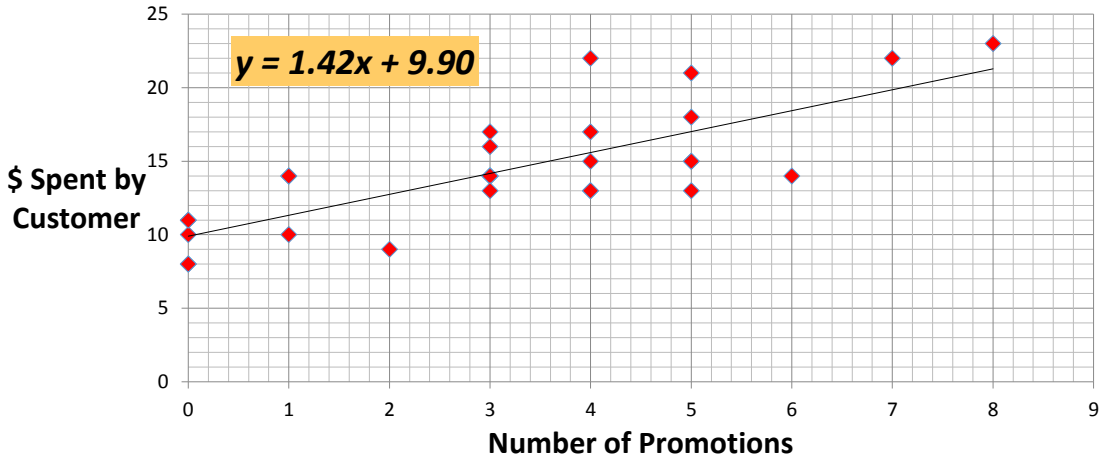
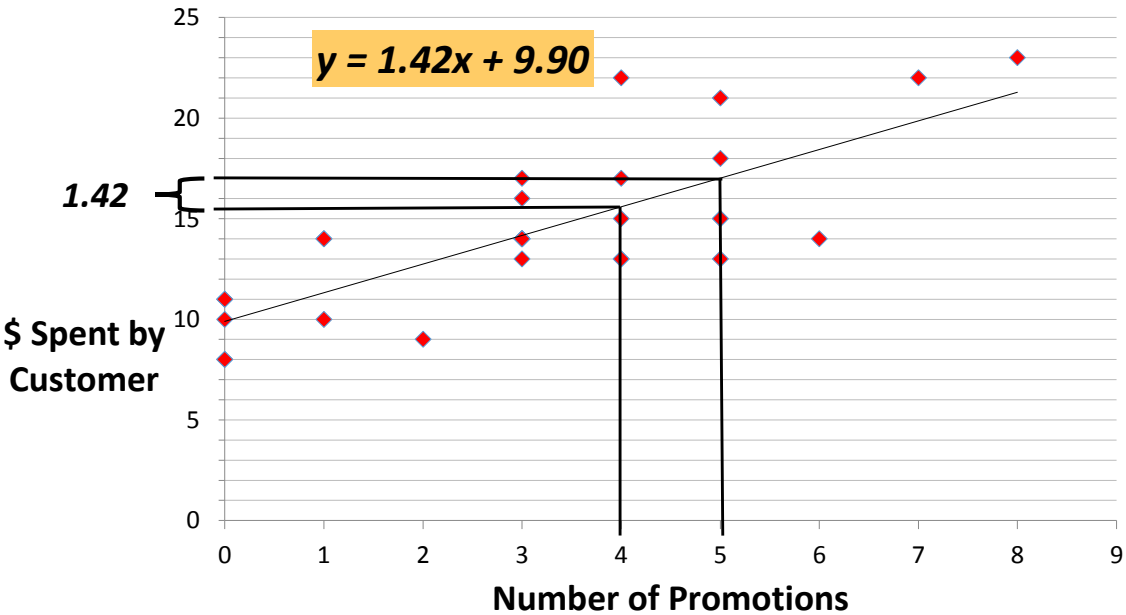
Regression Basics

Introduction

- Regression is an important part of an analytics tool kit that allows us to understand how two variables are related.
- In this module we will
  - Discuss how to interpret regression outputs
  - Explore confounding effects and the biases introduced by missing variables
  - Distinguish between economic and statistical significance
- At the end of this module, you will be able to make inferences about customer behavior from regressions and connect them to business decisions.

