Short packets over a massive random-access channel

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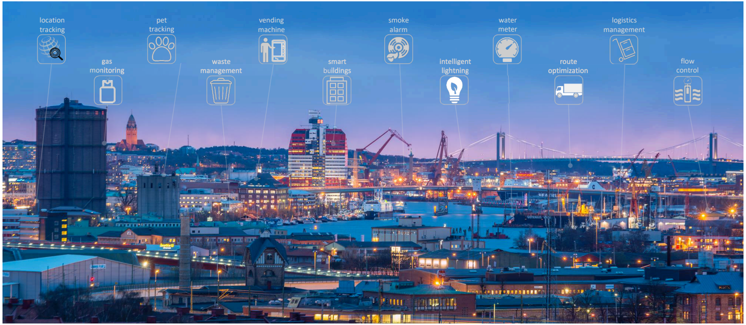
Joint work with H. K. Ngo, A. Lancho, A. Graell i Amat, P. Popovski, A. Kalør, B. Soretz

CHALMERS

Wireless connectivity enables new services



source: IoTpool

Challenges

- Collect data from a massive number of low-cost sensors
- Communicate reliably critical information

Massive and critical wireless connectivity

massive machine-type comm. (mMTC)

- Uplink mostly
- High energy efficiency
- Great commercial interest
- LPWAN, satellite

ultra-reliable low-latency comm. (URLLC)

- Bidirectional
- Low latency, high reliability
- Limited commercial interest (so far)
- Private 5G network

Some characteristics

mMTC

- Small information payload (100 bits)
- High user density (10^7 devices/Km^2)
- Sporadic TX (less that 1 per minute)

  $\Rightarrow 120$ dof per user at $B = 20\,\mathrm{MHz}$

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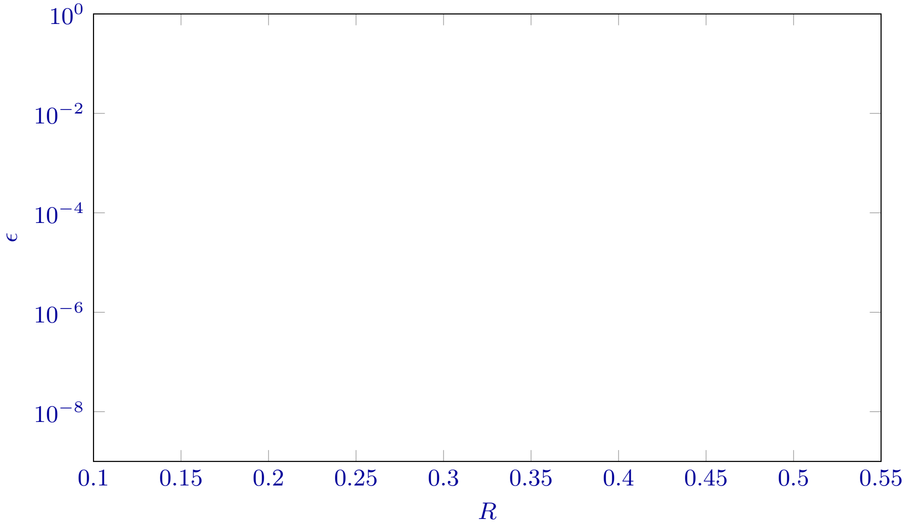
Key design tool

Finite-blocklength information theory

Finite-blocklength IT for URLLC



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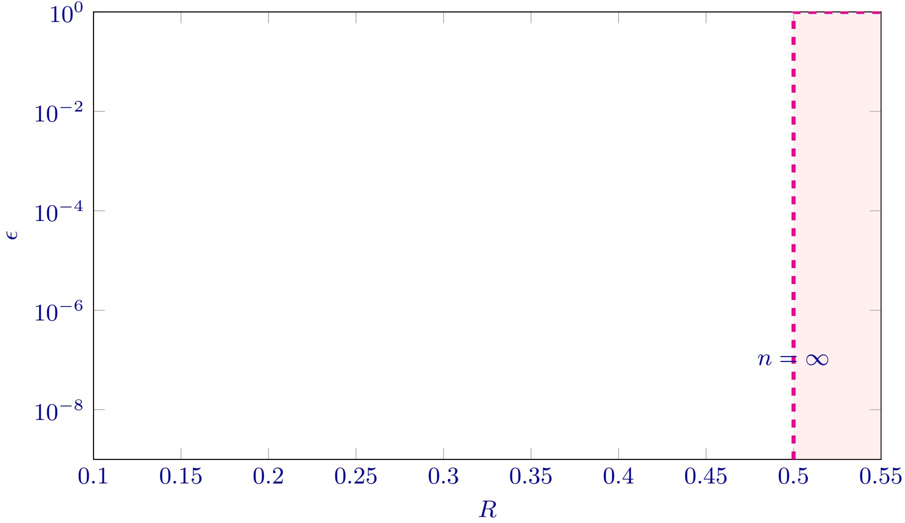

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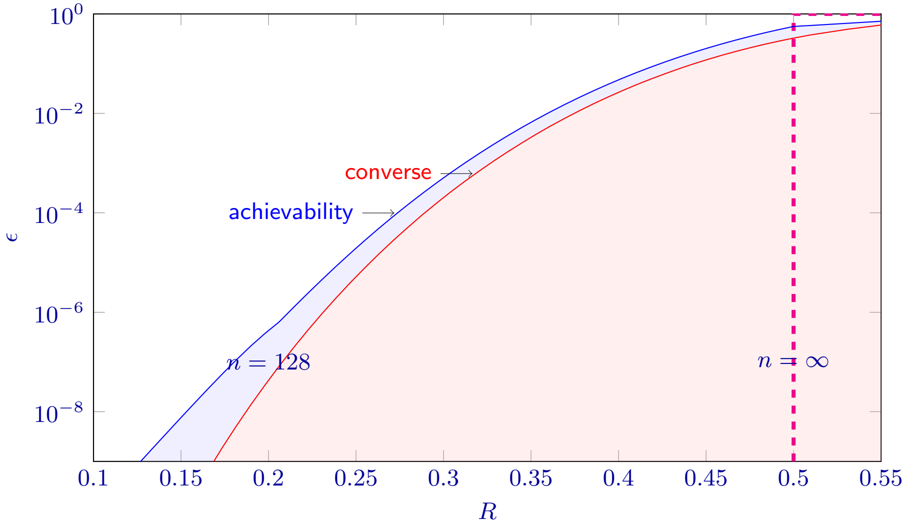

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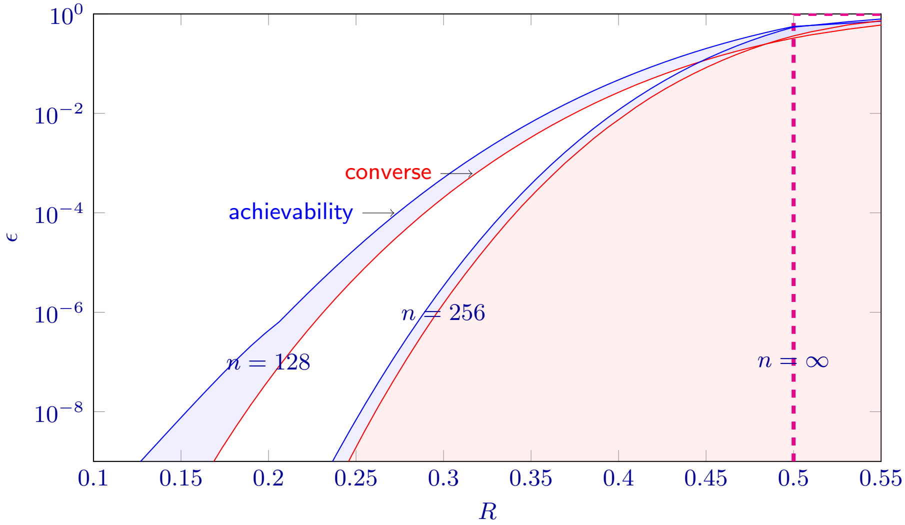

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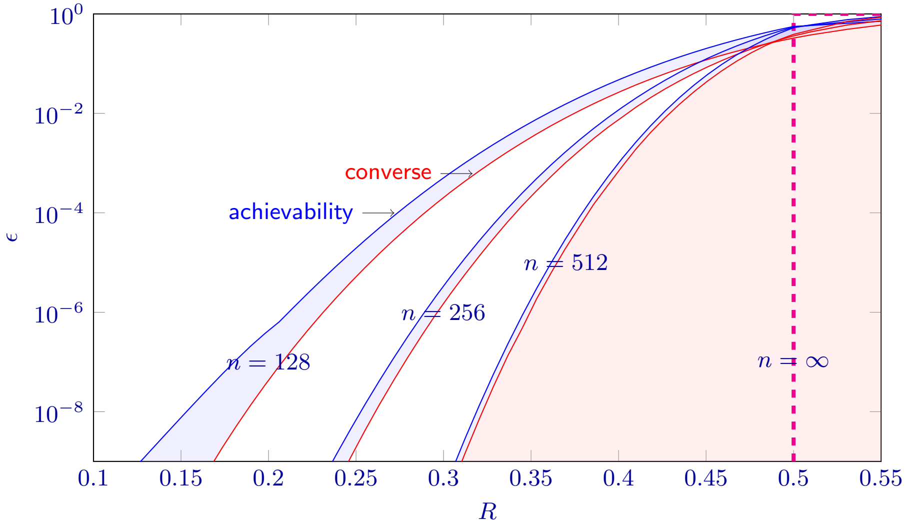

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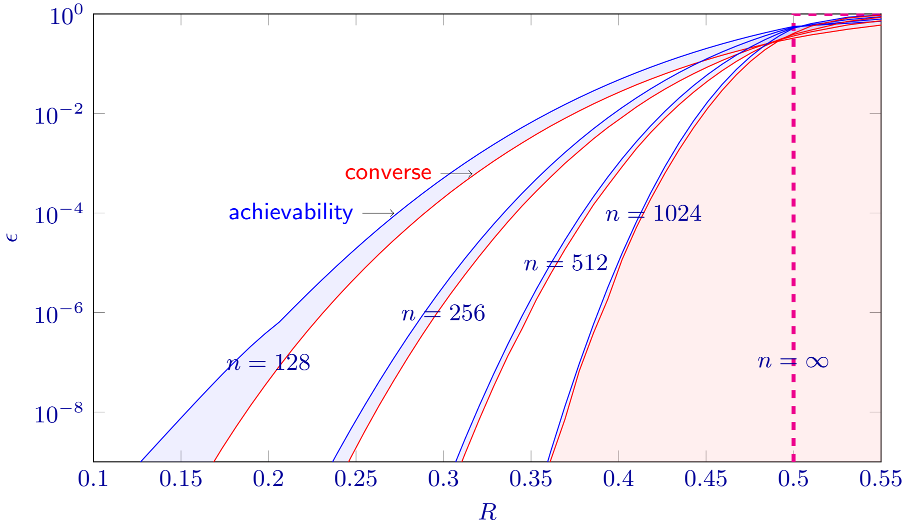

