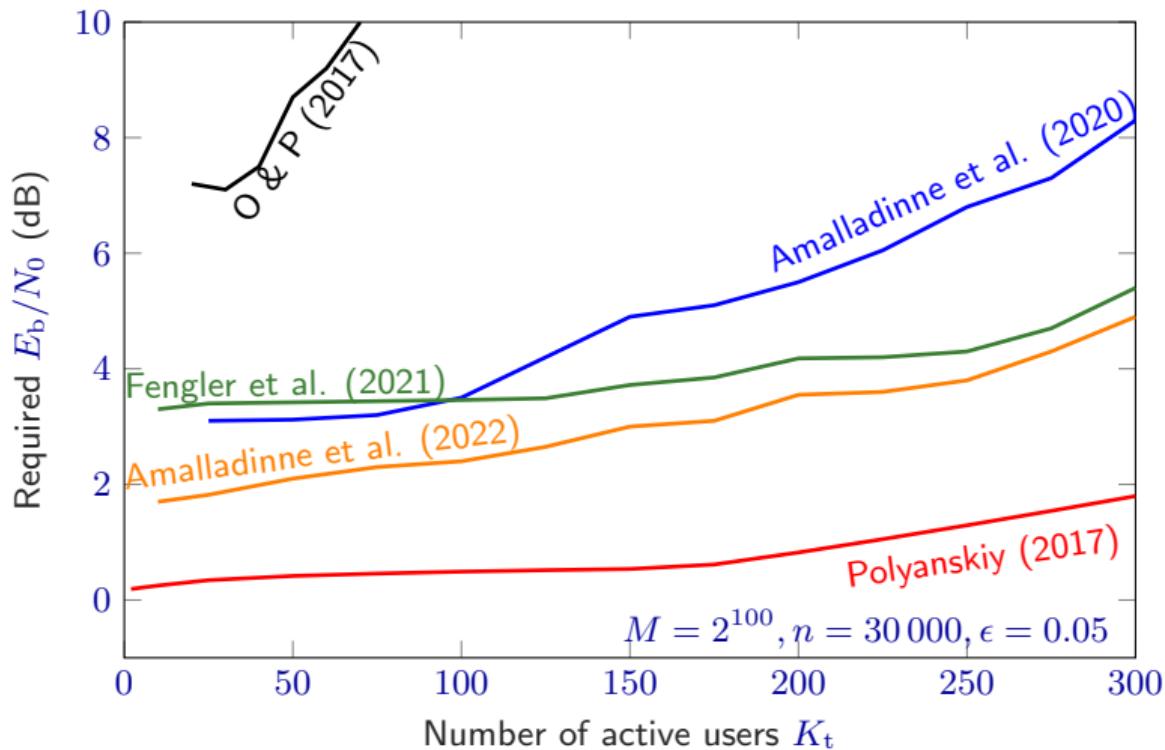
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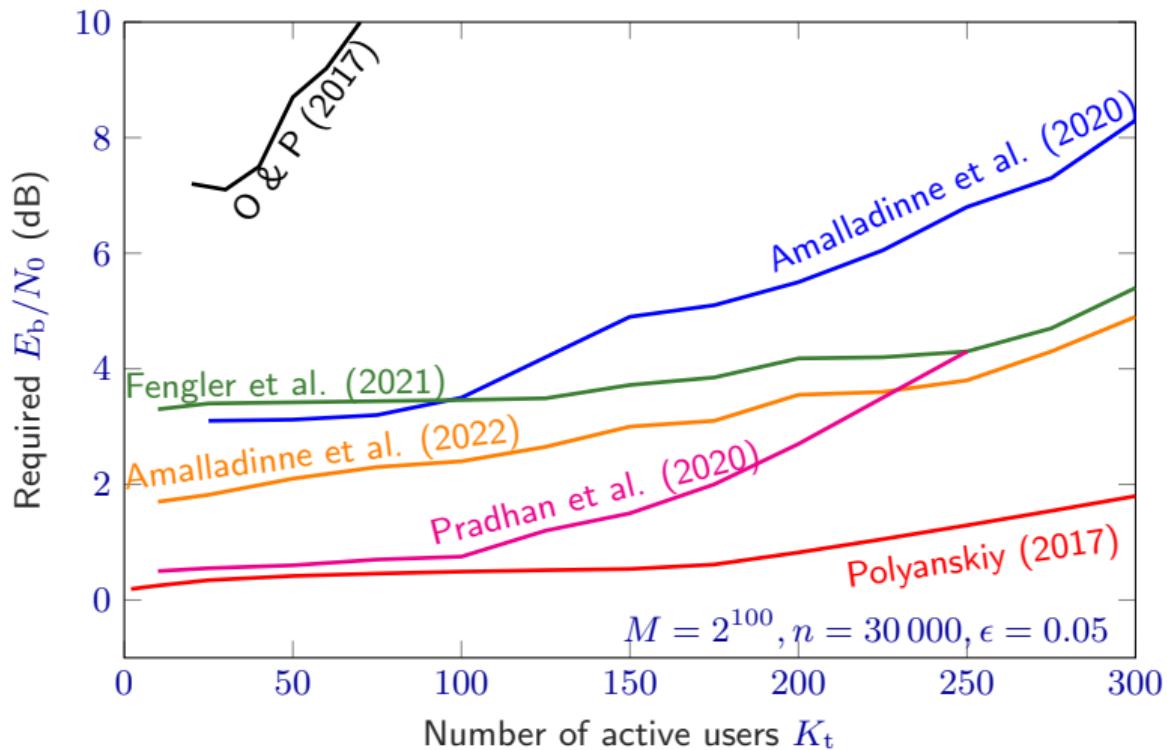
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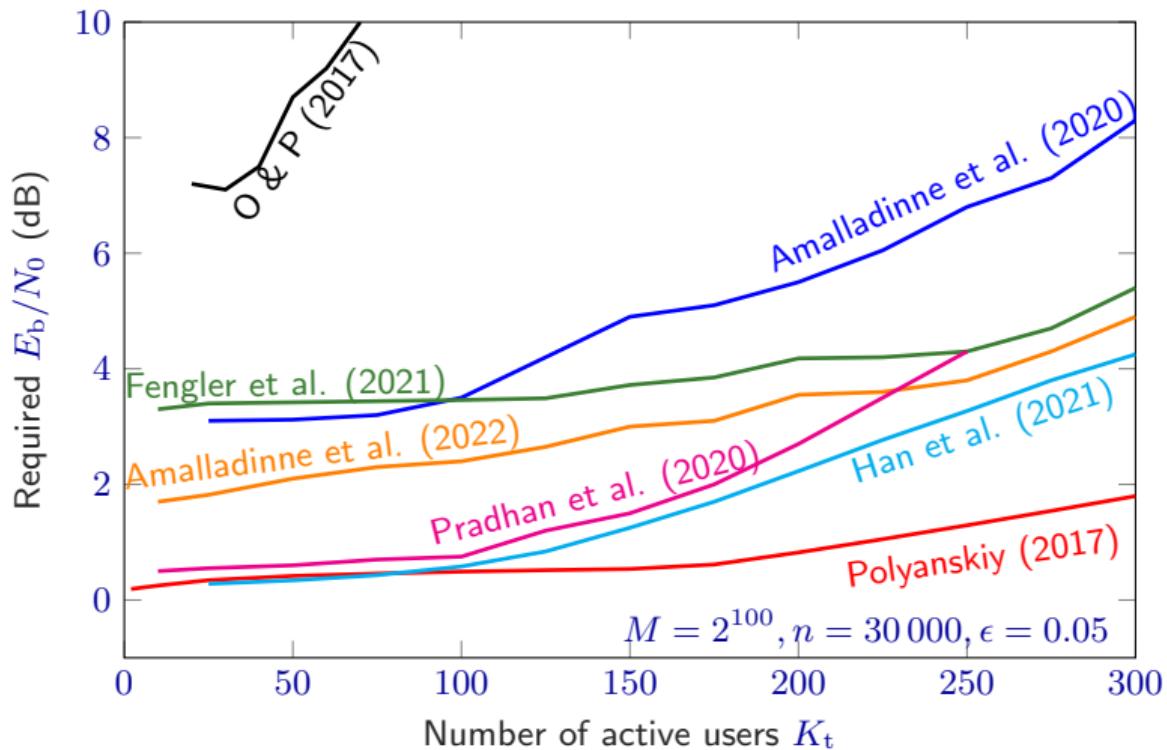
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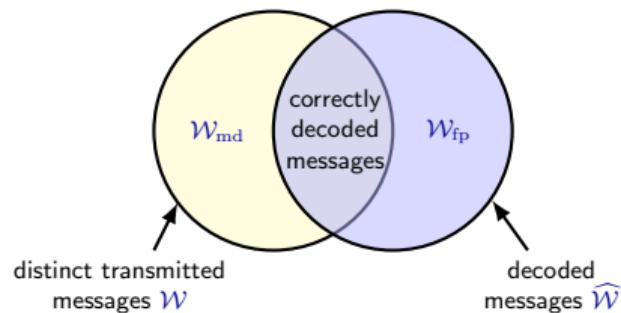
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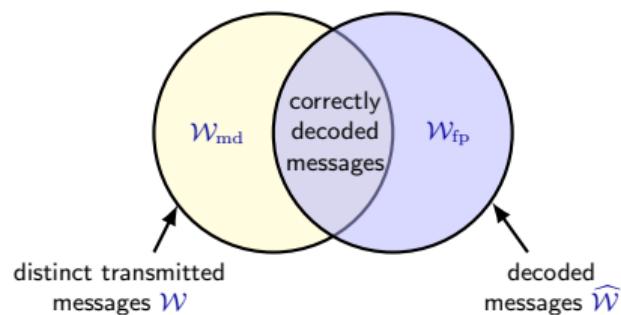


K_t is random and not known to the receiver [Ngo et al. '23a]

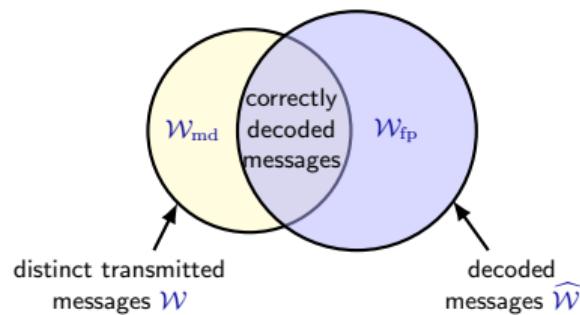


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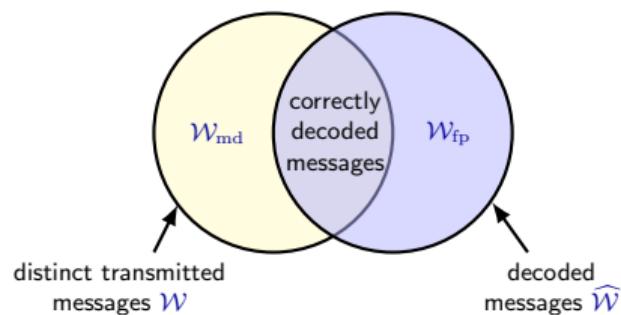


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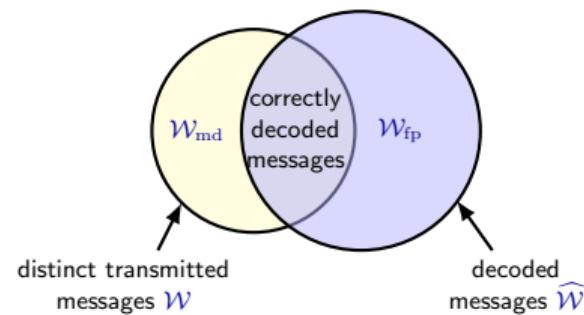


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Pragmatic mismatched approach