Diagnosing Market Response: Regression Analysis





Diagnosing Market Response: Regression Analysis

Regression Statistics	
Multiple R	0.775
R-Squared	0.601
Adjusted R-Squared	0.586
Standard Error	2.566
Observations	29

ANOVA

	df	SS	MS	F	Sig F
Regression	1	267.28	267.28	40.60	0.00
Residual	27	177.75	6.58		
Total	28	445.03			

	Coefficients	Standard Error	t Stat	P-value
Intercept	9.90	0.85	11.60	0.00
Number of Promotions	1.42	0.22	6.37	0.00

Example: Simulated Shopper Card Data

Units purchased = a+b1*price paid + b2*feature + b3*display + error

Customer	Price Paid	Feature	Display	Units Purchased
1	1.50	0	0	3
1	2.56	1	1	1
1	1.62	1	0	3
2	2.41	1	0	1
2	2.37	0	1	1
2	2.23	0	1	1
2	2.65	0	0	0
2	2.06	1	0	2
2	2.12	1	1	2
3	2.31	0	1	1
3	1.69	1	1	3
3	1.37	1	1	4
3	1.82	0	0	2
3	1.54	0	1	3
3	1.29	1	1	4
3	1.96	1	0	2
3	2.20	0	0	1
3	1.55	1	0	3
3	2.01	0	1	2
4	2.07	0	1	2
4	2.79	1	0	0
4	2.15	0	0	1
4	2.50	1	0	1

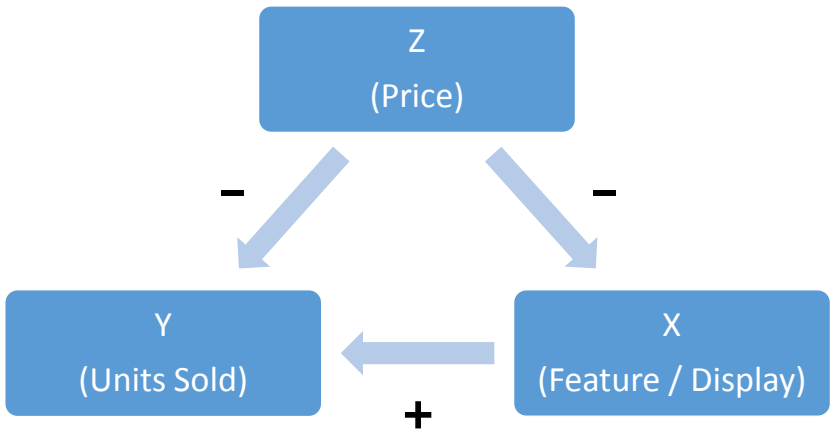
- **Feature and Display:**
  - 1 = Yes
  - 0 = No

Simulated Shopper Card Data: Regression Output

	True Model	Estimated Model
Intercept	6.28	1.34
Price	-2.31	—
Feature	0.38	0.822
Display	0.48	0.687
R-Squared	0.93	0.188

Why are the coefficients of feature and display different in the true and estimated models?

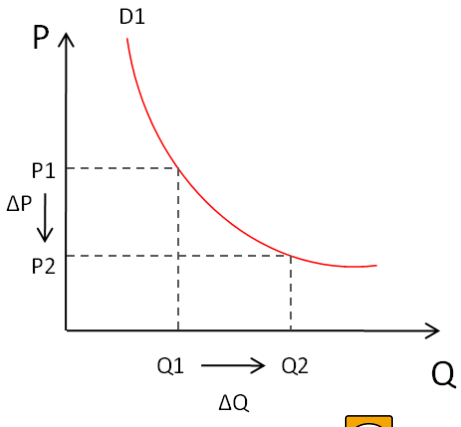
Omitted Variable Bias



Simulated Shopper Card Data: Correlation Matrix

	Price	Feature	Display	Units Purchased
Price	1	-0.25	-0.24	-0.98
Feature		1	-0.09	0.45
Display			1	0.32
Units Purchased				1

Price Elasticity



Price elasticity can be derived as the ratio of change in quantity demanded (%ΔQ) and percentage change in price (%ΔP).