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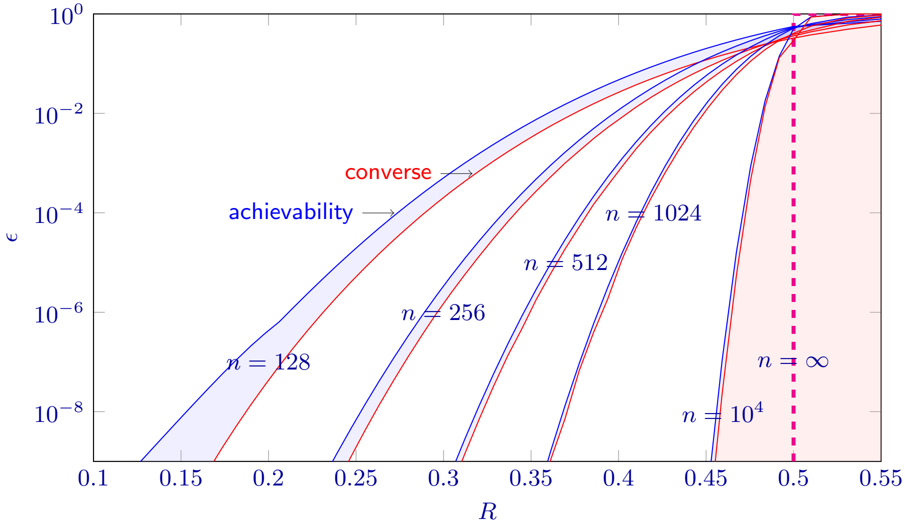

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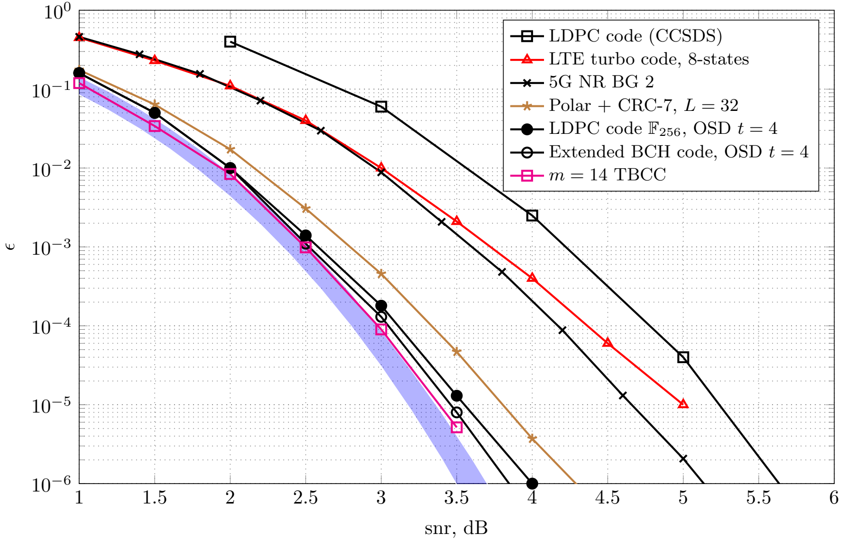
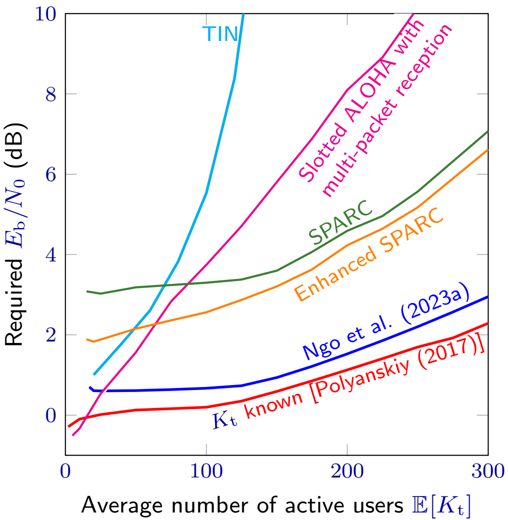
Guidelines for optimal design ($R = 1/2$ bit/channel use)



This talk

- FBL-IT bounds for mMTC [Polyanskiy '17]
- Coding schemes approaching the bound
- Extension to
  - Unknown number of active users [Ngo et al. 2023a]
  - Heterogeneous traffic [Ngo et al. 2023b]
- Further extensions and open problems



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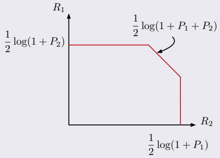

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- Each user is given a different codebook
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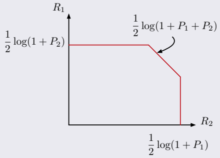

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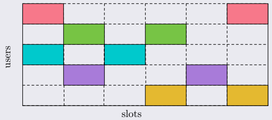

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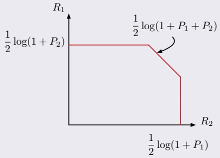

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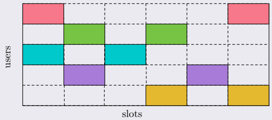

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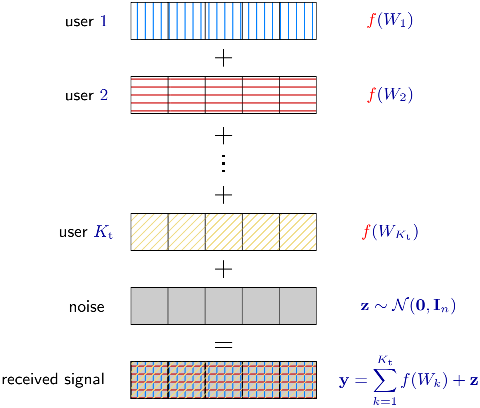
Addressing these limitations

- Noiseless adder channel (e.g., [Bar-David et al., '97])
- More general information-theoretic perspective [Polyanskiy '17]

Unsourced GMAC model [Polyanskiy '17]



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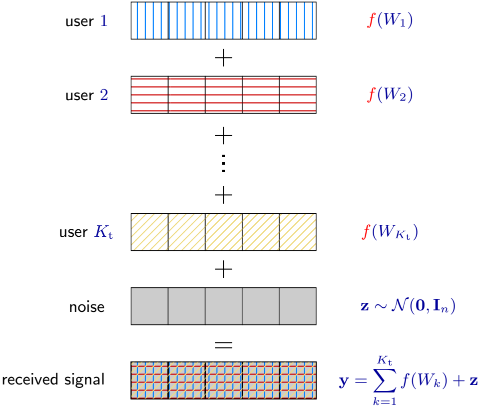

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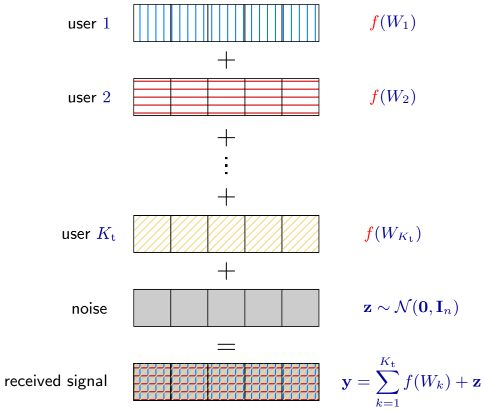

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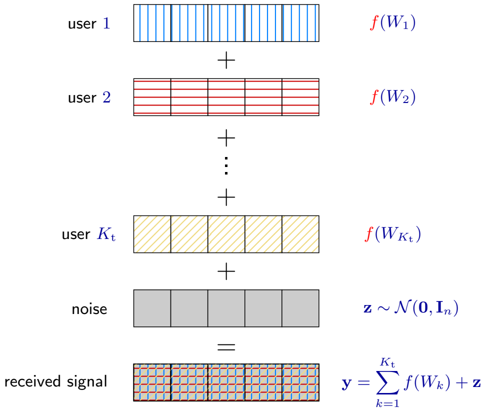

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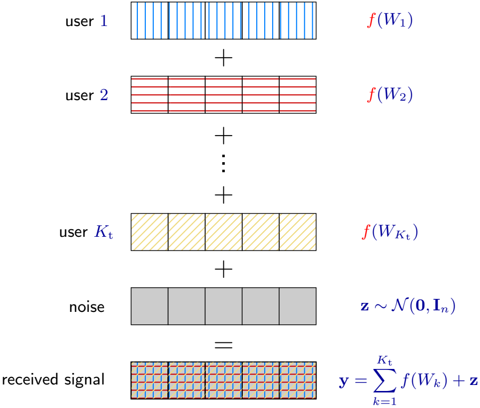

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- K_{t} known to the decoder
- ! Per-user error-probability

$$P_{\mathrm{e}} = \frac{1}{K_{\mathrm{t}}} \sum_{k=1}^{K_{\mathrm{t}}} \mathbb{P}\left[W_k \notin \widehat{\mathcal{W}}\right]$$

Random coding achievability bound

(M, n, ϵ) code for K_{t}-user unsourced GMAC with power constraint P

It consists of a pair of possibly randomized encoder and decoder satisfying $P_{\mathrm{e}} \leq \epsilon$

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Random-coding achievability bound [Polyanskiy '17]

For every $P' < P$, there exists an (M, n, ϵ) code for the K_t-user unsourced GMAC with power constraint P satisfying

$$\epsilon \leq \sum_{k=1}^{K_t} \frac{k}{K_t} \min\{p_k, q_k\} + p_0, \quad \text{where}$$

$$p_0 = \frac{\binom{K_t}{2}}{M} + K_t \, \mathbb{P}\left[\frac{1}{n}\sum_{j=1}^{n} z_j^2 > \frac{P}{P'}\right]$$

$$p_k = e^{-E(t)}$$

$$E(t) = \max_{0 \leq \rho_1, \rho_2 \leq 1} -\rho_1\rho_2 k R_1 - \rho_2 R_2 + E_0(\rho_1, \rho_2)$$

$E_0(\rho_1, \rho_2)$: complicated expression in ρ_1, ρ_2, k, P'

$$q_k = \inf_{\gamma} \mathbb{P}[I_k \leq \gamma] + e^{n(kR_1 + R_2) - \gamma}$$

I_k : related to inf. dens.

$$R_1 = \frac{1}{n}\log M - \frac{1}{nk}\log k!$$

$$R_2 = \frac{1}{n}\log\binom{K_t}{k}$$

Key ideas and steps in the proof

Random codebook generation and encoder

- Gaussian codebook: fix $P' < P$; generate M codewords $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P'\mathbf{I}_n)$
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Decoder

Unordered list $\widehat{\mathcal{W}}$ of decoded messages obtained by solving

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Change of measure

- Change measure from P to Q for which: message are distinct and codewords are i.i.d. Gaussian
- For every event $\mathcal{E}$, $\mathbb{E}_P[\mathcal{E}] \leq \mathbb{E}_Q[\mathcal{E}] + d_{\mathrm{TV}}(P, Q)$

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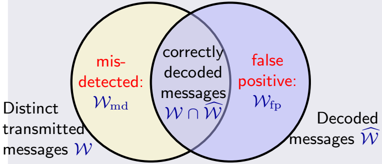

