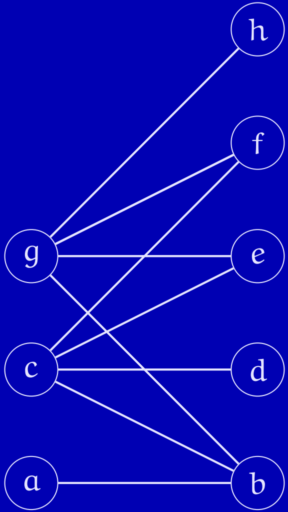
Bipartite graphs



Bipartite graphs

Lemma 1

Let $G = \langle V, E \rangle$ be a graph. Then G is bipartite if and only if it does not have any odd cycle.

Lemma 2

Let $G = \langle V, E \rangle$ be a graph and let M be its **incidence** matrix. Then G is bipartite if and only if M is totally unimodular.

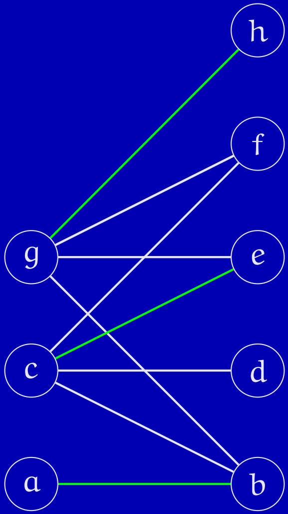
Max cardinality matching

Instance

An unweighted graph $G = \langle V, E \rangle$ with bipartition A, B

$$
\begin{aligned}
\max \sum_{(v,w)\in E} e_{v,w} & \quad \text{subject to} && (1)\\
\sum_{w\in B} e_{v,w} \leqslant 1 & \quad \forall v \in A && (2)\\
\sum_{v\in A} e_{v,w} \leqslant 1 & \quad \forall w \in B && (3)\\
e_{v,w} \in \{0,1\} & \quad \forall (v,w) \in E
\end{aligned}
$$

Max cardinality matching



Max weighted matching

Instance

An edge-weighted graph $G = \langle V, E \rangle$ with bipartition A, B, with weight function c

$$\max \sum_{(v,w)\in E} c(v,w)e_{v,w} \quad \text{subject to} \tag{4}$$

$$\sum_{w\in B} e_{v,w} \leqslant 1 \quad \forall v \in A \tag{5}$$

$$\sum_{v\in A} e_{v,w} \leqslant 1 \quad \forall w \in B \tag{6}$$

$$e_{v,w} \in \{0,1\} \quad \forall (v,w) \in E$$

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