

# Short packets over a massive random-access channel

Giuseppe Durisi

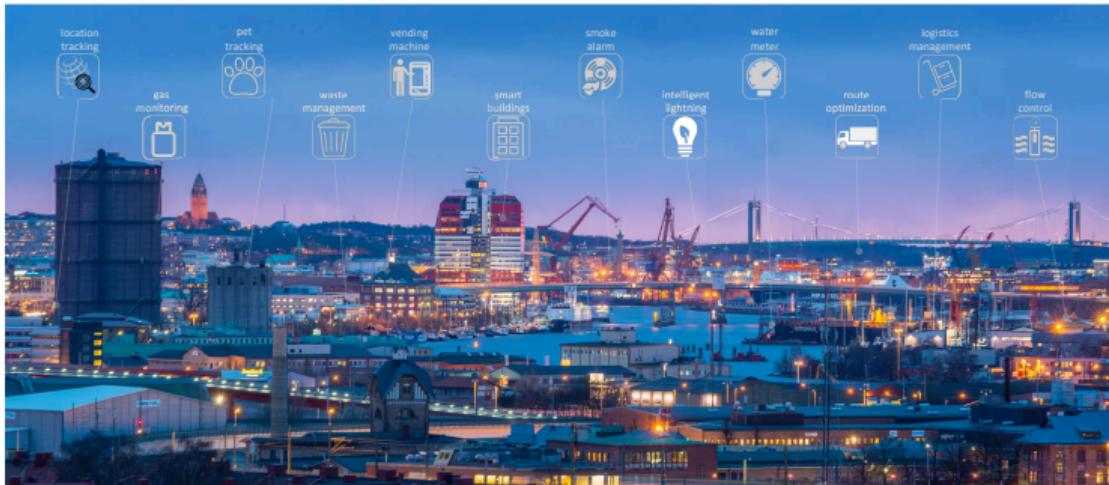
*Chalmers, Sweden*

LSIT, May, 2023



Joint work with H. K. Ngo, A. Lancho, A. Graell i Amat, P. Popovski, A. Kalør,  
B. Soretz

# Wireless connectivity enables new services



source: IoTpool

## Challenges

- 🔧 Collect data from a **massive number** of **low-cost sensors**
- 🔧 Communicate **reliably critical** information

# Massive and critical wireless connectivity

## massive machine-type comm. (**mMTC**)

- Uplink mostly
- High energy efficiency
- Great commercial interest
- LPWAN, satellite

## ultra-reliable low-latency comm. (**URLLC**)

- Bidirectional
- Low latency, high reliability
- Limited commercial interest (so far)
- Private 5G network

## Some characteristics

### mMTC

- Small information payload (100 bits)
- High user density ( $10^7$  devices/Km<sup>2</sup>)
- Sporadic TX (less than 1 per minute)  
⇒ 120 dof per user at  $B = 20$  MHz

### URLLC

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- Low error prob. (below  $10^{-5}$ )  
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How to transmit hundreds of bits in hundreds of dof per user (**short-packet regime**) under stringent constraints on reliability/latency/energy efficiency?

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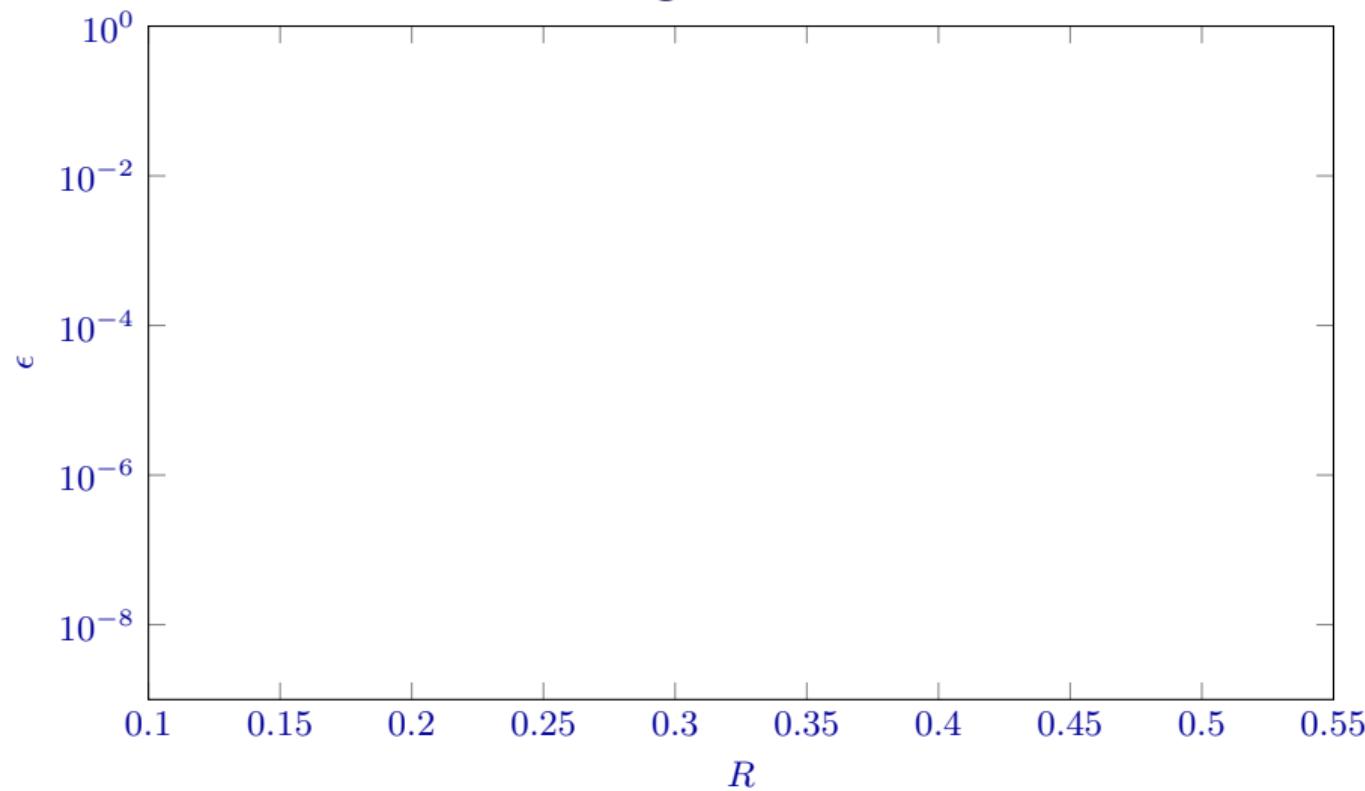
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## Key design tool