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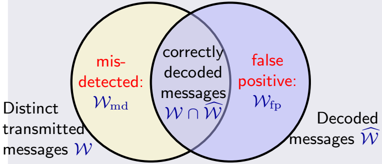

$$P_\mathrm{e} \le \sum_{k=1}^{K_\mathrm{t}} \frac{k}{K_\mathrm{t}} \mathbb{P}[|\mathcal{W}_\mathrm{md}| = |\mathcal{W}_\mathrm{fp}| = k] + p_0$$

$$\mathbb{P}[|\mathcal{W}_\mathrm{md}| = |\mathcal{W}_\mathrm{fp}| = k] = \mathbb{P}\left[\bigcup_{\substack{\mathcal{W}_\mathrm{md} \subset \mathcal{W} \\ |\mathcal{W}_\mathrm{md}| = k}} \bigcup_{\substack{\mathcal{W}_\mathrm{fp} \subset [1:M] \setminus \mathcal{W} \\ |\mathcal{W}_\mathrm{fp}| = k}} \|\mathbf{z} + \mathbf{c}(\mathcal{W}_\mathrm{md}) - \mathbf{c}(\mathcal{W}_\mathrm{fp})\| \le \|\mathbf{z}\|\right]$$

Three tools and their applications

Chernoff: for every random $\mathbf{u}$ and every $\lambda > 0$

$$\mathbb{P}[\|\mathbf{z}+\mathbf{u}\| \leq v] \leq e^{\lambda v^2}\, \mathbb{E}_{\mathbf{z}}\left[e^{-\lambda\|\mathbf{z}+\mathbf{u}\|^2}\right]$$

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- Assume that $\mathbb{P}[\mathcal{A}_j \,|\, \mathbf{z}] \leq e^{-nE(\mathbf{z})}, j = 1, \dots, m$
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Gallager's trick to tighten the union bound

- Assume that $\mathbb{P}[\mathcal{A}_j \,|\, \mathbf{z}] \leq e^{-nE(\mathbf{z})}, j = 1, \dots, m$
- Then for all $0 \leq \rho \leq 1$

$$\mathbb{P}\left[\bigcup_{j=1}^{m} \mathcal{A}_j\right] \leq m^\rho\, \mathbb{E}\left[e^{-n\rho E(\mathbf{z})}\right]$$

$$\mathbb{P}\left[\bigcup_{\mathcal{W}_{\mathrm{md}}} \bigcup_{\mathcal{W}_{\mathrm{fp}}} \|\mathbf{z} + \mathbf{c}(\mathcal{W}_{\mathrm{md}}) - \mathbf{c}(\mathcal{W}_{\mathrm{fp}})\| \leq \|\mathbf{z}\|\right]$$

1. Chernoff bound to evaluate

$$\mathbb{P}\left[\|\mathbf{z} + \mathbf{c}(\mathcal{W}_{\mathrm{md}}) - \mathbf{c}(\mathcal{W}_{\mathrm{fp}})\| \leq \|\mathbf{z}\| \,\middle|\, \mathbf{c}(\mathcal{W}_{\mathrm{md}}), \mathbf{z}\right]$$

Three tools and their applications

Chernoff: for every random $\mathbf{u}$ and every $\lambda > 0$

$$\mathbb{P}[\|\mathbf{z}+\mathbf{u}\| \le v] \le e^{\lambda v^2}\, \mathbb{E}_{\mathbf{z}}\left[e^{-\lambda\|\mathbf{z}+\mathbf{u}\|^2}\right]$$

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3. MGF to compute expectation over $\mathbf{c}(\mathcal{W}_{\mathrm{md}})$
4. Gallager's trick on $\bigcup_{\mathcal{W}_{\mathrm{md}}}$
5. MGF to compute expectation over $\mathbf{z}$

Random coding achievability bound [Polyanskiy '17]

For every $P' < P$, there exists an (M, n, ϵ) code for the K_t-user unsourced GMAC with power constraint P satisfying

$$\epsilon \le \sum_{k=1}^{K_t} \frac{k}{K_t} \min\{p_k, q_k\} + p_0, \quad \text{where}$$

$$p_0 = \frac{\binom{K_t}{2}}{M} + K_t\, \mathbb{P}\left[\frac{1}{n}\sum_{j=1}^{n} z_j^2 > \frac{P}{P'}\right]$$

$$p_k = e^{-E(t)}$$

$$E(t) = \max_{0 \le \rho_1, \rho_2 \le 1} -\rho_1\rho_2 k R_1 - \rho_2 R_2 + E_0(\rho_1, \rho_2)$$

$E_0(\rho_1, \rho_2)$: complicated expression in ρ_1, ρ_2, k, P'

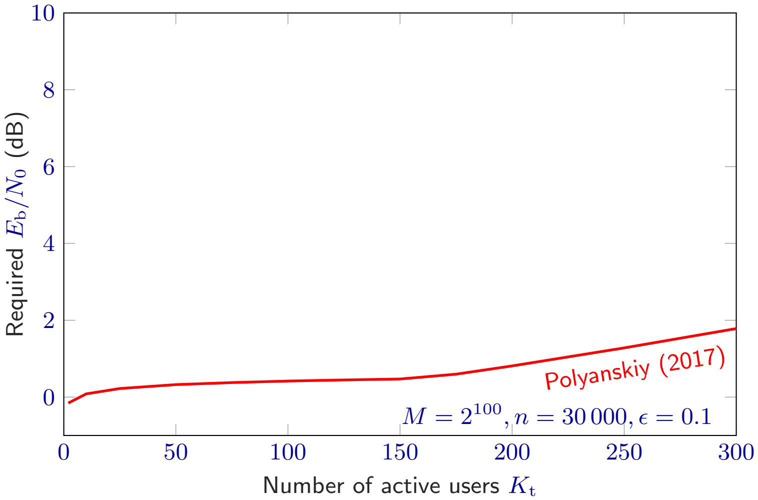
$$q_k = \inf_{\gamma} \mathbb{P}[I_k \le \gamma] + e^{n(kR_1 + R_2) - \gamma}$$

I_k : related to inf. dens.

$$R_1 = \frac{1}{n}\log M - \frac{1}{nk}\log k!$$

$$R_2 = \frac{1}{n}\log\binom{K_t}{k}$$

Numerical evaluation of the bound



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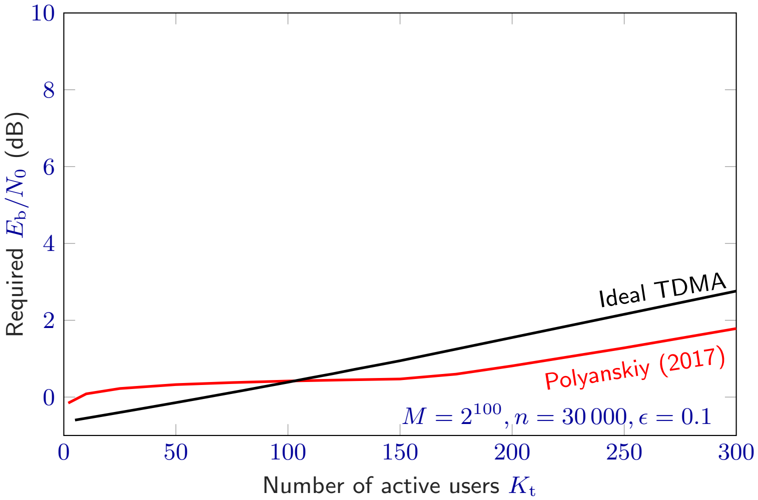

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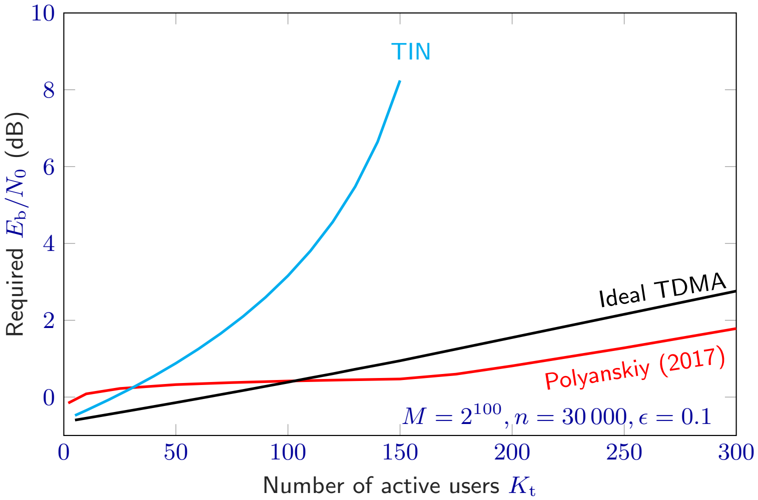

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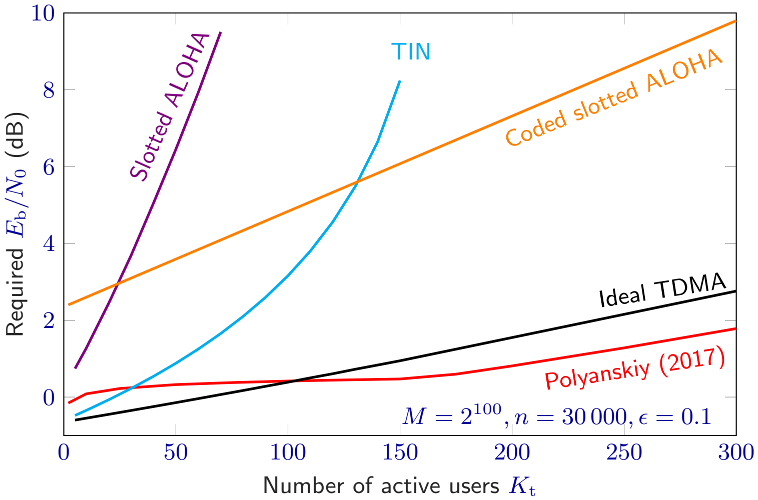
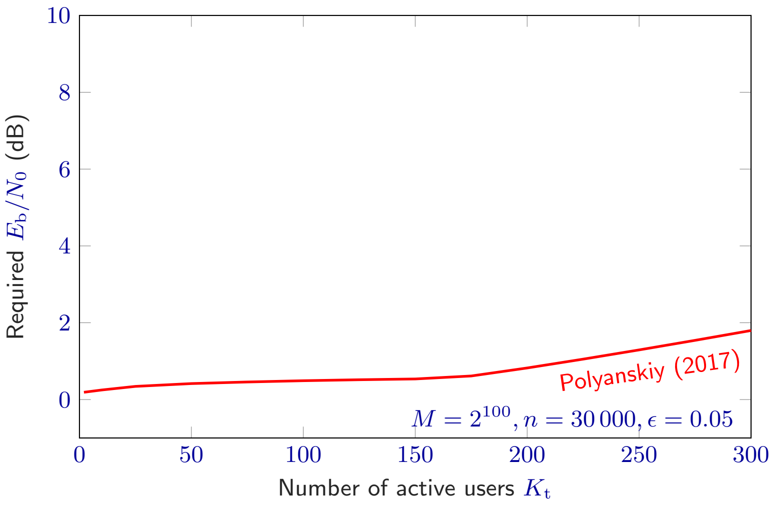


Novel coding schemes (2017–2022)