Estimate K_t and deploy a coding scheme that treats K_t as known

Performance metrics and definition of a code

Two performance metrics

$$P_{\text{md}} = \mathbb{E} \left[\frac{1}{|\mathcal{W}|} \sum_{i=1}^{|\mathcal{W}|} \mathbb{P}[W_i \notin \widehat{\mathcal{W}}] \right]$$

$$P_{\text{fp}} = \mathbb{E} \left[\frac{1}{|\widehat{\mathcal{W}}|} \sum_{i=1}^{|\widehat{\mathcal{W}}|} \mathbb{P}[\widehat{W}_i \notin \mathcal{W}] \right]$$

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It consists of a pair of possibly randomized encoder and decoder satisfying $P_{\text{md}} \leq \epsilon_{\text{md}}$ and $P_{\text{fp}} \leq \epsilon_{\text{fp}}$

A two-step decoder

Step 1

Obtain an estimate K'_t of K_t by maximizing a suitably chosen metric $m(\mathbf{y}, k)$

$$K'_t = \arg \max_k m(\mathbf{y}, k)$$

Examples: ML estimation, energy-based estimation

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Step 2

Find the list $\widehat{\mathcal{W}}$ of decoded messages as

$$\widehat{\mathcal{W}} = \arg \min_{\substack{\mathcal{W}' \subset [1:M] \\ |\mathcal{W}'| = K'_t}} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|$$

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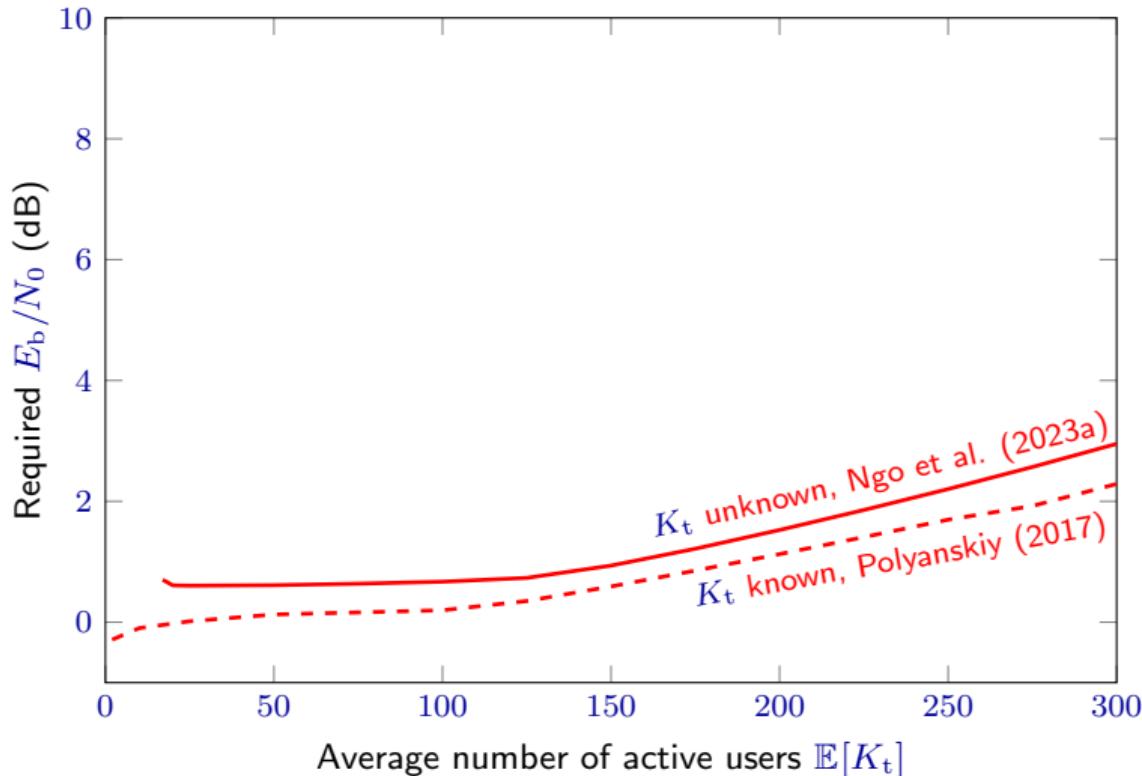
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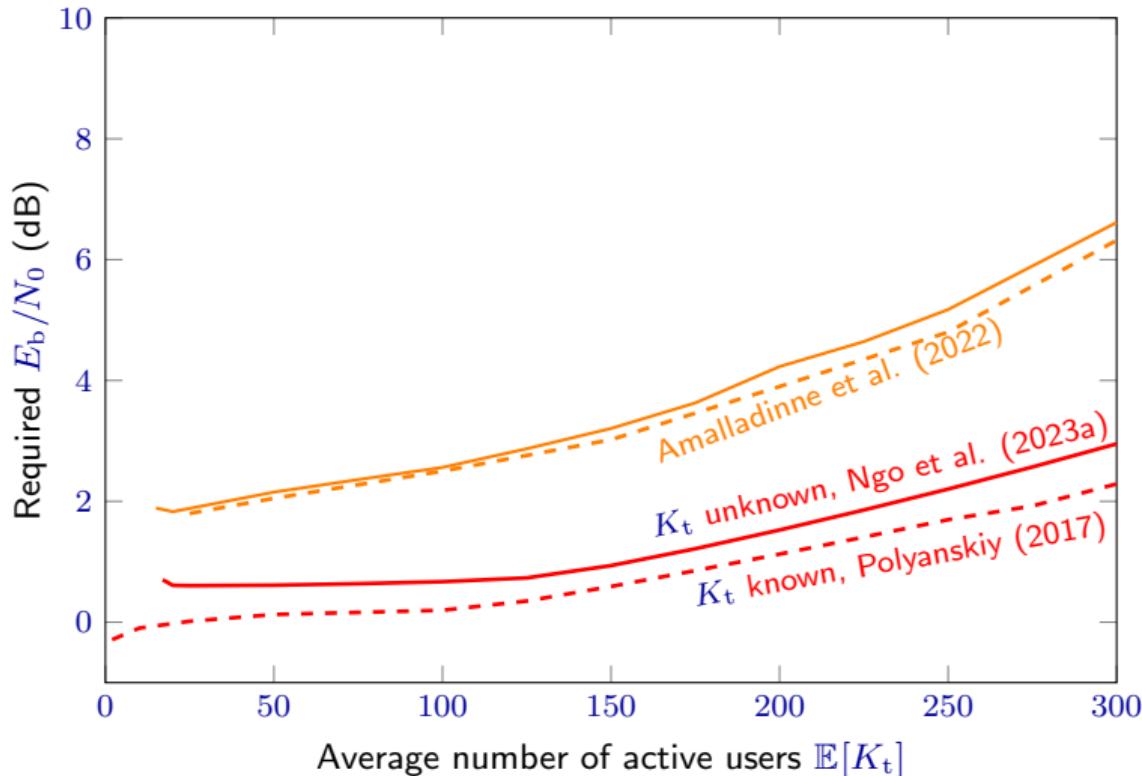
$$\widehat{\mathcal{W}} = \arg \min_{\substack{\mathcal{W}' \subset [1:M] \\ K'_t - r \leq |\mathcal{W}'| \leq K'_t + r}} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|$$

$r > 0$: decoding radius (cannot be chosen too large)

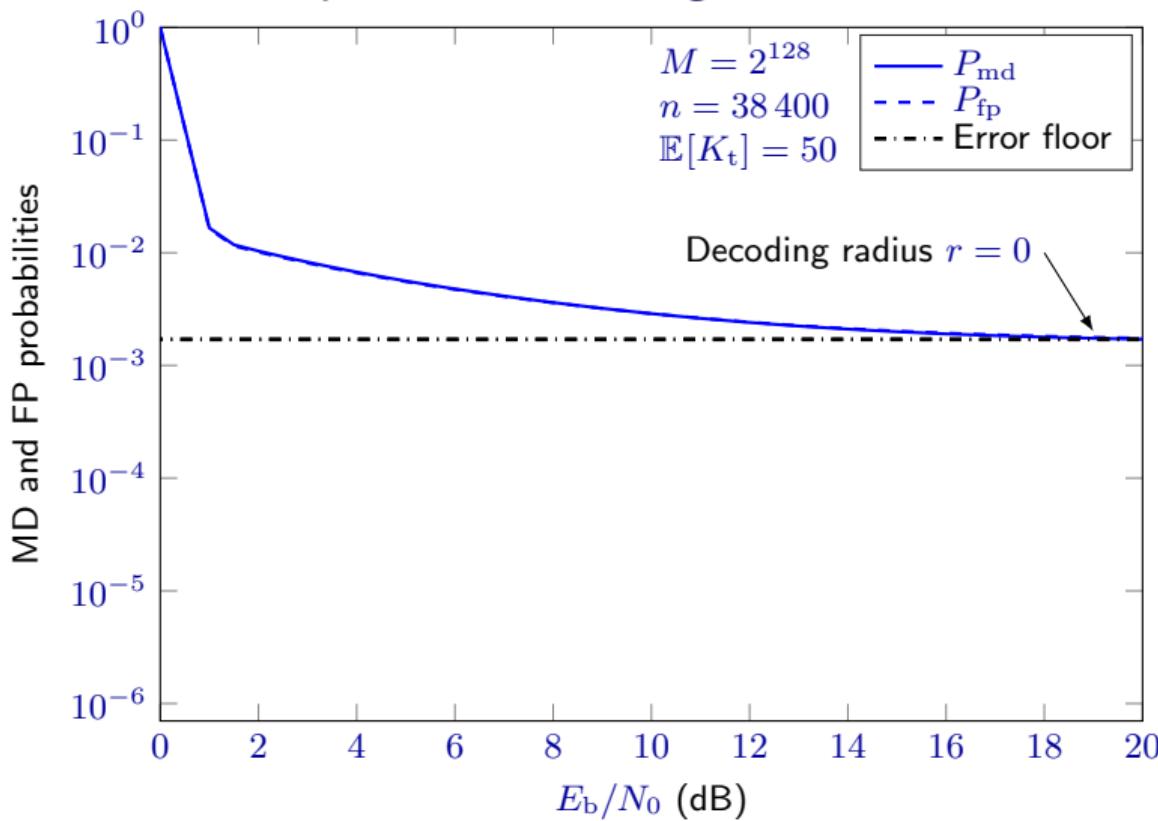
$M = 2^{128}$, $n = 38\,400$, $K_t \sim \text{Poisson}$, $\epsilon_{\text{md}} = \epsilon_{\text{fp}} = 0.1$