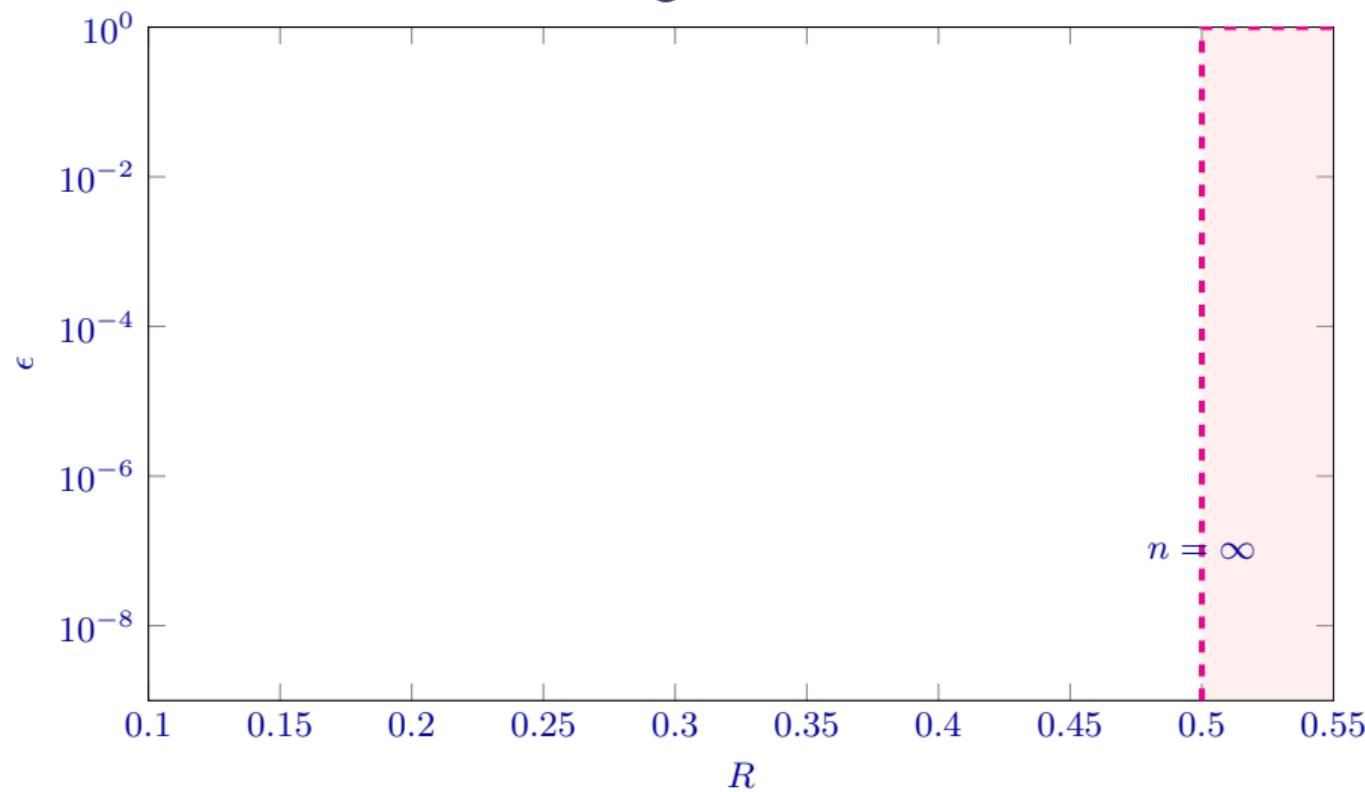
Finite-blocklength information theory

# Finite-blocklength IT for URLLC



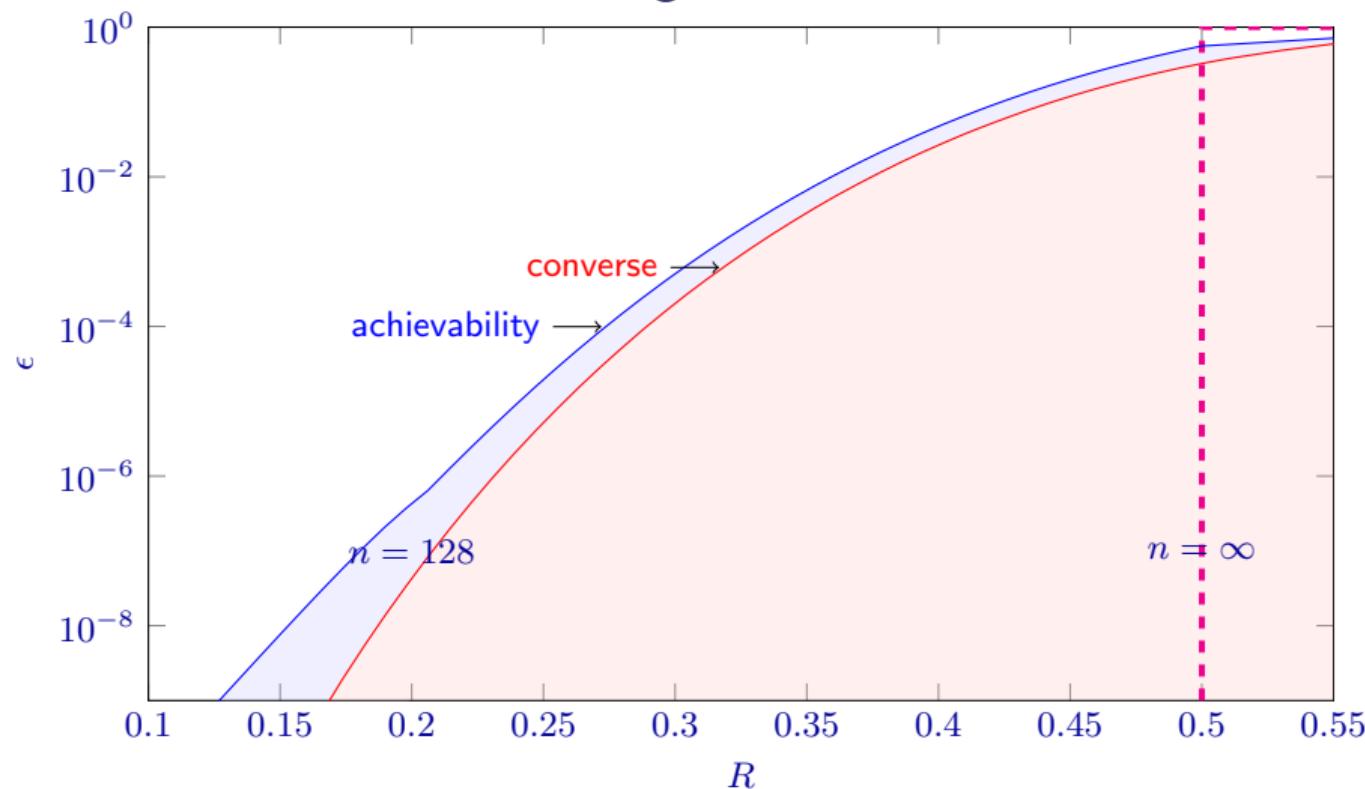
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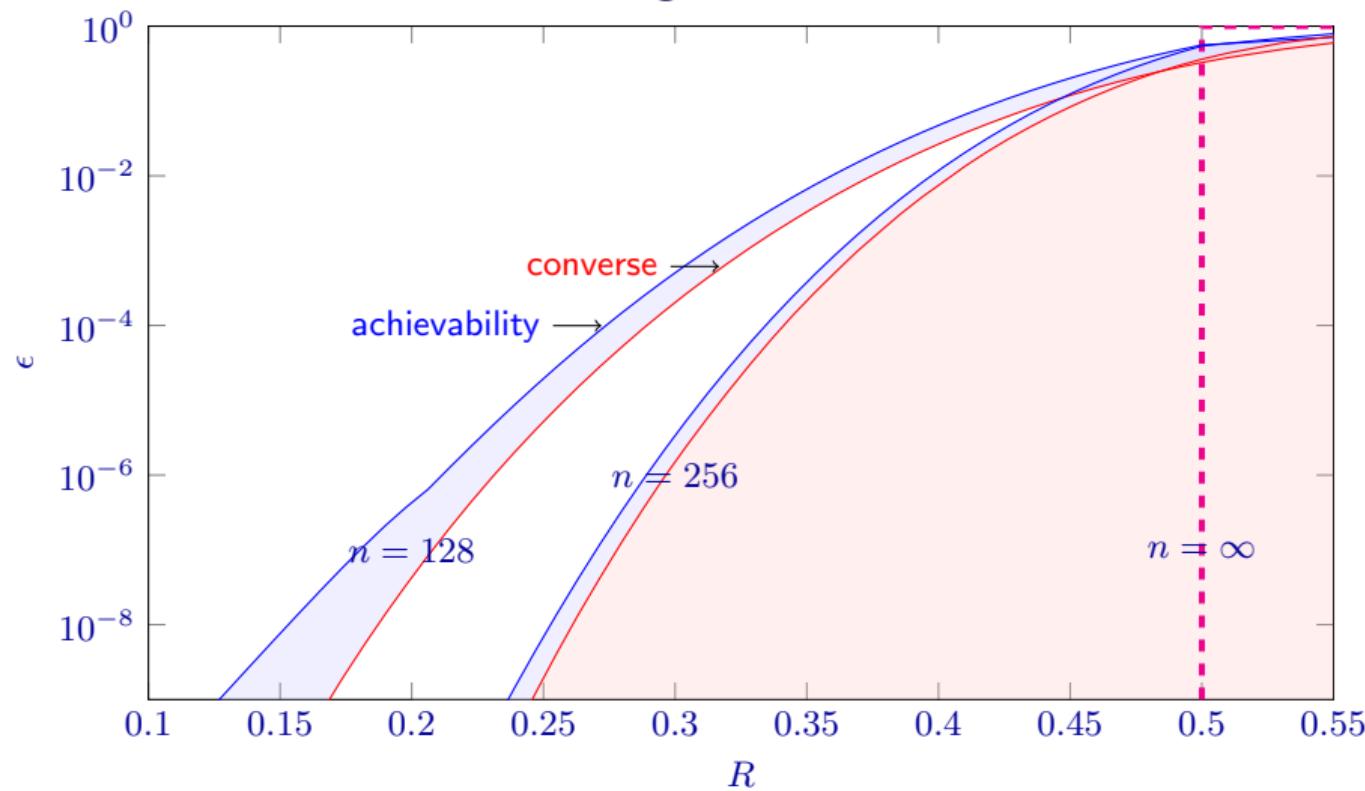
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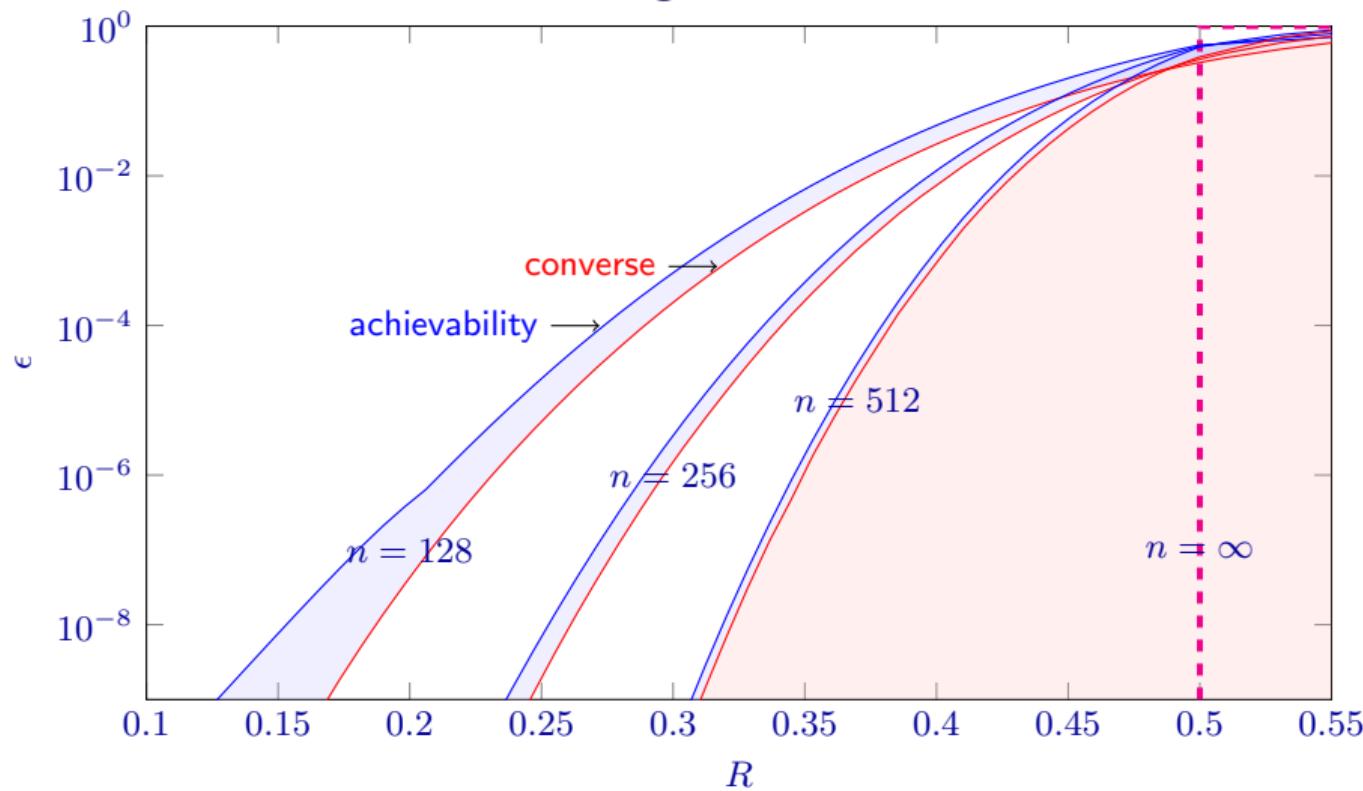
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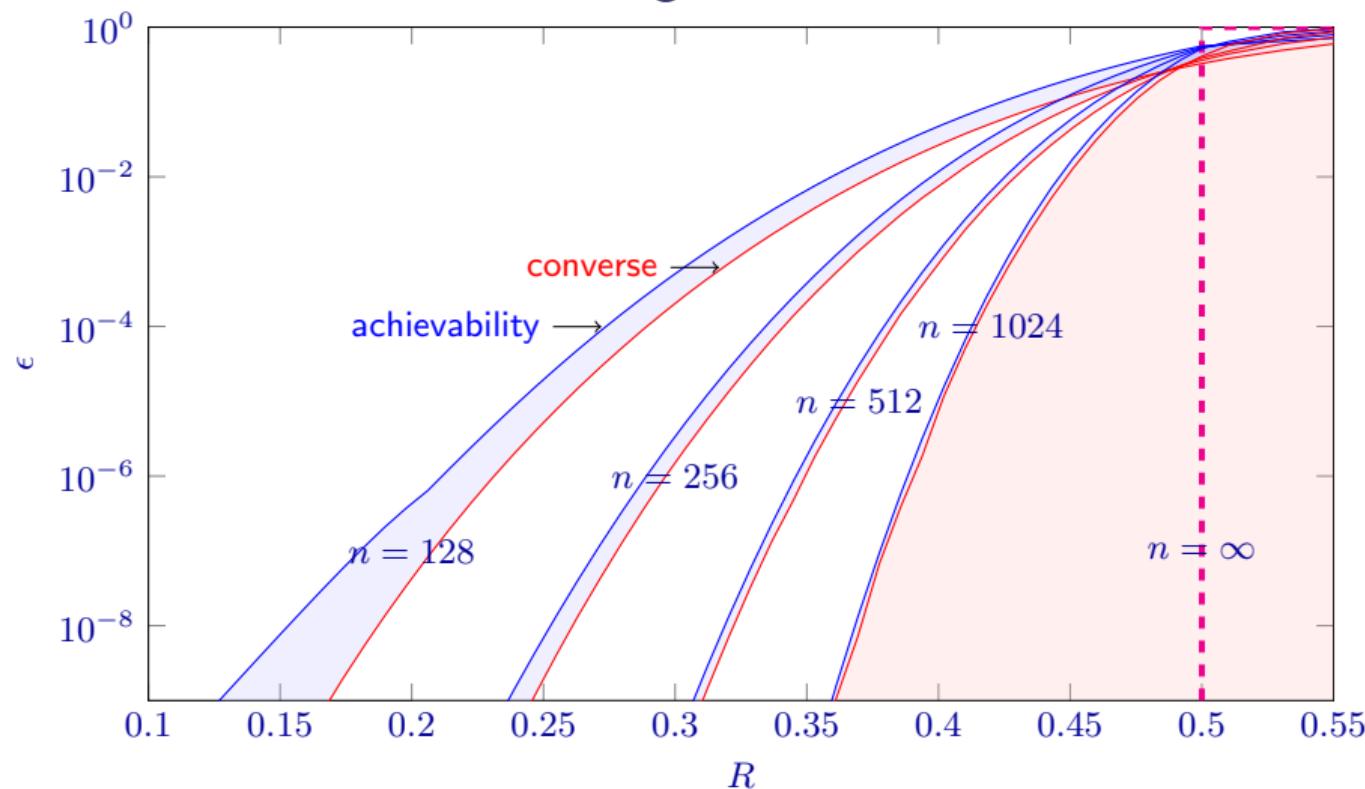
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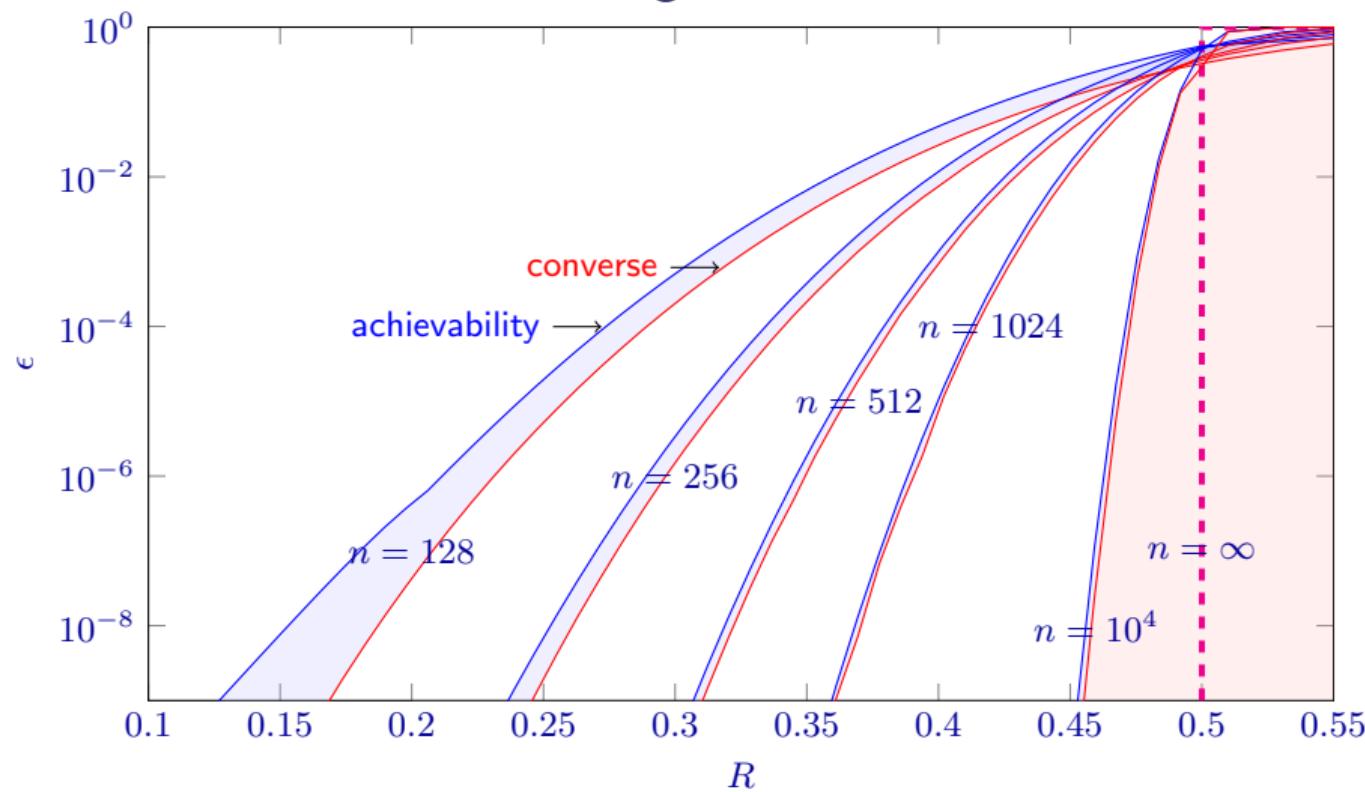
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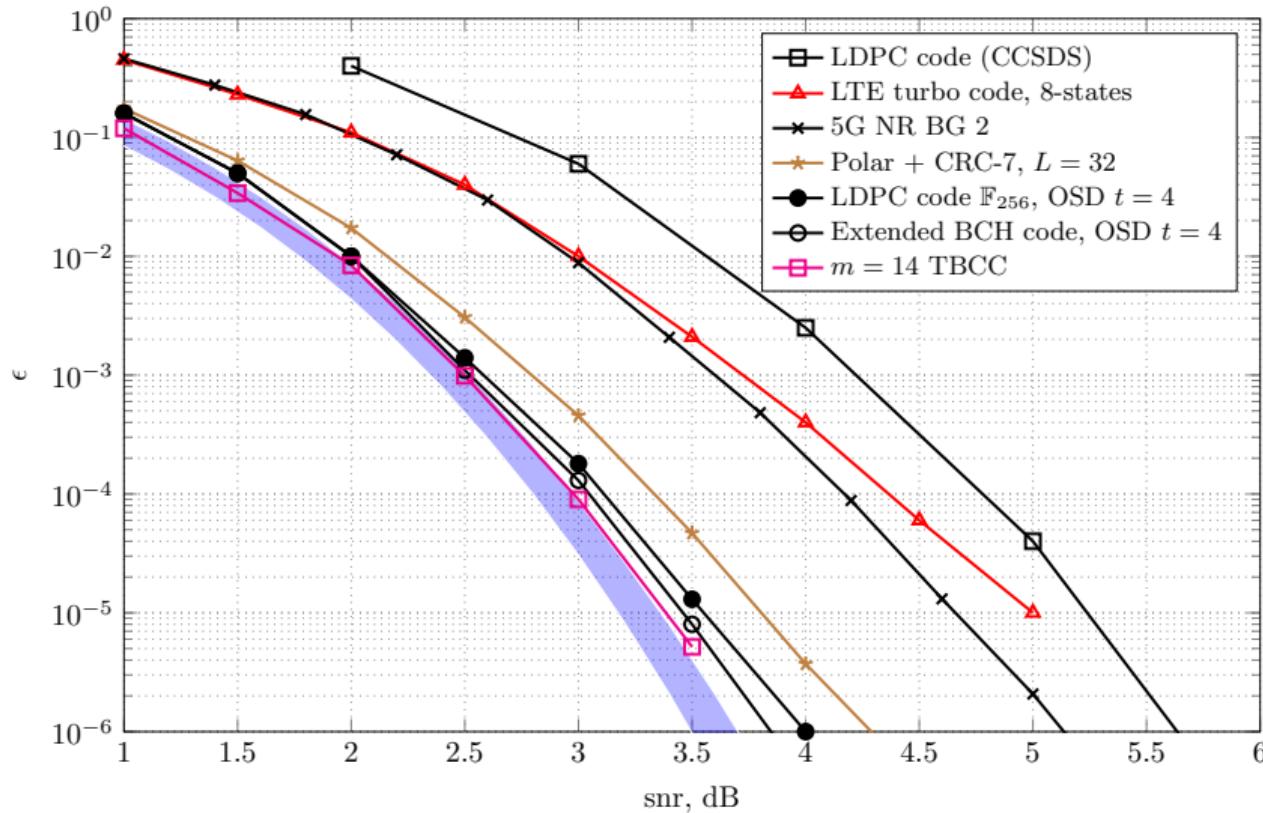
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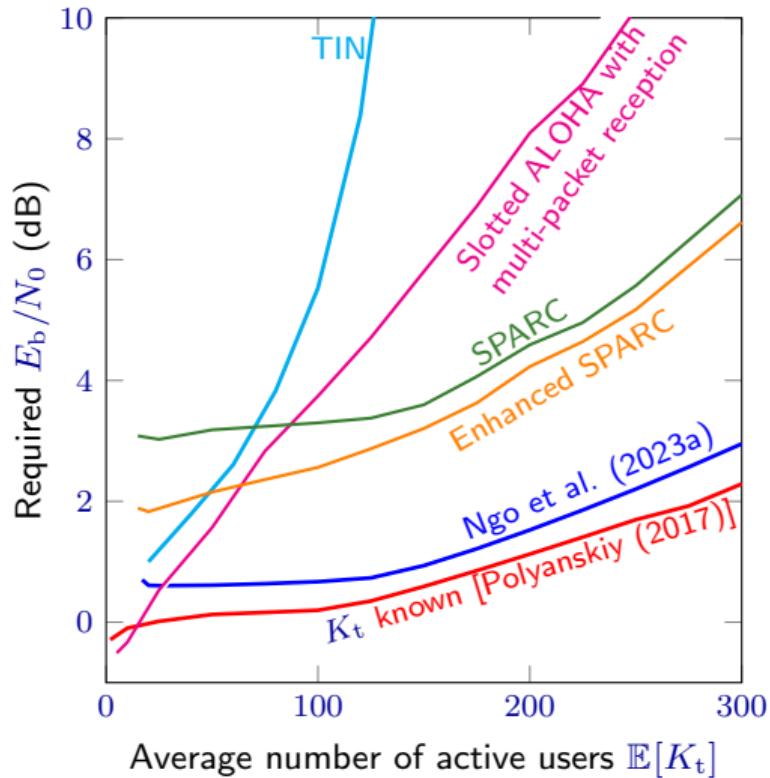
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# Guidelines for optimal design ( $R = 1/2$ bit/channel use)



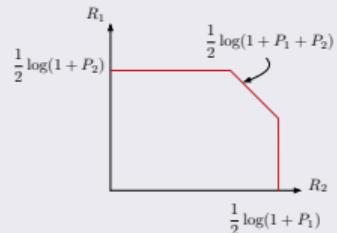
# This talk

- FBL-IT bounds for mMTC [Polyanskiy '17]
- Coding schemes approaching the bound
- Extension to
  - Unknown number of active users [Ngo et al. 2023a]
  - Heterogeneous traffic [Ngo et al. 2023b]
- Further extensions and open problems



# Traditional multiple access models and their limitations [Gallager '85]

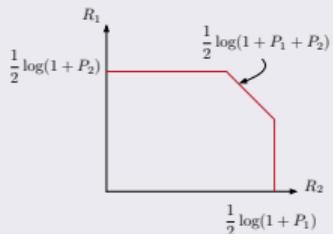
Multiaccess IT [Cover '75, Wyner '74]



- ✖ All user **active** (no sporadicity)
  - Each user is given a **different codebook**
- ✖ Not feasible for mMTC (**overhead** too large)

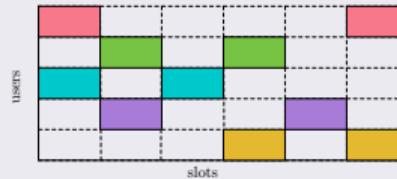
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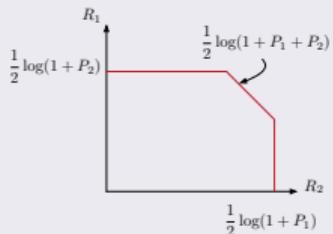
## Collision resolution [Abramson '70, Roberts' 72, Liva '11]



- ✓ Infinitely many, **sporadically** active users
- ✗ Crude modeling of communication aspects
  - De-facto **standard** for mMTC

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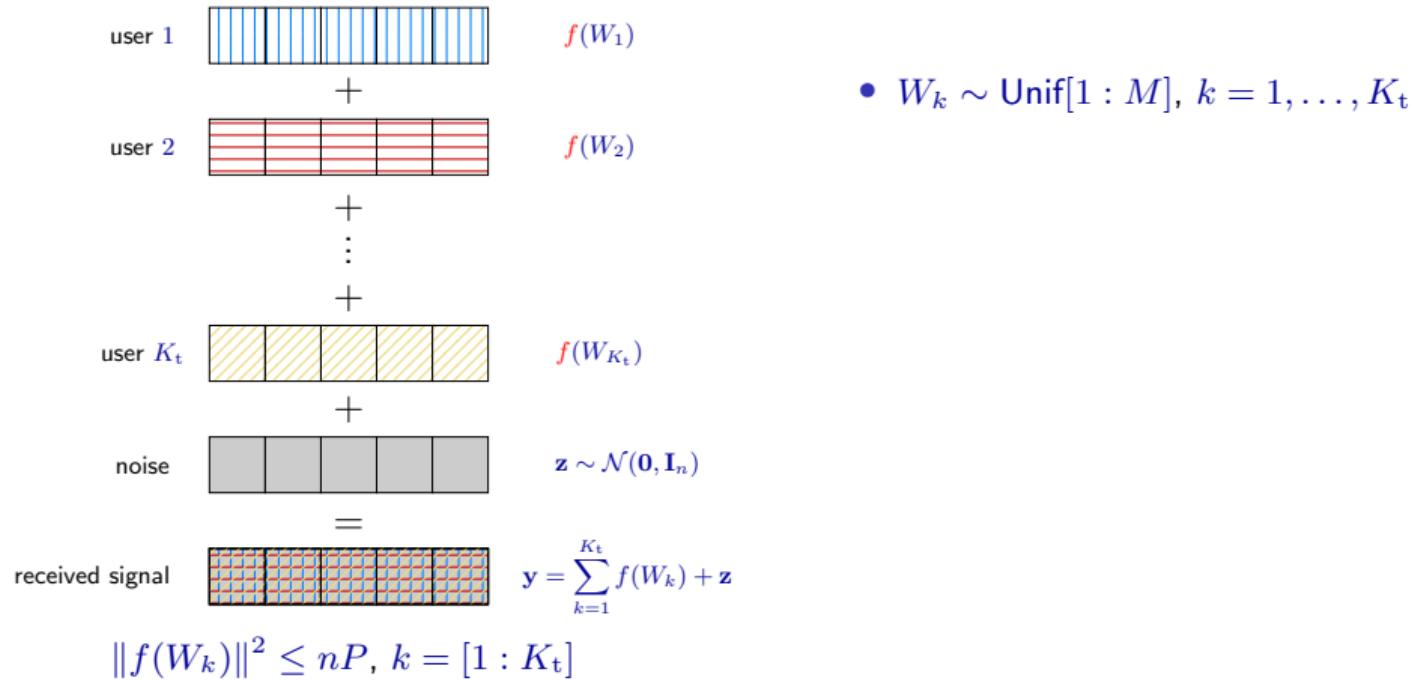
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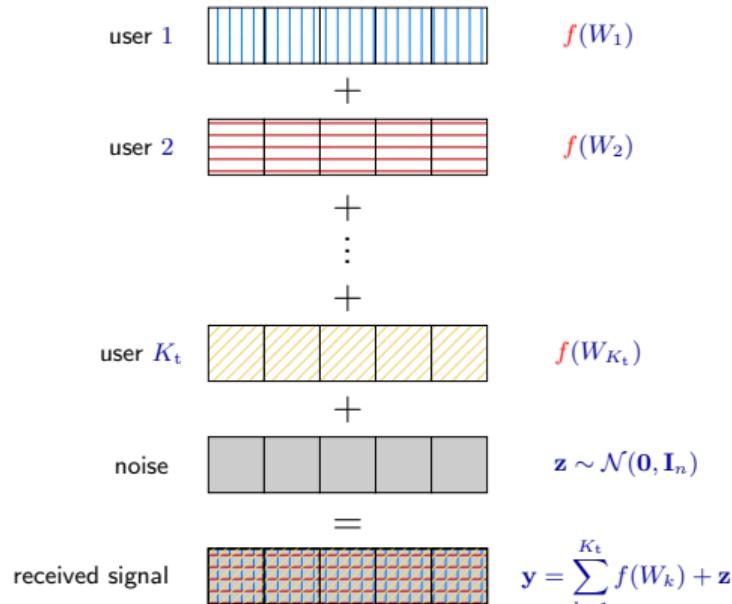
## Addressing these limitations

- Noiseless adder channel (e.g., [Bar-David et al., '97])
- More general information-theoretic perspective [Polyanskiy '17]

# Unsourced GMAC model [Polyanskiy '17]



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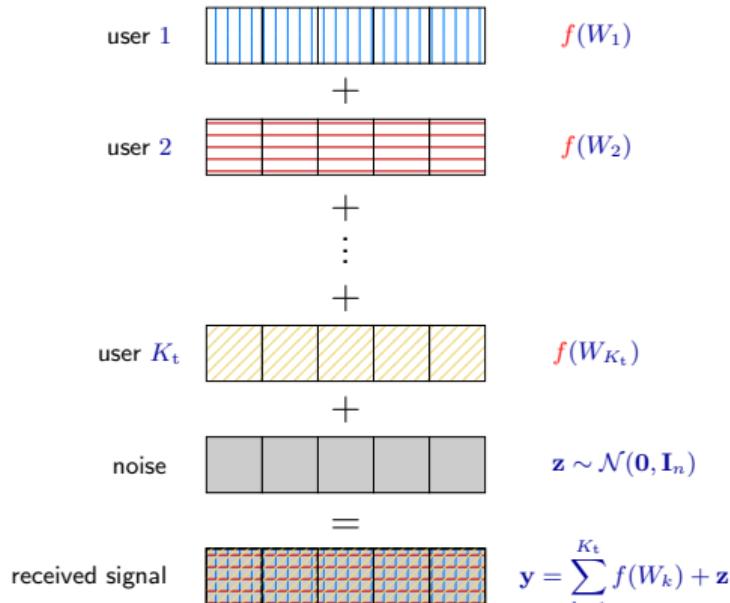