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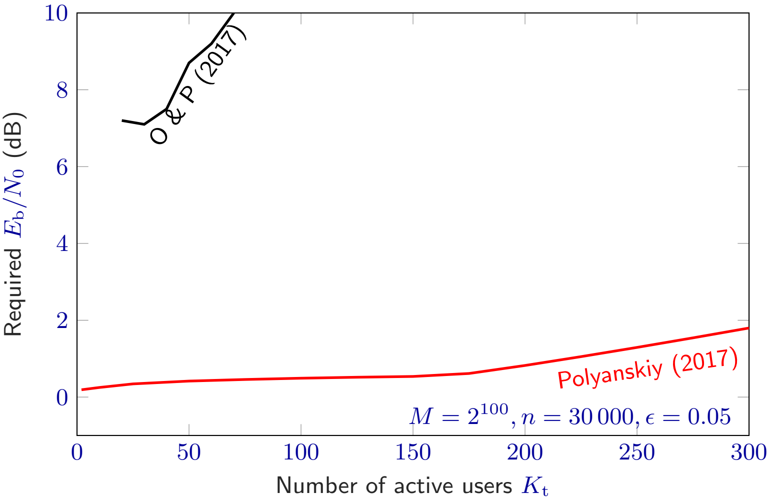

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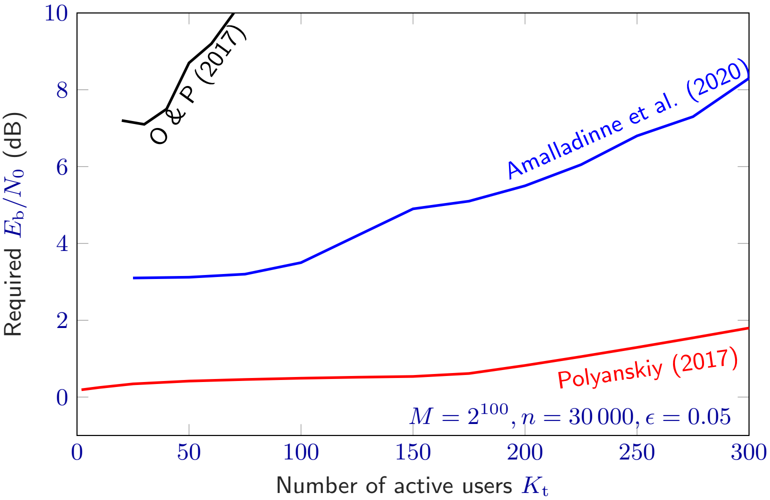

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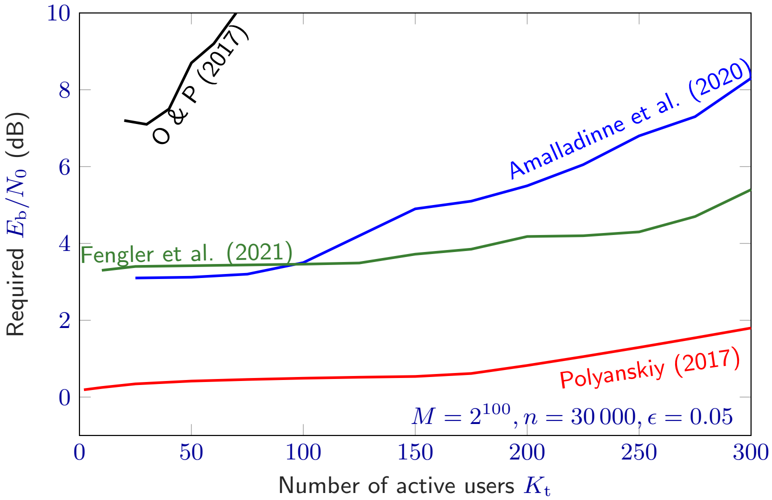

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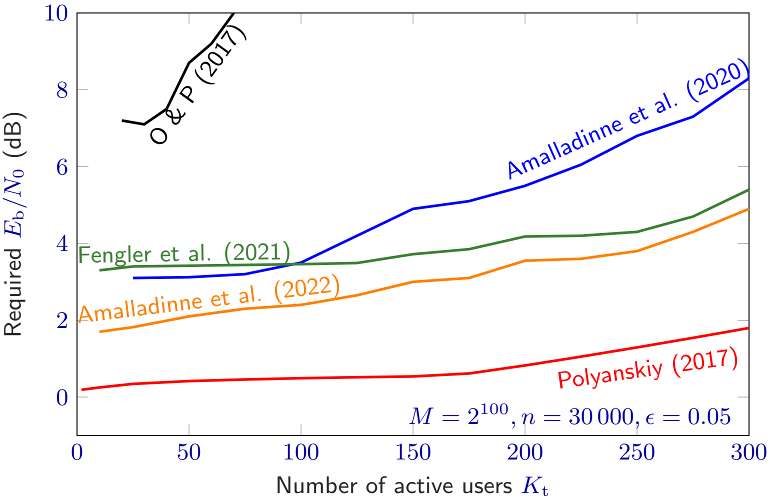

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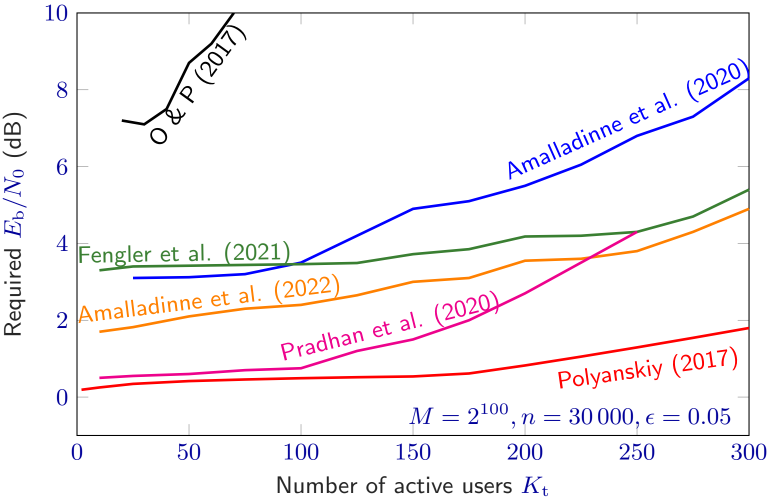

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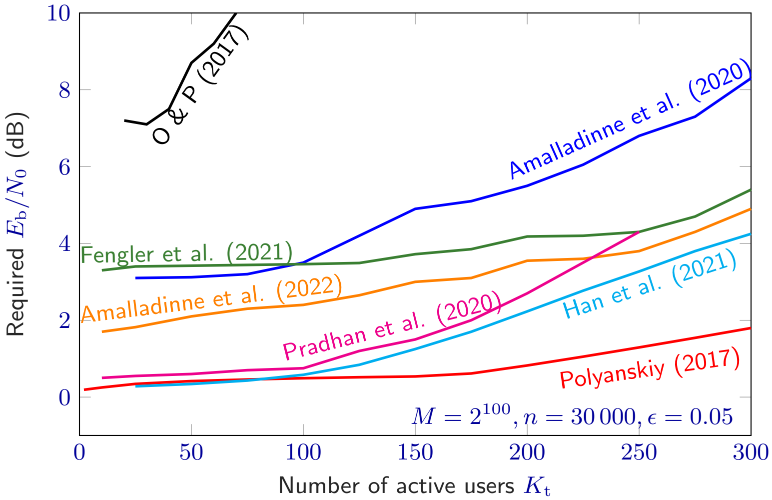
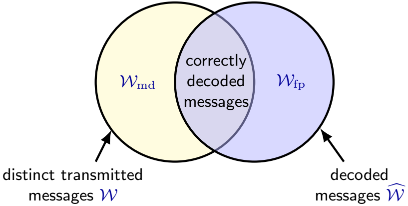


$K_{\rm t}$ is random and not known to the receiver [Ngo et al. '23]



$K_{\rm t}$ known $\Rightarrow |\mathcal{W}_{\rm md}| = |\mathcal{W}_{\rm fp}|$

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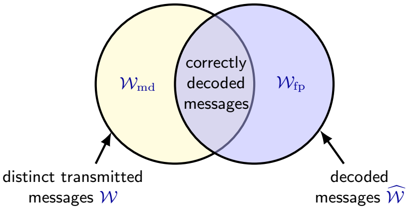

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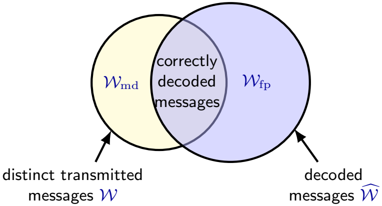

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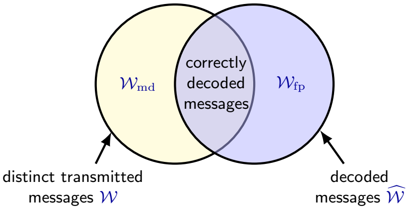

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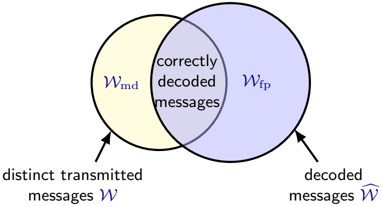

K_t unknown $\Rightarrow |\mathcal{W}_\mathrm{md}| \neq |\mathcal{W}_\mathrm{fp}|$

Pragmatic mismatched approach

Estimate K_t and deploy a coding scheme that treats K_t as known

Performance metrics and definition of a code

Two performance metrics

$$P_{\mathrm{md}} = \mathbb{E}\left[\frac{1}{|\mathcal{W}|}\sum_{i=1}^{|\mathcal{W}|}\mathbb{P}\left[W_i \notin \widehat{\mathcal{W}}\right]\right]$$

$$P_{\mathrm{fp}} = \mathbb{E}\left[\frac{1}{\left|\widehat{\mathcal{W}}\right|}\sum_{i=1}^{|\widehat{\mathcal{W}}|}\mathbb{P}\left[\widehat{W}_i \notin \mathcal{W}\right]\right]$$

Performance metrics and definition of a code

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$(M, n, \epsilon_{\mathrm{md}}, \epsilon_{\mathrm{fp}})$ code for unsourced GMAC with power constraint P and random, unknown number of transmitting users

It consists of a pair of possibly randomized encoder and decoder satisfying $P_{\mathrm{md}} \le \epsilon_{\mathrm{md}}$ and $P_{\mathrm{fp}} \le \epsilon_{\mathrm{fp}}$

A two-step decoder

Step 1

Obtain an estimate K'_{t} of K_{t} by maximizing a suitably chosen metric $m(\mathbf{y}, k)$

$$K'_{\mathrm{t}} = \arg\max_{k} m(\mathbf{y}, k)$$

Examples: ML estimation, energy-based estimation

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Step 2

Find the list $\widehat{\mathcal{W}}$ of decoded messages as

$$\widehat{\mathcal{W}} = \arg\min_{\substack{\mathcal{W}' \subset [1:M] \\ |\mathcal{W}'| = K_{\mathrm{t}}'}} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|$$