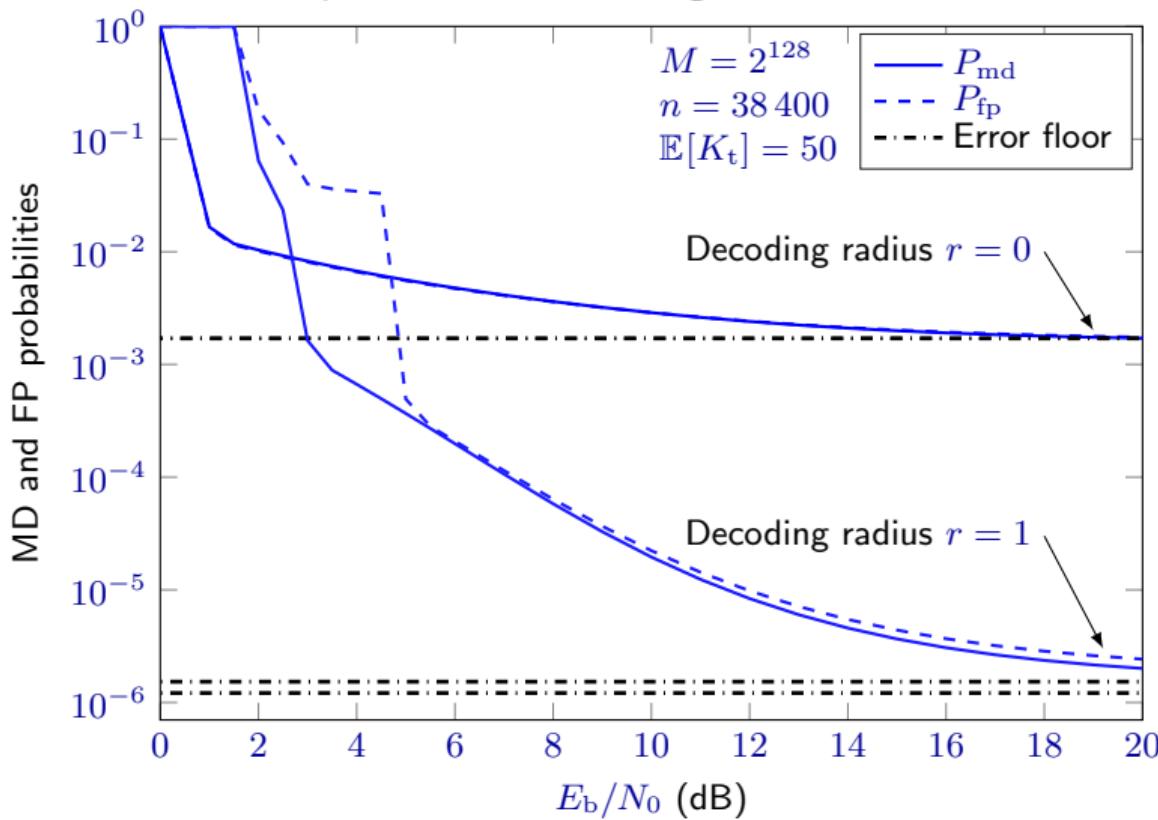
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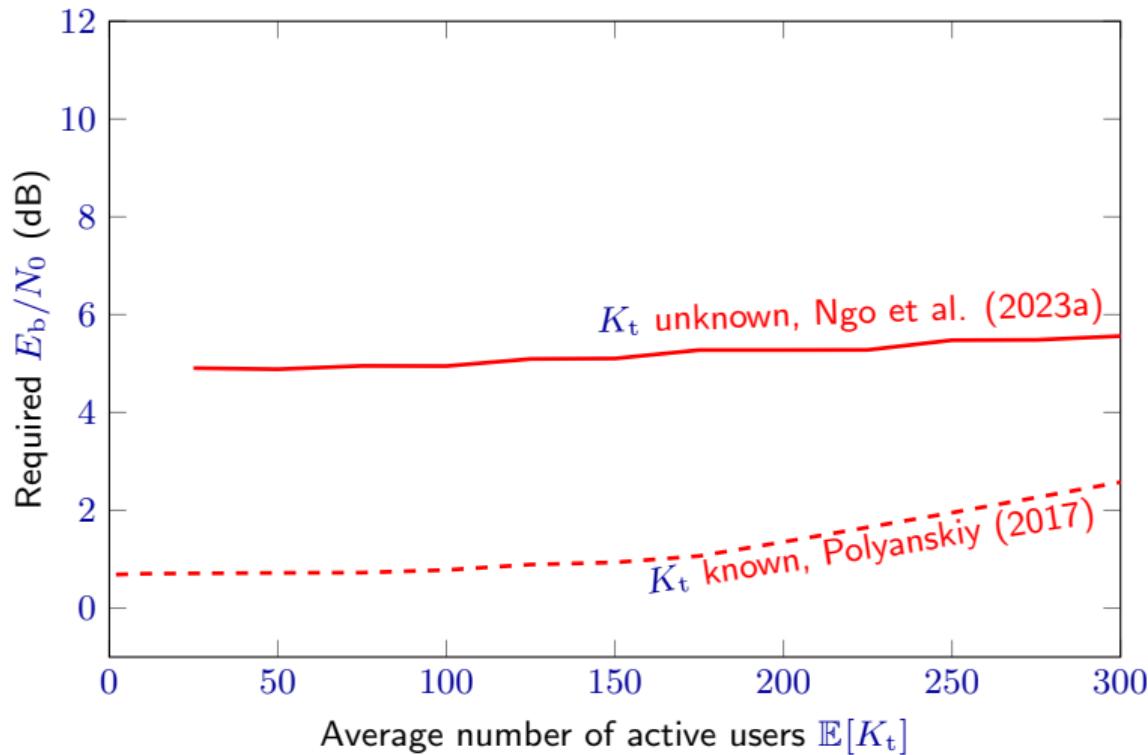
Impact of decoding radius r



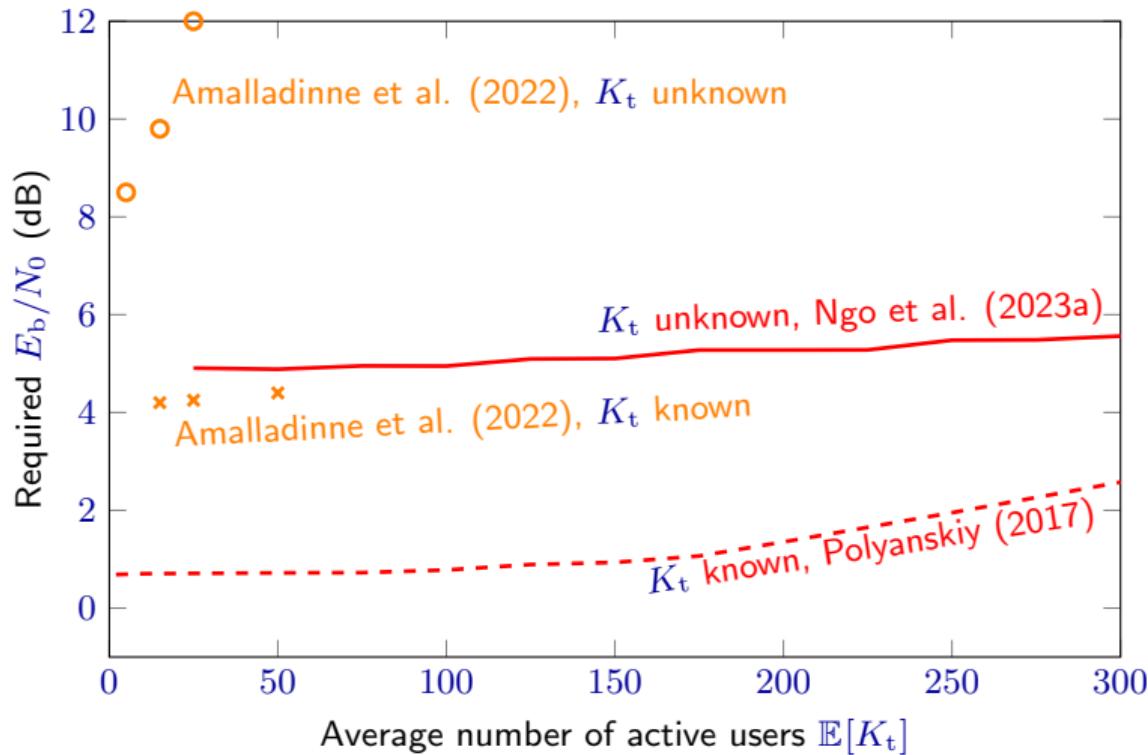
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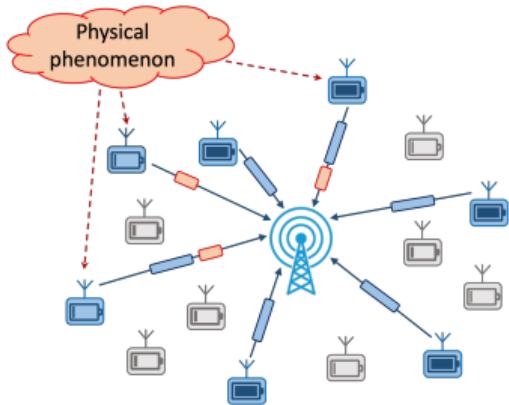
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Heterogeneous traffic: massive and critical IoTs [Ngo et al. '23b]



- K devices
- K_s devices transmit a standard msg.
- K_a devices transmit an alarm msg.

- Device k has a standard message $W_s \in [1 : M_s]$ with prob. ρ_s

$$K_s \sim \text{Bin}(K, \rho_s)$$
- If an alarm occurs, all devices transmit the same alarm message $W_a \in [1 : M_a]$, with probability $\rho_a \leq \rho_{a,\max}$

$$K_a \sim \text{Bin}(K, \rho_a)$$
- Each device can transmit a standard message, an alarm message, both, or none

Performance metrics

- \mathcal{A} : alarm event
- \mathcal{W} : set of transmitted standard messages
- $\widehat{\mathcal{W}}$: set of decoded standard messages
- $W_a \in [1 : M_a]$: transmitted alarm message
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For $\mathcal{B} \in \{\mathcal{A}, \bar{\mathcal{A}}\}$

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$$P_{\text{amd}} = \mathbb{P} \left[\widehat{W}_a \neq W_a \mid \mathcal{A} \right], \quad P_{\text{afp}} = \mathbb{P} \left[\widehat{W}_a \neq 0 \mid \bar{\mathcal{A}} \right]$$

Definition of code and coexistence strategies

$(M_a, M_s, n, \epsilon_{\text{smd}}, \epsilon_{\text{sfp}}, \epsilon_{\text{amd}}, \epsilon_{\text{afp}})$ code for unsourced GMAC with standard and alarm messages

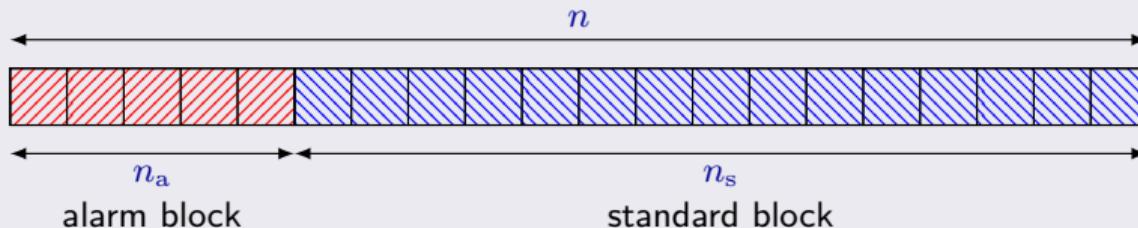
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Orthogonalization

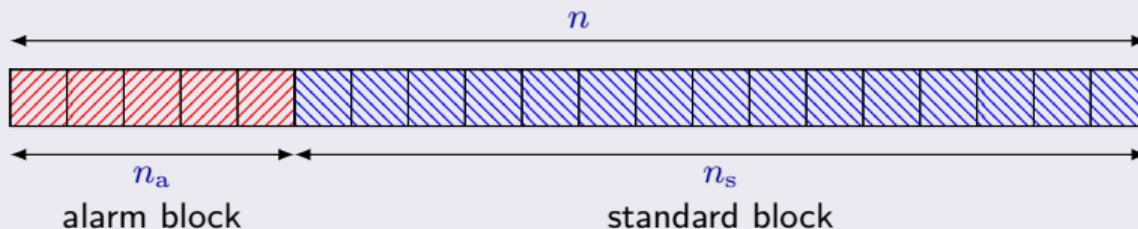


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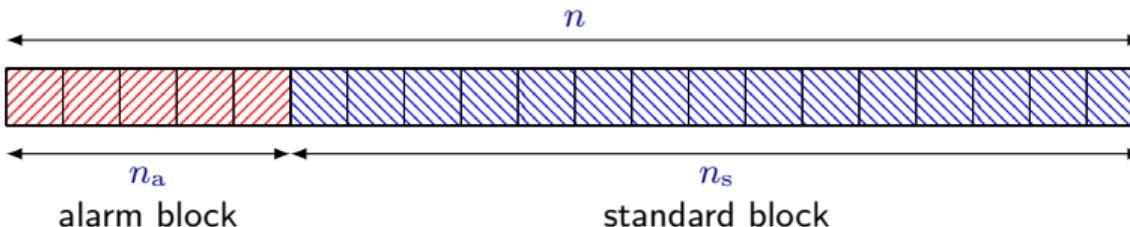
Orthogonalization