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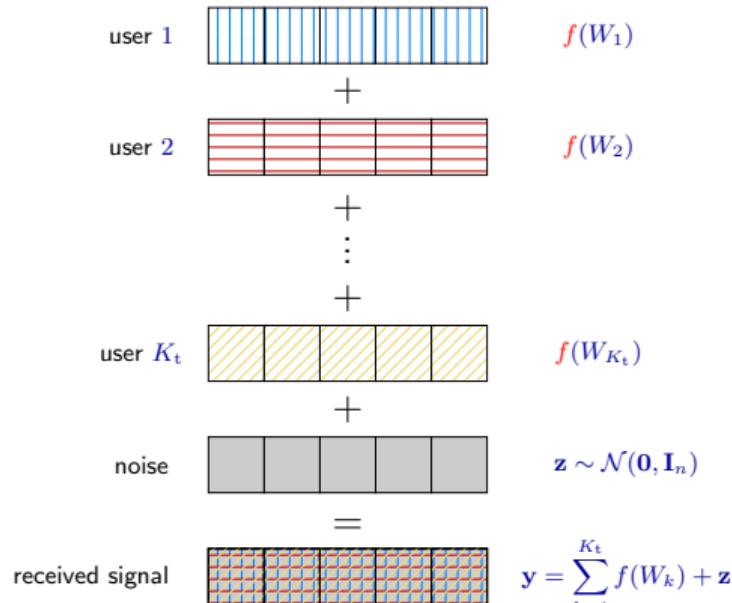
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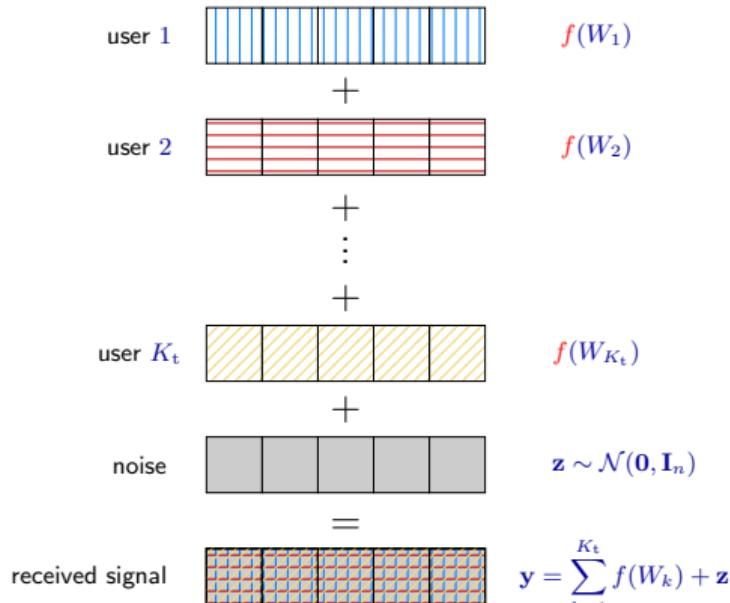
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- $K_t$  known to the decoder
- ! Per-user error-probability

$$P_e = \frac{1}{K_t} \sum_{k=1}^{K_t} \mathbb{P}\left[W_k \notin \widehat{\mathcal{W}}\right]$$

## Random coding achievability bound

$(M, n, \epsilon)$  code for  $K_t$ -user unsourced GMAC with power constraint  $P$

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Random-coding achievability bound [Polyanskiy '17]

For every  $P' < P$ , there exists an  $(M, n, \epsilon)$  code for the  $K_t$ -user unsourced GMAC with power constraint  $P$  satisfying

$$\epsilon \leq \sum_{k=1}^{K_t} \frac{k}{K_t} \min\{p_k, q_k\} + p_0, \quad \text{where}$$

$$p_0 = \frac{\binom{K_t}{2}}{M} + K_t \mathbb{P} \left[ \frac{1}{n} \sum_{j=1}^n z_j^2 > \frac{P}{P'} \right]$$

$$p_k = e^{-E(t)}$$

$$E(t) = \max_{0 \leq \rho_1, \rho_2 \leq 1} -\rho_1 \rho_2 k R_1 - \rho_2 R_2 + E_0(\rho_1, \rho_2)$$

$E_0(\rho_1, \rho_2)$  : complicated expression in  $\rho_1, \rho_2, k, P'$

$$q_k = \inf_{\gamma} \mathbb{P}[I_k \leq \gamma] + e^{n(kR_1 + R_2) - \gamma}$$

$I_k$  : related to inf. dens.

$$R_1 = \frac{1}{n} \log M - \frac{1}{nk} \log k!$$

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# Key ideas and steps in the proof

## Random codebook generation and encoder

- **Gaussian codebook:** fix  $P' < P$ ; generate  $M$  codewords  $\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P' \mathbf{I}_n)$
- **Encoder:** User  $k$  transmits  $\mathbf{c}_{W_k} \mathbb{1}\{\|\mathbf{c}_{W_k}\|^2 \leq nP'\}$

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## Decoder

Unordered list  $\widehat{\mathcal{W}}$  of decoded messages obtained by solving

$$\widehat{\mathcal{W}} = \arg \min_{\mathcal{W}' \subset [1:M], |\mathcal{W}'| = K_t} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|, \quad \text{with} \quad \mathbf{c}(\mathcal{W}') = \sum_{w \in \mathcal{W}'} \mathbf{c}_w$$

# Analysis of per-user error probability

$$P_e = \frac{1}{K_t} \mathbb{E}_P \left[ \sum_{k=1}^{K_t} \mathbb{1} \left\{ W_k \notin \widehat{\mathcal{W}} \right\} \right]$$

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## Change of measure

- Change measure from  $P$  to  $Q$  for which: message are distinct and codewords are i.i.d. Gaussian
- For every event  $\mathcal{E}$ ,  $\mathbb{E}_P[\mathcal{E}] \leq \mathbb{E}_Q[\mathcal{E}] + d_{\text{TV}}(P, Q)$

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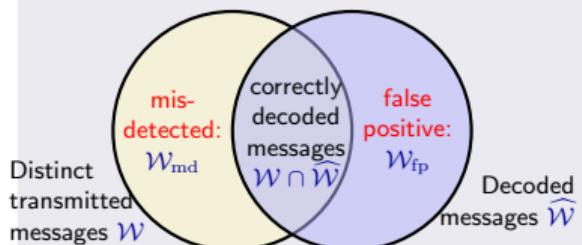
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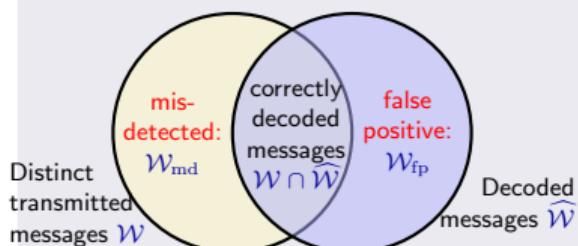
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$$\mathbb{P}[|\mathcal{W}_{\text{md}}| = |\mathcal{W}_{\text{fp}}| = k] = \mathbb{P} \left[ \bigcup_{\substack{\mathcal{W}_{\text{md}} \subset \mathcal{W} \\ |\mathcal{W}_{\text{md}}|=k}} \bigcup_{\substack{\mathcal{W}_{\text{fp}} \subset [1:M] \setminus \mathcal{W} \\ |\mathcal{W}_{\text{fp}}|=k}} \right]$$

$$\left. \|\mathbf{z} + \mathbf{c}(\mathcal{W}_{\text{md}}) - \mathbf{c}(\mathcal{W}_{\text{fp}})\| \leq \|\mathbf{z}\| \right]$$

# Three tools and their applications

Chernoff: for every random  $\mathbf{u}$  and every  $\lambda > 0$

$$\mathbb{P}[\|\mathbf{z} + \mathbf{u}\| \leq v] \leq e^{\lambda v^2} \mathbb{E}_{\mathbf{z}}[e^{-\lambda \|\mathbf{z} + \mathbf{u}\|^2}]$$

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**Gallager's trick** to tighten the union bound

- Assume that  $\mathbb{P}[\mathcal{A}_j | \mathbf{z}] \leq e^{-nE(\mathbf{z})}, j = 1, \dots, m$
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1. Chernoff bound to evaluate

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$$\mathbb{P}[\|\mathbf{z} + \mathbf{u}\| \leq v] \leq e^{\lambda v^2} \mathbb{E}_{\mathbf{z}}[e^{-\lambda \|\mathbf{z} + \mathbf{u}\|^2}]$$

**MGF of Gaussian vector:** if  $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$

$$\mathbb{E}_{\mathbf{z}}[e^{-\lambda \|\mathbf{z} + \mathbf{u}\|^2}] = \frac{e^{-\frac{\lambda \|\mathbf{u}\|^2}{1+2\sigma^2\lambda}}}{(1+2\sigma^2\lambda)^{n/2}}$$

**Gallager's trick to tighten the union bound**

- Assume that  $\mathbb{P}[\mathcal{A}_j | \mathbf{z}] \leq e^{-nE(\mathbf{z})}, j = 1, \dots, m$
- Then for all  $0 \leq \rho \leq 1$

$$\mathbb{P}\left[\bigcup_{j=1}^m \mathcal{A}_j\right] \leq m^\rho \mathbb{E}[e^{-n\rho E(\mathbf{z})}]$$

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5. MGF to compute expectation over  $\mathbf{z}$

# Random coding achievability bound [Polyanskiy '17]

For every  $P' < P$ , there exists an  $(M, n, \epsilon)$  code for the  $K_t$ -user unsourced GMAC with power constraint  $P$  satisfying

$$\epsilon \leq \sum_{k=1}^{K_t} \frac{k}{K_t} \min\{p_k, q_k\} + p_0, \quad \text{where}$$

$$p_0 = \frac{\binom{K_t}{2}}{M} + K_t \mathbb{P}\left[\frac{1}{n} \sum_{j=1}^n z_j^2 > \frac{P}{P'}\right]$$

$$p_k = e^{-E(t)}$$

$$E(t) = \max_{0 \leq \rho_1, \rho_2 \leq 1} -\rho_1 \rho_2 k R_1 - \rho_2 R_2 + E_0(\rho_1, \rho_2)$$

$E_0(\rho_1, \rho_2)$  : complicated expression in  $\rho_1, \rho_2, k, P'$

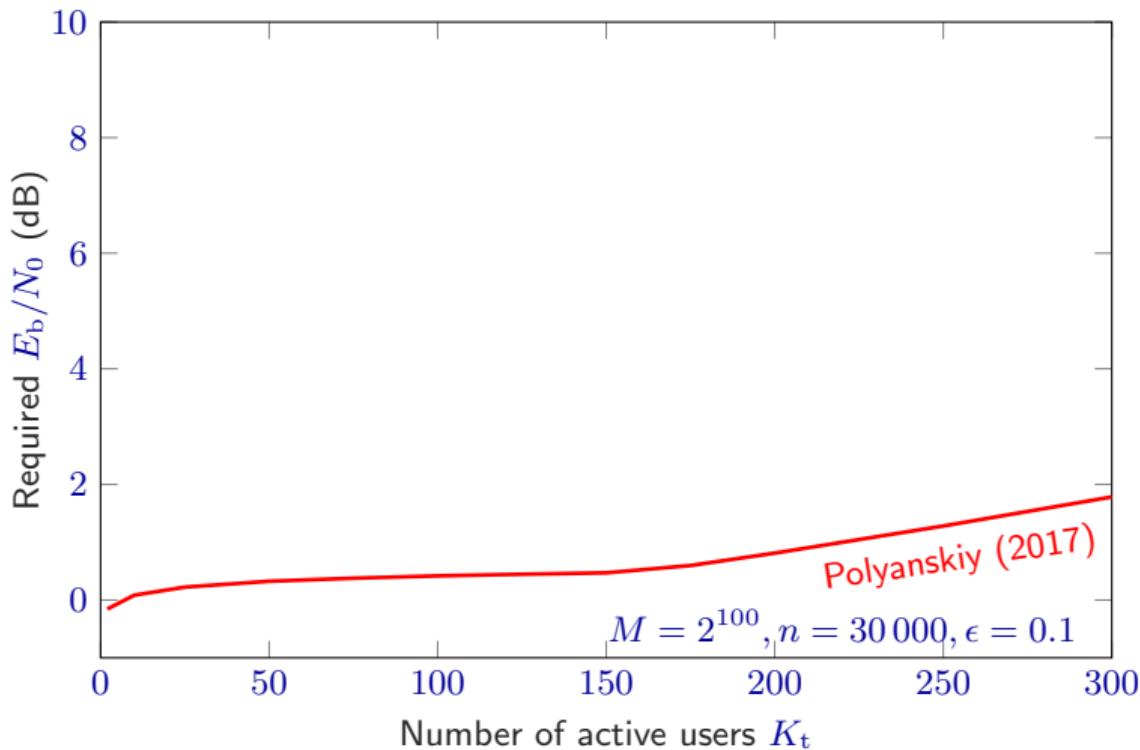
$$q_k = \inf_{\gamma} \mathbb{P}[I_k \leq \gamma] + e^{n(kR_1 + R_2) - \gamma}$$

$I_k$  : related to inf. dens.

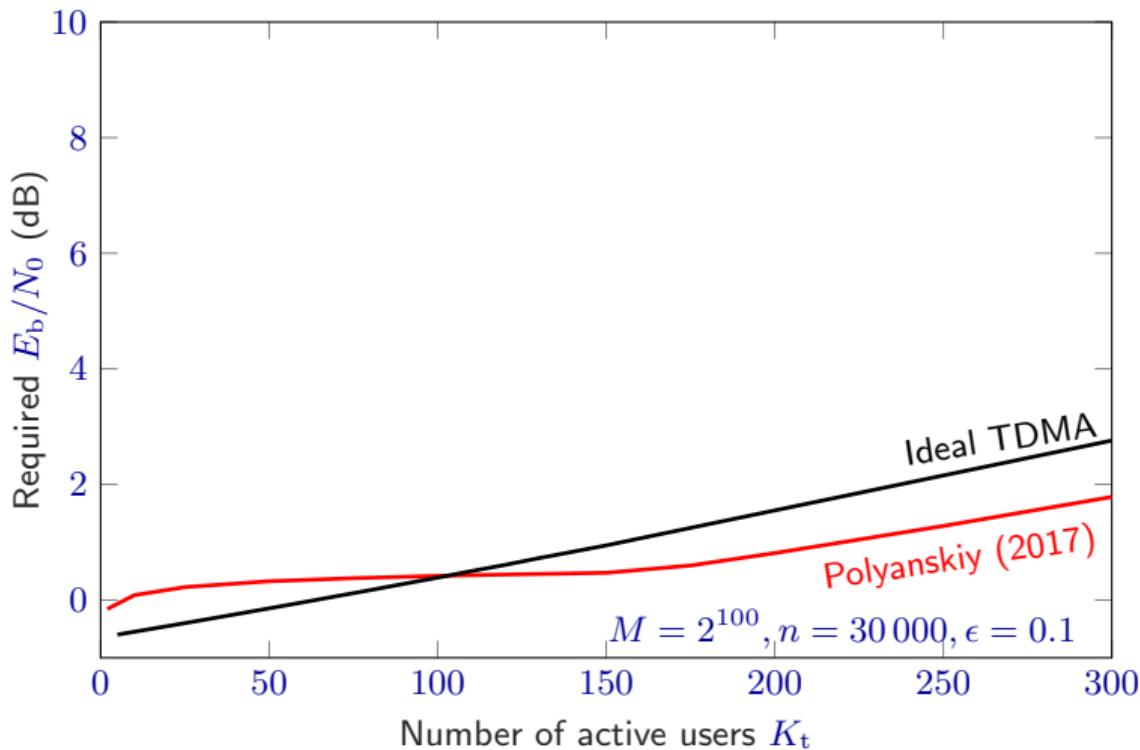
$$R_1 = \frac{1}{n} \log M - \frac{1}{nk} \log k!$$