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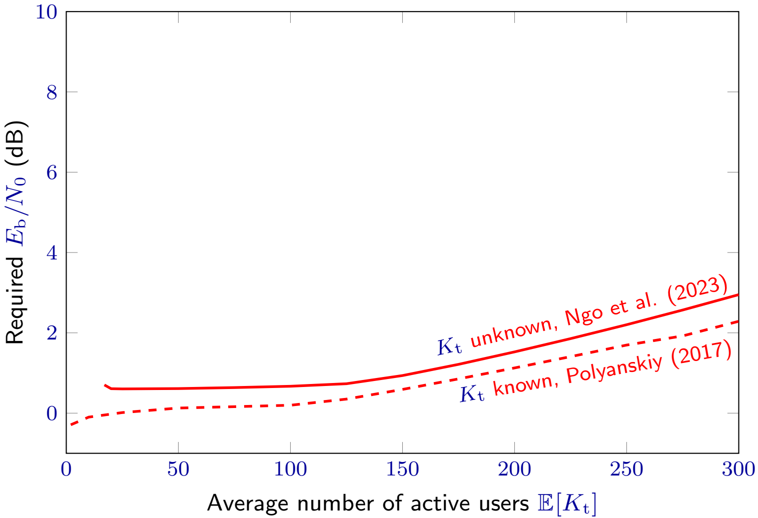
$r > 0$: decoding radius (cannot be chosen too large)

$M = 2^{128}$, $n = 38\,400$, $K_\mathrm{t} \sim$ Poisson, $\epsilon_\mathrm{md} = \epsilon_\mathrm{fp} = 0.1$



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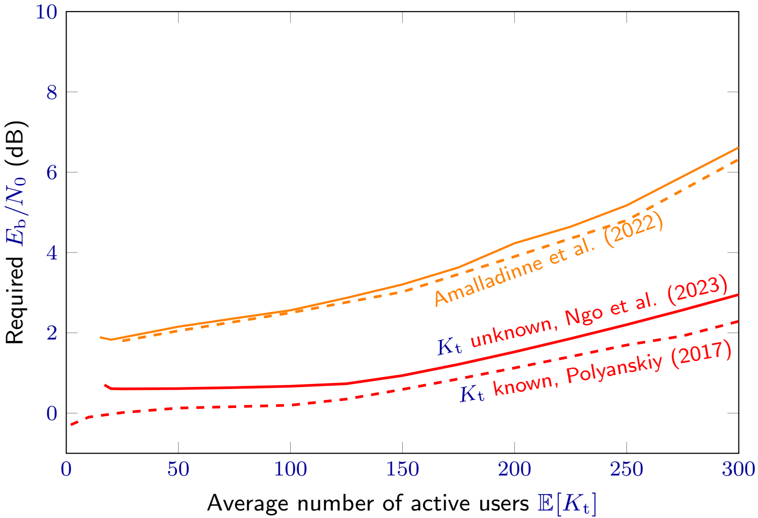
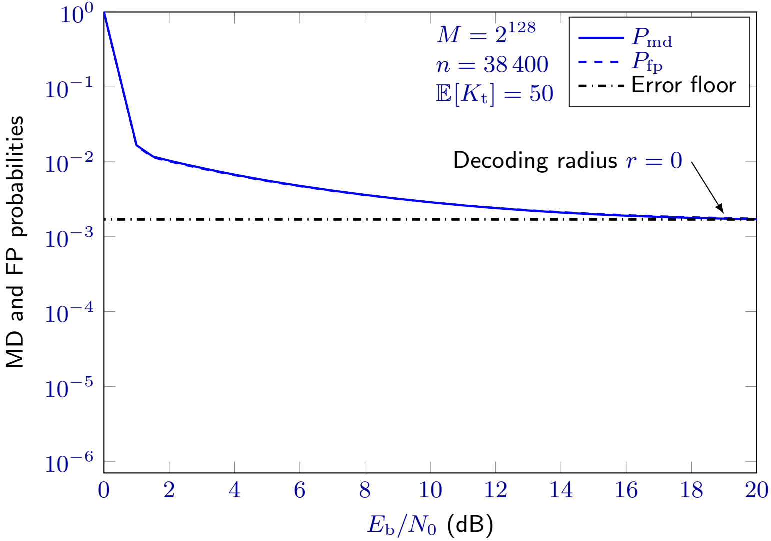


Impact of decoding radius r



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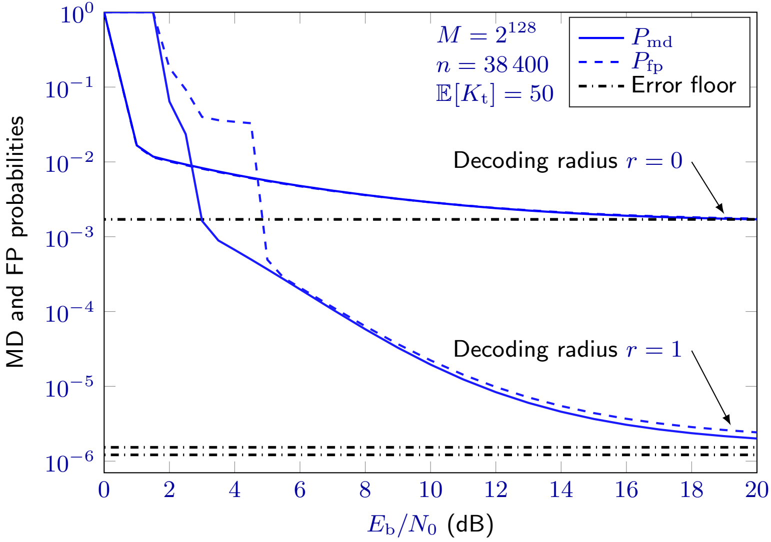

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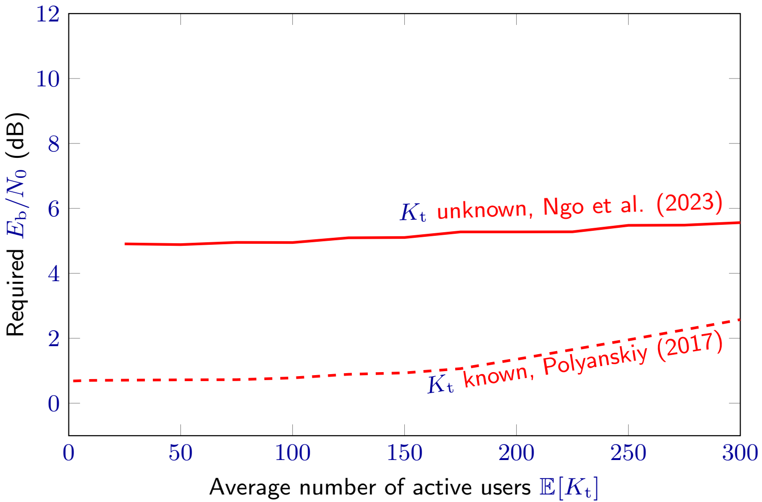
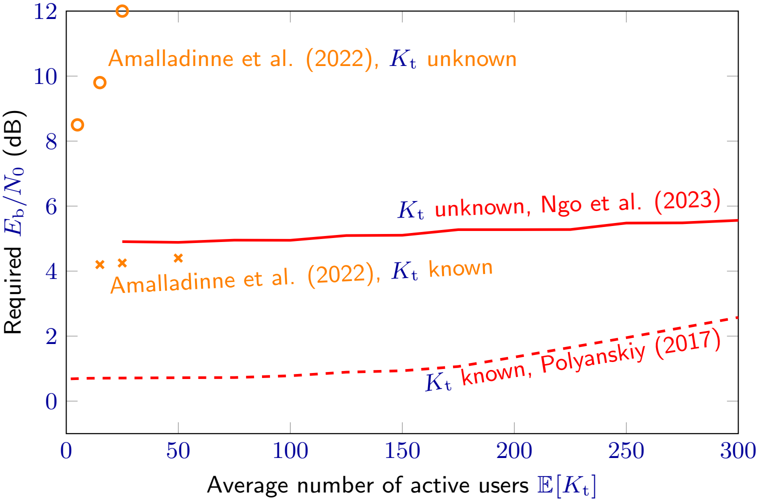
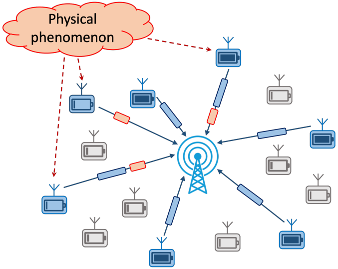


$$M = 2^{128},\ n = 38\,400,\ K_{\mathrm{t}} \sim \text{Poisson},\ \epsilon_{\mathrm{md}} = \epsilon_{\mathrm{fp}} = 10^{-3}$$



Heterogeneous traffic: massive and critical IoTs [Ngo et al. '23]



- K devices
- K_{s} devices transmit a standard msg.
- K_{a} devices transmit an alarm msg.

- Device k has a standard message $W_{\mathrm{s}} \in [1:M_{\mathrm{s}}]$ with prob. ρ_{s}

$$K_{\mathrm{s}} \sim \mathrm{Bin}(K, \rho_{\mathrm{s}})$$

- If an alarm occurs, all devices transmit the same alarm message $W_{\mathrm{a}} \in [1:M_{\mathrm{a}}]$, with probability $\rho_{\mathrm{a}} \leq \rho_{\mathrm{a,max}}$

$$K_{\mathrm{a}} \sim \mathrm{Bin}(K, \rho_{\mathrm{a}})$$

- Each device can transmit a standard message, an alarm message, both, or none

Performance metrics

- $\mathcal{A}$: alarm event
- $\mathcal{W}$: set of transmitted standard messages
- $\widehat{\mathcal{W}}$: set of decoded standard messages
- $W_{\mathrm{a}} \in [1 : M_{\mathrm{a}}]$: transmitted alarm message
- $\widehat{W}_{\mathrm{a}} \in [0 : M_{\mathrm{a}}]$: decoded alarm message ($0 \Rightarrow$ no alarm)

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For $\mathcal{B} \in \{\mathcal{A}, \bar{\mathcal{A}}\}$

$$P_{\mathsf{smd}\,|\,\mathcal{B}} = \mathbb{E}\left[\frac{1}{|\mathcal{W}|}\sum_{i=1}^{|\mathcal{W}|}\mathbb{P}\left[W_i \notin \widehat{\mathcal{W}} \,|\, \mathcal{B}\right]\right], \quad P_{\mathsf{sfp}\,|\,\mathcal{B}} = \mathbb{E}\left[\frac{1}{|\widehat{\mathcal{W}}|}\sum_{i=1}^{|\widehat{\mathcal{W}}|}\mathbb{P}[W_i \notin \mathcal{W} \,|\, \mathcal{B}]\right]$$

Performance metrics

- $\mathcal{A}$: alarm event
- $\mathcal{W}$: set of transmitted standard messages
- $\widehat{\mathcal{W}}$: set of decoded standard messages
- $W_\mathrm{a} \in [1 : M_\mathrm{a}]$: transmitted alarm message
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For $\mathcal{B} \in \{\mathcal{A}, \bar{\mathcal{A}}\}$

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$$P_\mathrm{amd} = \mathbb{P}\left[\widehat{W}_\mathrm{a} \neq W_a \,|\, \mathcal{A}\right], \quad P_\mathrm{afp} = \mathbb{P}\left[\widehat{W}_\mathrm{a} \neq 0 \,|\, \bar{\mathcal{A}}\right]$$