Superposition: exploiting reliability diversity [Popovski et al., 2018]



Orthogonalization



Standard block

$$\mathbf{y}_s = \sum_{k=1}^{K_s} \mathbf{x}_k + \mathbf{z}_s, \quad \mathbf{z}_s \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{n_s})$$

- $\|\mathbf{x}_k\|^2 \leq n_s P_s$
- $(E_b/N_0)_s = \frac{n_s P_s}{2 \log_2 M_s}$
- unsourced GMAC with random and unknown number of active users
- Can be analyzed as before

Alarm block

$$\mathbf{y}_a = K_a \mathbf{x}_a + \mathbf{z}_a, \quad \mathbf{z}_a \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{n_a})$$

- $\|\mathbf{x}_a\|^2 \leq n_a P_a$
- $(E_b/N_0)_a = \frac{n_a P_a \rho_a K}{2 \log_2 M_a}$
- Single-user AWGN channel with random, unknown SNR $K_a^2 P_a$ (coherent combining)
- Can be analyzed with FBL tools

Analysis of alarm block

Random codebook generation and encoder

- Gaussian codebook: fix $P'_a < P_a$; generate M_a codewords $\mathbf{c}_1, \dots, \mathbf{c}_{M_a} \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P'_a \mathbf{I}_{n_a})$
- Encoder: User k transmits $\mathbf{c}_{W_0} \mathbb{1}\{\|\mathbf{c}_{W_0}\|^2 \leq nP_a\}$ with probability ρ_a

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Two-step mismatch decoder

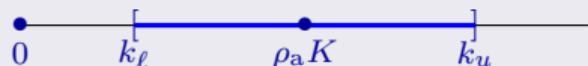
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Two-step mismatch decoder

- Detection of alarm event and number of devices: $K'_a = \arg \max_{k \in [k_\ell : k_u] \cup \{0\}} p(\mathbf{y}_a | k)$



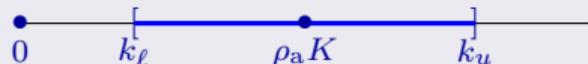
Analysis of alarm block

Random codebook generation and encoder

- **Gaussian codebook:** fix $P'_a < P_a$; generate M_a codewords $\mathbf{c}_1, \dots, \mathbf{c}_{M_a} \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P'_a \mathbf{I}_{n_a})$
- **Encoder:** User k transmits $\mathbf{c}_{W_0} \mathbb{1}\{\|\mathbf{c}_{W_0}\|^2 \leq nP_a\}$ with probability ρ_a

Two-step mismatch decoder

- Detection of alarm event and number of devices: $K'_a = \arg \max_{k \in [k_\ell : k_u] \cup \{0\}} p(\mathbf{y}_a | k)$



- Decoding of W_a : if $K'_a = 0$ return $\widehat{W}_a = 0$; otherwise

$$\{\widehat{W}_a, \widehat{K}_a\} = \arg \min_{w \in [0:M_a], k \in [k_l:k_u] \cup \{0\}} \|\mathbf{y}_a - k\mathbf{c}_w\|^2$$

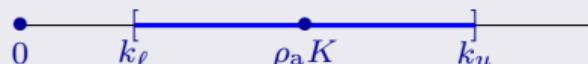
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- Bounds on P_{amd} and P_{afp} via random coding union bound with parameter s

Numerical experiment

- $n = 30\,000$, $(M_a, M_s) = (2^3, 2^{100})$, $1000 \leq K \leq 30\,000$, $\rho_s = 0.01$
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constr.

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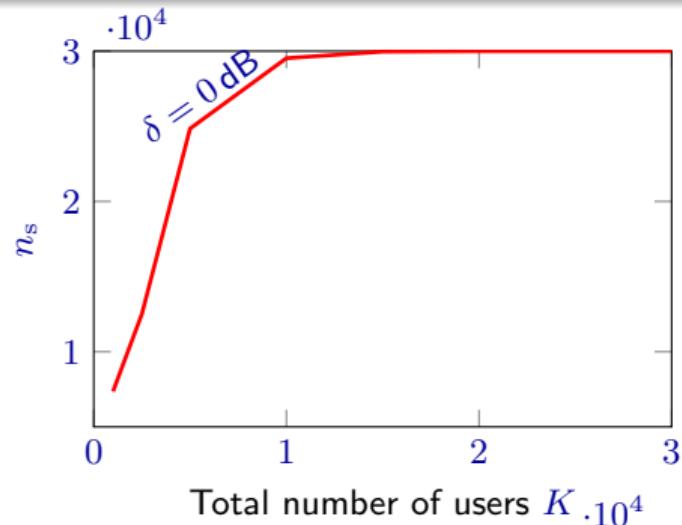
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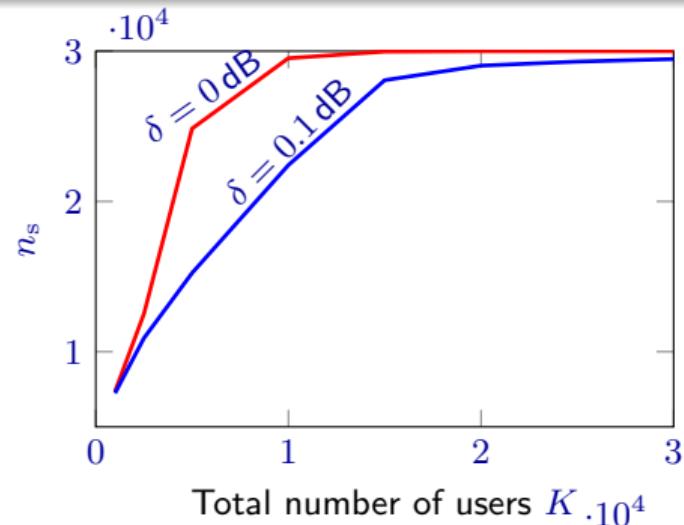


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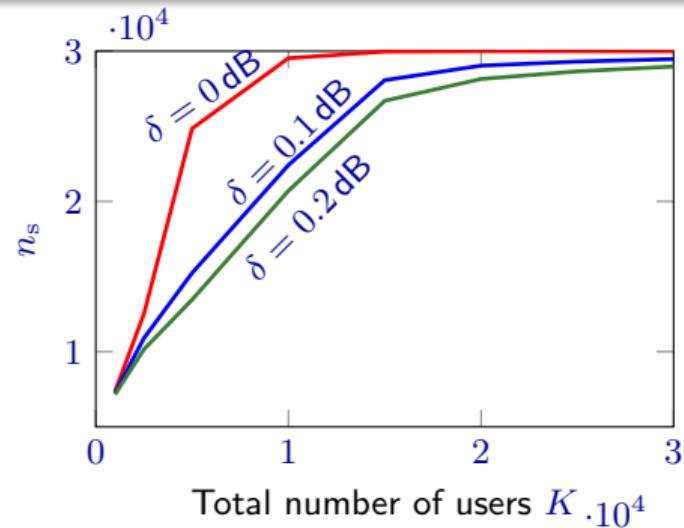


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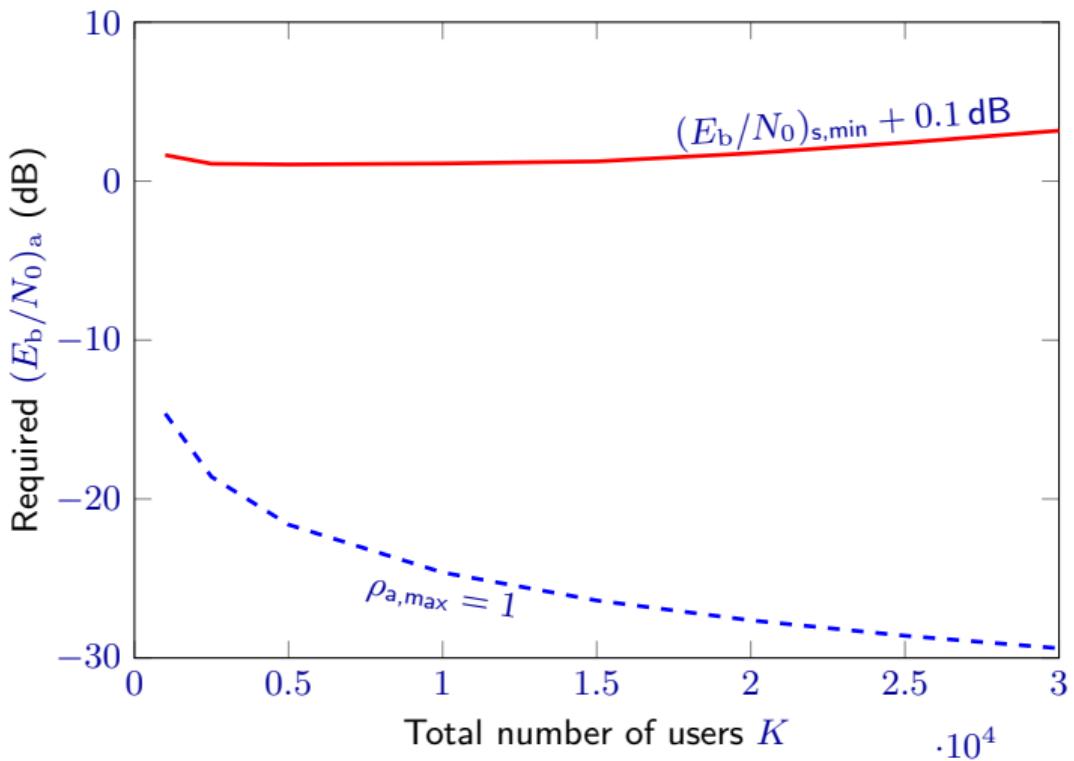
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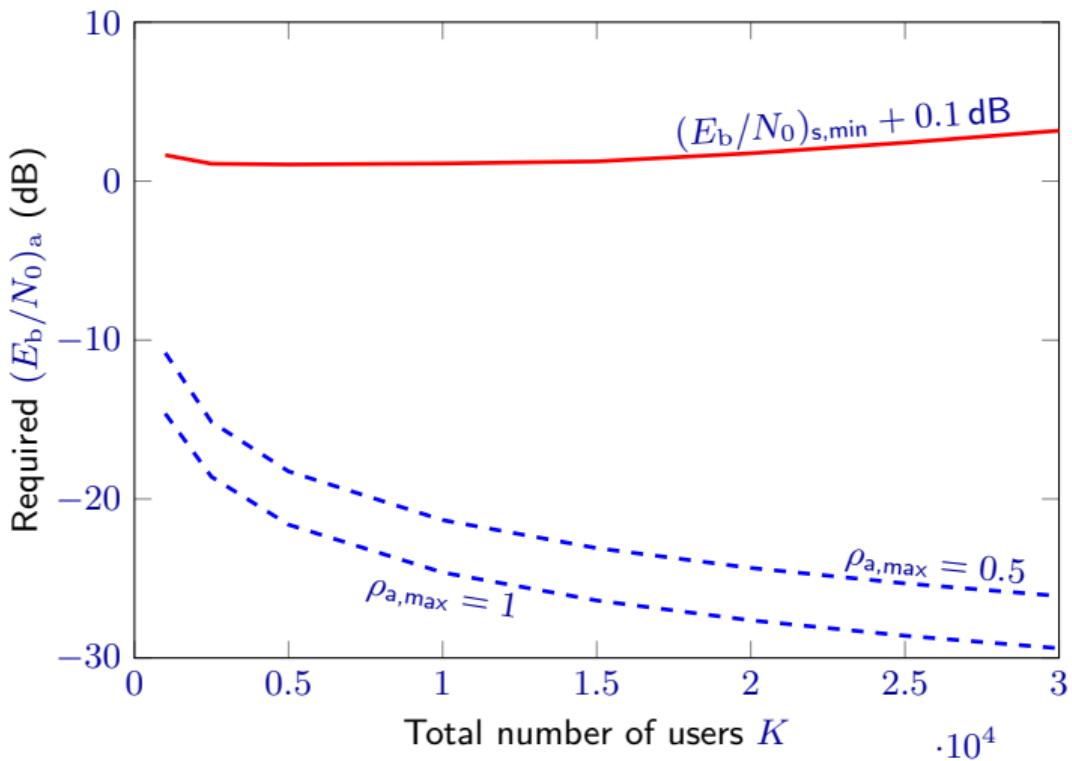
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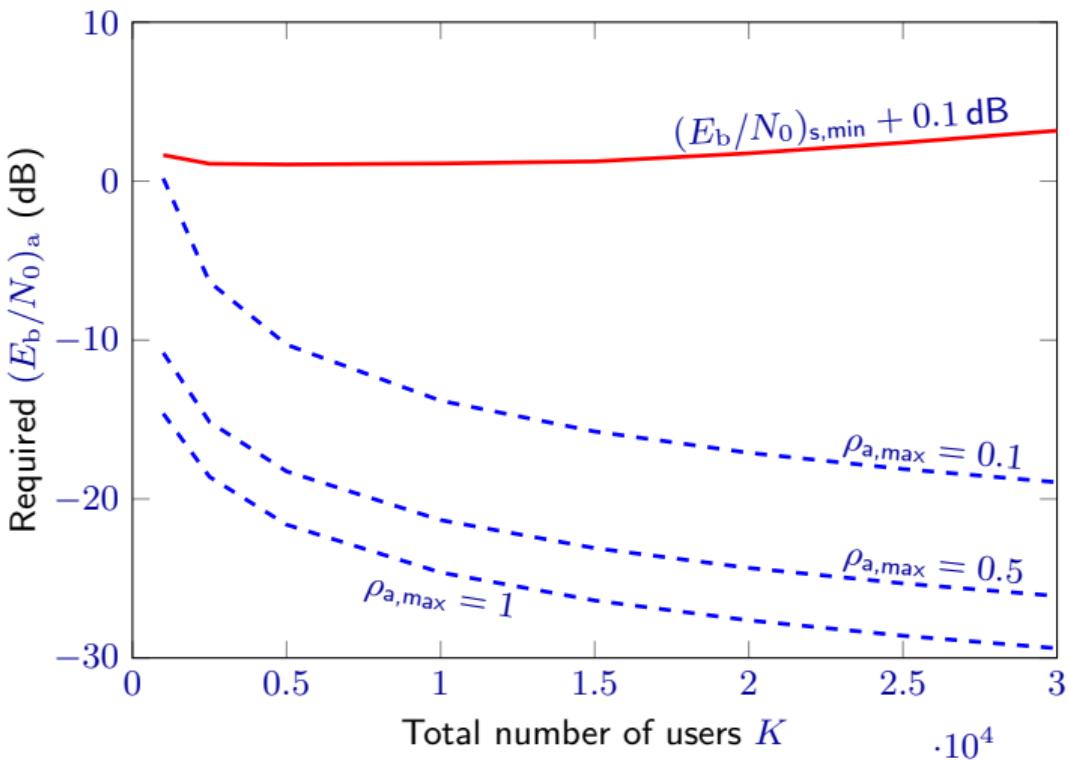
Role of device sensitivity $\rho_{a,\max}$