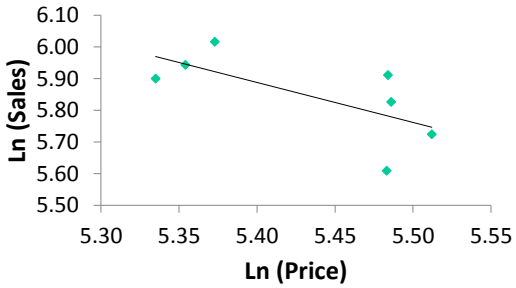
PED = [Change in Sales/Change in Price] × [Price/Sales] = (ΔQ/ΔP) × (P/Q)

Belvedere Vodka

Year	Sales (units)	Ln(Sales)	Price (US dollars)	Ln(Price)	Advertising (US dollars)	Ln(Advertising)
2007	410	6.016	215.44	5.373	20486.1	9.93
2006	381	5.943	211.45	5.354	2923.5	7.98
2005	365	5.900	207.45	5.335	4826.3	8.48
2004	369	5.911	240.87	5.484	13726.6	9.53
2003	339	5.826	241.33	5.486	10330.2	9.24
2002	306	5.724	247.55	5.512	13473.6	9.51
2001	273	5.609	240.48	5.483	9264.6	9.13

Belvedere Price Elasticity

Regression Statistics	
Multiple R	0.67536
R-Squared	0.45611
Adjusted R	
Square	0.34733
Observations	7

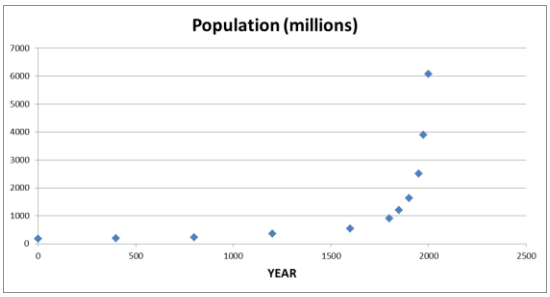


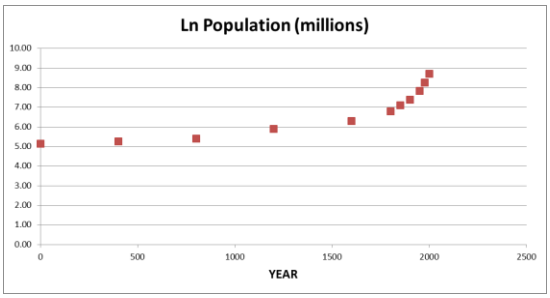
	Coefficients	Standard Error	t Stat	P-value
Intercept	12.686	3.340	3.798	0.013
Ln(Price)	-1.259	0.615	-2.048	0.096

What Is Log?

Year	Population (millions)	Ln(population)
1	170	5.14
400	190	5.25
800	220	5.39
1200	360	5.89
1600	545	6.30
1800	900	6.80
1850	1200	7.09
1900	1625	7.39
1950	2500	7.82
1975	3900	8.27
2000	6080	8.71

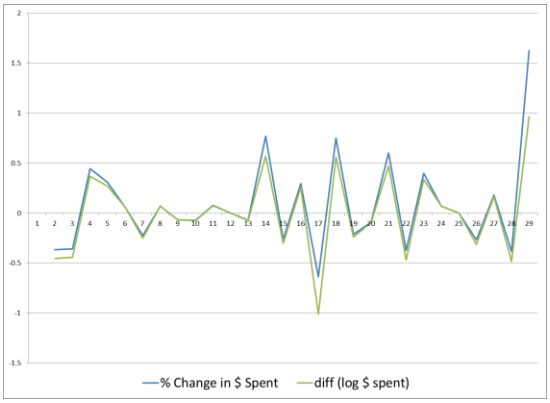
What Is Log?