Definition of code and coexistence strategies

$(M_\mathrm{a}, M_\mathrm{s}, n, \epsilon_\mathrm{smd}, \epsilon_\mathrm{sfp}, \epsilon_\mathrm{amd}, \epsilon_\mathrm{afp})$ code for unsourced GMAC with standard and alarm messages

It consists of a pair of possibly randomized encoder and decoder satisfying $P_{\mathsf{smd}\,|\,\mathcal{B}} \leq \epsilon_\mathrm{smd}$, $P_{\mathsf{sfp}\,|\,\mathcal{B}} \leq \epsilon_\mathrm{sfp}$, for $\mathcal{B} \in \{\mathcal{A}, \bar{\mathcal{A}}\}$ and $P_\mathrm{amd} \leq \epsilon_\mathrm{amd}$, $P_\mathrm{amd} \leq \epsilon_\mathrm{amd}$

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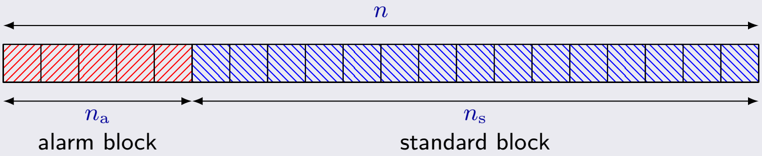
Orthogonalization

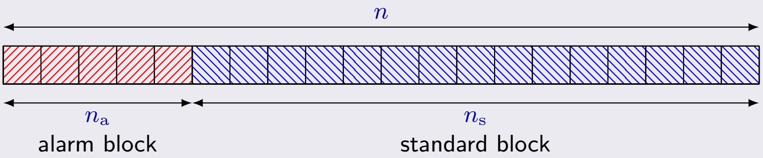


Definition of code and coexistence strategies

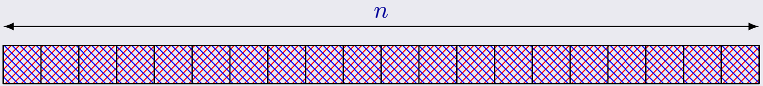
$(M_\mathrm{a}, M_\mathrm{s}, n, \epsilon_\mathrm{smd}, \epsilon_\mathrm{sfp}, \epsilon_\mathrm{amd}, \epsilon_\mathrm{afp})$ code for unsourced GMAC with standard and alarm messages

It consists of a pair of possibly randomized encoder and decoder satisfying $P_{\mathsf{smd}\,|\,\mathcal{B}} \le \epsilon_\mathrm{smd}$, $P_{\mathsf{sfp}\,|\,\mathcal{B}} \le \epsilon_\mathrm{sfp}$, for $\mathcal{B} \in \{\mathcal{A}, \bar{\mathcal{A}}\}$ and $P_\mathrm{amd} \le \epsilon_\mathrm{amd}$, $P_\mathrm{amd} \le \epsilon_\mathrm{amd}$

Orthogonalization



Superposition: exploiting reliability diversity [Popovski et al., 2018]



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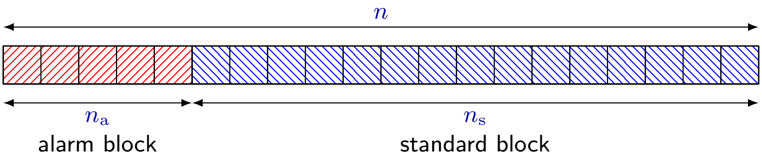

Standard block

$$\mathbf{y}_{\mathrm{s}} = \sum_{k=1}^{K_{\mathrm{s}}} \mathbf{x}_k + \mathbf{z}_{\mathrm{s}}, \quad \mathbf{z}_{\mathrm{s}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{n_{\mathrm{s}}})$$

- $\|\mathbf{x}_k\|^2 \le n_{\mathrm{s}} P_{\mathrm{s}}$
- $(E_{\mathrm{b}}/N_0)_{\mathrm{s}} = \frac{n_{\mathrm{s}} P_{\mathrm{s}}}{2 \log_2 M_{\mathrm{s}}}$
- unsourced GMAC with random and unknown number of active users
- Can be analyzed as before

Alarm block

$$\mathbf{y}_{\mathrm{a}} = K_{\mathrm{a}} \mathbf{x}_{\mathrm{a}} + \mathbf{z}_{\mathrm{a}}, \quad \mathbf{z}_{\mathrm{a}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{n_a})$$

- $\|\mathbf{x}_{\mathrm{a}}\|^2 \le n_{\mathrm{a}} P_{\mathrm{a}}$
- $(E_{\mathrm{b}}/N_0)_{\mathrm{a}} = \frac{n_{\mathrm{a}} P_{\mathrm{a}} \rho_{\mathrm{a}} K}{2 \log_2 M_{\mathrm{a}}}$
- Single-user AWGN channel with random, unknown SNR $K_{\mathrm{a}}^2 P_{\mathrm{a}}$ (coherent combining)
- Can be analyzed with FBL tools

Analysis of alarm block

Random codebook generation and encoder

- Gaussian codebook: fix $P' < P$; generate M_a codewords $\mathbf{c}_1, \dots, \mathbf{c}_{M_\mathrm{a}} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P'\mathbf{I}_{n_\mathrm{a}})$
- Encoder: User k transmits $\mathbf{c}_{W_0} \mathbb{1}\{\|\mathbf{c}_{W_0}\|^2 \le nP\}$ with probability ρ_a

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Two-step mismatch decoder

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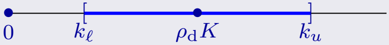
Two-step mismatch decoder

- Detection of alarm event and number of devices: $K'_\mathrm{a} = \underset{k \in [k_\ell : k_u] \cup \{0\}}{\arg\max}\ p(\mathbf{y}_\mathrm{a} \,|\, k)$



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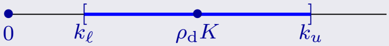

- Decoding of W_{a}: if $K'_{\mathrm{a}} = 0$ return $\widehat{W}_{\mathrm{a}} = 0$; otherwise

$$\{\widehat{W}_{\mathrm{a}}, \widehat{K}_{\mathrm{a}}\} = \underset{w \in [0:M_{\mathrm{a}}], k \in [k_l : k_u] \cup \{0\}}{\arg\min} \|\mathbf{y}_{\mathrm{a}} - k\mathbf{c}_w\|^2$$

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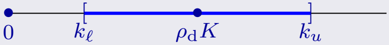

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- Bounds on P_{amd} and P_{afp} via random coding union bound with parameter s

Numerical experiment

- $n = 30\,000, \qquad (M_\mathrm{a}, M_\mathrm{s}) = (2^3, 2^{100}), \qquad 1000 \le K \le 30\,000, \qquad \rho_\mathrm{s} = 0.01$
- $\max\{\epsilon_\mathrm{smd}, \epsilon_\mathrm{sfp}\} \le 10^{-1}, \qquad \max\{\epsilon_\mathrm{amd}, \epsilon_\mathrm{afp}\} \le 10^{-5}$

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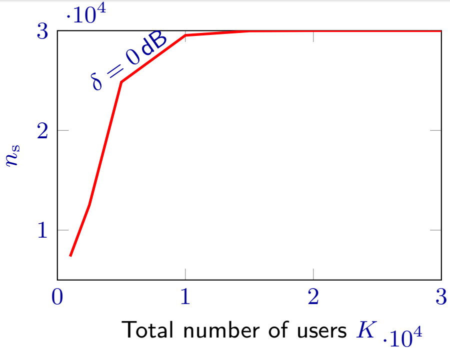

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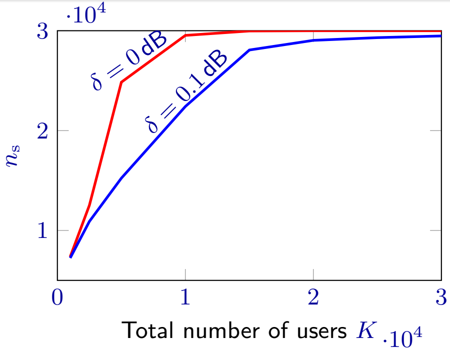

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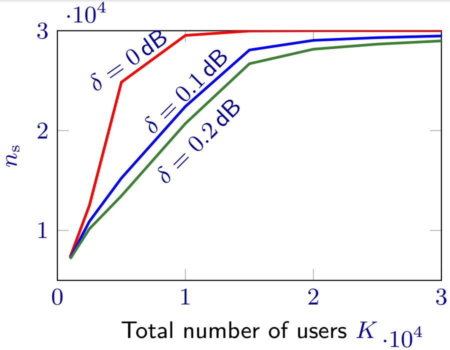
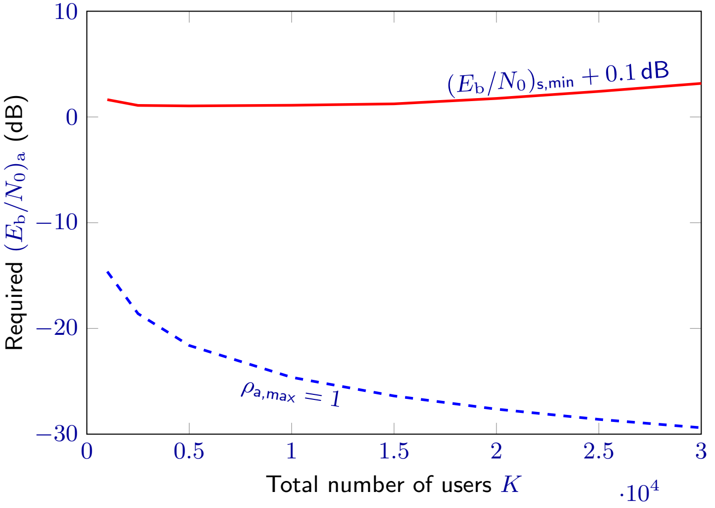


Role of device sensitivity $\rho_{\mathsf{a,max}}$



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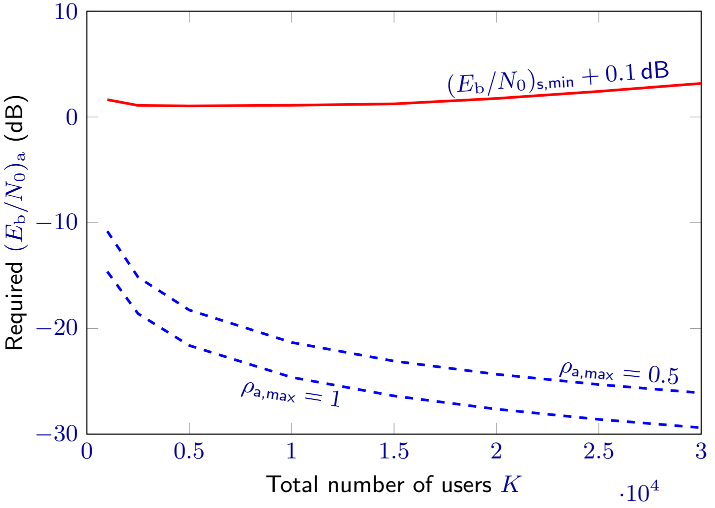