$$R_2 = \frac{1}{n} \log \binom{K_t}{k}$$

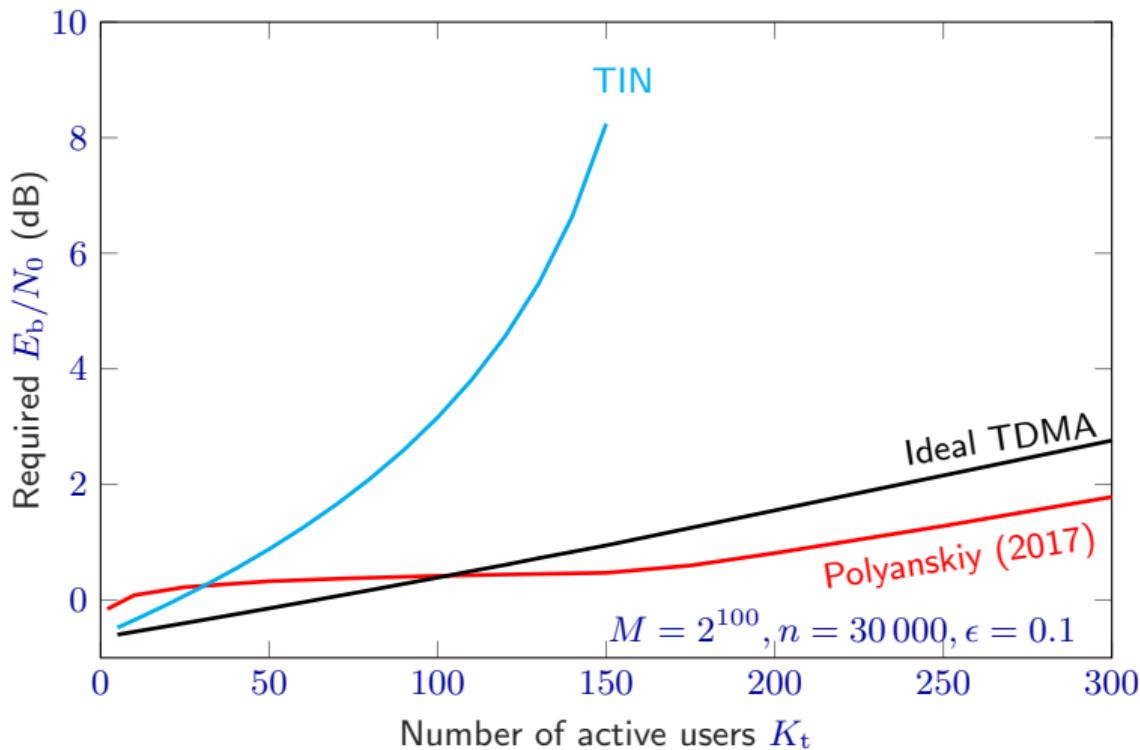
## Numerical evaluation of the bound



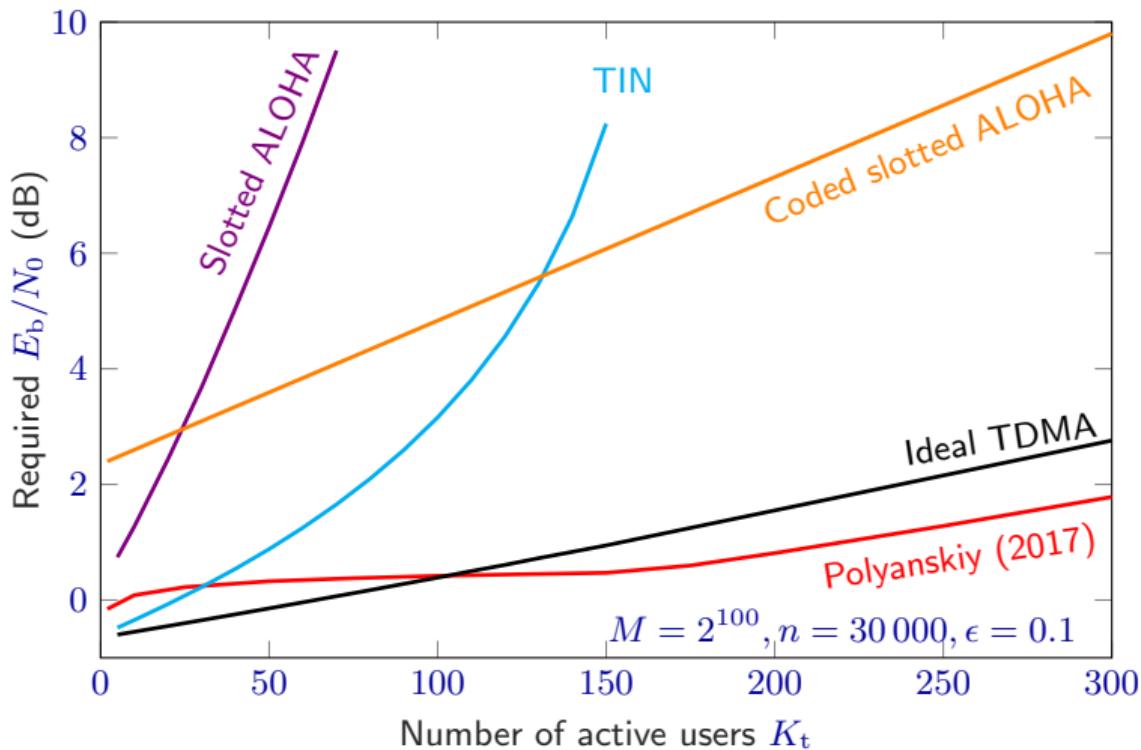
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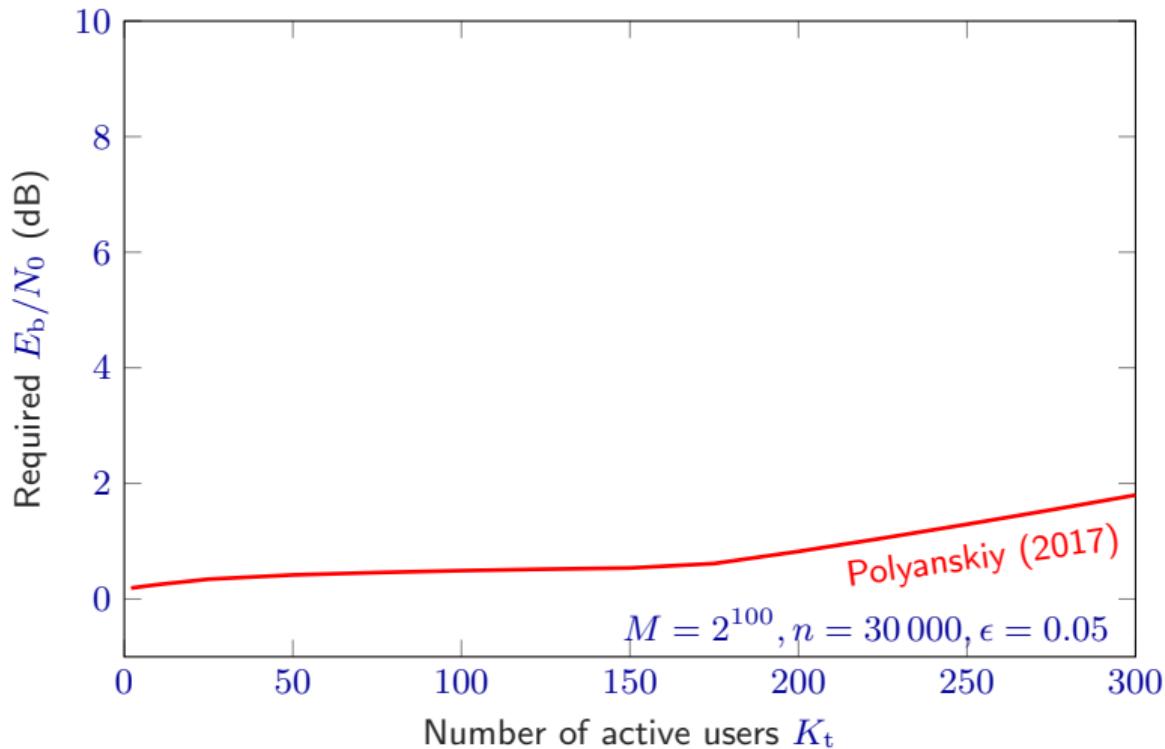
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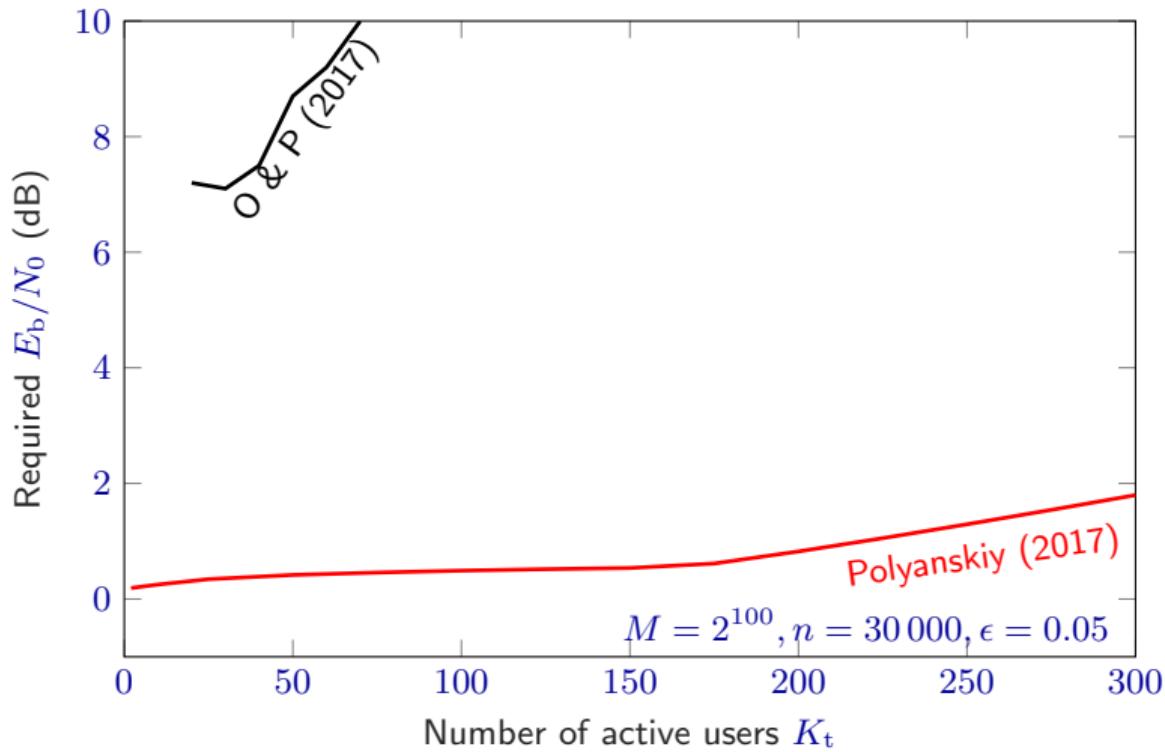
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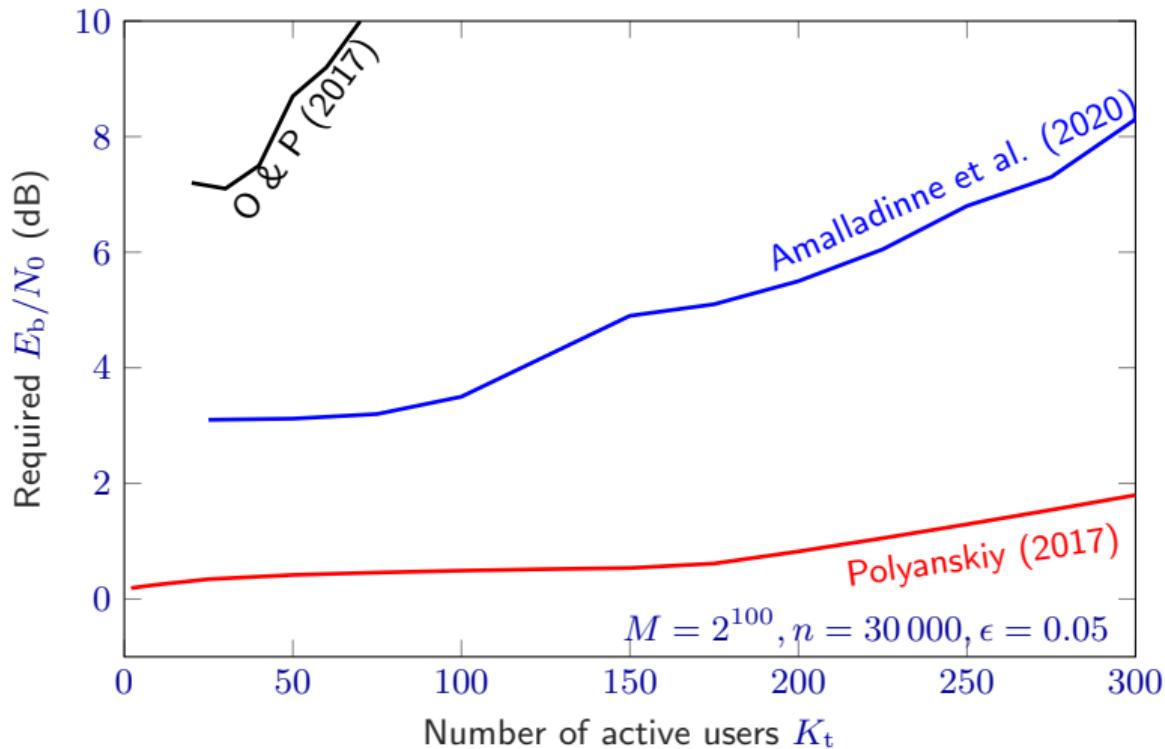
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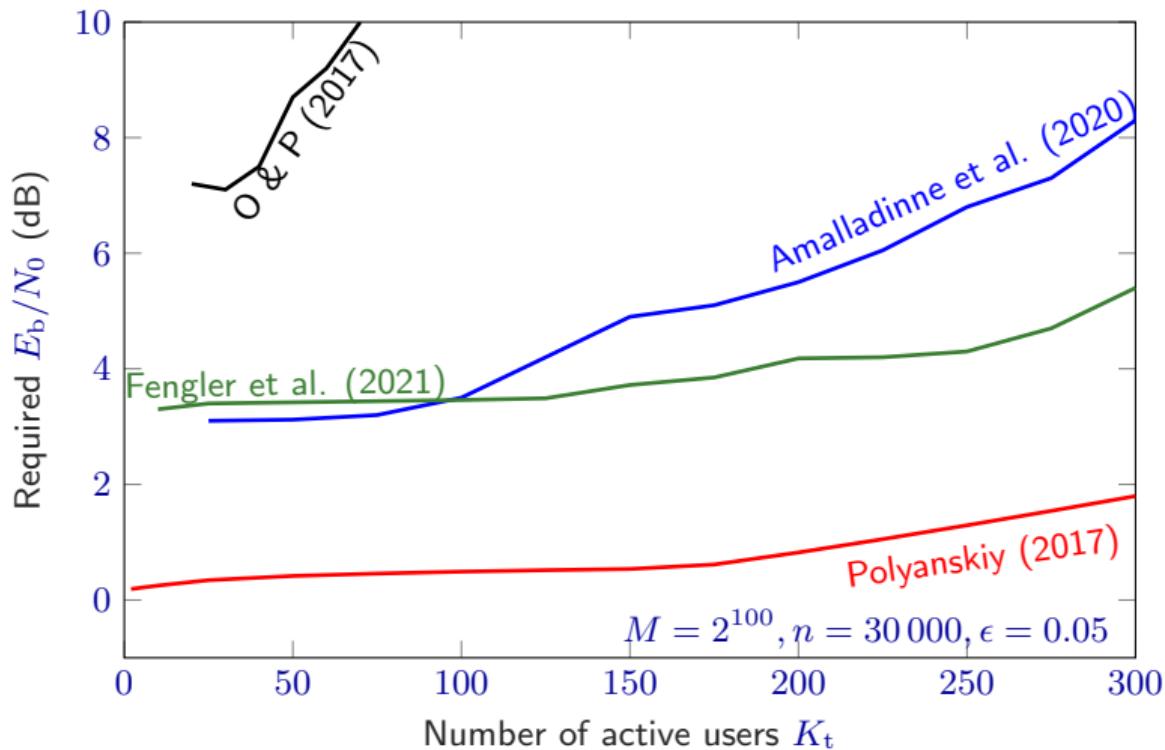
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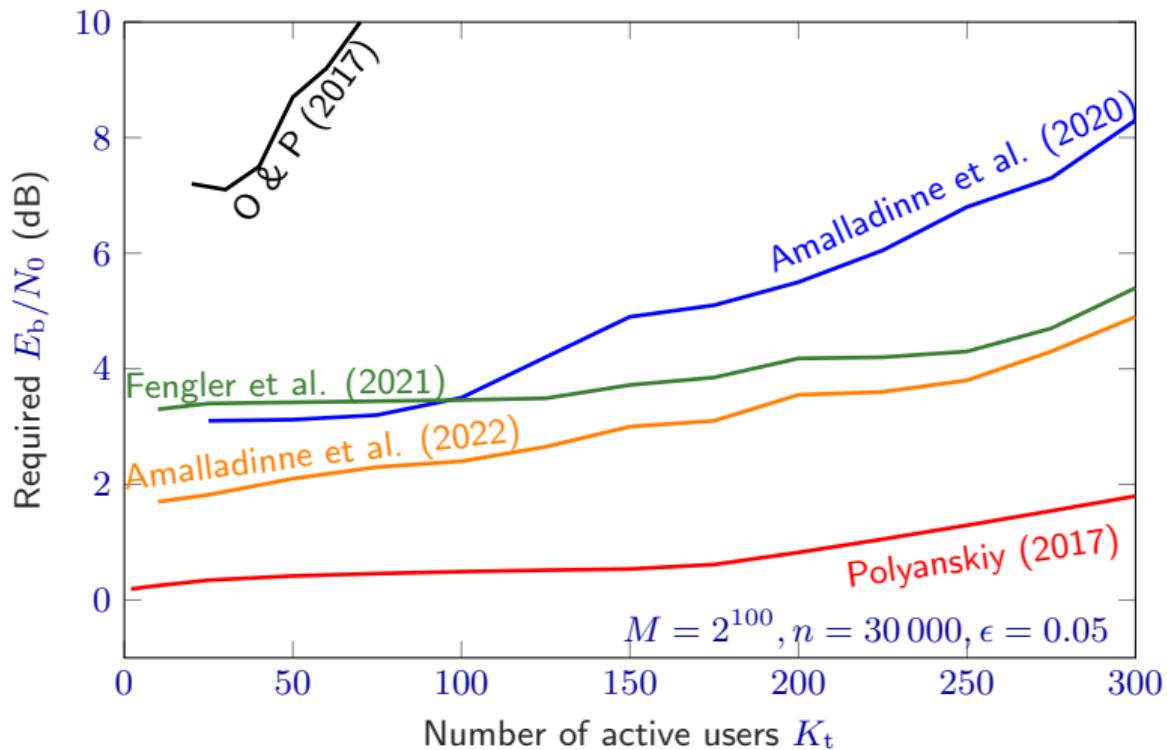
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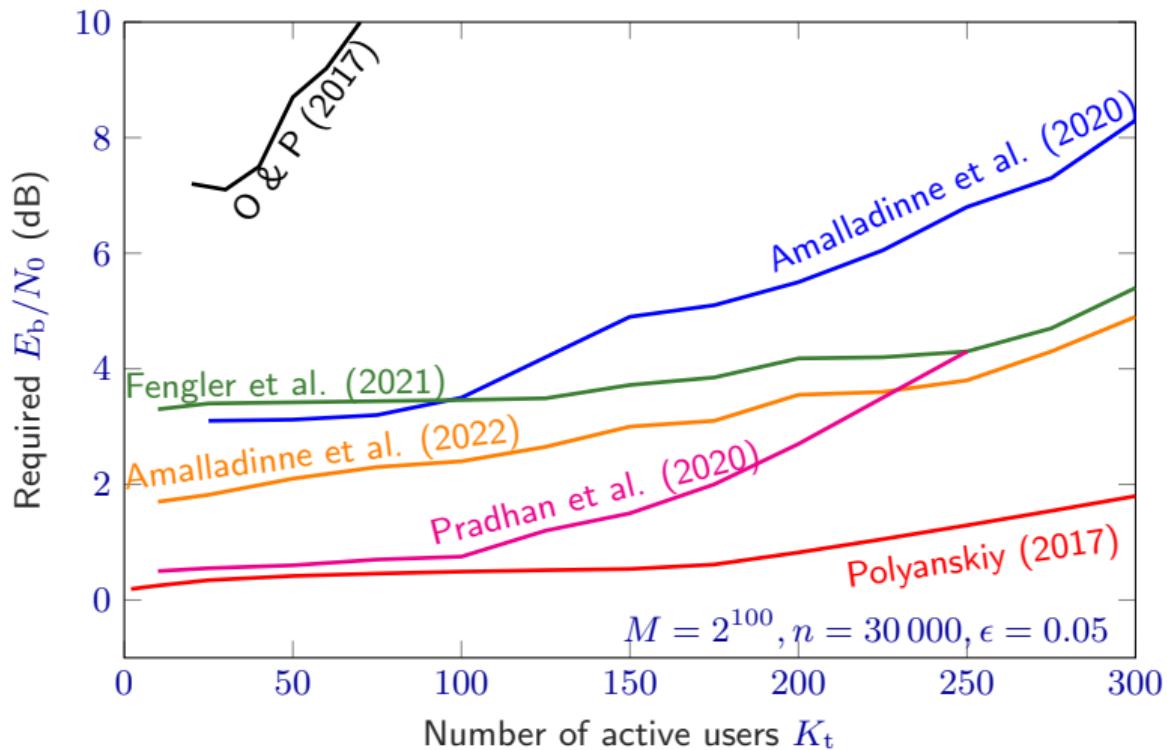
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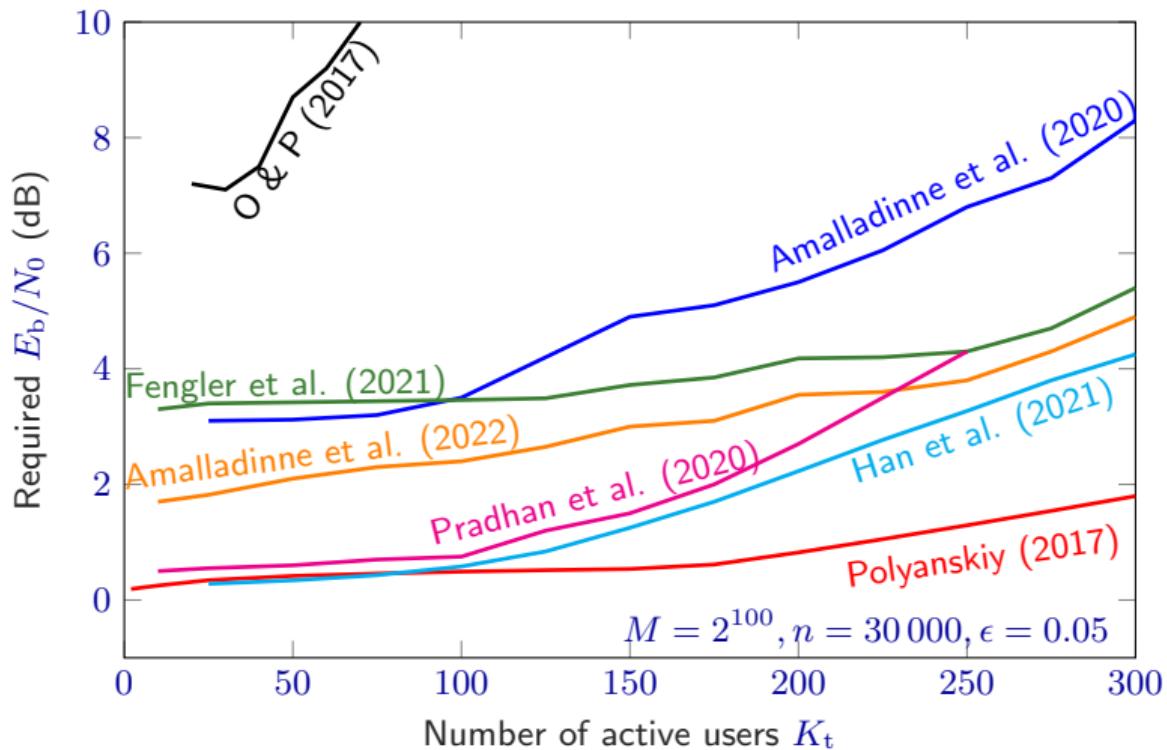
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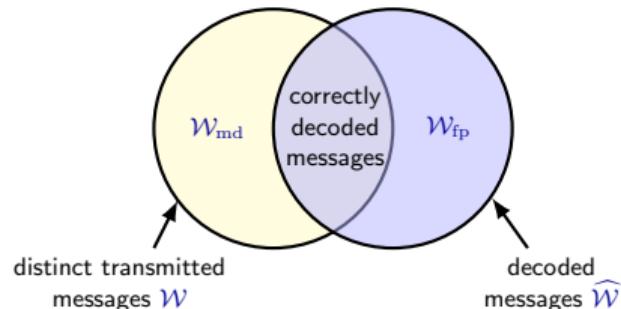
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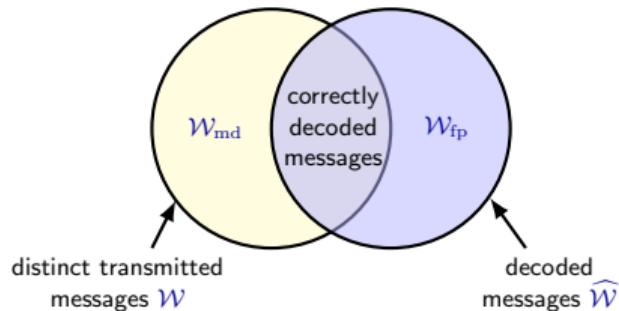


$K_t$  is random and not known to the receiver [Ngo et al. '23]

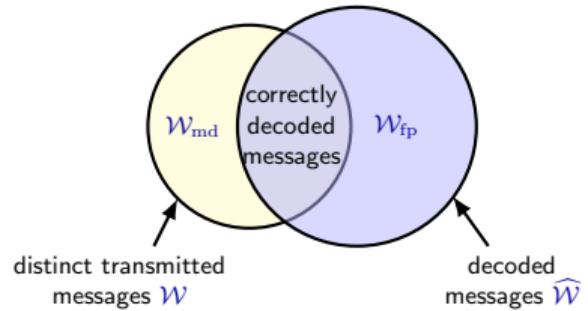


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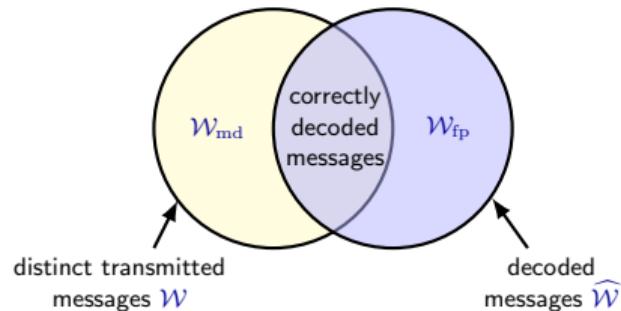


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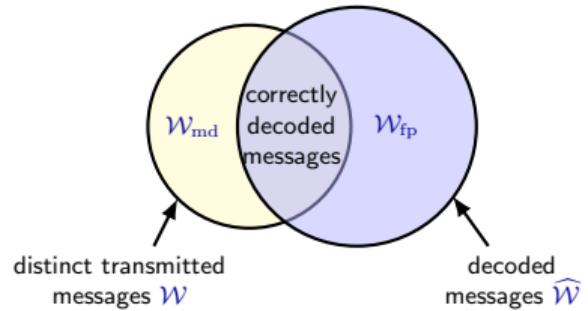


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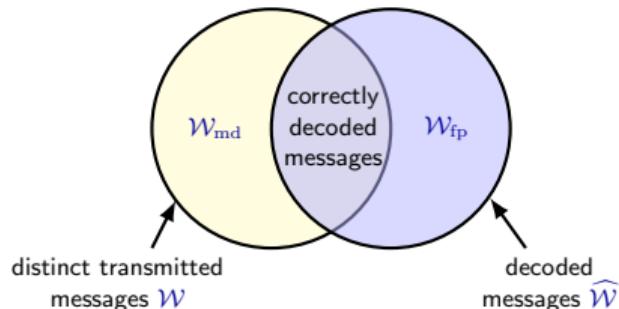


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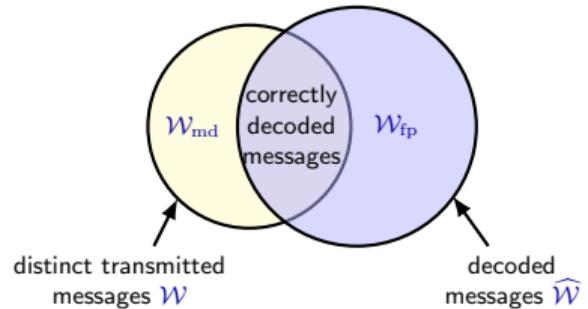
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Estimate  $K_t$  and deploy a coding scheme that treats  $K_t$  as known

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### Performance?