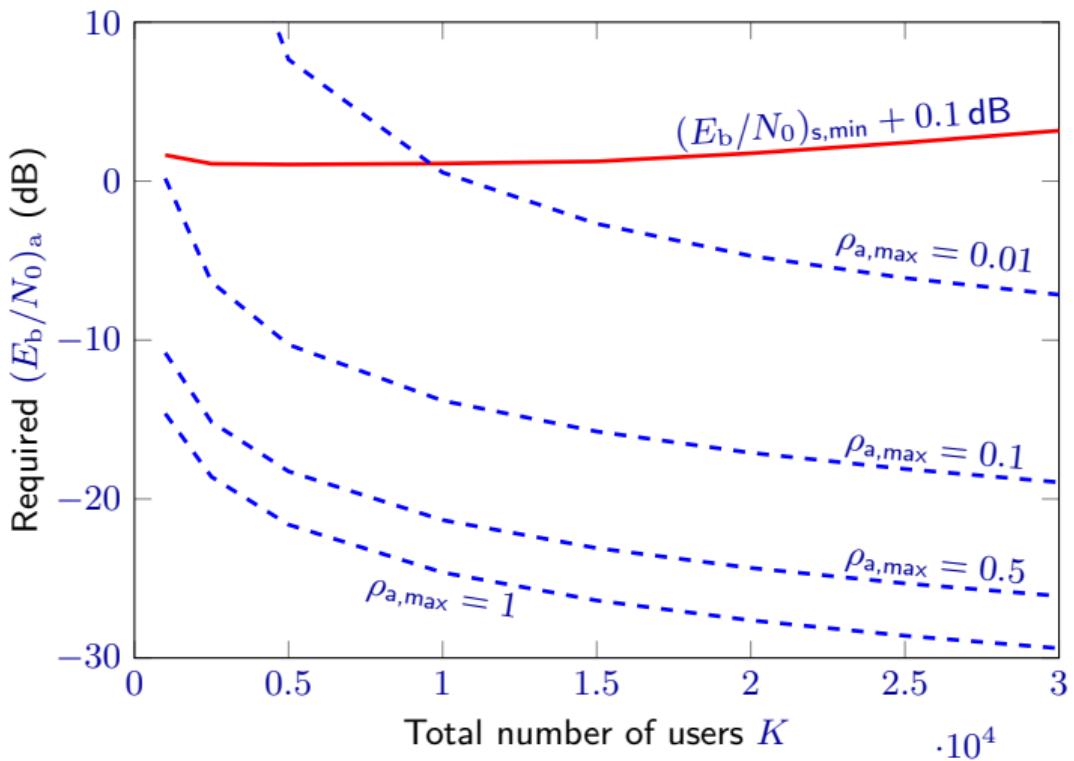
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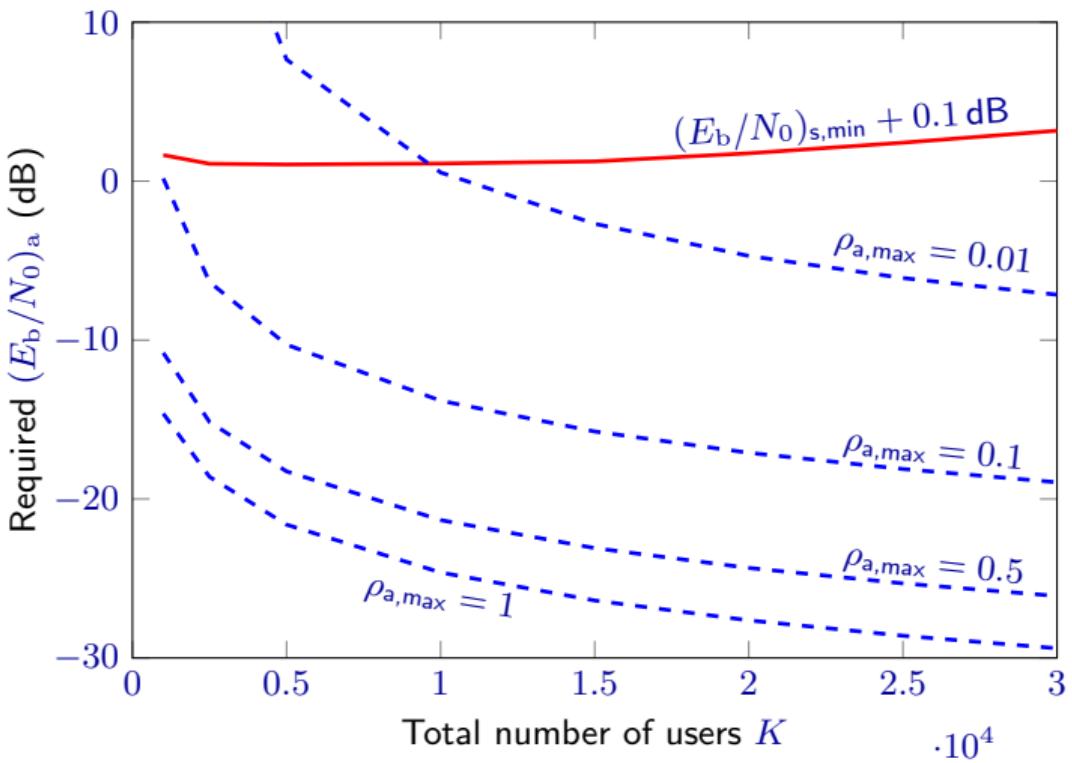
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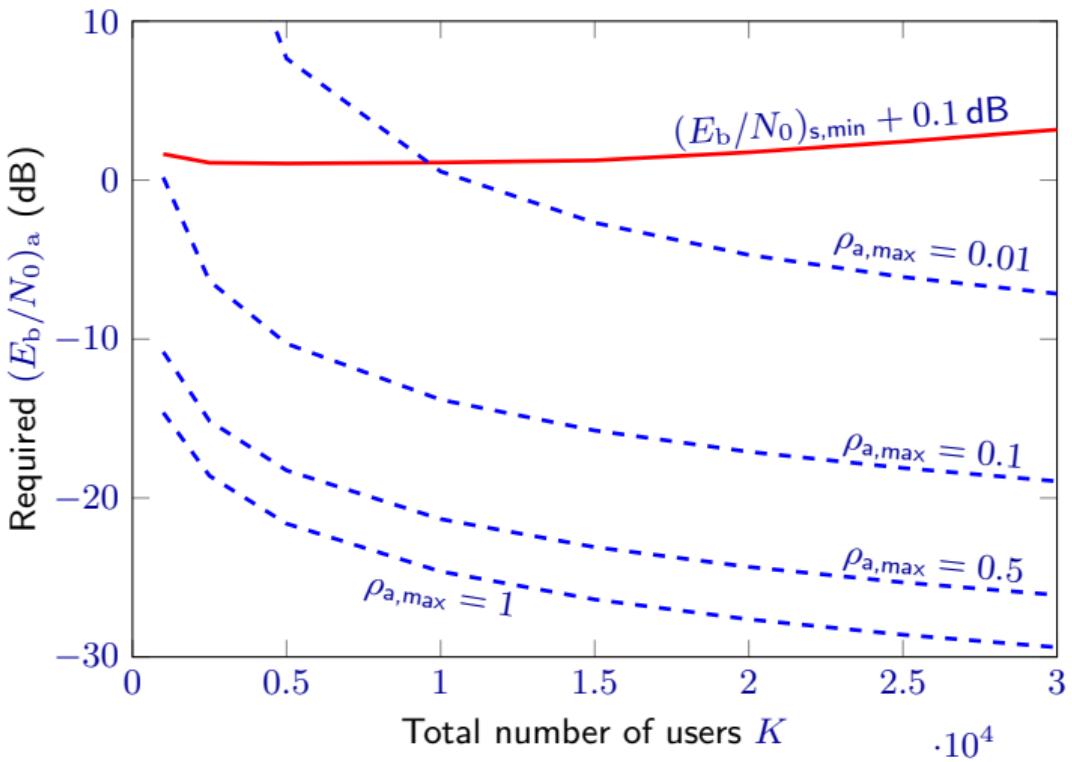
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Optimal choice of parameters