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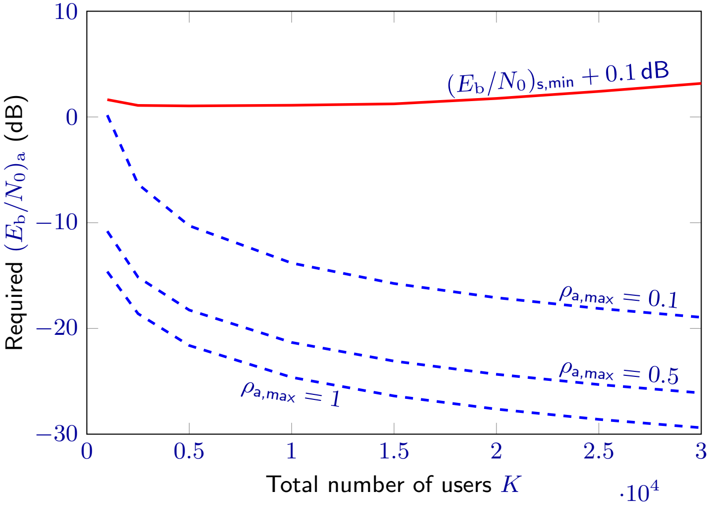

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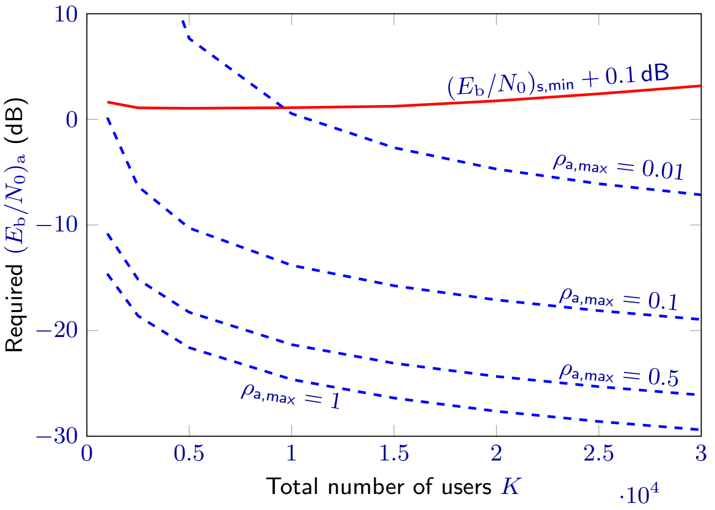

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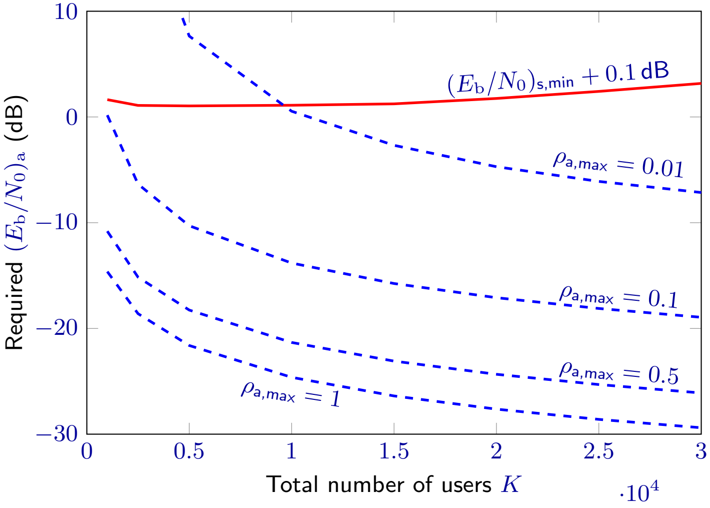

Optimal choice of parameters

- $\rho_a = \rho_{\text{a,max}}$
- n_a and P_a as low as possible

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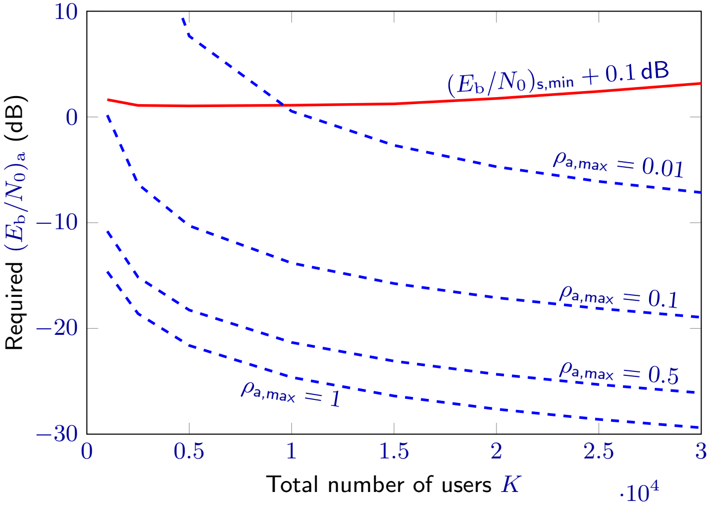

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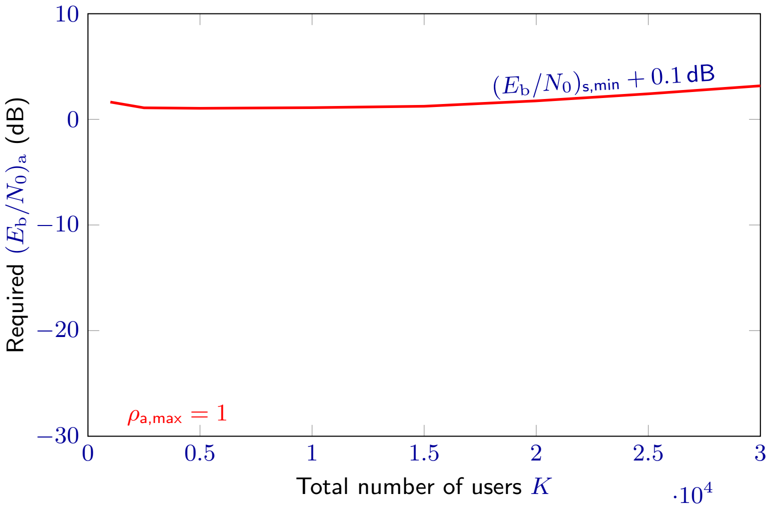
Issue: dynamic range

$$\psi = P_s/P_a \in [30:70]\,\text{dB}$$

Role of dynamic range ψ



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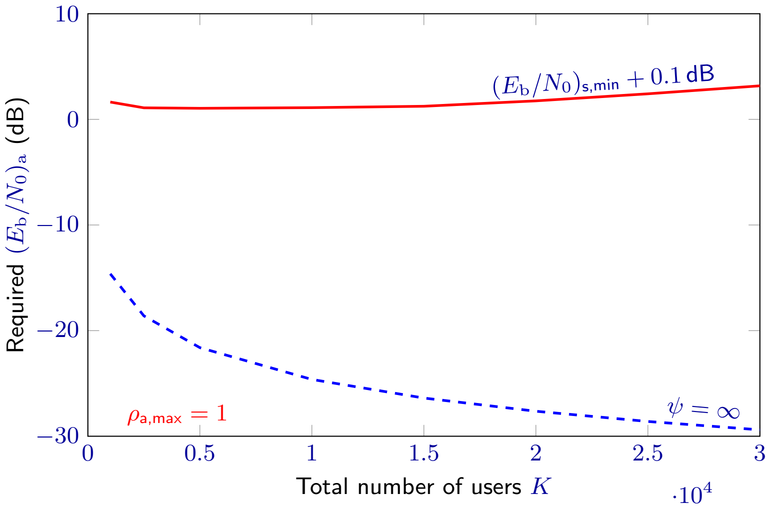

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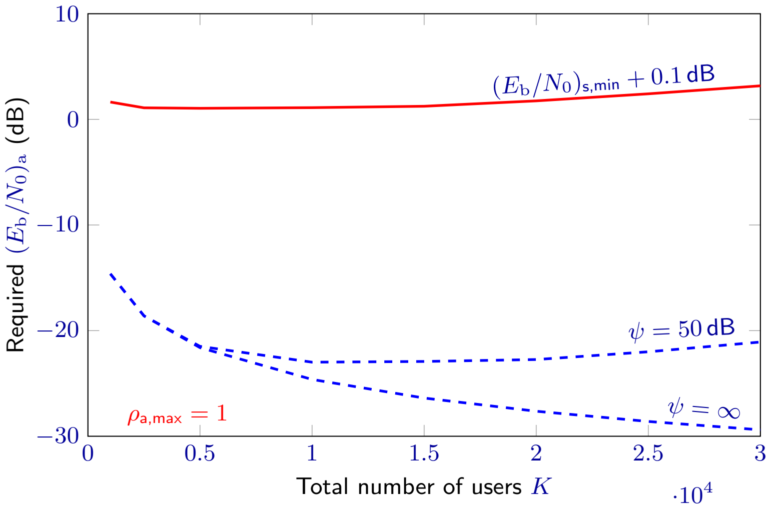

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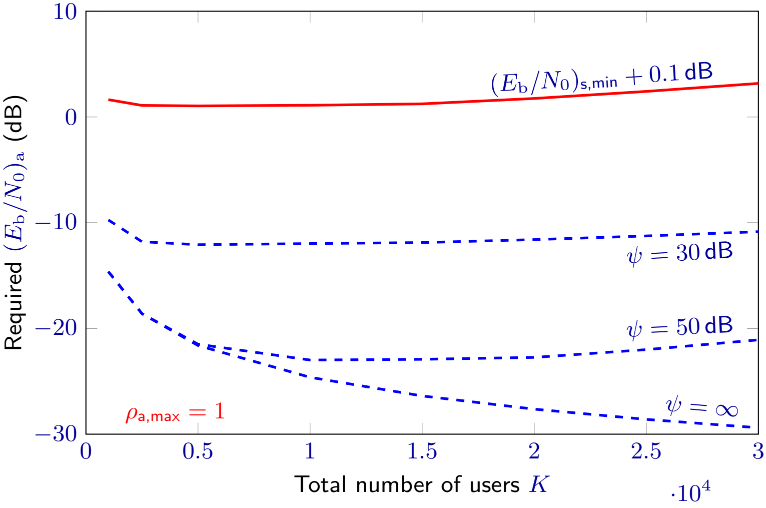

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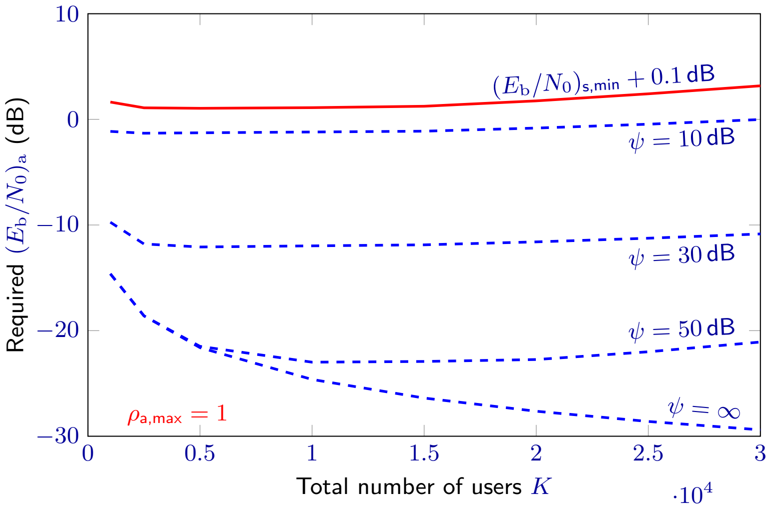