Good if reliability requirements are mild

# Performance metrics and definition of a code

Two performance metrics

$$P_{\text{md}} = \mathbb{E} \left[ \frac{1}{|\mathcal{W}|} \sum_{i=1}^{|\mathcal{W}|} \mathbb{P}[W_i \notin \widehat{\mathcal{W}}] \right]$$

$$P_{\text{fp}} = \mathbb{E} \left[ \frac{1}{|\widehat{\mathcal{W}}|} \sum_{i=1}^{|\widehat{\mathcal{W}}|} \mathbb{P}[\widehat{W}_i \notin \mathcal{W}] \right]$$

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$(M, n, \epsilon_{\text{md}}, \epsilon_{\text{fa}})$  code for unsourced GMAC with power constraint  $P$  and random, unknown number of transmitting users

It consists of a pair of possibly randomized encoder and decoder satisfying  $P_{\text{md}} \leq \epsilon_{\text{md}}$  and  $P_{\text{fp}} \leq \epsilon_{\text{fp}}$

# A two-step decoder

## Step 1

Obtain an estimate  $K'_t$  of  $K_t$  by minimizing a suitably chosen metric  $m(\mathbf{y}, k)$

$$K'_t = \arg \max_k m(\mathbf{y}, k)$$

Examples: ML estimation, energy-based estimation

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## Step 2

Find the list  $\widehat{\mathcal{W}}$  of decoded messages as

$$\widehat{\mathcal{W}} = \arg \min_{\substack{\mathcal{W}' \subset [1:M] \\ |\mathcal{W}'| = K'_t}} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|$$

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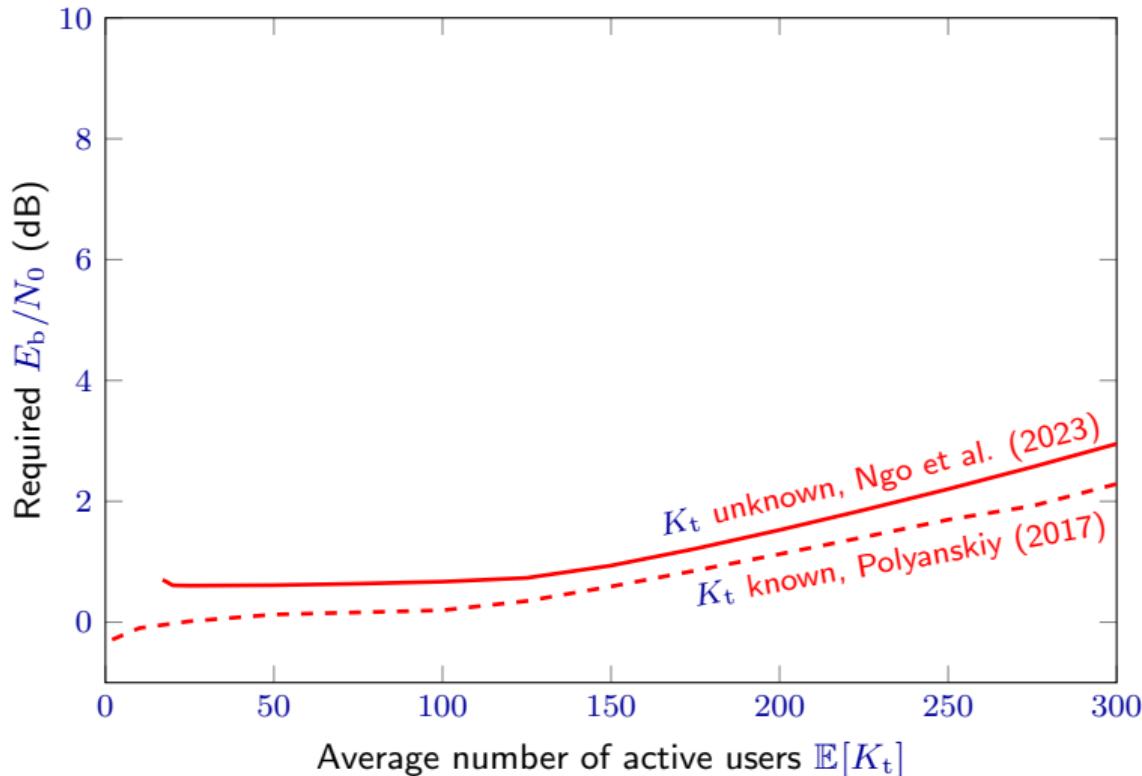
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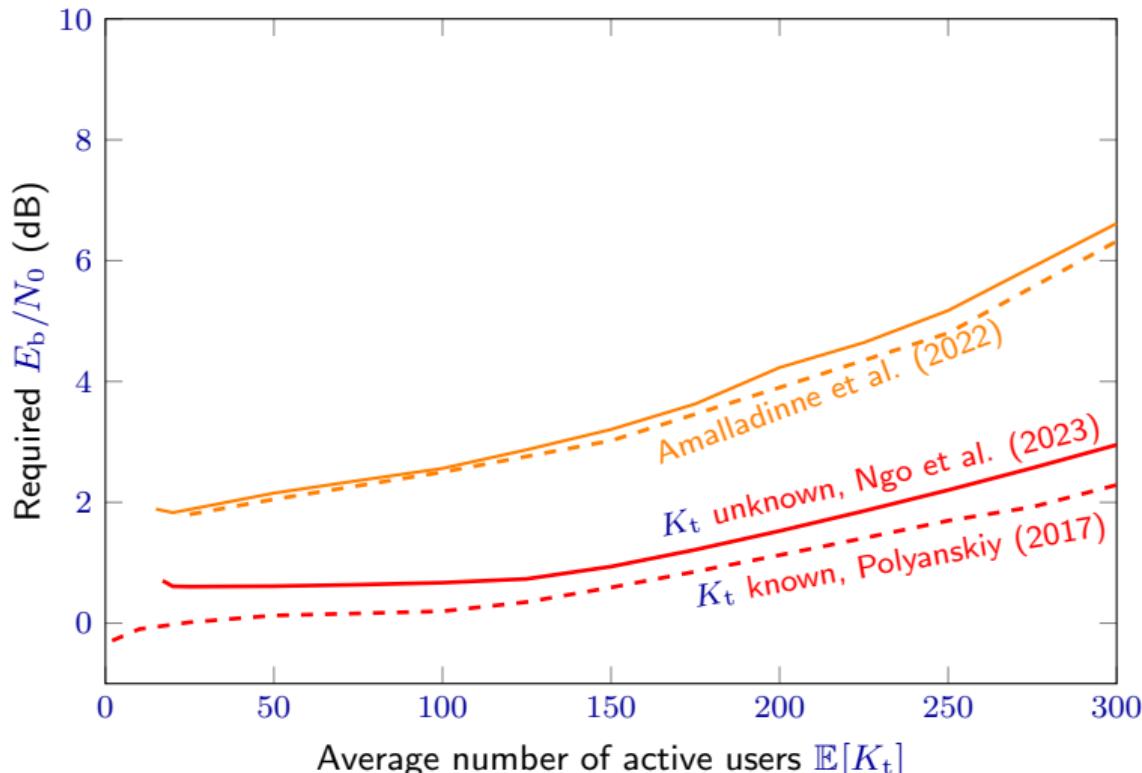
$$\widehat{\mathcal{W}} = \arg \min_{\substack{\mathcal{W}' \subset [1:M] \\ K'_t - r \leq |\mathcal{W}'| \leq K'_t + r}} \|\mathbf{y} - \mathbf{c}(\mathcal{W}')\|$$

$r > 0$ : decoding radius (cannot be chosen too large)

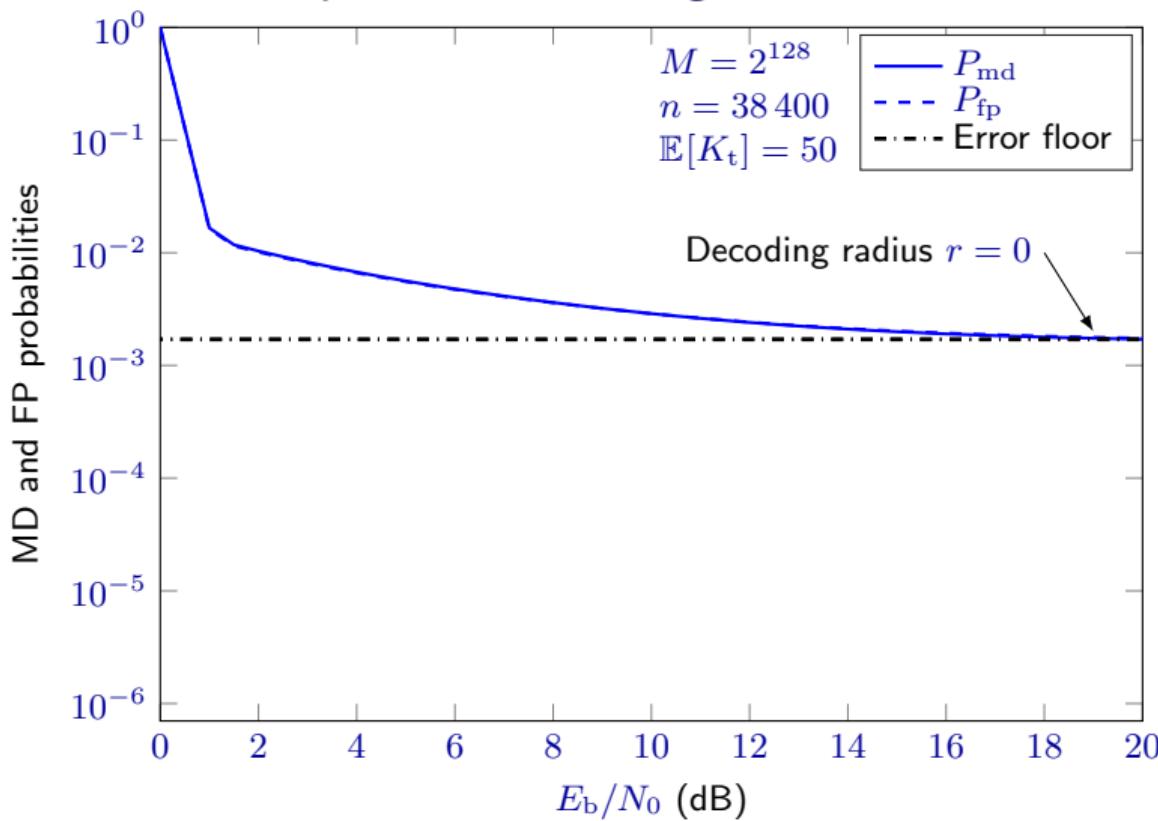
$M = 2^{128}$ ,  $n = 38\,400$ ,  $K_t \sim \text{Poisson}$ ,  $\epsilon_{\text{md}} = \epsilon_{\text{fa}} = 0.1$



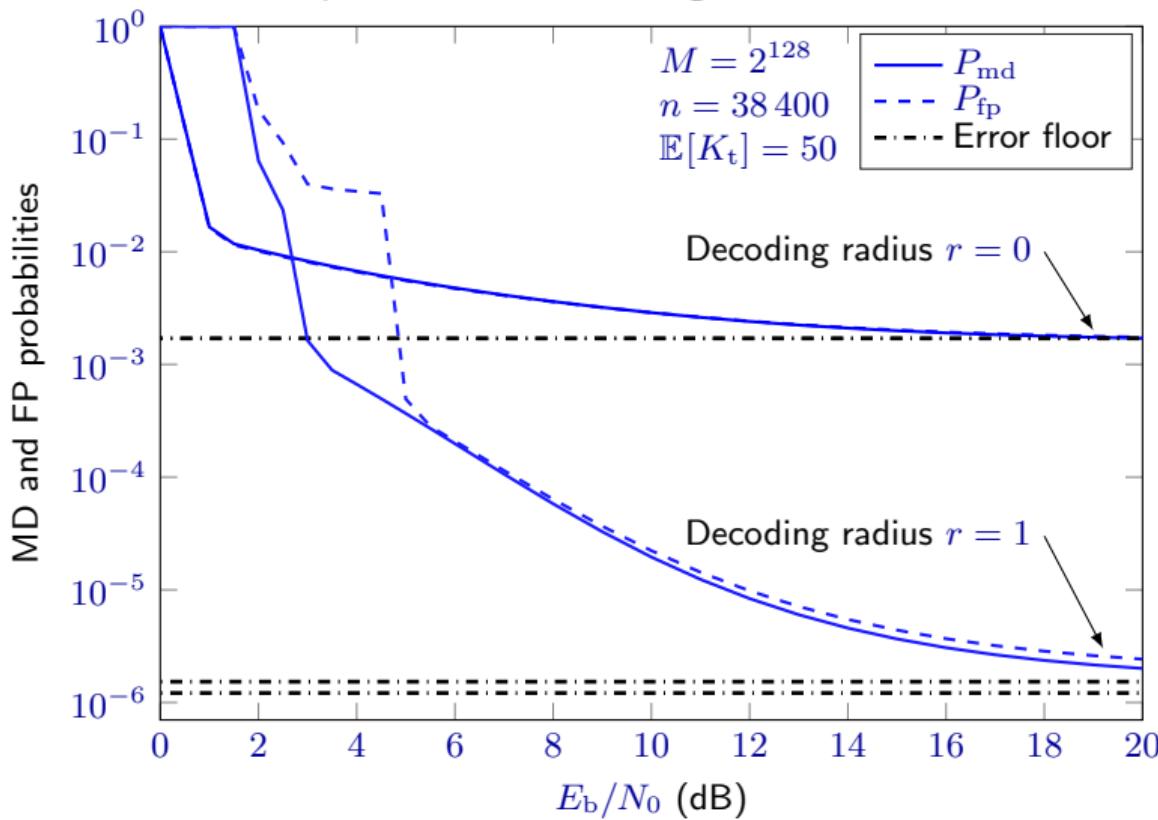
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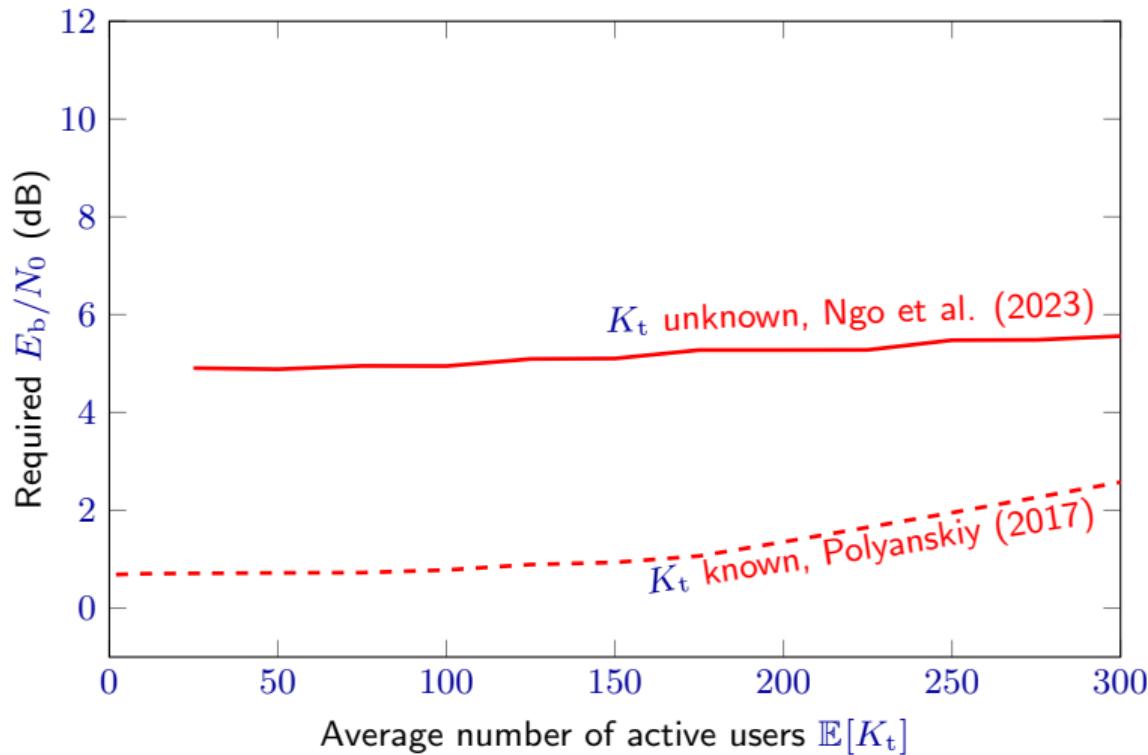
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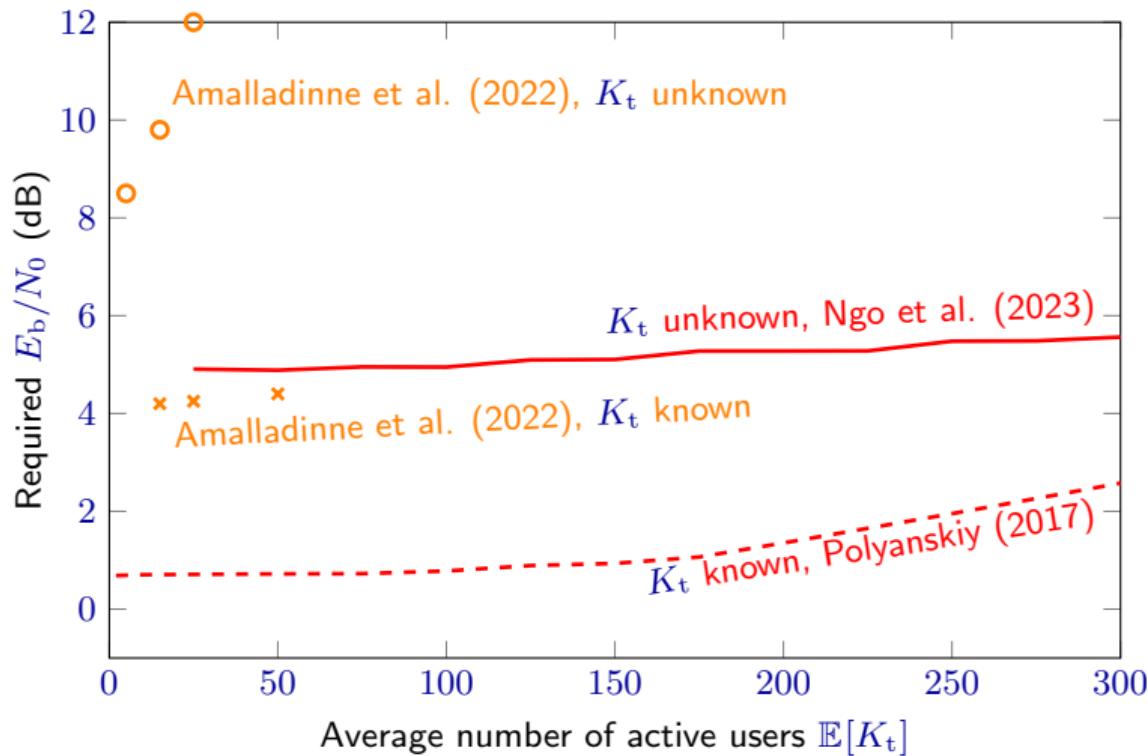
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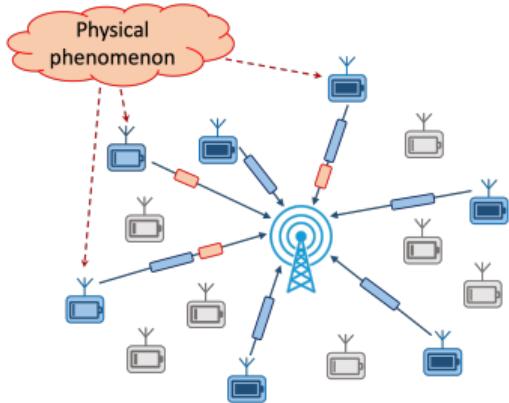
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# Heterogeneous traffic: massive and critical IoTs [Ngo et al. '23]



- $K$  devices
- $K_s$  devices transmit a standard msg.
- $K_a$  devices transmit an alarm msg.