- $\rho_a = \rho_{a,\max}$
- n_a and P_a as low as possible

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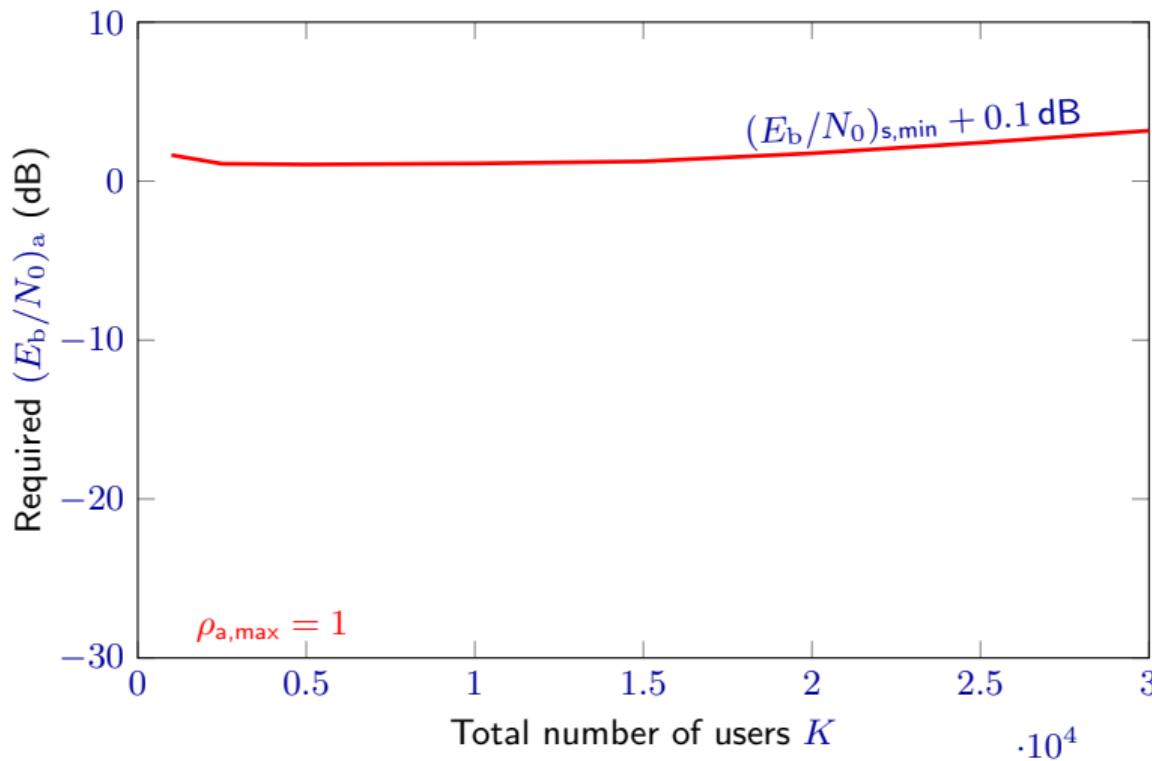
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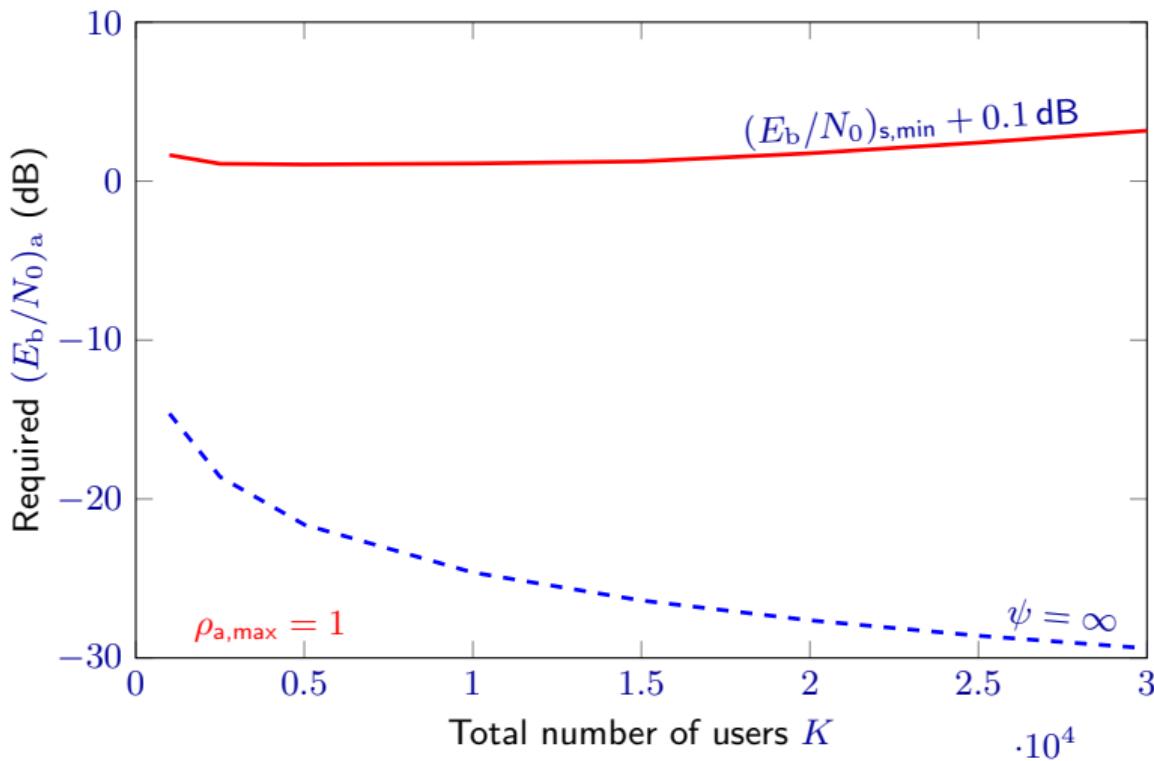
Issue: dynamic range

