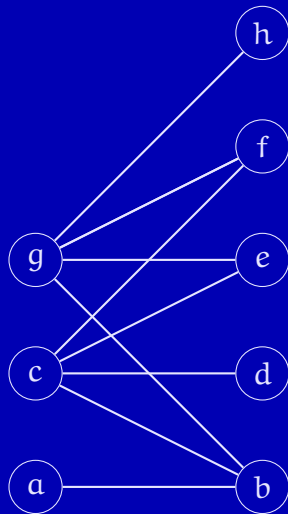


Bipartite graphs



Bipartite graphs

Lemma 1

Let $G = \langle V, E \rangle$ be a graph. Then G is bipartite if and only if it does not have any odd cycle.

Lemma 2

Let $G = \langle V, E \rangle$ be a graph and let M be its **incidence** matrix. Then G is bipartite if and only if M is totally unimodular.

Max cardinality matching

Instance

An unweighted graph $G = \langle V, E \rangle$ with bipartition A, B

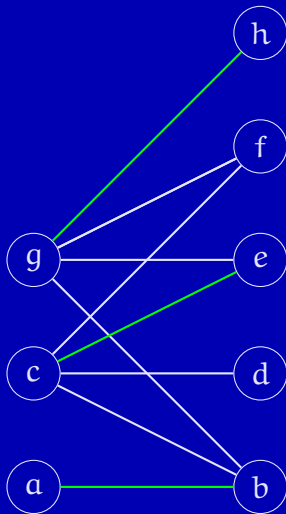
$$\max \sum_{(v,w) \in E} e_{v,w} \quad \text{subject to} \quad (1)$$

$$\sum_{w \in B} e_{v,w} \leq 1 \quad \forall v \in A \quad (2)$$

$$\sum_{v \in A} e_{v,w} \leq 1 \quad \forall w \in B \quad (3)$$

$$e_{v,w} \in \{0, 1\} \quad \forall (v, w) \in E$$

Max cardinality matching



Max weighted matching

Instance

An edge-weighted graph $G = \langle V, E \rangle$ with bipartition A, B , with weight function c

$$\max \sum_{(v,w) \in E} c(v,w) e_{v,w} \quad \text{subject to} \quad (4)$$

$$\sum_{w \in B} e_{v,w} \leq 1 \quad \forall v \in A \quad (5)$$

$$\sum_{v \in A} e_{v,w} \leq 1 \quad \forall w \in B \quad (6)$$

$$e_{v,w} \in \{0, 1\} \quad \forall (v, w) \in E$$

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