- Device  $k$  has a standard message  $W_s \in [1 : M_s]$  with prob.  $\rho_s$

$$K_s \sim \text{Bin}(K, \rho_s)$$

- If an alarm occurs, all users transmit the same alarm message  $W_a \in [1 : M_a]$ , with probability  $\rho_d \leq \rho_{d,\max}$

$$K_a \sim \text{Bin}(K, \rho_a)$$

- Each user can transmit a standard message, an alarm message, both, or none

## Performance metrics

- $\mathcal{A}$ : alarm event
- $\mathcal{W}$ : set of transmitted standard messages
- $\widehat{\mathcal{W}}$ : set of decoded standard messages
- $W_a \in [1 : M_a]$ : transmitted alarm message
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For  $\mathcal{B} \in \{\mathcal{A}, \bar{\mathcal{A}}\}$

$$P_{\text{smd} \mid \mathcal{B}} = \mathbb{E} \left[ \frac{1}{|\mathcal{W}|} \sum_{i=1}^{|\mathcal{W}|} \mathbb{P}[W_i \notin \widehat{\mathcal{W}} \mid \mathcal{B}] \right], \quad P_{\text{sfp} \mid \mathcal{B}} = \mathbb{E} \left[ \frac{1}{|\widehat{\mathcal{W}}|} \sum_{i=1}^{|\widehat{\mathcal{W}}|} \mathbb{P}[W_i \notin \mathcal{W} \mid \mathcal{B}] \right]$$

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$$P_{\text{amd}} = \mathbb{P}[\widehat{W}_a \neq W_a | \mathcal{A}], \quad P_{\text{afp}} = \mathbb{P}[\widehat{W}_a \neq 0 | \bar{\mathcal{A}}]$$

## Definition of code and coexistence strategies

$(M, n, \epsilon_{\text{smd}}, \epsilon_{\text{sfp}}, \epsilon_{\text{amd}}, \epsilon_{\text{afp}})$  code for unsourced GMAC with standard and alarm messages

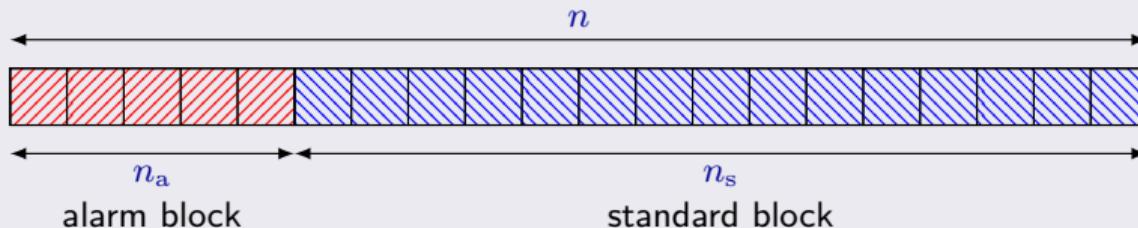
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## Orthogonalization

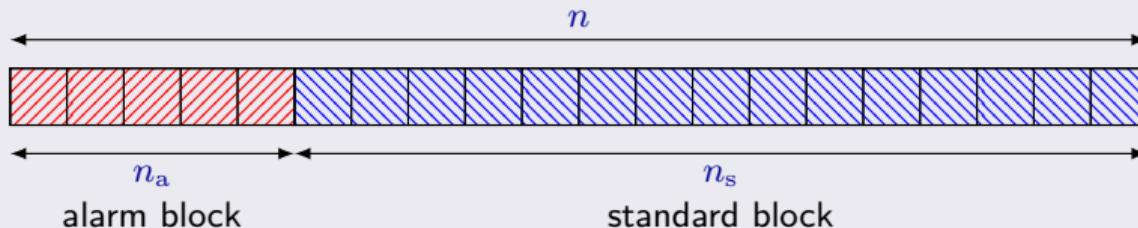


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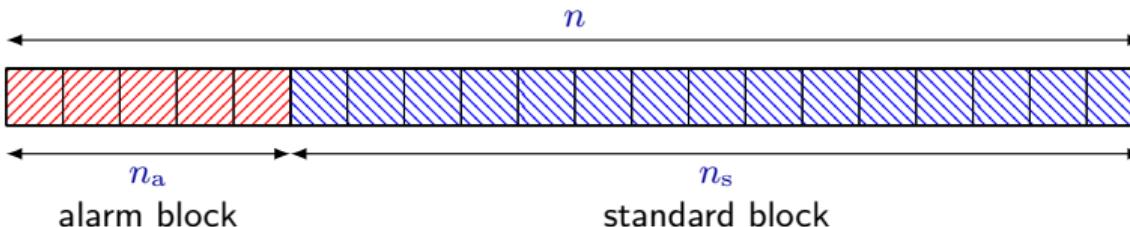
## Orthogonalization



Superposition: exploiting reliability diversity [Popovski et al., 2018]



# Orthogonalization



## Standard block

$$\mathbf{y}_s = \sum_{k=1}^{K_s} \mathbf{x}_k + \mathbf{z}_s, \quad \mathbf{z}_s \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{n_s})$$

- $\|\mathbf{x}_k\|^2 \leq n_s P_s$
- $(E_b/N_0)_s = \frac{n_s P_s}{2 \log_2 M_s}$
- unsourced GMAC with random and unknown number of active users
- Can be analyzed as before

## Alarm block

$$\mathbf{y}_a = K_a \mathbf{x}_a + \mathbf{z}_a, \quad \mathbf{z}_a \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{n_a})$$

- $\|\mathbf{x}_a\|^2 \leq n_a P_a$
- $(E_b/N_0)_a = \frac{n_a P_a \rho_a K}{2 \log_2 M_a}$
- Single-user AWGN channel with random, unknown SNR  $K_a^2 P_a$  (coherent combining)
- Can be analyzed with FBL tools

# Analysis of alarm block

Random codebook generation and encoder

- Gaussian codebook: fix  $P' < P$ ; generate  $M_a$  codewords  $\mathbf{c}_1, \dots, \mathbf{c}_{M_a} \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, P' \mathbf{I}_{n_a})$
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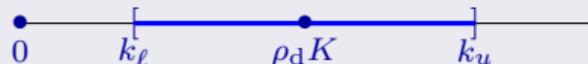
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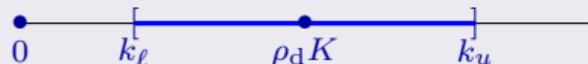
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- Bounds on  $P_{\text{amd}}$  and  $P_{\text{afp}}$  via random coding union bound with parameter  $s$

## Numerical experiment

- $n = 30\,000$ ,  $(M_a, M_s) = (2^3, 2^{100})$ ,  $1000 \leq K \leq 30\,000$ ,  $\rho_s = 0.01$
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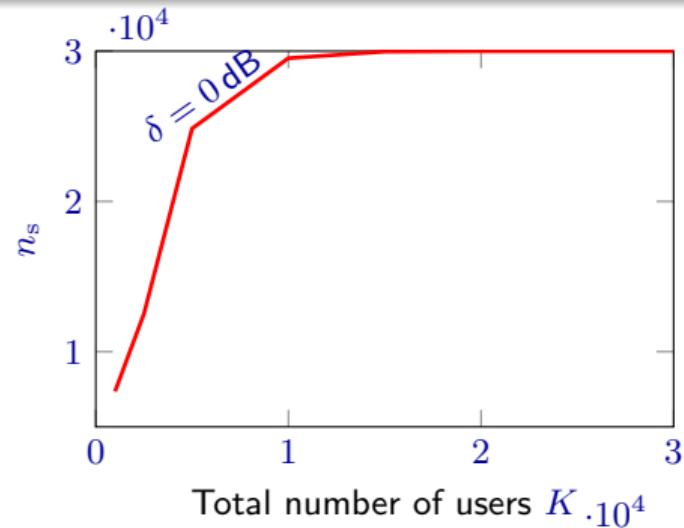
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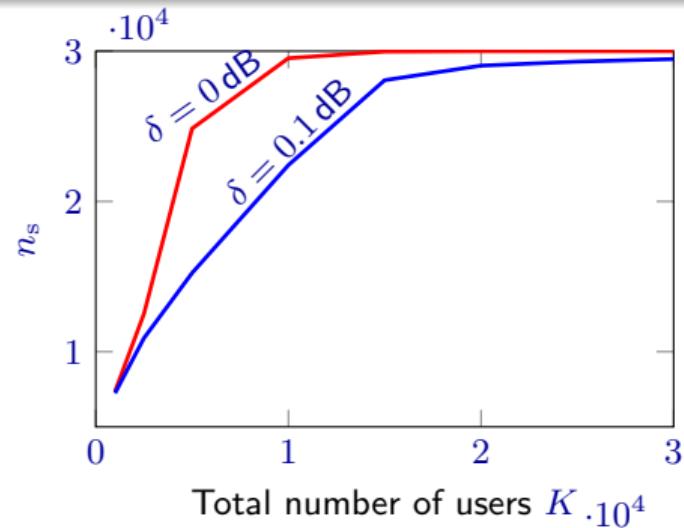


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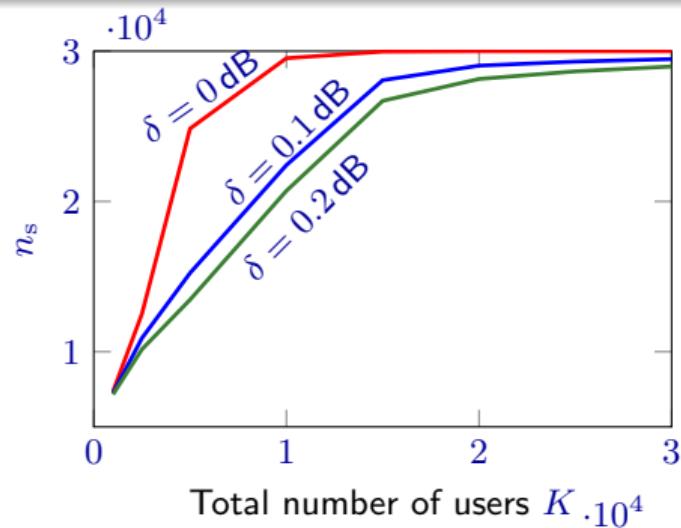


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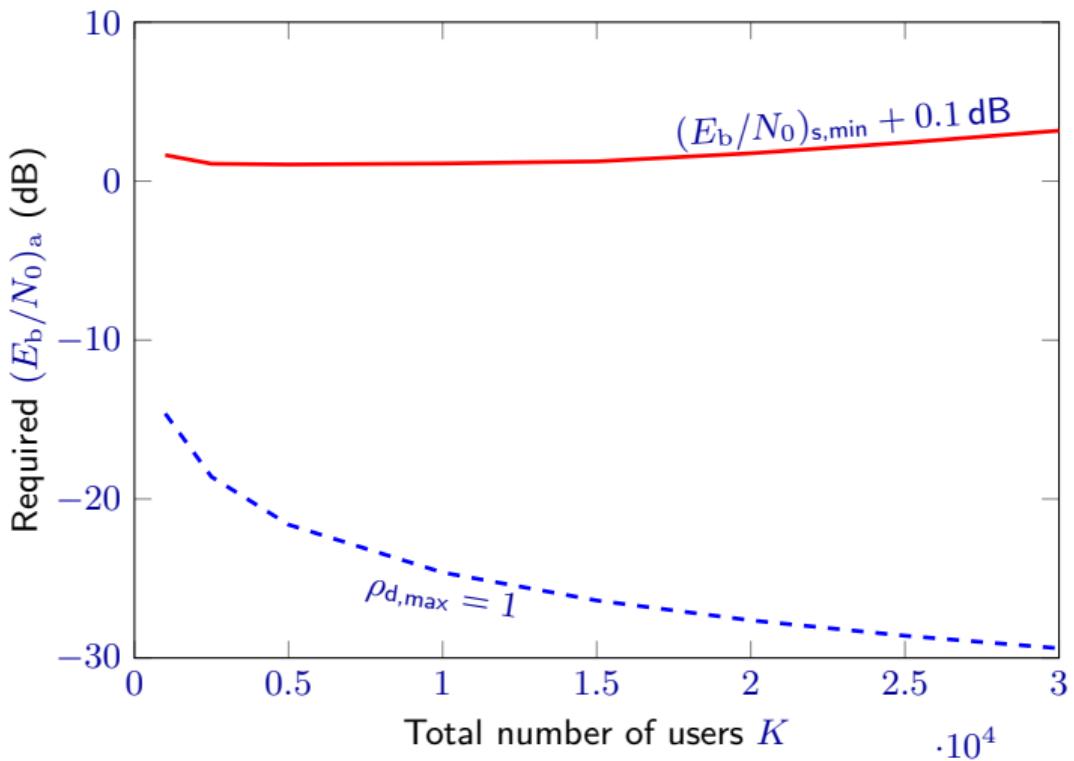
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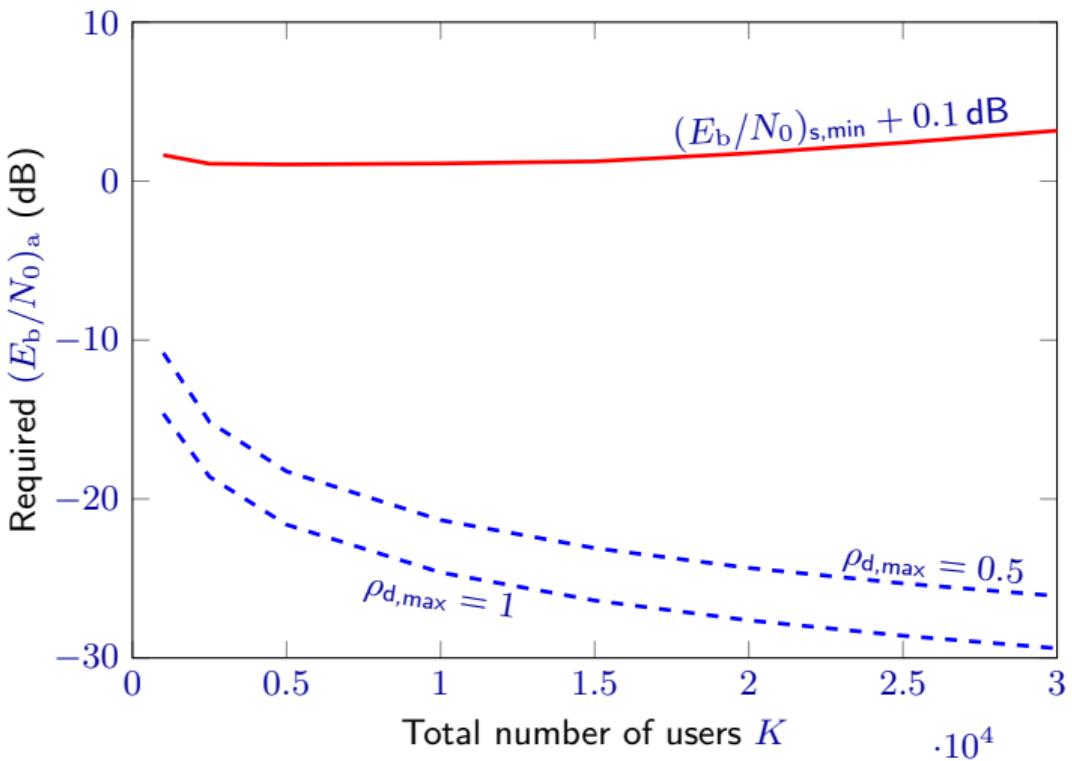
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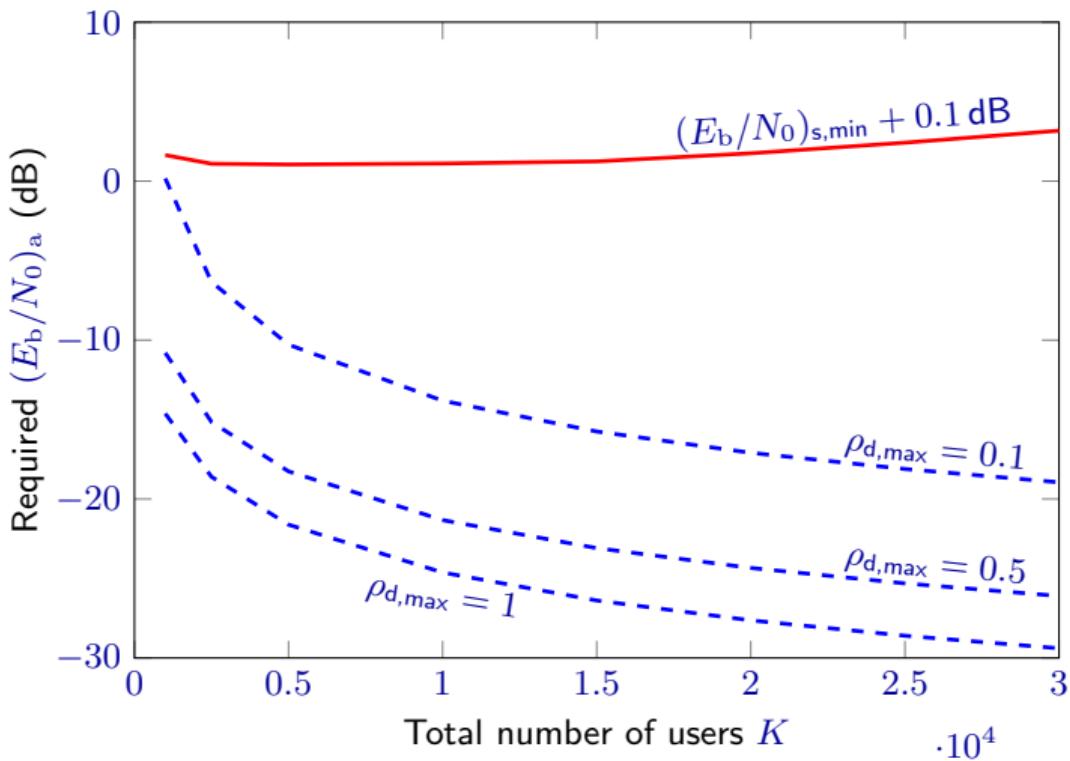
## Role of device sensitivity $\rho_{d,\max}$



