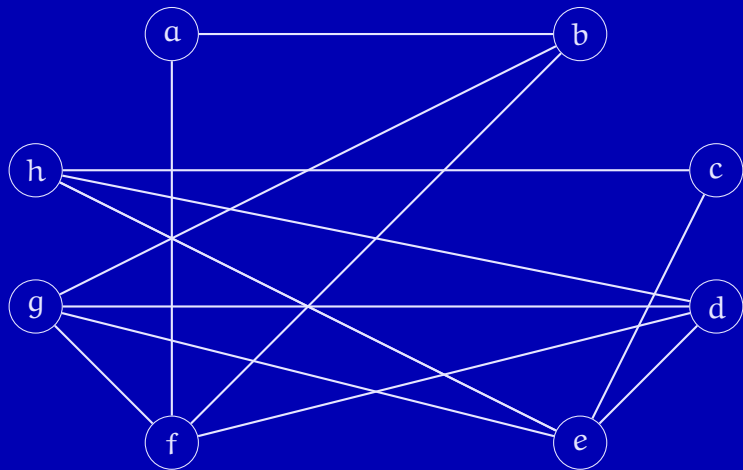


Lossless Graph Compression



Lossless Graph Compression

Main ideas

- Exploit repetitions
- Exploit distribution of values

Huffman coding

Data: Set O of objects, each object o_i has probability p_i

if $|O| > 2$ **then**

 Pick two objects o_i, o_j with smallest probability;

$x \leftarrow$ new object with probability $p_i + p_j$;

$h \leftarrow \text{Huffman}(O \setminus \{o_i, o_j\} \cup \{x\})$;

$h(o_i) \leftarrow h(x)0$;

$h(o_j) \leftarrow h(x)1$;

 Remove $h(x)$;

else

$h(o_1) \leftarrow 0$;

$h(o_2) \leftarrow 1$;

return h ;

Elias γ code

binary code for $x \geq 1$

- 1 $N = \lfloor \log_2 x \rfloor$
- 2 N zeroes \cdot 1 one \cdot binary representation of x , omitting the leading bit
- 3 uses $2\lfloor \log_2 x \rfloor + 1$ bits

Elias δ code

binary code for $x \geq 1$

- 1 $N = \lfloor \log_2 x \rfloor$
- 2 $\gamma(N + 1) \cdot$ binary representation of x , omitting the leading bit
- 3 uses $\lfloor \log_2 x \rfloor + 2\lfloor \log_2 (\lfloor \log_2 x \rfloor + 1) \rfloor + 1$ bits

Variable-length nibble code

binary code for $x \geq 1$

- 1 $p \leftarrow$ the binary representation of n , left-padded with zeroes, so that its length is a multiple of 3
- 2 Split p into 3-bit blocks
- 3 prepend each block with a zero, replace the leading 0 of the last block with a one.

Minimal binary code

binary code for $0 \leq x \leq z - 1$

- 1 $s = \lceil \log_2 z \rceil$
- 2 $p \leftarrow 2^s - z$
- 3 If $x < p$ then output the x -th binary word of length $s - 1$
- 4 If $x \geq p$ then output the $(x - z + 2^s)$ -th binary word of length s

ζ_k code

binary code for $2^{hk} \leq x \leq 2^{(h+1)k} - 1$

- 1 k : shrinking factor
- 2 $h+1$ in unary \cdot minbincode of $x - 2^{hk}$, with $z = 2^{(h+1)k} - 2^{hk} - 1$

Move-to-front transform

- 1 Maintain the list L of recently used objects
- 2 Encode an object as its index in L
- 3 Move the object to the head of L

Run-length encoding

AAAAAABBBB \rightarrow (A,6)(B,4)

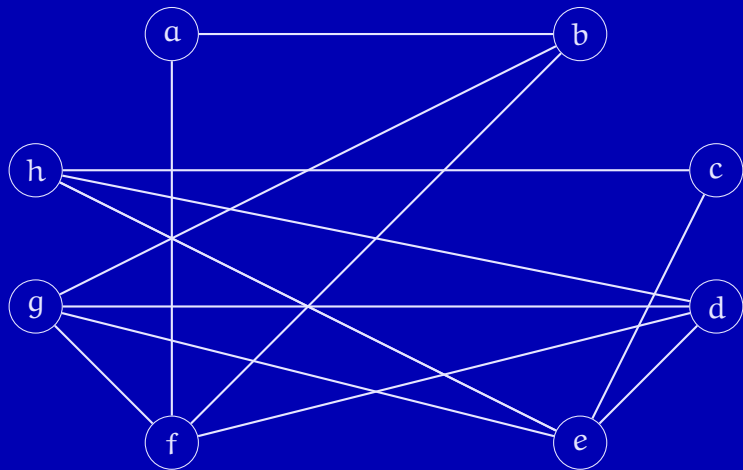
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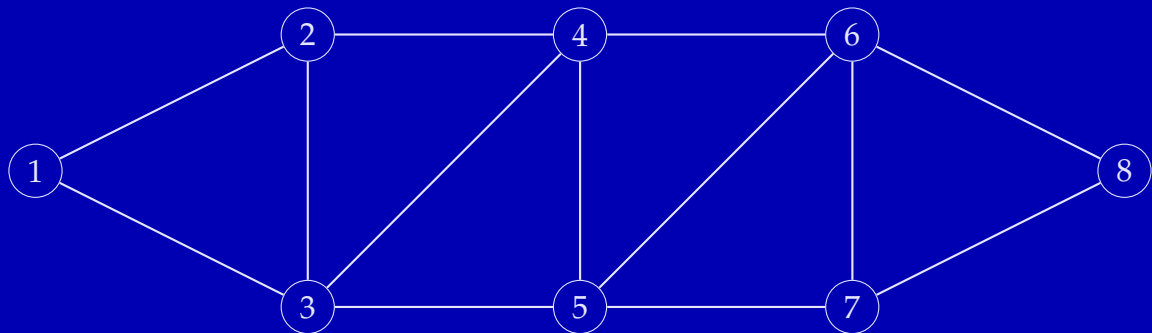
Binary alphabet

- 0-runs and 1-runs alternate
- start with 1-run (0-length if 0-run)
- first, number of runs
- then run-lengths (lengths decremented by 1)

Lossless Graph Compression



Lossless Graph Compression



Gap representation

Delta

- 1 adjacency list of v
- 2 store difference with previous vertex
- 3 store difference with v

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Example $\text{adj}(3)$

-2, 1, 2, 1

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Assumptions?

Gap representation

Delta

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- 2 store difference with previous vertex
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Example $\text{adj}(3)$

-2, 1, 2, 1

Assumptions?

Neighbors of a vertex are close to the vertex.

Reference compression

$N(x)$ and $N(y)$

- 1 $N(y)$ is a previous vertex
- 2 which elements of $N(x)$ are not in $N(y)$?
- 3 $N(x) \setminus N(y)$

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Example $\text{adj}(4)$

$\text{adj}(2) = [1, 3, 4]; \text{adj}(4) = [2, 3, 5, 6] \rightarrow \langle 2, 101_2, [1, 1] \rangle$

using the triple \langle previous vertex, characteristic vector of the vertices of $N(y)$ that are not in $N(x)$, the encoding of $N(x) \setminus N(y) \rangle$

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Assumptions?

$N(x)$ and $N(y)$ are almost identical

Interval encoding

Interval encoding

$[b, e]$

- $[b, e - b]$
- if all intervals are longer than a threshold $L \Rightarrow$ decrement all lengths by L

Figures

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