$$\psi = P_s/P_a \in [30 : 70] \text{ dB}$$

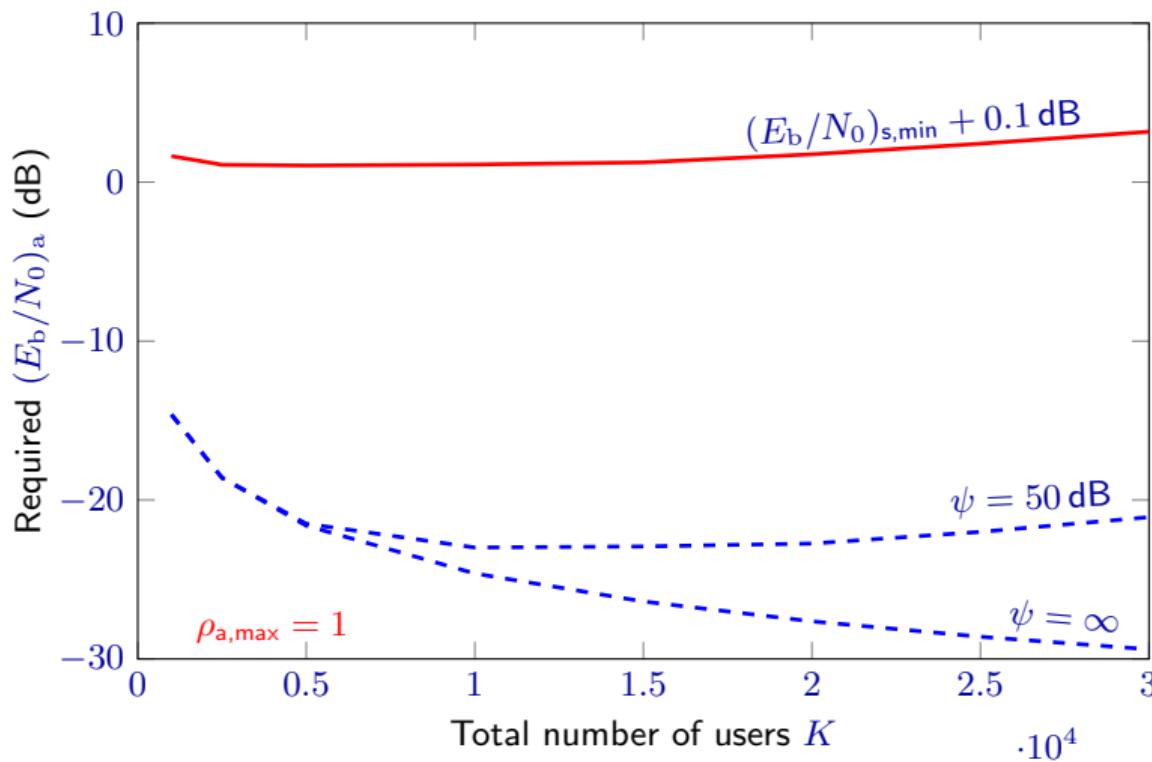
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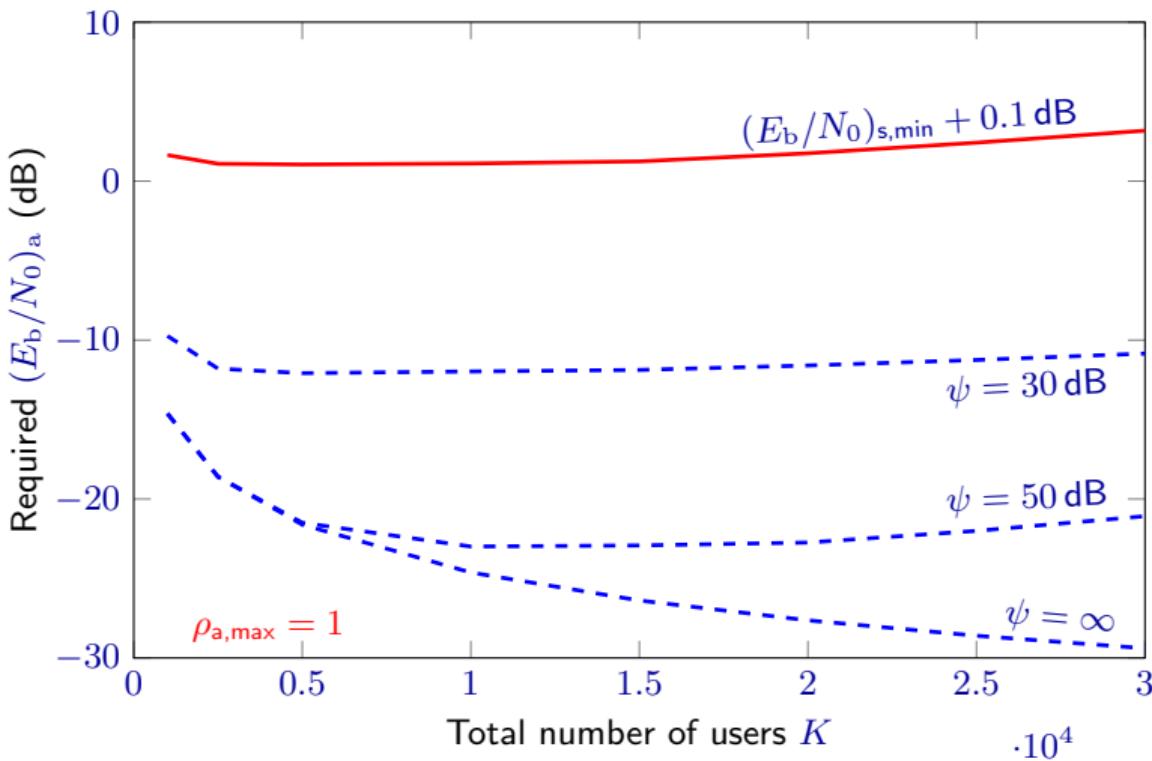
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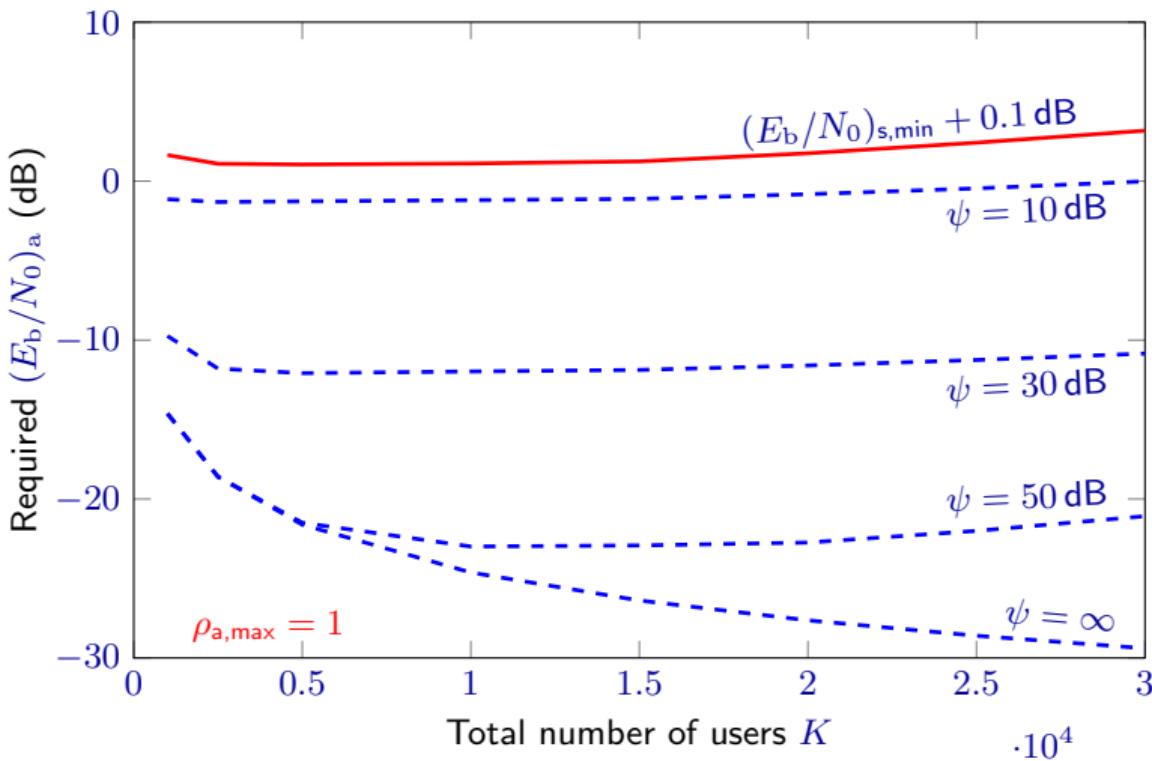
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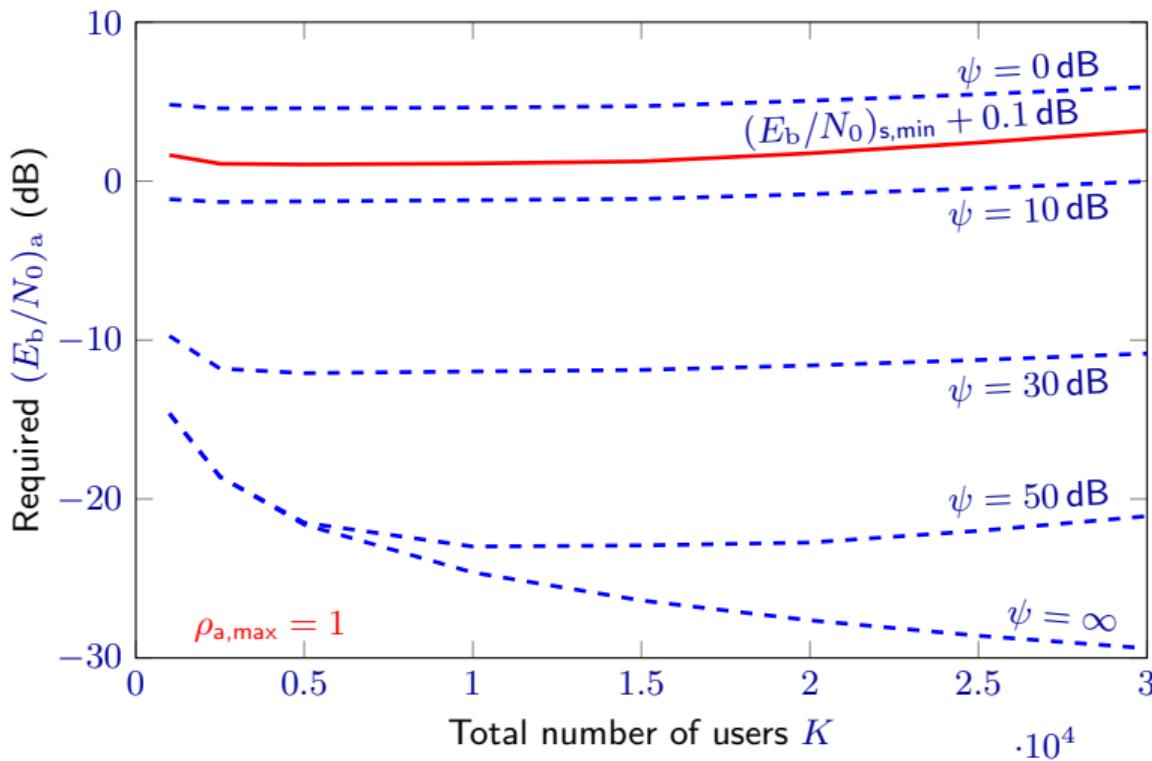
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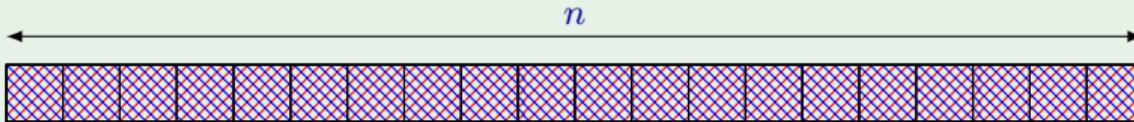
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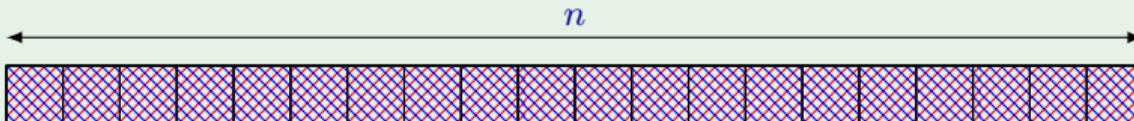


Superposition



$$\mathbf{y} = \sum_{k=1}^{K_s} \mathbf{x}_k + K_a \mathbf{x}_a + \mathbf{z}$$

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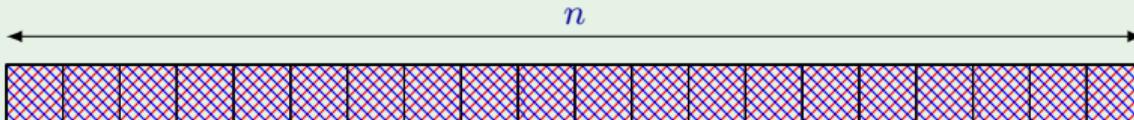


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Decoder: reliability diversity

- Estimate K_a and \mathbf{W}_a ; treat $\sum_{k=1}^{K_s} \mathbf{x}_k$ as noise
- Interference cancellation $\mathbf{y}_{ic} = \mathbf{y} - \hat{K}_a \hat{\mathbf{x}}_a$
- Estimate K_s and \mathcal{W} from \mathbf{y}_{ic} ; residual interference $K_a \mathbf{x}_a - \hat{K}_a \hat{\mathbf{x}}_a$

Superposition



$$\mathbf{y} = \sum_{k=1}^{K_s} \mathbf{x}_k + K_a \mathbf{x}_a + \mathbf{z}$$

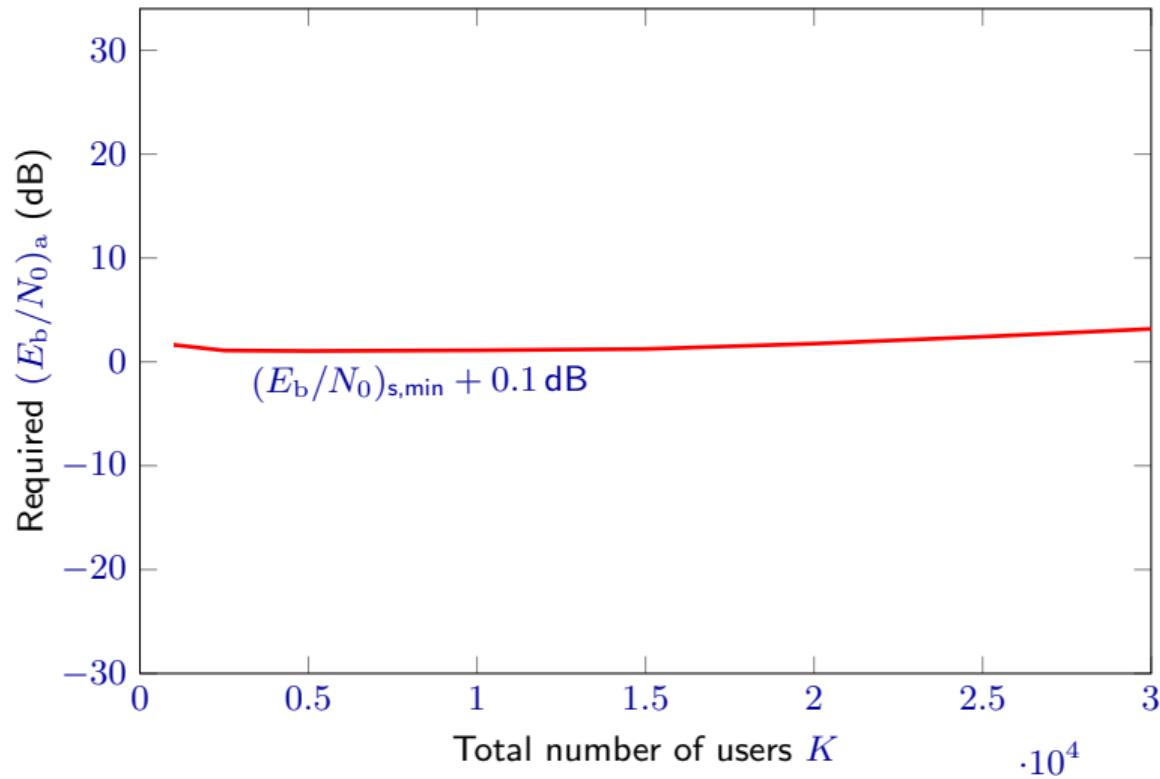
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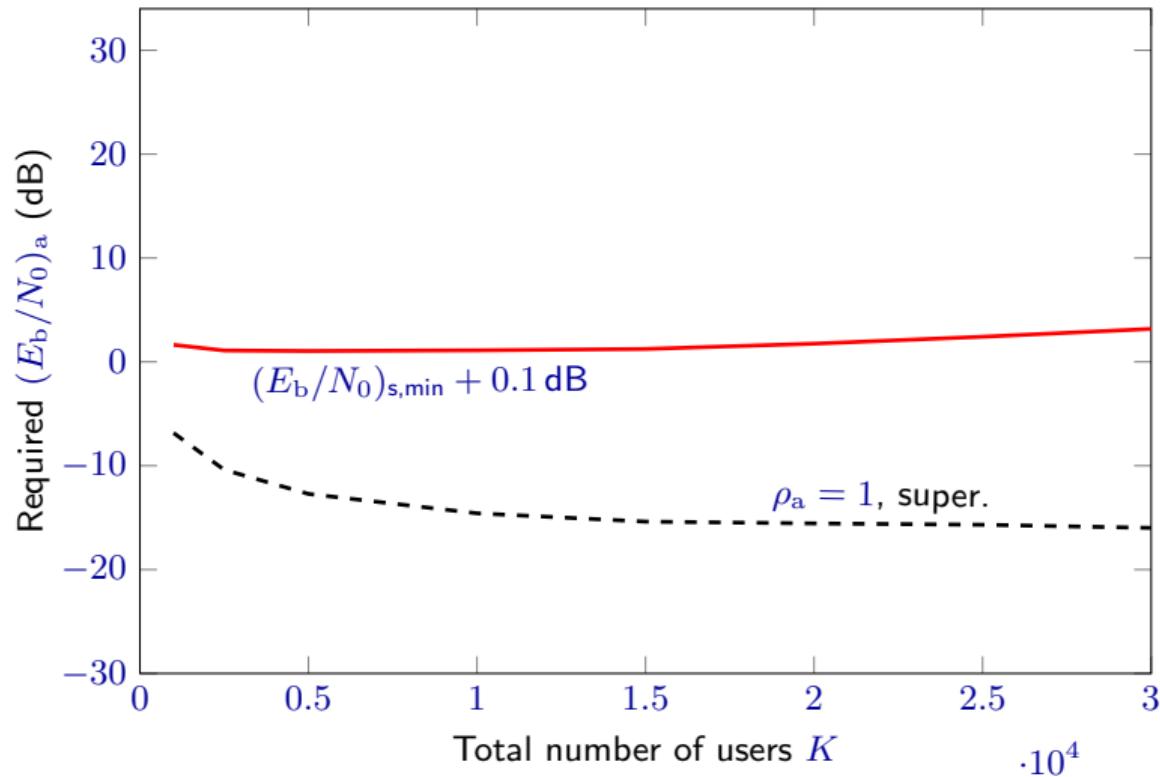
Issue