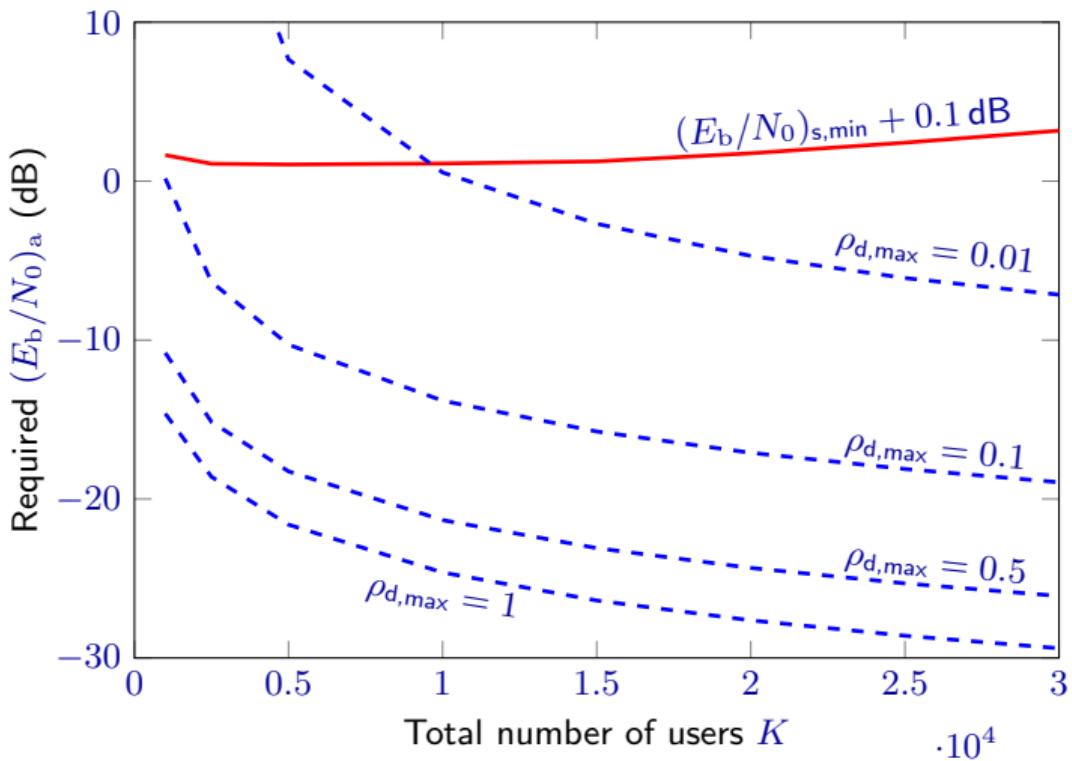
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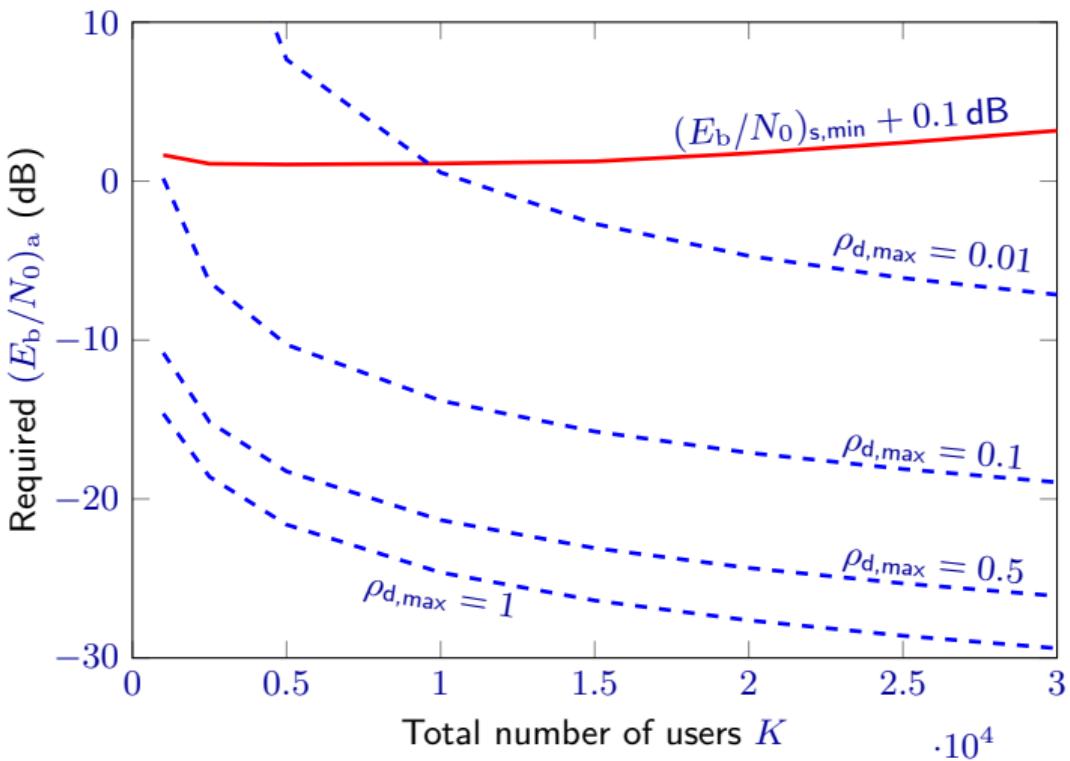
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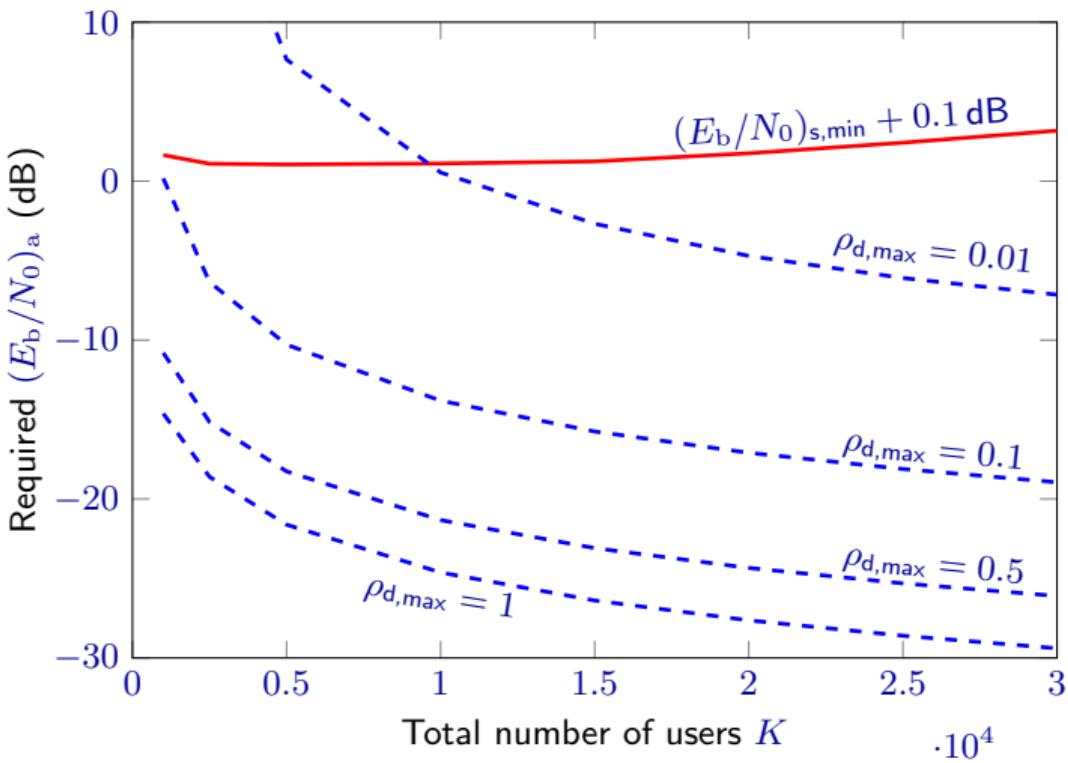
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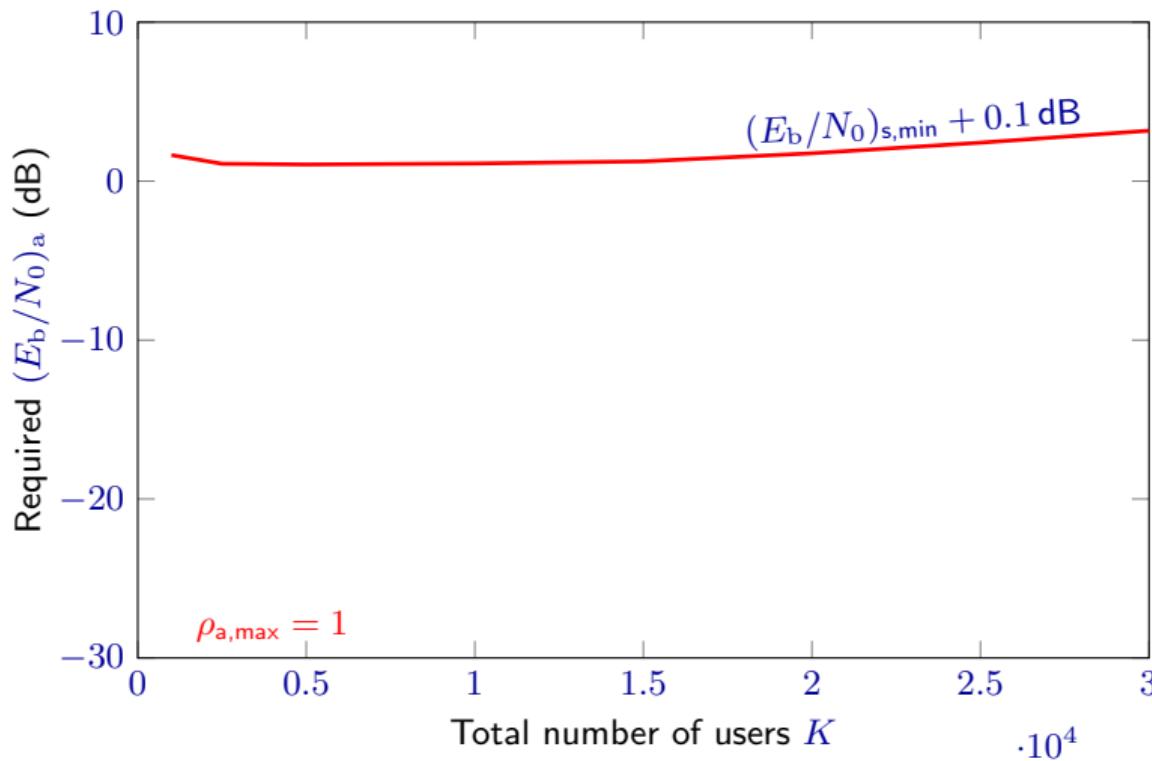
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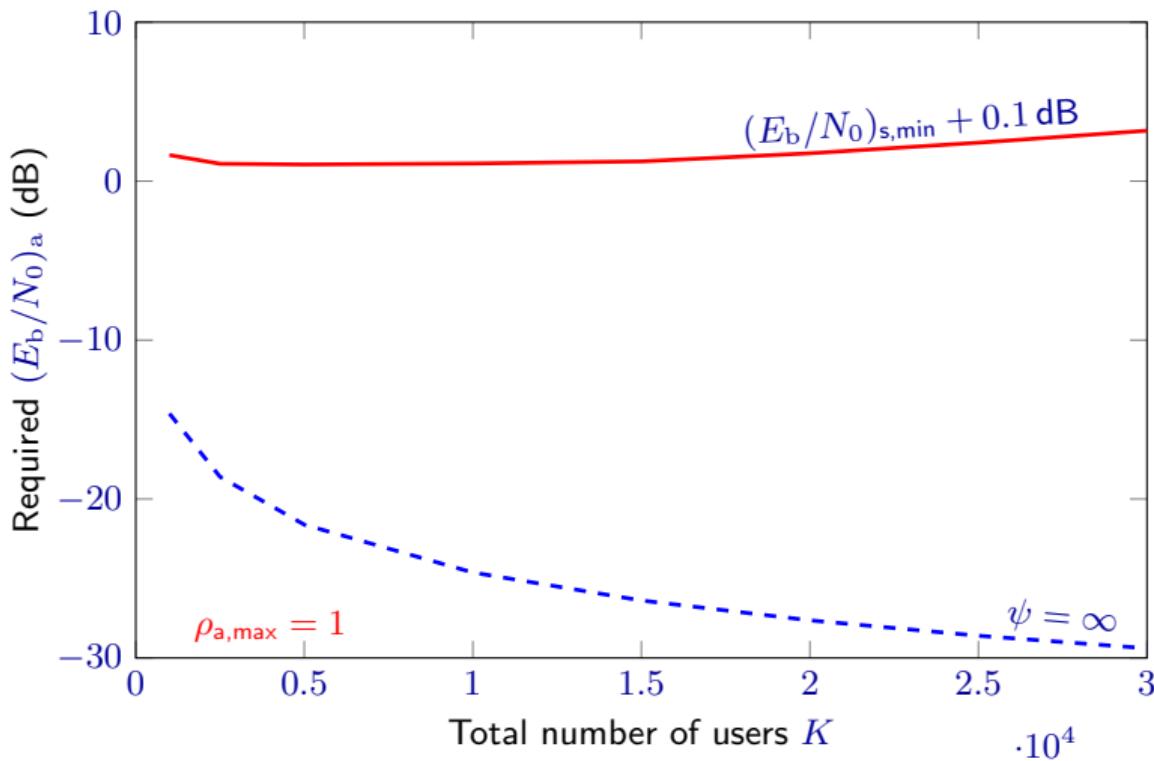
Issue: dynamic range

$$\psi = P_s/P_a \in [30 : 70] \text{ dB}$$

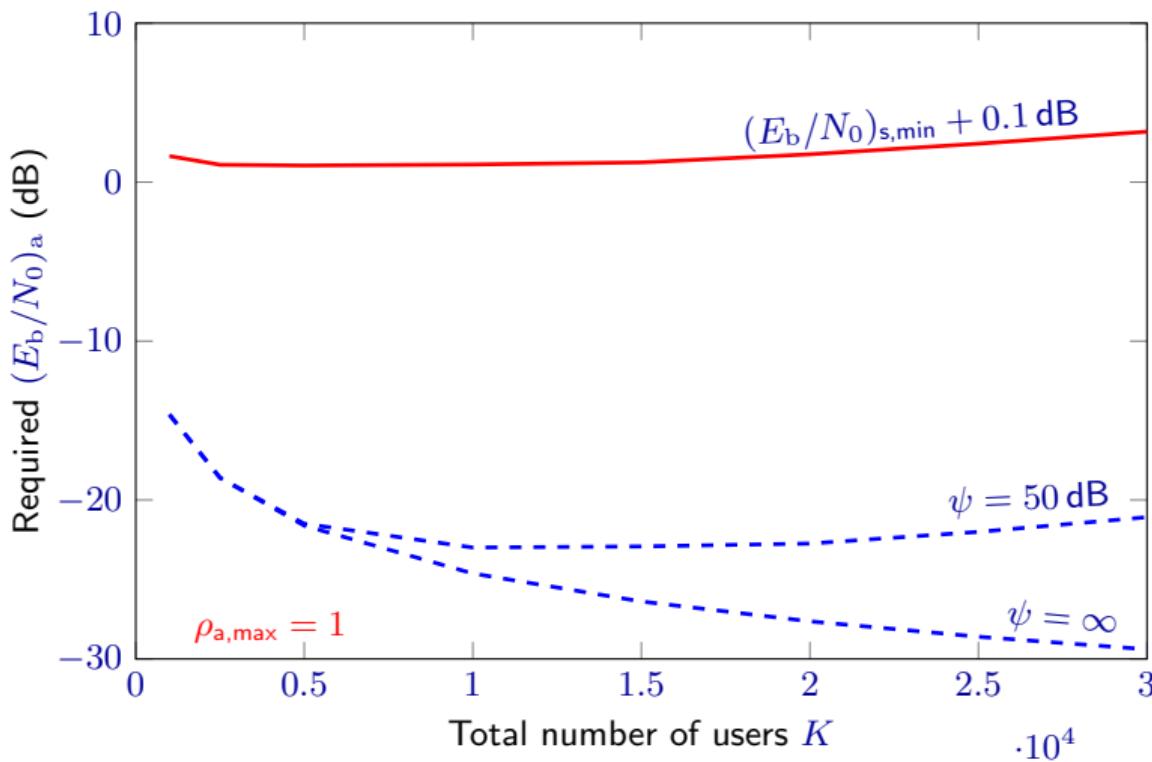
## Role of dynamic range $\psi$



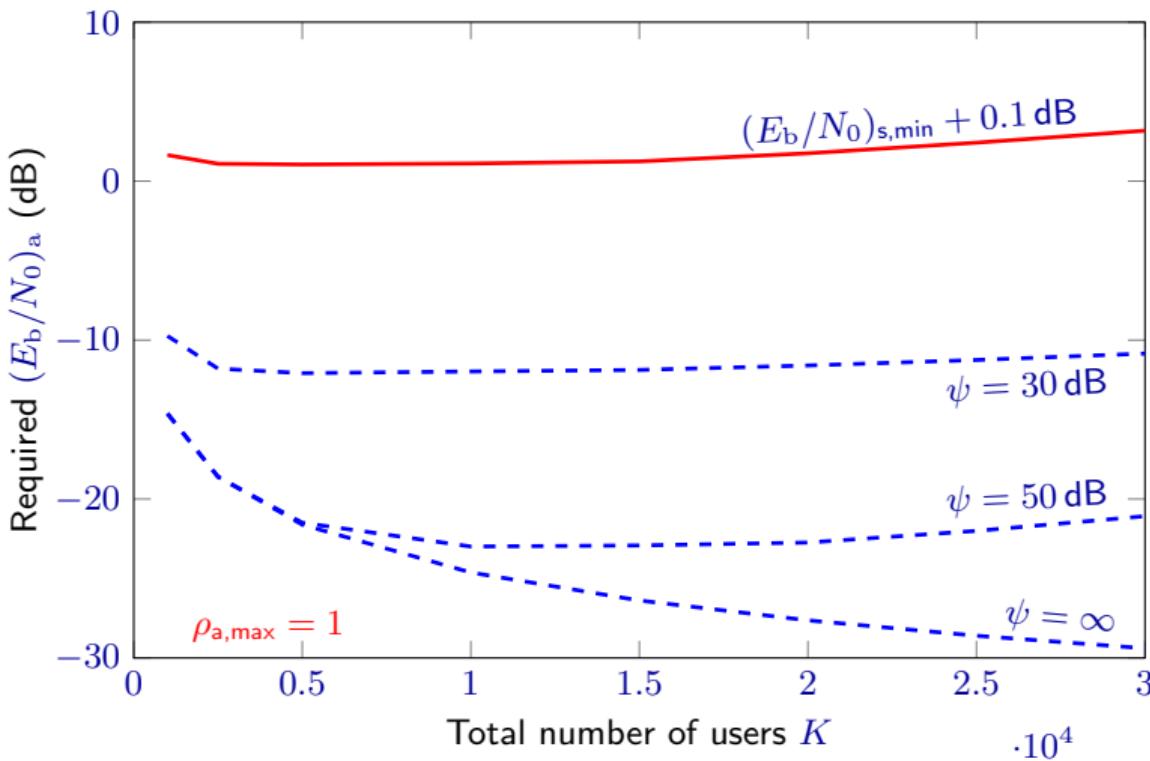
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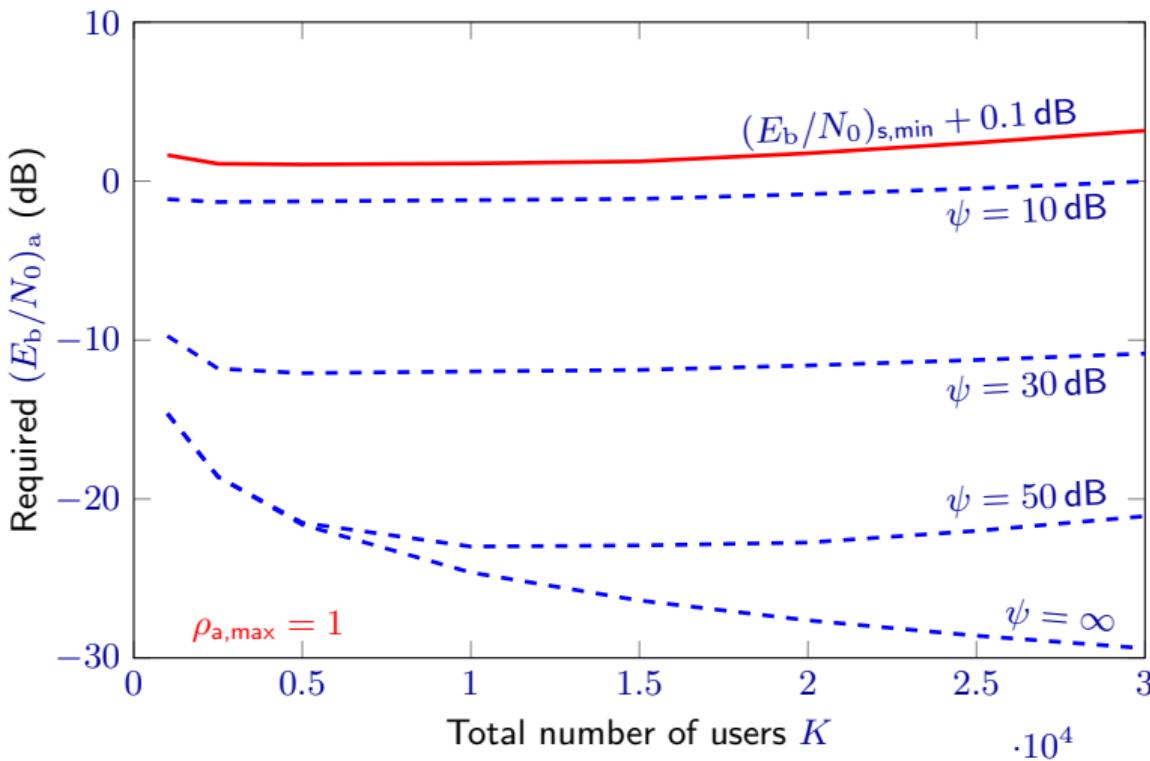
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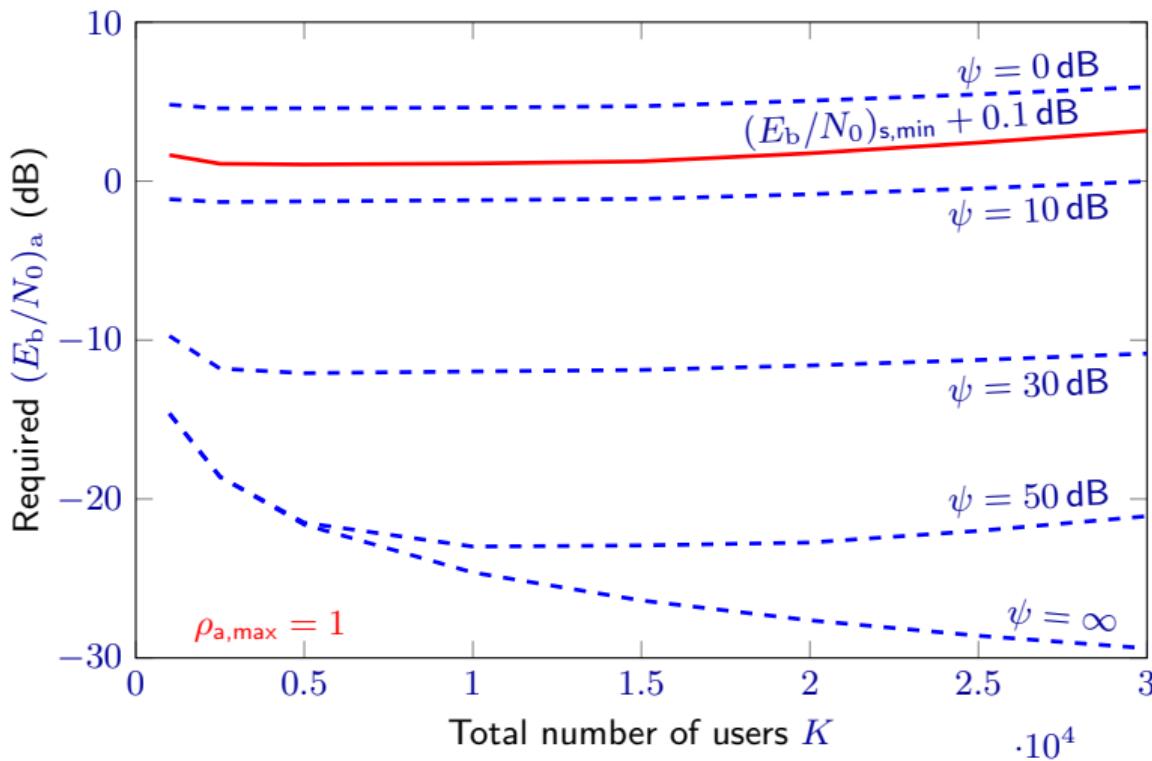
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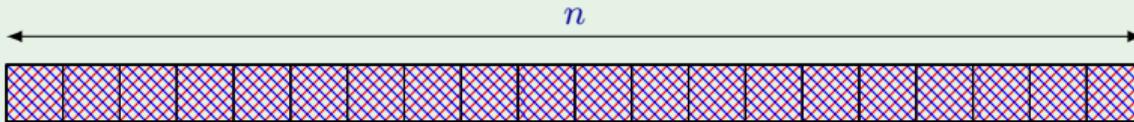
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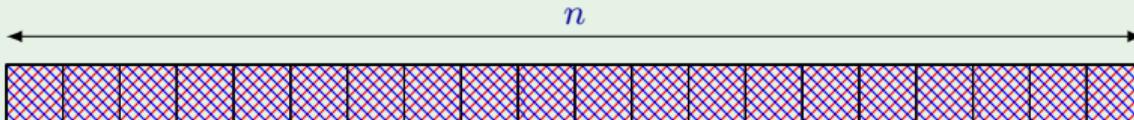