Log and Percentage Change

- First difference of natural LOG = percentage change:
  - Logging converts absolute differences into relative (i.e., percentage) differences.
  - The series DIFF(LOG(Y)) represents the percentage change in Y from period to period.



Elasticity – Log/Log Models

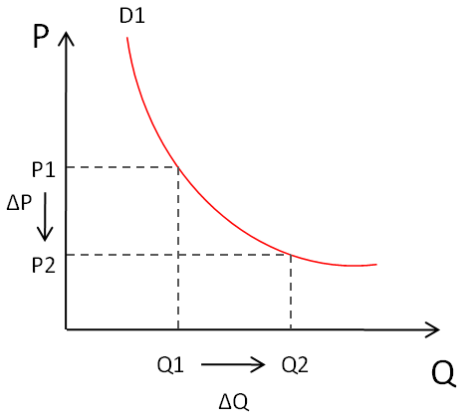
Dependent Variable : ln ($ Spent)

	Coefficients	Standard Error	t Stat	P-value
Intercept	2.24	0.07	32.06	0.00
log(num promo + 1)	0.32	0.05	6.44	0.00

0.317 = change in log ($ spent) when log(num promo) increases by 1 unit

log($ spent) when log(num promo) is 0 = 2.236 (1)
log($ spent) when log(num promo) is 1 = 2.553 (2)
(1) – (2) = 0.317

Price Elasticity



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Common Variables to Consider in Marketing Mix Models

Factor	Bias in Price Elasticity	Bias in Advertising Elasticity
Product quality	+	
Distribution	–	
Brand life cycle – early	+	
Absolute sales	+	
Time series	–	–
Include carryover		+

Statistical and Economic Significance

- **Statistical significance:**
  - Is the relationship observed in the sample likely to be observed in the population as well
  - Look for p–value < .10 for the coefficient of interest
- **Economic significance:**
  - Does the benefit from a marketing intervention (i.e., the size of the coefficient) justify the expense?

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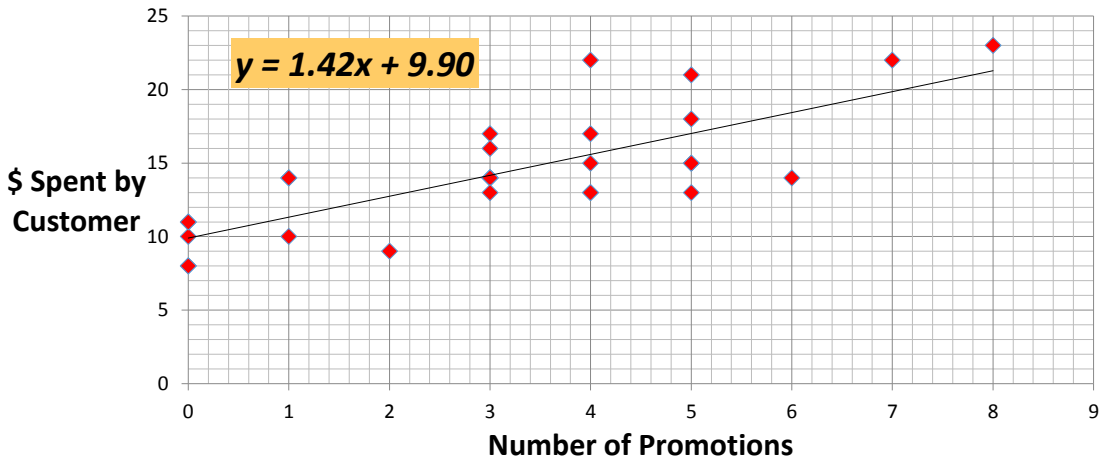

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Economic Significance

- A unit increase in number of promotions increases units purchased by 1.42
- Assume gross profit per unit is $5
- Cost of promotion is $0.50
- Profit = (units purchased * gross profit) – (cost of promotion * number of promotions)
- Profit = (1.42 * 5 - 0.50 * 1) = (7.1 - 0.5) = 6.6

Conclusion

- Regressions are about what you include and also what you DON'T include in the model.
- Logarithm is a useful transformation for calculating elasticity from regression.
- Connecting regression to business decisions requires understanding economic significance.