Expectation Argument

Lemma

Let X be a random variable on a discrete space S such that $E(X) = \mu$. Then $Pr[X \geqslant \mu] > 0$ and $Pr[X \leqslant \mu] > 0$.

Max cut

Data: graph $G = \langle V, E \rangle$
foreach *vertex* $v \in V$ **do**
  Assign v to A or to B with probability $1/2$
return (A, B)

Lemma 1 (to prove)

The expected value of the number of edges in the cut (A, B) is at least $m/2$

Lemma 2 (to prove)

The probability that the cut (A, B) has at least $m/2$ edges is $\geqslant \frac{1}{m/2+1}$

Sample and modifying

Lemma 1 (to prove)

Let $G = \langle V, E \rangle$ be a undirected graph. Then $G = \langle V, E \rangle$ has an independent set with at least $\frac{n^2}{4m}$ vertices.

Hint 1: $d = \frac{2m}{n}$ is the average degree.
Hint 2: the proof is a probabilistic algorithm

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