# Superposition



$$\mathbf{y} = \sum_{k=1}^{K_s} \mathbf{x}_k + K_a \mathbf{x}_a + \mathbf{z}$$

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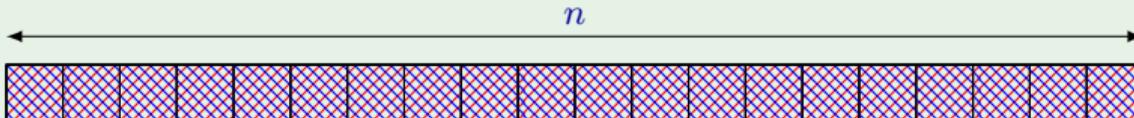


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Decoder: reliability diversity

- Estimate  $K_a$  and  $\mathbf{W}_a$ ; treat  $\sum_{k=1}^{K_s} \mathbf{x}_k$  as noise
- Interference cancellation  $\mathbf{y}_{ic} = \mathbf{y} - \hat{K}_a \hat{\mathbf{x}}_a$
- Estimate  $K_s$  and  $\mathcal{W}$  from  $\mathbf{y}_{ic}$ ; residual interference  $K_a \mathbf{x}_a - \hat{K}_a \hat{\mathbf{x}}_a$

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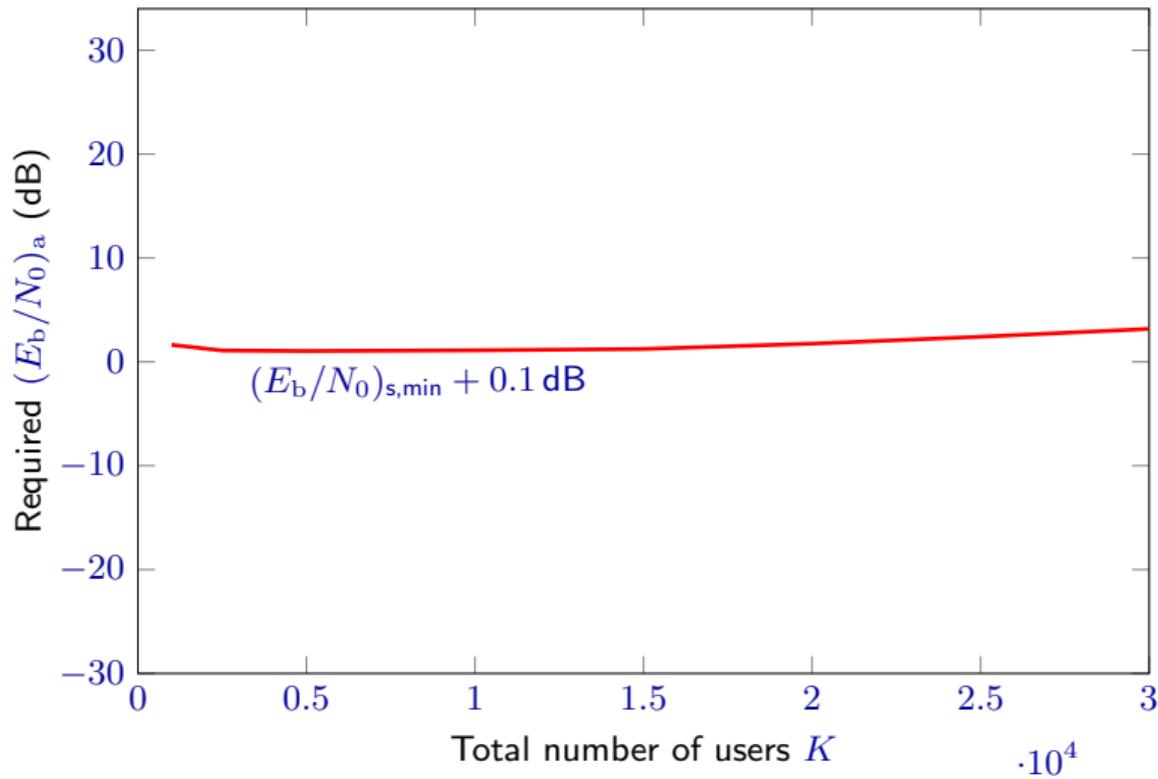
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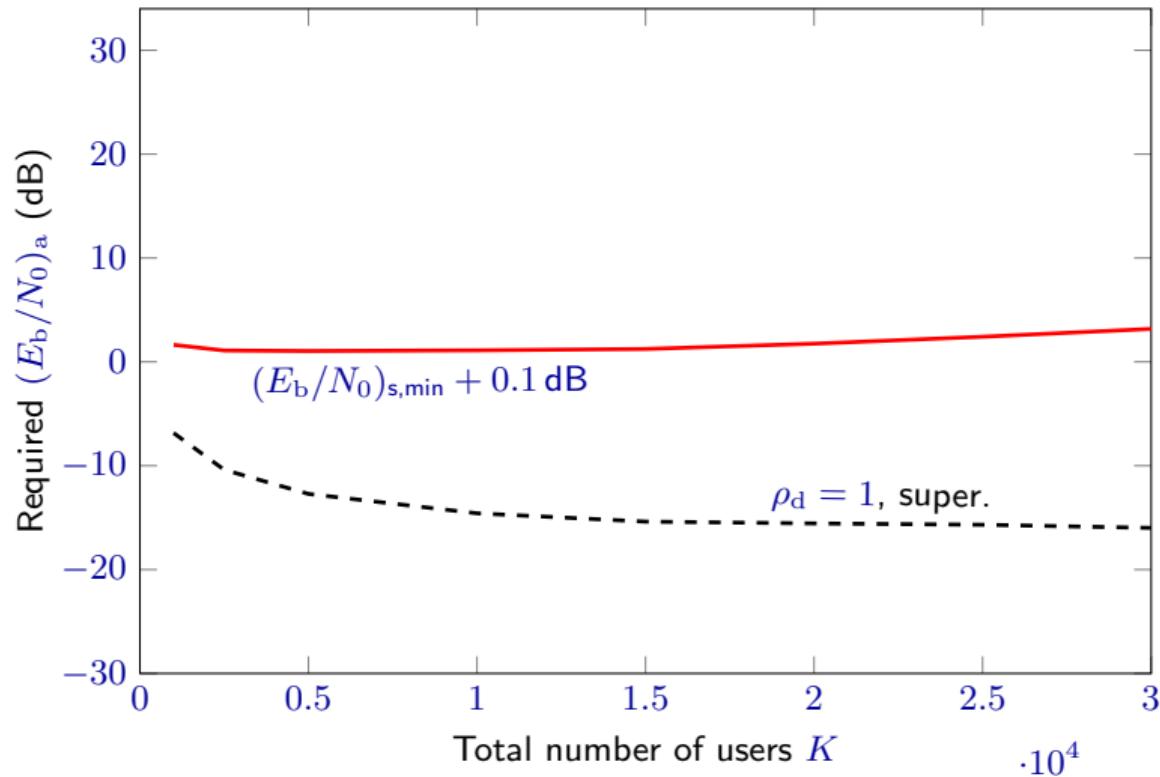
## Issue

Difficult to estimate  $K_a$  reliably in the presence of noise; imperfect interference cancellation

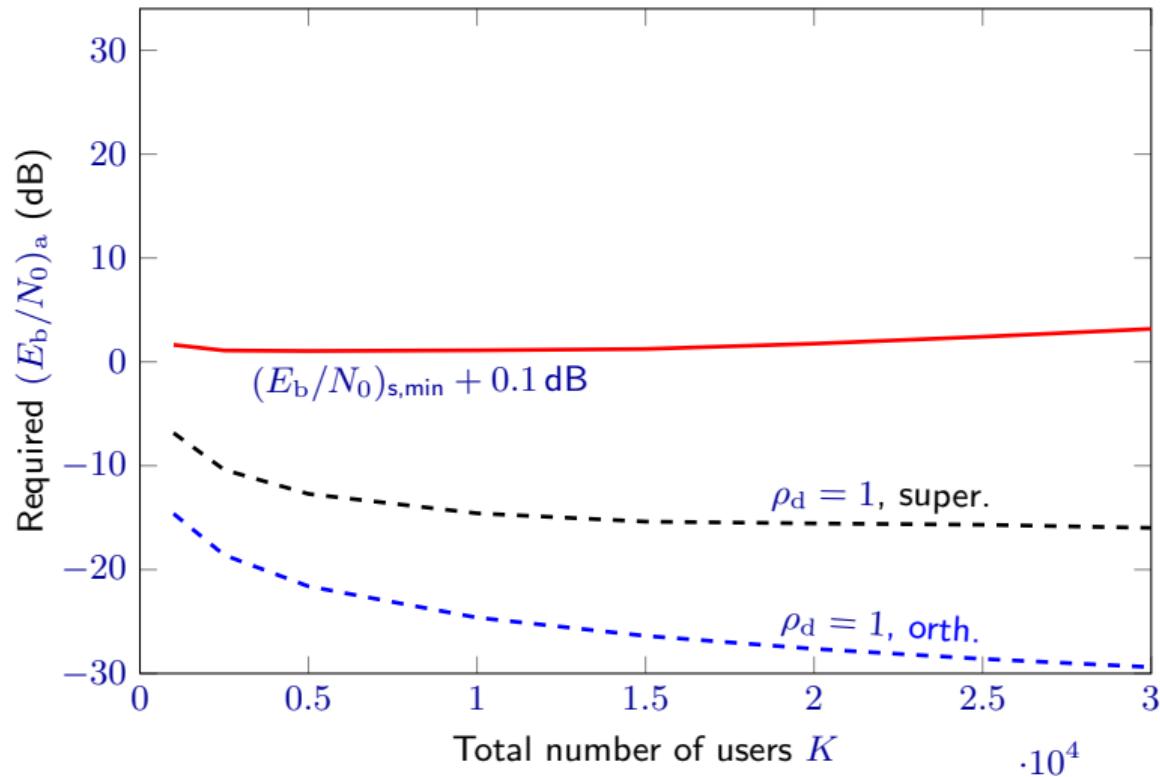
## Performance: orthogonalization vs. superposition



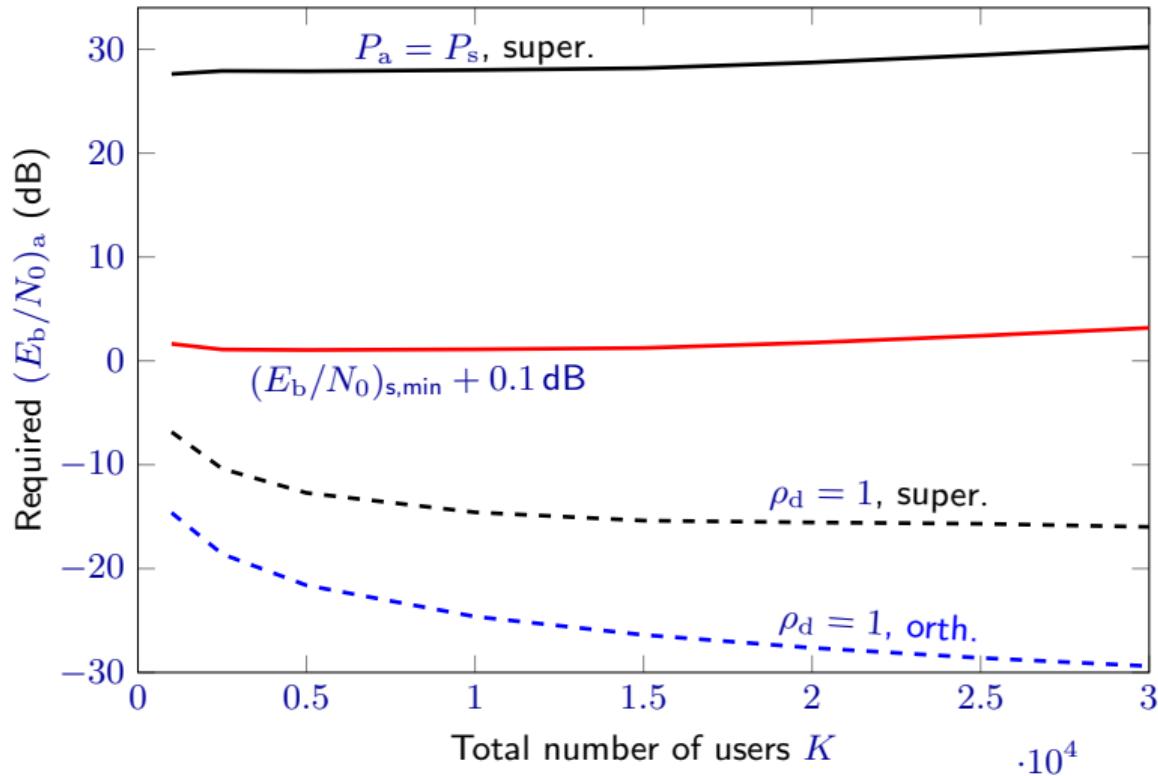
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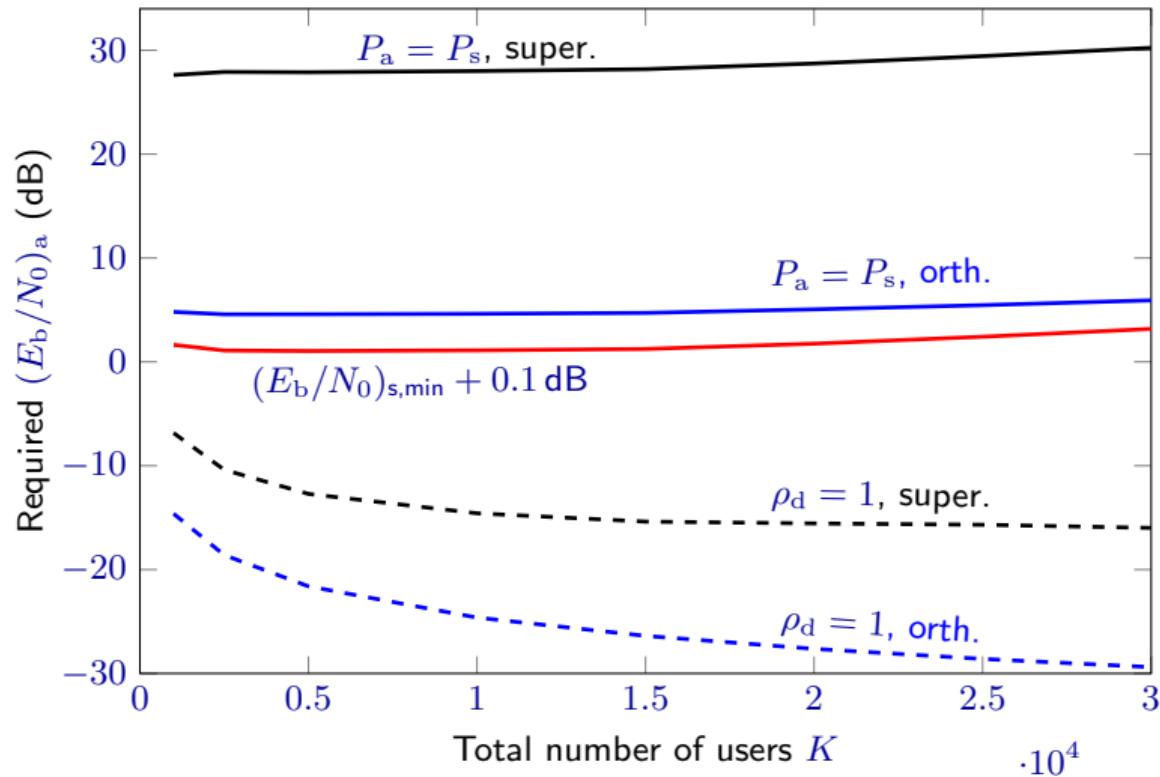
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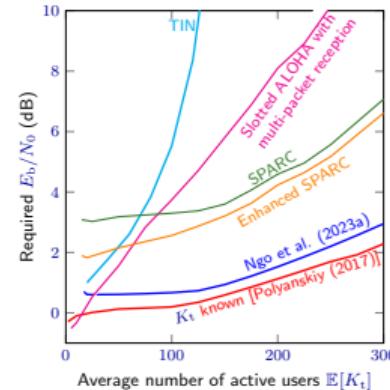


## Performance: orthogonalization vs. superposition



# Summary and further extensions

- Overview of information-theoretic bounds for mMTC
- Extension to unknown number of active users and heterogeneous traffic



## Further extensions and open problems

- Fading, massive MIMO, cell-free [Kowshik & Polyanskiy, 2021; Fengler et al. 2022; Decurninge et al. 2021; Gkagkos et al. 2023]
- Variable-length codes with stop feedback [Yavas et al. 2021]
- Imperfect synchronization [Decurninge et al. 2022 , Fengler et al. 2023]
- Age of information [Munari 2021, Munari et al., 2023]
- Energy harvesting [Demirhan & Duman, 2019]