Difficult to estimate K_a reliably in the presence of noise; imperfect interference cancellation

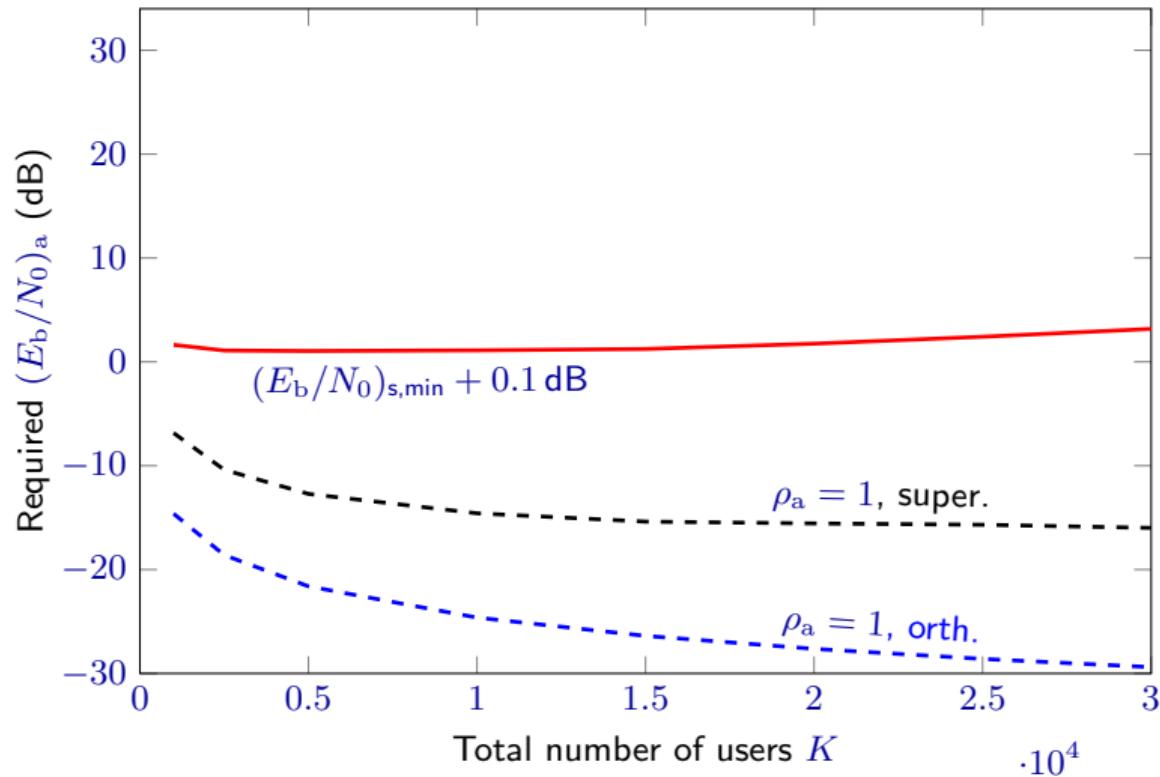
Performance: orthogonalization vs. superposition



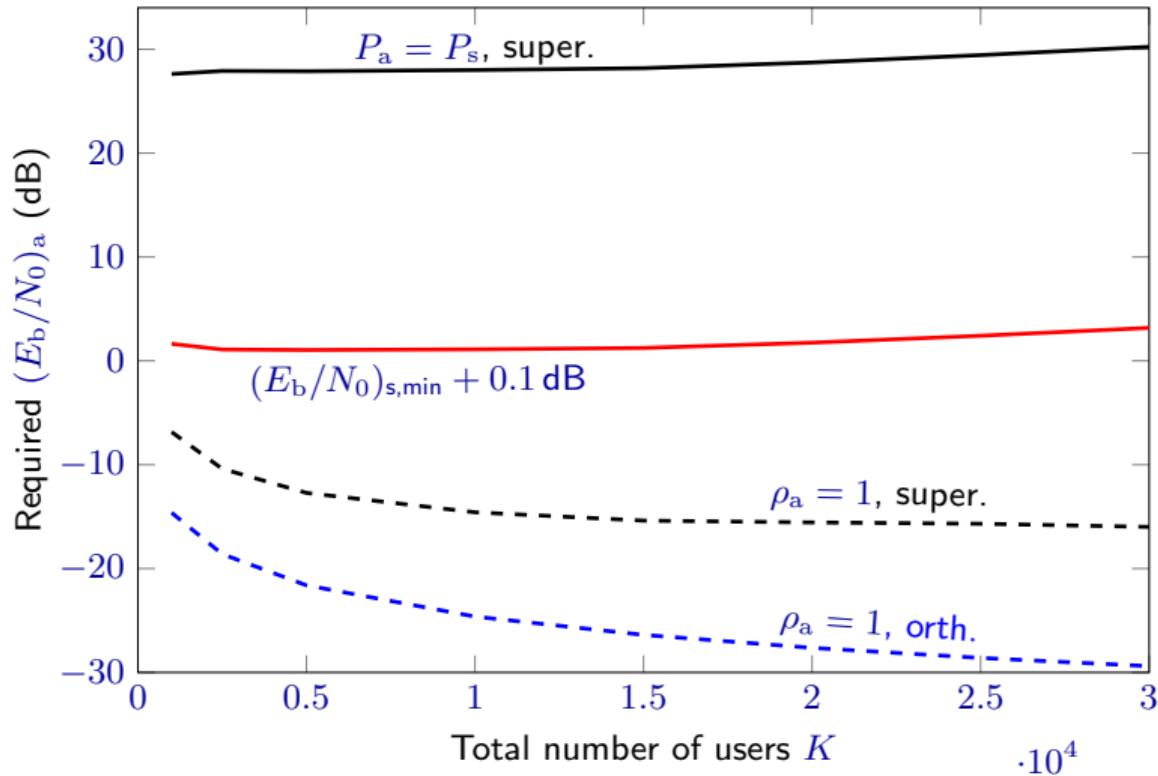
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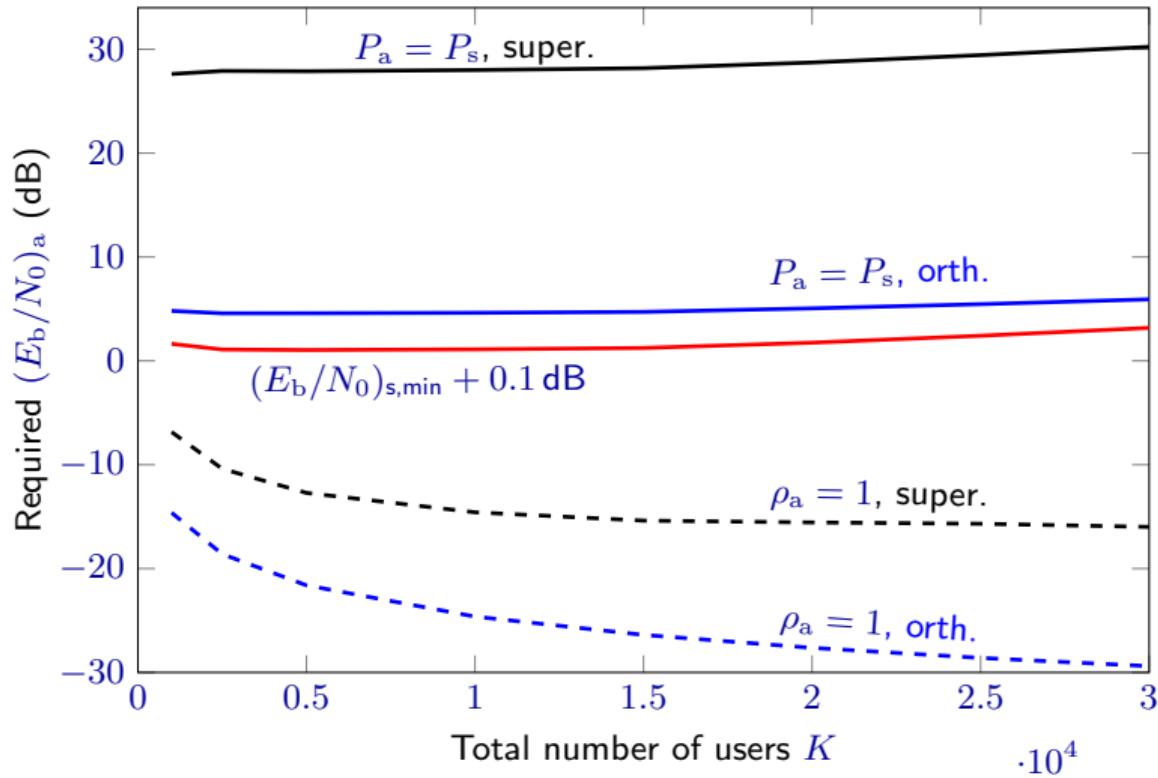
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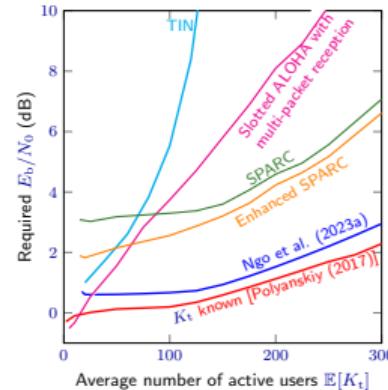


Performance: orthogonalization vs. superposition



Summary and further extensions

- Overview of information-theoretic bounds for mMTC
- Generalization to unknown number of active users and heterogeneous traffic



Further extensions and open problems

- Fading, massive MIMO, cell-free [Kowshik & Polyanskiy, 2021; Fengler et al. 2022; Decurninge et al. 2021; Gkagkos et al. 2023]
- Variable-length codes with stop feedback [Yavas et al. 2021]
- Imperfect synchronization [Decurninge et al. 2022 , Fengler et al. 2023]
- Age of information [Munari 2021, Munari et al., 2023]
- Energy harvesting [Demirhan & Duman, 2019]