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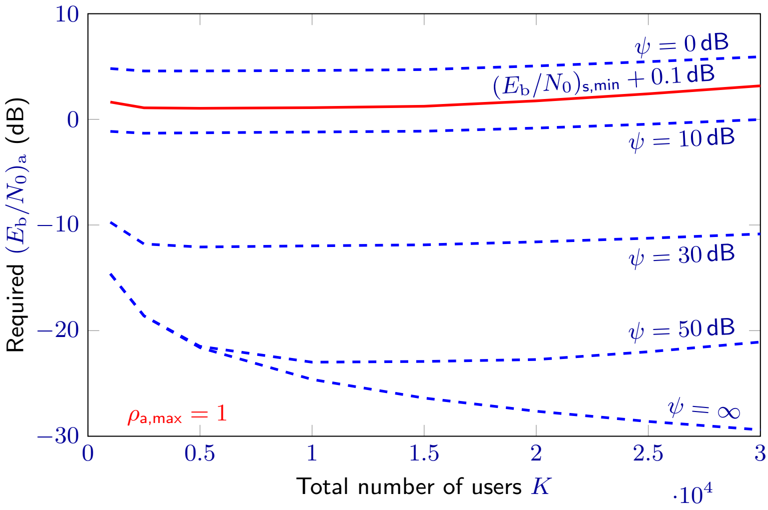
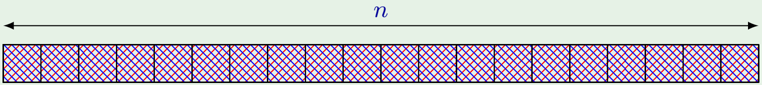
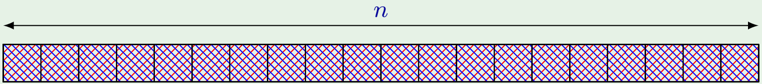


Superposition



$$\mathbf{y} = \sum_{k=1}^{K_{\mathrm{s}}} \mathbf{x}_k + K_{\mathrm{a}} \mathbf{x}_{\mathrm{a}} + \mathbf{z}$$

Superposition



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Decoder: reliability diversity

- Estimate K_a and W_a; treat $\sum_{k=1}^{K_\mathrm{s}} \mathbf{x}_k$ as noise
- Interference cancellation $\mathbf{y}_\mathrm{ic} = \mathbf{y} - \widehat{K}_\mathrm{a}\widehat{\mathbf{x}}_\mathrm{a}$
- Estimate K_s and $\mathcal{W}$ from $\mathbf{y}_\mathrm{ic}$; residual interference $K_\mathrm{a}\mathbf{x}_\mathrm{a} - \widehat{K}_\mathrm{a}\widehat{\mathbf{x}}_\mathrm{a}$

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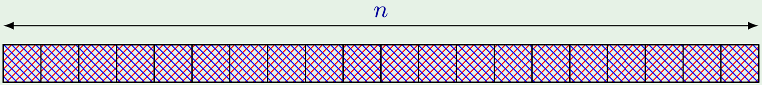

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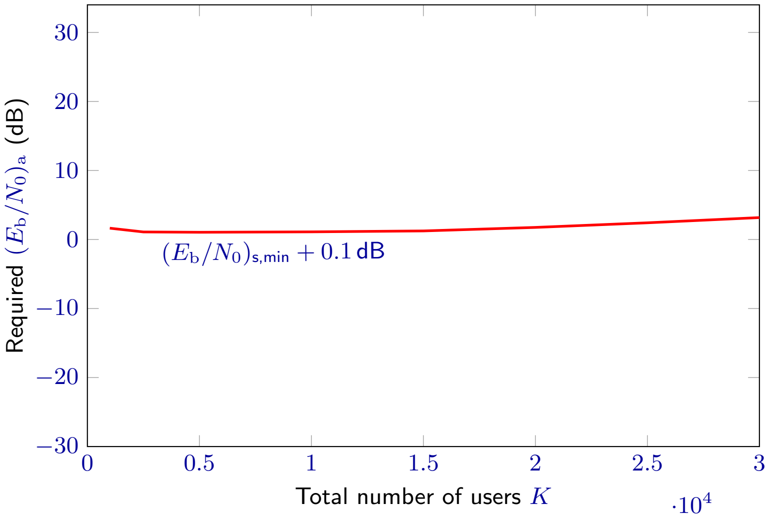
Issue

Difficult to estimate K_a reliably in the presence of noise; imperfect interference cancellation

Performance: orthogonalization vs. superposition



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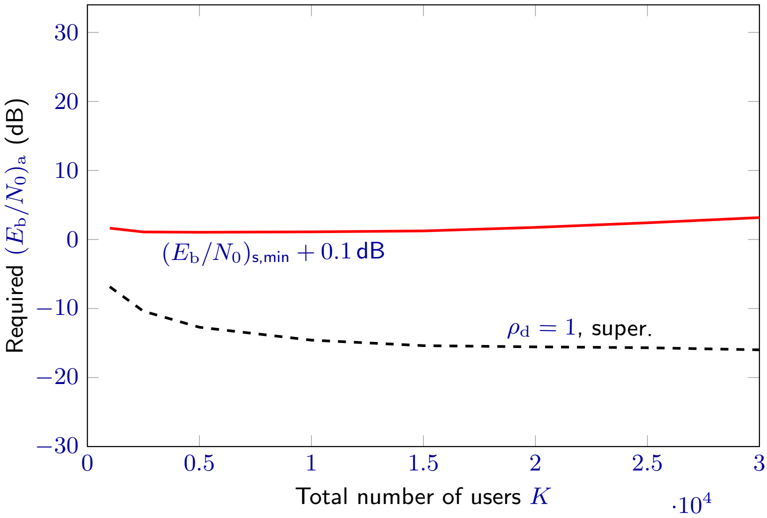

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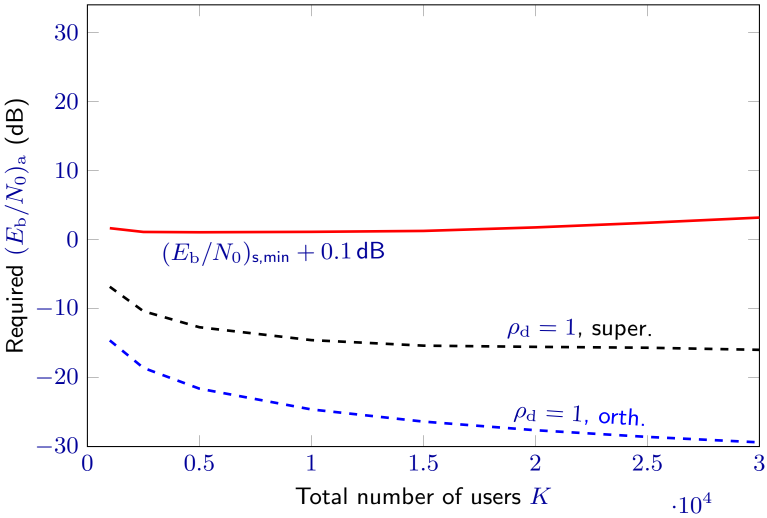

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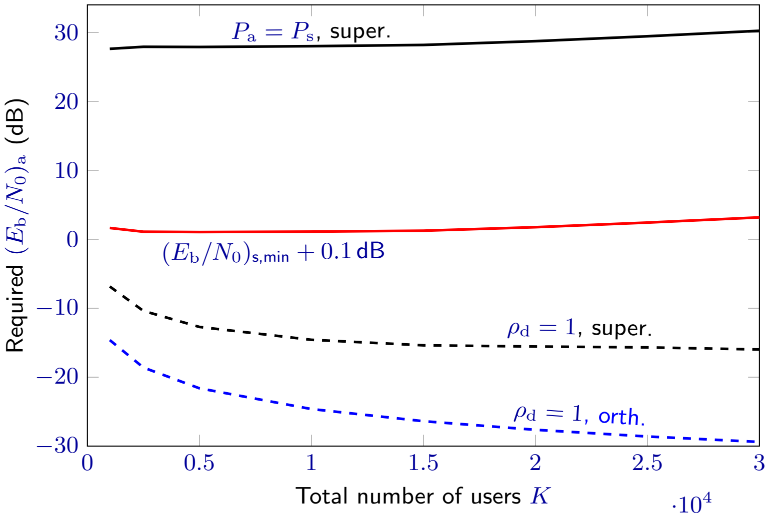

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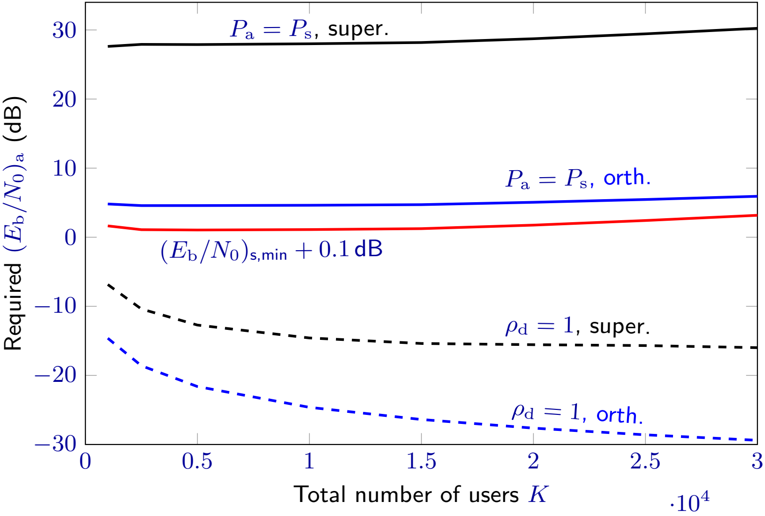
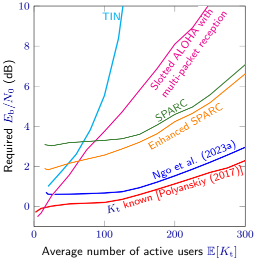


Summary and further extensions

- Overview of information-theoretic bounds for mMTC
- Generalization to unknown number of active users and heterogeneous traffic



Further extensions and open problems

- Fading, massive MIMO, cell-free [Kowshik & Polyanskiy, 2021; Fengler et al. 2022; Decurninge et al. 2021; Gkagkos et al. 2023]
- Variable-length codes with stop feedback [Yavas et al. 2021]
- Imperfect synchronization [Decurninge et al. 2022 , Fengler et al. 2023]
- Age of information [Munari 2021, Munari et al., 2023]
- Energy harvesting [Demirhan & Duman, 2019]