

The background of the slide is a complex, abstract composition. It features a dark, muted purple or brownish background. Overlaid on this are several geometric and data-like elements. In the upper and lower portions, there are intricate networks of thin, light-colored lines forming a mesh or web-like structure. Scattered throughout these areas are numerous small, colored dots in shades of green, blue, and yellow. On the left side, there is a vertical strip containing a grid of small, light-colored plus signs. In the center, a large, white, angular shape, resembling a stylized 'V' or a folded piece of paper, serves as a backdrop for the title. The overall aesthetic is technical and modern, suggesting themes of data science, geometry, or network theory.

Session 3: Null Invariance Measures

Interestingness Measures & Null-Invariance

- ❑ **Null invariance**: Value does not change with the # of null-transactions
- ❑ A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant
$\chi^2(A, B)$	$\sum_{i,j=0,1} \frac{(e(a_i b_j) - o(a_i b_j))^2}{e(a_i b_j)}$	$[0, \infty]$	No
$Lift(A, B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$AllConf(A, B)$	$\frac{s(A \cup B)}{\max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A, B)$	$\frac{s(A \cup B)}{s(A) + s(B) - s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A, B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A, B)$	$\frac{1}{2} \left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)} \right)$	$[0, 1]$	Yes
$MaxConf(A, B)$	$\max\left\{ \frac{s(A)}{s(A \cup B)}, \frac{s(B)}{s(A \cup B)} \right\}$	$[0, 1]$	Yes

χ^2 and lift are not null-invariant

Jaccard, cosine, AllConf, MaxConf, and Kulczynski are null-invariant measures

Null Invariance: An Important Property

□ Why is null invariance crucial for the analysis of massive transaction data?

□ Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	<i>milk</i>	$\neg milk$	Σ_{row}
<i>coffee</i>	<i>mc</i>	$\neg mc$	<i>c</i>
$\neg coffee$	<i>m</i> $\neg c$	$\neg m$ $\neg c$	$\neg c$
Σ_{col}	<i>m</i>	$\neg m$	Σ

□ Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!

□ Many measures are not null-invariant!

Null-transactions
w.r.t. *m* and *c*

Data set	<i>mc</i>	$\neg mc$	<i>m</i> $\neg c$	$\neg m$ $\neg c$	χ^2	<i>Lift</i>
<i>D</i> ₁	10,000	1,000	1,000	100,000	90557	9.26
<i>D</i> ₂	10,000	1,000	1,000	100	0	1
<i>D</i> ₃	100	1,000	1,000	100,000	670	8.44
<i>D</i> ₄	1,000	1,000	1,000	100,000	24740	25.75
<i>D</i> ₅	1,000	100	10,000	100,000	8173	9.18
<i>D</i> ₆	1,000	10	100,000	100,000	965	1.97